Optical-Model Partial-Wave Analysis of 1-GeV Proton-Nucleus Elastic Scattering*

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We have performed partial-wave calculations using the Klein-Gordon and the Dirac equations for the 1-GeV proton scattering experiments of Palevsky *et al.* Without the explicit introduction of spin in the optical potential, the two equations give closely similar results. Compared with the eikonal approximation the differential cross sections of the partial-wave analysis show considerable filling in of the diffraction minima. We compare the simple Watson theoretical optical-potential predictions with phenomenological fits to the data, and reach substantially the same conclusions as Palevsky *et al.* The optical potentials used in this work are either the fourth component of a four-vector or a scalar potential.

INTRODUCTION

Differential cross sections for the elastic scattering of 1-GeV protons from hydrogen, helium, carbon, and oxygen were measured some time ago by Palevsky *et al.*[1] These authors also reported optical-model fits to the data, using E. H. Auerbach's ABACUS-2 optical-model code, a partial-wave analysis adapted for this purpose to the Klein-Gordon equation. A Woods-Saxon shape was chosen for the optical potential. The general conclusions reported in Ref. 1 were that the ^{12}C and ^{16}O data could be fitted well with an optical potential whose shape parameters resembled those found from electron scattering experiments,[2,3] while for the ^{4}He data, complete agreement could not be obtained in this manner.[4,5]

We have been working on a complete partial-wave analysis for particle-nucleus scattering for some time, and in connection with this project we decided to reexamine the analysis of Ref. 1. We will first discuss the results obtained when the nuclear potential is taken to be the fourth part of a four-vector rather than the scalar potential used in the analysis of Ref. 1. Then we shall present our results when a scalar nuclear potential is assumed. Our general conclusions regarding the agreement with electron scattering experiments are substantially the same as Ref. 1. Since accurate analysis of scattering data in this energy range does require a complete partial-wave analysis rather than the frequently used eikonal approximation, our presentation of these results at this time may also provide a check point for other similar calculations.

CALCULATION

In our optical-model analysis, we use Klein-Gordon or Dirac equations with a Coulomb potential obtained from the charge distributions determined by electron scattering. The basis for the theoretical nuclear optical-model potential used here is described by Watson.[6] In its simplest form, it relates the optical potential to the nucleon-nucleon forward scattering amplitude and the matter density of the nucleus. The potential has the form

$$V_{\rm opt}(r) = -A\frac{\sigma_T\, k(i+\alpha_0)}{2E}\rho(r)\,, \tag{1}$$

where A is the number of nucleons in the target, α_0 is the ratio of real to imaginary parts of the nucleon-nucleon scattering amplitude, σ_T is the average nucleon-nucleon total cross section, k and E are the projectile momentum and total energy taken in the nucleon-nucleus center-of-mass system, and for $\rho(r)$ we take the nuclear charge-density function[7] normalized to unity.[8] The optical potential of Eq. (1) is inserted as the fourth component of a four-vector in either the Dirac or the Klein-Gordon equation. The parameters obtained are given in the first row of Tables I–III. In calculating the corresponding scalar optical potential, we multiply the above expression for $V_{\rm opt}$ by E/m, where m is the rest mass of the proton. This pro-

TABLE I. Parameters used in the optical-model potential for p-^{4}He elastic scattering at 1.696 GeV/c incident lab momentum. The experimental reaction cross section is given by G. J. Igo, J. L. Freides, H. Palevsky, R. Sutter, G. Bennett, W. D. Simpson, D. M. Corley, and R. L. Stearns, Nucl. Phys. B3, 181 (1967) as 111 ± 10 mb. The optical potential is taken to be the fourth component of a four-vector for the first three rows, and a scalar potential for the last two rows.

