Mass of the Second Excited State of $^{12}C^{\dagger}$

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By the simultaneous measurement of the magnetic rigidity of the α_0^{++} and α_2^{+} groups from the $^{15}N(p,\alpha)^{12}C$ reaction, using a double-focusing spectrometer, the excitation energy of the second excited state of ^{12}C is found to be 7654.2 ± 1.6 keV.

INTRODUCTION

The second excited state of ^{12}C (at 7.6 MeV) is particularly significant in astrophysics, since its properties determine the rate of helium burning in red giant stars. After a star on the main sequence has exhausted the supply of hydrogen in its core, gravitational contraction will raise the central temperature and density until a new energy source proceeds at the necessary rate for stabilization. The conversion of three α particles into ^{12}C, enhanced by the resonance corresponding to the 7.6-MeV state, can supply the needed energy at temperatures of $10^8\,°K$ and densities of 10^5 g/cm^3.

The rate for this reaction, known as the triple-α process, depends strongly on the mass difference between $^{12}C^*(7.6$ MeV) and three α particles, viz.,[1]

$$\text{rate} \propto e^{-Q/kT},$$

where

$$Q=(M_{^{12}C^*}-3M_\alpha)c^2.$$

Using the most precise measurement of Q prior to 1970, performed by Cook, Fowler, Lauritsen, and Lauritsen (CFLL),[2] the rate of the triple-α process becomes, at $T=10^8\,°K$,

$$\text{rate} \propto \exp[-43(1+\delta Q/Q_{\text{CFLL}})],$$

where the effect of the error δQ is explicitly displayed. CFLL populated the second excited state of carbon-12 by the β decay of boron-12, produced by $^{11}B(d,p)^{12}B$, and measured the energy of the subsequent α decay to the ground state of ^8Be. Although the measurement of CFLL was the most precise prior to 1970, other measurements[3-7] had given consistently higher values for the mass difference Q as summarized in Table I. The data have been revised through the use of the 1971 Mass Tables of Wapstra and Gove,[8] where appropriate. The average of the measurements prior to 1970, excluding the one by CFLL, gives $E_x=7658\pm5$ keV and $Q=384\pm5$ keV, to be compared with $Q_{\text{CFLL}}=370\pm4$ keV.

PROCEDURE

The reaction $^{15}N(p,\alpha)^{12}C^*(7.6)$ was used to measure the excitation energy of the second excited state of ^{12}C. The protons were accelerated by the Office of Naval Research–California Institute of Technology tandem Van de Graaff, and the α particles were analyzed in a 61-cm 180° magnetic spectrometer. The laboratory angle of the spectrometer was about $\Theta=140°$ with entrance aperture $\Delta\Theta=0.4°$ and $\Delta\varphi=6.6°$. In order to calibrate the spectrometer, the α particles to the ground state of ^{12}C were detected simultaneously with the α particles to the second excited state. That is, the energy of the incoming protons and the angle of the spectrometer were chosen so that the doubly charged α_0 group had nearly the same rigidity as the singly charged α_2 group, and then the spectrometer field was varied in this neighborhood to determine the precise difference between the α_0 and α_2 rigidities at corresponding points of the yield curves.

An 11-μm silicon detector and a 1.6-mm slit at the spectrometer image allowed separation of α_0^{++} and α_2^{+} by energy at fixed rigidity. Such a

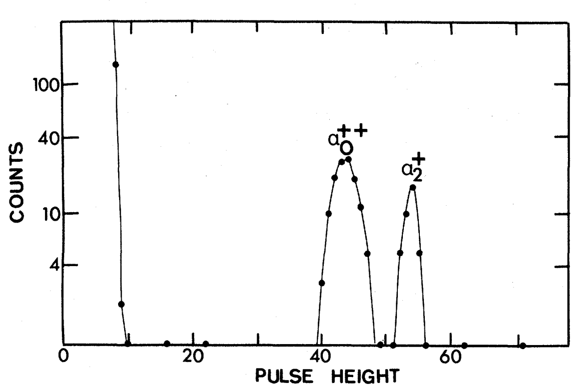

FIG. 1. Pulse-height spectrum taken with the 11-μm silicon detector at the spectrometer image for the reaction $^{15}N(p,\alpha)^{12}C$. The higher energy α_0^{++} appears to the *left* of the α_2^{+} because less than one fourth of the energy (6.8 MeV) of the α_0^{++} is deposited in the thin detector.