V (MeV)	W (MeV)	c (fm)	z (fm)	w	rms radius (fm)	Volume integral (fm^3)	σ_R (mb)	χ^2/deg freedom	λ
16.09	−80.45	1.008	0.327	0.445	1.717	16.81	98.6	19.1	1.43
36.53	−111.25	1.008	0.327	0.445	1.717	16.81	116.4	9.3	1.23
27.80	−156.19	1.008	0.286	0.445	1.527	13.57	109.7	1.8	0.98
25.71	−128.56	1.008	0.327	0.445	1.717	16.81	99.3	17.5	1.43
40.41	−243.07	1.008	0.288	0.445	1.538	13.73	110.9	1.6	0.98

cedure insures that the eikonal-approximation phase shifts are the same to first order in the potential for the two cases. The parameters for the scalar potential are given in the fourth row of Tables I–III.

We also compare the results of the simple potential of Eq. (1) with a purely phenomenological potential defined by

$$V(r) = (V + iW) f(r), \qquad (2)$$

where V and W are the real and imaginary strength parameters of the potential and $f(r)$ is the shape function which gives the variation of the potential with radius. We have in all cases taken $f(r)$ to have the same analytic form as the charge distribution determined from electron scattering experiments.

With regard to the dependence of the optical potential on the spin of the nucleon, there are two separate questions. The simpler question is one of the effect of nucleon spin on the partial-wave analysis itself. We examine this effect, and provide an independent verification of our calculations by making a partial-wave computation using the four-component Dirac equation. As with the Klein-Gordon equation, the potential is inserted either as the fourth component of a four-vector or as a scalar. We find that there is no appreciable difference in the results of the two calculations and that the same phenomenological optical potential may be determined by fitting the data with either equation. In addition, we note that the shape function $f(r)$ is essentially independent of the assumed character of the potential although, as would be expected, the strength parameters V and W differ. The second question regarding spin relates to the possible addition of an actual spin-orbit interaction. Because we do not consider polarization, we make no statement about this second question; it is, however, a very appreciable effect. It has been shown by Franco[9] and by Kujawski, Sachs, and Trefil[10] that the inclusion of spin and isospin improves the Glauber cross-section agreement with experiment. Kujawski[11] and Lambert and Feshbach[12] have also examined the use of spin-dependent optical potentials, and find the effect of spin to be important.

RESULTS FOR VECTOR POTENTIAL

For the p-^{4}He experiments, the simple optical-model potential of Eq. (1) does not fit the data well. We have used the charge density of ^{4}He determined from electron scattering by Frosch *et al.*,[4] namely, the parabolic Fermi shape

$$\rho(r) = \rho_0 f_{czw}(r), \qquad f_{czw}(r) = \frac{1 + wr^2/c^2}{1 + e^{(r-c)/z}} \qquad (3)$$

with $c = 1.008$ fm, $z = 0.326$ fm, $w = 0.445$ in determining $V_{\text{opt}}(r)$ from Eq. (1). In addition, we take $\sigma_T = 43.91$ mb[13] and $\alpha_0 = -0.2$.[14] That the fit to the p-^{4}He data is poor perhaps may be expected. For example, calculations by Feshbach, Gal, and Hüff-

TABLE II. Parameters used in the optical-model potential for p-^{12}C elastic scattering at 1.696 GeV/c incident lab momentum. The experimental reaction cross section is given by Igo *et al.* as 258 ± 17 mb. The optical potential is taken to be the fourth component of a four-vector for the first three rows, and a scalar for the last row.

V (MeV)	W (MeV)	R (fm)	α	rms radius (fm)	Volume integral (fm^3)	σ_R (mb)	χ^2/deg freedom	λ
11.71	−58.58	1.71	1.12	2.494	74.62	255.2	9.5	1.17
17.50	−49.17	1.71	1.12	2.494	74.62	233.1	4.7	1.23
16.73	−58.48	1.662	1.12	2.424	68.51	237.3	3.5	1.07
21.65	−108.23	1.71	1.12	2.494	74.62	256.6	9.4	1.15

ner,[15] and Kujawski[11] indicate an improved agreement with experiment if correlations are included. The calculations of Lambert and Feshbach[12] where spin and isospin effects were included in the optical potential, as well as the spin-dependent calculation of Kujawski,[11] indicate these effects to be important. In addition, we have neglected nuclear recoil, and have approximated crudely the momentum-transfer dependence of the nucleon-nucleon scattering amplitude.[7] All of these effects are expected to become more important as the scattering angle increases. Figure 1 shows the results of the simple optical potential of Eq. (1), indicated as Method (a), and one can see that the discrepancy becomes greater as the scattering angle increases.

In the phenomenological optical potential given by Eq. (2), we have adopted two successively more flexible fitting methods. In the first, Method (b), V and W are treated as free parameters and $f(r)$ is fixed as the electron scattering shape of Eq. (3). The resulting parameters are in the second row of Table I. In the second [labeled Method (c) in the figures] V, W, and the z parameter of Eq. (3) are allowed to vary. The third row of Table I gives the parameters in this case. The least-squares fitting procedure used in obtaining the parameters is designed to allow for the over-all 20% uncertainty in the absolute value of the cross section,[1] as well as the statistical uncertainty in the cross section measured at each angle.[16] Method (b) improves significantly the agreement with experiment as can be seen in Table I, and Method (c) results in further improvement. As is shown in Fig. 1, the agreement of Method (c) with experiment is very acceptable. In Table I, we give the value of the rms radius of $f(r)$ and the volume integral of $f(r)$ as these quantities are commonly determined by optical-model analysis of low-energy experiments.

In Fig. 2, we show the cross section calculated from the optical-model potential of Method (c) using both Dirac and Klein-Gordon equations for the partial-wave analysis. These independent calculations agree very well as mentioned previously.[17]

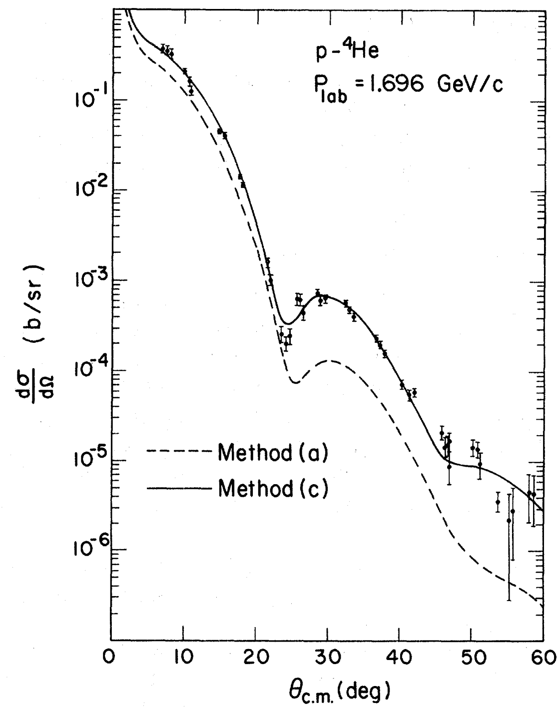

FIG. 1. The differential cross section in the center-of-mass system for elastic p-^{4}He scattering at incident lab momentum 1.696 GeV/c. The dashed curve is the result of Method (a), and the solid curve is the result of Method (c). The potential is taken to be the fourth component of a four-vector. The experiemental cross sections are from Ref. 1.

For comparison, the eikonal-approximation cross section for this optical-model potential has been calculated. In the forward direction, at angles beyond the Coulomb interference region,[18] the eikonal approximation agrees well with the exact result; however, the distortion of the wave implicit in the partial-wave analysis clearly results in a filling in of the diffraction minima. We note that essentially the same behavior is exhibited when the calculations are repeated using a scalar optical potential found using Method (c) to fit the data. In this case, the Dirac cross sections lie

TABLE III. Parameters used in the optical-model potential for p-^{16}O elastic scattering at 1.696 GeV/c incident lab momentum. The experimental reaction cross section is given by Igo *et al.* as 296 ± 50 mb. The optical potential is taken to be the fourth component of a four-vector for the first three rows, and a scalar for the last row.