TABLE I. Prior work is listed here if the error is less than 40 keV. Data have been revised through use of 1971 Mass Tables and other calibrations where appropriate.

Group	Reaction	E_x (keV)	Q (keV)
Dunbar *et al.* [a] (Ref. 3)	$^{14}N(d, \alpha)^{12}C$	7692 ±25	417 ±25
Pauli (Ref. 4)	$^{14}N(d, \alpha)^{12}C$	7665 ±15	390 ±15
Ahnlund [b] (Ref. 5)	$^{14}N(d, \alpha)^{12}C$	7660 ±13	385 ±13
Cook *et al.* (Ref. 2)	$^{12}C(\alpha)^{8}Be$	7645 ±4	370 ±4
Jaidar *et al.* [c] (Ref. 6)	$^{14}N(d, \alpha)^{12}C$	7653 ±10	378 ±10
Browne, Dorenbusch, and Erskine [d] (Ref. 7)	$^{10}B(^3He, p)^{12}C$	7657 ±6	382 ±6
Austin, Trentleman, and Kashy (Ref. 11)	$^{12}C(p, p')^{12}C$	7656.2±2.1	381.4±2.2
Stocker, Rollefson, and Browne (Ref. 12)	$^{12}C(p, p')^{12}C$ $^{12}C(^3He, {}^3He')^{12}C$	7655.9±2.5	381.1±2.6
Present work	$^{15}N(p, \alpha)^{12}C$	7654.2±1.6	379.4±1.7
Weighted average		7654.6±1.1	379.8±1.3

[a] Corrected using Q = 9135 keV for $^{14}N(d, \alpha_1)$.
[b] Corrected using Q = 3840 keV for $^{17}O(d, p_1)$.
[c] Corrected using Q = 13575 keV for $^{14}N(d, \alpha_0)$.
[d] Corrected using Q = 19695 keV for $^{10}B(^3He, p_0)$.

thin detector was used so that any elastically scattered protons reaching the detector would give pulse heights well below the two α groups. A typical spectrum is shown in Fig. 1. Not all the energy of the α_0^{++} was deposited in this thin detector, so that the lower-energy α_2^+ particles appear at higher pulse heights than those of the α_0^{++} group.

The measurement depends critically on four parameters: the energy E_1 of the incoming protons, the angle Θ of the spectrometer, the magnetometer frequency F_0 (proportional to the α_0 rigidity), and the magnetometer frequency F_2 (proportional to the α_2 rigidity). The value of the excitation energy was calculated through the use of relativistic kinematics, with appropriate corrections for target thickness[9] and ionic-charge states. An approximate nonrelativistic expression is given for reference in Table II, where the mass of the proton is denoted by M_1, the mass of ^{15}N by M_2, the mass of the α particle by M_3, and the mass of the ground state of ^{12}C by M_4, and where the energy of the α_0 group is denoted as E_0 (E_0 is determined from the measured E_1, Θ, and 1971 masses). Also in Table II are the evaluations of the partial derivatives of the relativistic equations (including target thickness) for the parameters used.

Two measurements were made, one using a thin target, 2 $\mu g/cm^2$ of $Ti^{15}N$, the other using a semithick target of $Ti^{15}N$. The thickness of the thin target was determined from the γ-ray yield of the $^{15}N(p, \alpha_1)^{12}C^*(4.43)$ reaction at E_p = 1140 keV with a 7.6-cm × 7.6-cm NaI(Tl) scintillation detector, using the data of Gorodetzky *et al.*[10] In order to make the two measurements as independent as possible, it was decided to calibrate E_1 and Θ differently for each target. However, both targets were made in the same way, by evaporating titanium onto 250-μm tungsten blanks and then

TABLE II. A nonrelativistic approximation to the expression used in data reduction, and values of the partial derivatives of the relativistic equation for the parameters used. See text for definition of symbols.

$$E_x = \frac{(M_1+M_2)(4F_0^2-F_2^2)}{4M_4F_0^2}\,E_0 - \frac{(M_1M_3E_1E_0)^{1/2}(2F_0-F_2)}{M_4F_0}\cos\theta$$

$\frac{\partial E_x}{\partial E_1} = 0.64$ $\frac{\partial E_x}{\partial M_1} = 0.73$ $\frac{\partial E_x}{\partial M_2} = 0.73$ $\frac{\partial E_x}{\partial M_3} = -0.73$

$\frac{\partial E_x}{\partial \theta} = -5.6$ keV/deg $\frac{\partial E_x}{\partial F_0} = 0.22$ keV/kHz $\frac{\partial E_x}{\partial F_2} = -0.22$ keV/kHz

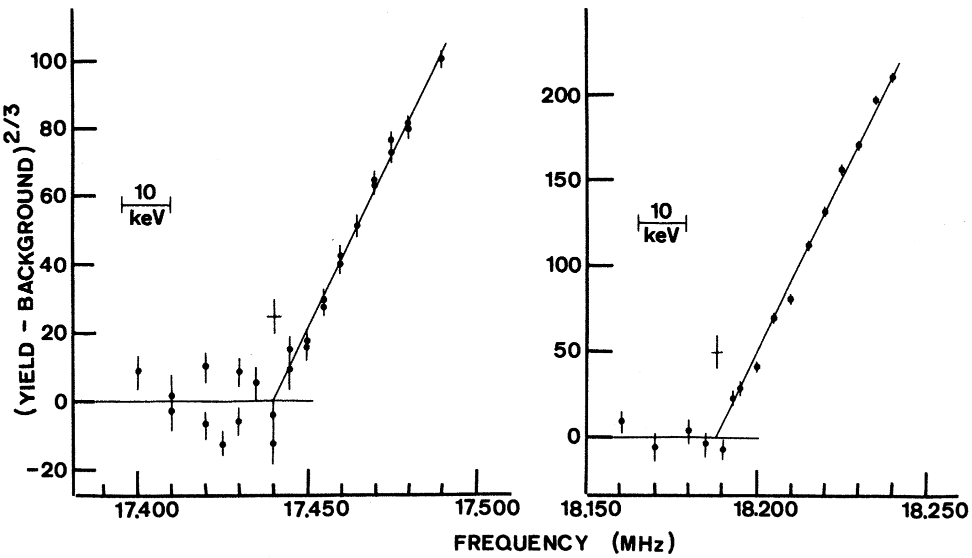

FIG. 2. Neutron yield vs magnetometer frequency of the beam-analyzing magnet in the calibration runs. The data at left are for the $^{14}N(\alpha,n)^{17}F$ threshold, and at the right for $^{6}Li(\alpha,n)^{9}B$. The straight-line fits by least squares are drawn in each case, and the standard deviations of the intercepts are indicated. For the $^{6}Li(\alpha,n)$ threshold, a prior similar run, with intercept at 18.1926± 0.0020, was averaged with the data shown here to give the final value used, 18.190 ± 0.002 MHz.

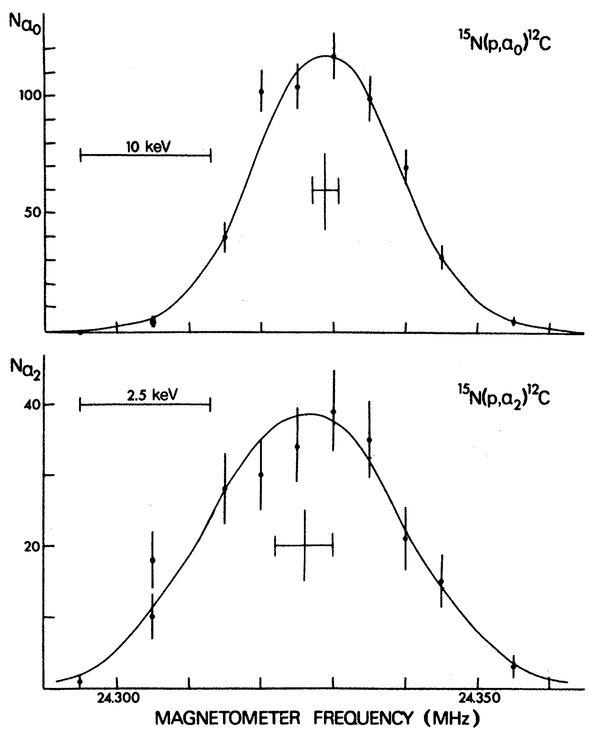

FIG. 3. The α-particle yields vs magnetometer frequency of the 61-cm magnetic spectrometer for the thin-target measurement. The curves are Gaussian fits with the indicated centroids and errors (for 1% confidence limits).