V (MeV)	W (MeV)	R (fm)	α	rms radius (fm)	Volume integral (fm^3)	σ_R (mb)	χ^2/deg freedom	λ
10.31	−51.56	1.82	1.60	2.703	114.14	316.3	4.1	1.38
13.69	−51.66	1.82	1.60	2.703	114.14	316.1	4.0	1.34
12.90	−61.88	1.763	1.60	2.618	103.75	317.1	3.3	1.24
19.52	−97.63	1.82	1.60	2.703	114.14	318.0	4.0	1.37

slightly above the Klein-Gordon cross sections for large angles.

In the case of ^{12}C and ^{16}O, the simple model produces quite good agreement with experiment. In these calculations, we use the charge density determined from electron scattering[2] given by

$$\rho(r) = \rho_0 f_{R\alpha}(r) ,$$
$$f_{R\alpha}(r) = (1 + \alpha r^2/R^2)e^{-r^2/R^2} , \qquad (4)$$

with $\alpha = 1.12$, $R = 1.71$ fm for ^{12}C and $\alpha = 1.6$, $R = 1.82$ fm for ^{16}O.[19] The results of these calculations are shown in Figs. 3 and 4. Applying Method (b), and allowing V and W to be determined by fitting the data, we improve the fit to experiment; the results are given in Tables II and III. In Figs. 3 and 4, we also show the phenomenological fit to the data obtained when the shape parameter R is allowed to vary as well as V and W, Method (c). For ^{12}C and ^{16}O, the shape functions needed to fit the data are not very different from the charge distributions from electron scattering. This is reasonable, as one would not expect correlation or recoil effects to be as important for these nuclei as for the ^4He nucleus. The calculations of Feshbach, Gal, and Hüffner[15] for ^4He and ^{16}O, and Kujawski[11] for ^4He, ^{12}C, and ^{16}O indicate that this is the case. We have used these potentials in the eikonal approximation and find the same behavior as for the p-^4He case, and we find the same kind of agreement between the Dirac and Klein-Gordon calculations.

In carbon and oxygen where the simple theory of Eq. (1) appears to work reasonably well, the fitting procedure adopted in Method (b) can be regarded as an experimental determination of α_0, the real to imaginary part of the nucleon-nucleon forward-scattering amplitude, and of an effective average σ_T in the nucleus. For oxygen, we find $\alpha_0 = -0.265$ and $\sigma_{T,\mathrm{eff}} = 51$ mb.

RESULTS FOR SCALAR POTENTIAL

As in the case discussed above for p-^4He, the simple optical model of Eq. (1), modified by E/m as discussed previously, does not fit the data well. It would be surprising if changing the character of the optical potential improved the agreement with experiment, as the same reasons for the lack of fit presumably apply to both cases. As in Method (c), we allow V, W, and z to vary in our phenom-

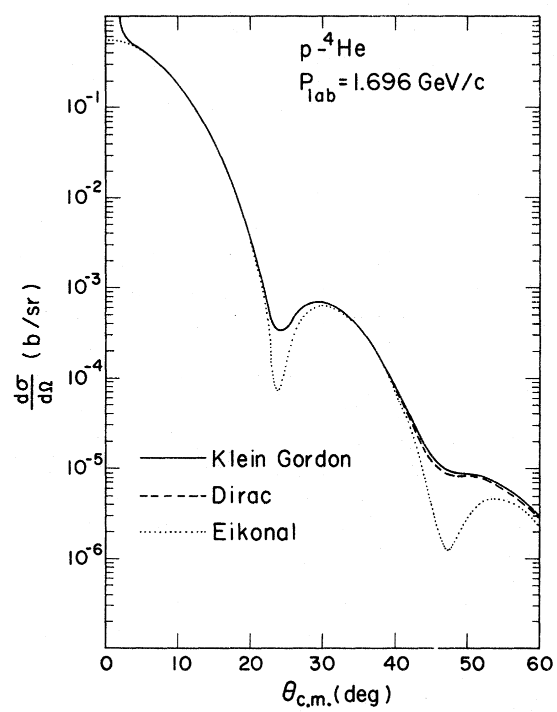

FIG. 2. The differential cross section in the center-of-mass system for elastic p-^4He scattering at incident lab momentum 1.696 GeV/c for the potential of Eq. (1) used in the Klein-Gordon equation (solid curve), the Dirac equation (dashed curve), and the eikonal approximation (dotted curve).

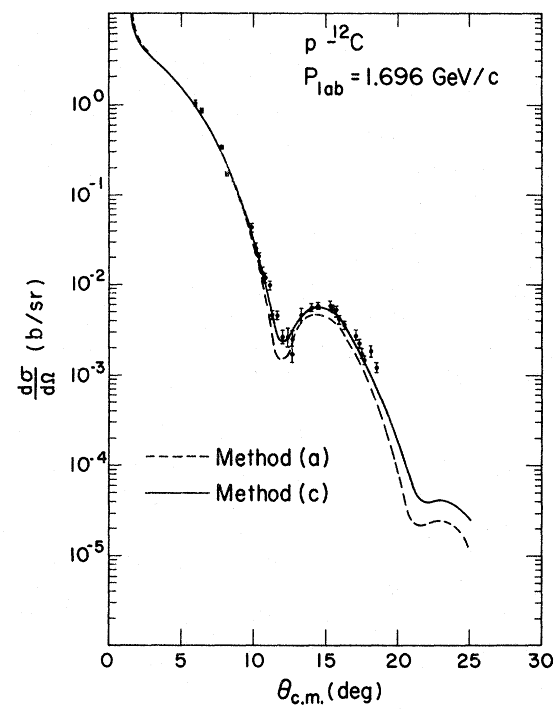

FIG. 3. The differential cross section in the center-of-mass system for elastic p-^{12}C scattering at incident lab momentum 1.696 GeV/c. The dashed curve is the result of Method (a), and the solid curve is the result of Method (c). The potential is taken to be the fourth component of a four-vector. The experimental cross sections are from Ref. 1.

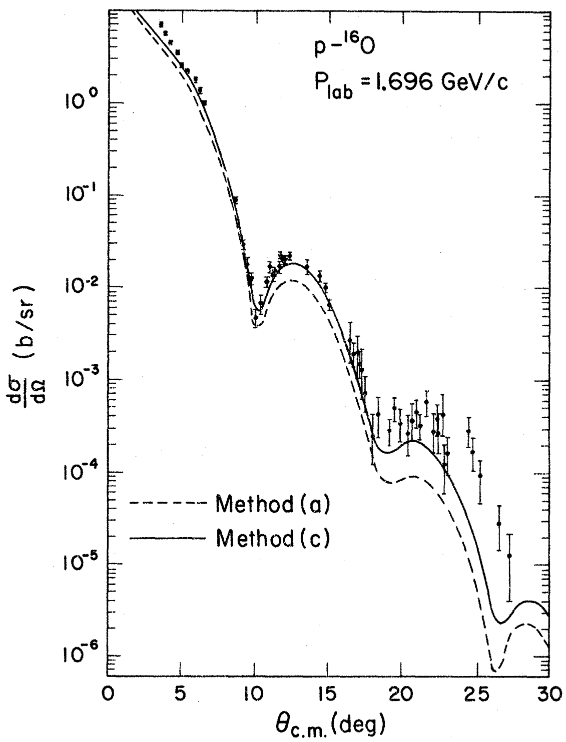

FIG. 4. The differential cross section in the center-of-mass system for elastic p-^{16}O scattering at incident lab momentum 1.696 GeV/c. The dashed curve is the result of Method (a), and the solid curve is the result of Method (c). The potential is taken to be the fourth component of a four-vector. The experimental cross sections are from Ref. 1.