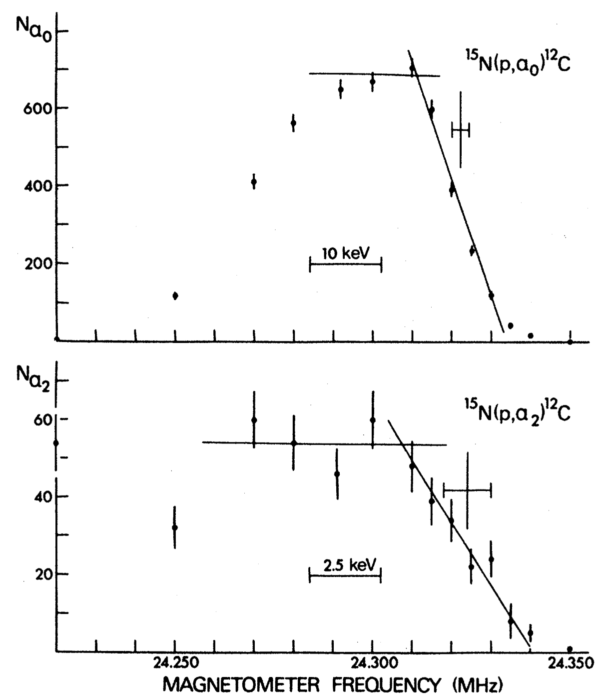

FIG. 4. The α-particle yields vs magnetometer frequency of the 61-cm magnetic spectrometer for the thick-target measurement. Standard deviations of the leading-edge midpoints are shown for the indicated straight-line fits.

heating the blanks in a ^{15}N-ammonia atmosphere (about 100 Torr pressure) in an induction furnace. During bombardment, a liquid-nitrogen cold trap was located about 5 cm from the target, and the target was maintained at red heat by the beam to minimize carbon deposits. Repeated runs showed no significant energy shift, and no visible contamination appeared on the target.

In order to know the value of the incident energy as accurately as possible, calibration reactions were chosen that could be run near the incident energy used in the $^{15}N(p, \alpha)$ reaction, such that the magnetic field in the 90° beam-analyzing magnet had to be changed by less than 2% from that for $^{15}N(p, \alpha)$. For the thin-target measurement, the $^{6}Li(\alpha, n)^{9}B$ reaction, which has a threshold[8] at 6621.4 ± 2.0 keV, was used with consistent results both before and after the $^{15}N(p, \alpha)$ runs. To calibrate the semithick-target measurements, the $^{14}N(\alpha, n)^{17}F$ reaction, which has a threshold[8] at 6089.0 ± 0.8, was used before the $^{15}N(p, \alpha)$ runs. For both calibration reactions, the neutron yield at 0° was measured with a slow-neutron detector (NE 402) as a function of incoming particle energy. The data (Fig. 2) were fitted by using a power law corresponding to s-wave emission. The beam-magnet constants derived from the two calibrations agreed well within 1 standard deviation, and were also in good agreement with other calibrations in recent years.

The second critical parameter was the angle of the 61-cm spectrometer. The measurement for the thin-target experiment was performed by using a precision reduction gear, which was marked in tenths of a degree, to move a straight edge around the inside of the scattering chamber in a circle centered at the spectrometer object point. With the detection system set to observe the elastically scattered protons and with the magnetic field only slightly changed from its value in the $^{15}N(p, \alpha)$ runs, the proton count rate versus angle of the straight edge was recorded, as the straight edge was moved first to intercept the incoming beam and then to intercept the protons scattered into the spectrometer acceptance angle. Care was taken to insure that the beam was centered in the scattering chamber when the $^{15}N(p, \alpha)$ reaction was carried out. The angle obtained through the use of the straight-edge calibration agreed within 0.1° with the markings at the base of the spectrometer which had been machined during the original installation and alignment of the spectrometer. For the thick-target experiment the value for the angle was taken from these markings.

TABLE III. Values and errors of parameters used.