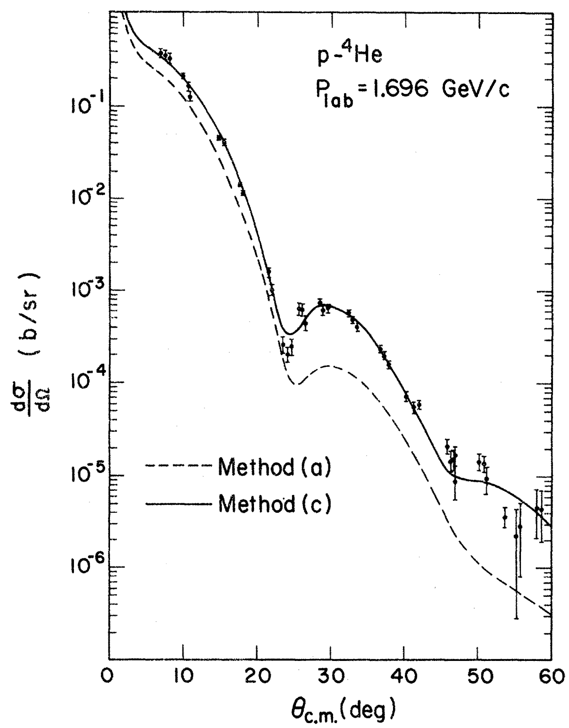

FIG. 5. The differential cross section in the center-of-mass system for elastic p-^{4}He scattering at incident lab momentum 1.696 GeV/c. The dashed curve is the result using Method (a), and the solid curve is the result of Method (c). The potential is taken to be a scalar. The experimental cross sections are from Ref. 1.

enological optical potential and obtain the good fit to the data shown in Fig. 5. The resulting parameters and fitting results are given in Table I.

If we calculate the differential scattering cross section using the Woods-Saxon scalar optical potential[20] given by Ref. 1, we do not obtain a reasonable fit to the data. We find that the χ^2 per degree of freedom is 39, a very poor fit. Figure 1 of Ref. 1 shows this optical potential to fit the data well. We feel that this discrepancy may be due to different treatment of recoil effects in the two calculations. The Klein-Gordon equation is intrinsically a one-body equation, and thus there is no recoil taken into account, and we use the proton mass in our calculations. If the reduced mass of the p-^{4}He system is used in an attempt to include recoil effects, then a repeat of the calculation with this modification results in reducing the χ^2 per degree of freedom from 39 to 22. Although this is an improvement, we still do not obtain the quality of fit with this shape which was obtained in Ref. 1.

In the case of ^{12}C and ^{16}O, the simple optical model of Eq. (1), modified by E/m, does fit the data well. Just as in the case of the vector potential, the shape function needed to fit the data is not very different from results given by electron scattering. When the Woods-Saxon potential, with the parameters given in Ref. 1 is used, we obtain good agreement with experiment; for ^{12}C the χ^2 per degree of freedom is 8.4, and for ^{16}O, the χ^2 per degree of freedom is 5.2. At this level of analysis, either the Woods-Saxon or harmonic-oscillator shape function is acceptable.

CONCLUSIONS

We come to substantially the same conclusion as that of Ref. 1 regarding the use of the simple optical model in analyzing proton scattering experiments at intermediate energies. We emphasize that at these energies accurate information about the nucleon distribution requires partial-wave analysis.

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[1]H. Palevsky, J. L. Friedes, R. J. Sutter, G. W. Bennett, G. J. Igo, W. D. Simpson, G. C. Phillips, D. M. Corley, N. S. Wall, R. L. Stearns, and B. Gottschalk, Phys. Rev. Letters 18, 1200 (1967). Numerical values are taken from BNL Report No. BNL-11360 and from an erratum to this report (private communication from G. J. Igo and R. Rolf).

[2]H. F. Ehrenburg, R. Hofstadter, U. Meyer-Berkhout, D. G. Ravenhall, and S. E. Sobottka, Phys. Rev. 113, 666 (1959). Recent electron scattering data for carbon and oxygen covering a larger range of recoil momenta has been reported by I. Sick and J. S. McCarthy, Nucl. Phys. A150, 631 (1970). The rms radii reported there are in agreement with the earlier work.

[3]The Woods-Saxon shapes used in Ref. 1 were chosen so that the rms radii were in close agreement with electron scattering. Reference 1 predicts rms radii of 2.42 and 2.62 fm for ^{12}C and ^{16}O, respectively. Reference 2, the analysis of electron scattering data, gives 2.49 and 2.69 fm for ^{12}C and ^{16}O, respectively.

[4]R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearin, Phys. Rev. 160, 874 (1967).

[5]The rms radius of the Woods-Saxon shape used in Ref. 1 is 1.67 fm, in good agreement with the early electron scattering experiments of McAllister and Hofstadter, Phys. Rev. 102, 851 (1956), of 1.6± 0.1 fm and the results of Ref. 4 and 1.71 fm.

[6]K. M. Watson, Phys. Rev. 89, 575 (1953); Phys. Rev. 105, 1388 (1957); see also A. L. Fetter and K. M. Watson, in *Advances in Theoretical Physics*, edited by K. A. Brueckner (Academic, New York, 1965), Vol. I, p. 115.

[7]As discussed by H. K. Lee and H. McManus, Phys. Rev. Letters 20, 337 (1968), the proton finite-size contribution to the charge density, and the correction to the optical model arising from the angular dependence of the nucleon-nucleon scattering amplitude, affect the cross sections in the same direction and to about the same extent. Thus, it is better to use the charge distribution rather than an unfolded matter distribution.

[8]Another commonly used optical potential has been given by A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959). In this formulation, the A in Eq. (1) is replaced by $A-1$. The scattering amplitude calculated, T', is related to the scattering amplitude of the original problem, T, by $T=A/(A-1)T'$. We have used this other form of Eq. (1) in calculating cross sections and note that there are no substantial differences; however, we choose Eq. (1) as given in the text as there are no uncertainties in the manner of inserting the Coulomb potential. Further, a reaction cross section may be calculated in an unambiguous way.

[9]V. Franco, Phys. Rev. Letters 21, 1360 (1968). The Glauber approximation calculation of Franco including spin and isospin may be compared with the spin-independent calculation of R. H. Bassel and C. Wilkin, Phys. Rev. Letters 18, 871 (1967).

[10]E. Kujawski, D. Sachs, and J. S. Trefil, Phys. Rev. Letters 21, 583 (1968).

[11]E. Kujawski, Phys. Rev. C 1, 1651 (1970).

[12]E. Lambert and H. Feshbach, Phys. Letters 38B, 487 (1972).

[13]D. V. Bugg, D. C. Salter, G. H. Stafford, R. F. George, K. F. Riley, and R. J. Tapper, Phys. Rev. 146, 980 (1966). Linear extrapolation gives $\sigma_{pp}=47.51$ mb, $\sigma_{pn}=40.32$ mb for the total cross sections at 1.696 GeV/c incident lab momenta. The averaged total cross section used in $\sigma_T=43.91$ mb.

[14]L. M. Dutton, R. J. W. Howells, J. D. Jafar, and H. B. Van Der Raay, Phys. Letters 25B, 245 (1967). They give $\alpha_p=0.1\pm0.16$ and $\alpha_n=-0.50\pm0.15$ at 1.69 GeV/c incident lab momentum for the ratio of real to imaginary parts of the proton-proton and proton-neutron scattering amplitudes, respectively. The average ratio is then −0.2.

[15]H. Feshbach, A. Gal, and J. Hüffner, Ann. Phys. 66, 20 (1971).

[16]We calculate a relative χ_2 defined by

$$\chi^2=\sum[(\lambda\sigma_c-\sigma_e)/\sigma_e]^2(\sigma_e/\Delta_e)^2\,,$$

where σ_c is the calculated cross section, σ_e the experimental cross section, Δ_e the statistical error, and λ an adjustable parameter used to minimize the total χ^2 defined by

$$\chi^2{}_T=\chi^2+(\sigma_e/\delta)^2(1-\lambda)^2\,.$$

Here δ is the over-all error in the absolute cross section. In λ we have a measure of the goodness of fit to the absolute cross section.

[17]In fact, for the vector potential case, the ratio of the Dirac to the Klein-Gordon cross sections is well represented numerically by the factor $1-\beta^2\sin^2\frac{1}{2}\theta$, the spin-overlap factor of the Born approximation.

[18]We do not include the Coulomb potential in our eikonal approximation calculations.

[19]See Table I, Pt. (c) of Ref. 3.

[20]We thank the referee of this paper for informing us that a scalar optical potential was used in Ref. 1. This fact is not mentioned in Ref. 1.