Parameter	Thick target		Thin target	
E_1 (keV)	6340.4	± 1.8	6334.8	± 2.4
θ (deg)	140.4	± 0.2	140.0	± 0.2
F_0 (MHz)	24.3221	± 0.0017	24.3293	± 0.0018
F_2 (MHz)	24.324	± 0.006	24.3262	± 0.0040
Target thickness ($\mu g/cm^2$)			2	± 1

RESULTS

The parameters used and their errors for each measurement are given in Table III. In the case of the thin-target measurements (Fig. 3), the values for the spectrometer frequencies F_0 and F_2 were taken at the centers of the Gaussian fits to the data. For the semithick-target measurements the half-height point of each curve's leading edge was taken to define the spectrometer frequency, as shown in Fig. 4.

The values obtained from the thin-target measurements were $E_x = 7653.3 \pm 2.4$ keV and $Q = 378.5 \pm 2.5$ keV, while the values of the semithick target were $E_x = 7654.9 \pm 2.2$ keV and $Q = 380.1 \pm 2.3$ keV. The average of these measurements gives our final result of 7654.2 ±1.6 keV for the excitation energy of the second excited state of ^{12}C and 379.4 ± 1.8 keV for the mass difference between this state and three α particles.

Two other recent measurements of the energy of the second excited state of ^{12}C have been reported. Austin, Trentleman, and Kashy,[11] using a magnetic spectrometer to analyze the outgoing protons in the inelastic scattering of protons from ^{12}C, obtained values of $E_x = 7656.2 \pm 2.1$ keV and $Q = 381.4 \pm 2.2$ keV. Stocker, Rollefson, and

TABLE IV. Empirical parameters of the 7.6-MeV state in ^{12}C.

Measured quantity	Best value	Reference
$\Gamma_{e\pm}$	64 ± 4 μeV	a
$\Gamma_{e\pm}/\Gamma$	$6.9 \pm 2.1 \times 10^{-6}$	b
Γ_{rad}/Γ	$2.9 \pm 0.3 \times 10^{-4}$	c
Q	379.8 ± 1.3 keV	Average from Table I

[a] Average of results of H. Crannell, T. A. Griffy, L. R. Suelzle, and M. R. Yearian, Nucl. Phys. A90, 152 (1967) and P. Strehl and Th. H. Schucan, Phys. Letters 27B, 641 (1968).

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Browne[12] report values of $E_x = 7655.9 \pm 2.5$ keV and $Q = 381.1 \pm 2.6$ keV from magnetic analysis of inelastic scattering of protons and ^{3}He. It will be noted that the recent experiments are in good agreement, and their weighted mean ($E_x = 7655.1 \pm 1.1$ keV and $Q = 380.3 \pm 1.4$ keV) is about 2 standard deviations higher than the value of CFLL. Using the weighted mean of all measurements ($E_x = 7654.6 \pm 1.1$ keV and $Q = 379.8 \pm 1.3$ keV), the triple-α process rate at $T = 10^8$ °K is reduced by a factor of 3, compared with the rate calculated from Q_{CFLL}. The error in the present best Q value leads to an error in the calculated rate of ±16%.

In order to calculate the rate of energy generation, $\epsilon_{3\alpha}$, in a stellar enviroment, it is necessary also to know the radiation width Γ_{rad} of the 7.6-MeV state. This width is known from the combination of three experimentally measured quantities, viz., the pair-emission width $\Gamma_{e^\pm}$ the ratio of $\Gamma_{e_\pm}$ to the total width Γ, and the ratio $\Gamma_{\mathrm{rad}}/\Gamma$. The presently known values of these quantities are included in Table IV, from which $\Gamma_{\mathrm{rad}} = 2.7 \pm 0.9$ meV may be calculated. Then the energy generation is[13]

$$\epsilon_{3\alpha} = 3.7 \times 10^{14} \frac{\rho_5^2 \chi_\alpha^3}{T_8^3} f e^{-44.08/T_8} \ \mathrm{W/g},$$

where ρ_5 is the density in 10^5 g cm^{-3}, χ_α is the ^{4}He mass fraction, $T_8 = 10^{-8} T$ is the temperature in 10^8 °K, and f is the electron-screening correction. The over-all uncertainty is now about 36%, mainly due to the 30% error in $\Gamma_{e^\pm}/\Gamma$.

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