Search for an Excited Rotational Band in ^{98}Mo with the (p, t) Reaction*

Harbans L. Sharma,† R. Seltz,‡ and Norton M. Hintz
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 10 January 1973)

The (p, t) reaction has been used to study the ^{98}Mo nucleus which is expected to show the characteristics of a shape-transitional nucleus. A search has been made for a rotational band built upon the 0^+ state at 737 keV. The incident energy was 19 MeV. Cross sections were measured to many states and a number of spins and parities were assigned from the (p, t) angular distribution. The results show no positive evidence for an axially symmetric rotational band built upon the 737-keV (0^+) state. The levels observed agree with those predicted from an axially asymmetric rotor in the model of Gneuss and Greiner. The present spin and parity assignments agree with the previous work for most of the states. However, a few discrepancies have been found.

I. INTRODUCTION

Recently, there has been considerable interest[1-9] in using (p, t) and (t, p) reactions as probes to study shape transitions in nuclei around $N = 88$, the main point of interest being the distribution of $L = 0$ strength to $J = 0^+$ states and the search for possible rotational excitations of these states. Another shape-transition region occurs around the heavy Zr-Pd isotopes.

A recent study of the low-lying levels in the even Mo isotopes has been made by Taketani *et al.*,[9] using the (p, t) reaction at 52 MeV with an over-all resolution of 80 keV full width at half maximum (FWHM). They observed strong excitation (24% of the ground state) of the first excited 0^+ state in ^{98}Mo. Their resolution was, however, insufficient to resolve the nearby 737- and 790-keV levels.

The ^{98}Mo nucleus is of particular interest because it is one of the few nuclei with a 0^+ first excited state showing considerable excitation strength in the (p, t) reaction. A high-resolution (p, t) reaction study of this nucleus will be of particular value in assigning spins and parities and in searching for excited rotational bands.

The even Mo isotopes have also been studied via Coulomb excitation,[10] inelastic scattering,[11-13] (n, γ),[14] decay of fission products,[15] β decay,[16] and (d, p)[17] reactions. These experiments have yielded considerable information on the level structure of the Mo isotopes but some of the reaction experiments were of low resolution. Furthermore, relatively few spins and parities have been assigned.

From the spectra of ^{104,106}Mo[15] it is evident that these nuclei show characteristic rotational spectra corresponding to deformed nuclei. A recent study of ^{102}Mo[18] by ^{100}Mo$(t, p)^{102}$Mo reaction shows that ^{102}Mo occupies a place between the spherical and deformed regions, agreeing with the recent calculations[19] which predict that deformations should develop more rapidly in the neutron rich Mo and Zr nuclei than Ru and Pd nuclei. Furthermore, the strong (p, t) and (t, p) transitions to low-lying 0^+ states around ^{100}Mo invite comparison with the Sm[7,8] and Nd[1] isotopes. The excited 0^+ state in ^{150}Sm, seen in (p, t), was interpreted as a deformed state and was found to be associated with 2^+ and 4^+ states with rotational properties.[7,8]

Thus, it is interesting to do the ^{100}Mo$(p, t)^{98}$Mo reaction to check transitions to possible excited rotational levels built on the 0^+ (735-keV) state. A search has also been made for other excited 0^+ states.

II. EXPERIMENTAL PROCEDURE

The reaction ^{100}Mo$(p, t)^{98}$Mo was studied using an MP tandem Van de Graaff and the Enge split-pole magnetic spectrometer with an incident proton energy of 19 MeV. The target was prepared by vacuum evaporation of Mo metal enriched to 97.42% in ^{100}Mo. Position-sensitive detectors were used to detect and identify the outgoing tritons. The over-all resolution in the experiment was 10–15 keV FWHM. Angular distributions were taken from $\theta_L = 10$ to $60°$ with the exception of the first three states for which θ_L ranged from 10 to $90°$.

A photographic plate run was made at $\theta_L = 32.5°$ for accurate energy calibration to about ±5 keV. ^{100}Mo elastic proton scattering data were taken from $\theta_L = 10$ to $\theta_L = 60°$ for target thickness and spectrometer solid-angle calibration. The experimental cross sections were compared with optical-model calculations to determine absolute cross sections. The target thickness so determined was 17.2 μg/cm^2.

The error in the absolute (p, t) cross sections is estimated to be 10–15%. Relative errors are somewhat better and are shown in the figures.

The plate spectrum at 32.5° is shown in Fig. 1. Angular distributions are shown in Figs. 2–7. The states observed are listed in Table III along with some of the previous work.

III. SPIN-PARITY ASSIGNMENTS AND DWBA CALCULATIONS

As is well known the (p, t) selection rules restrict the final spin and parity for even-even target nuclei to

$$J = L, \quad \pi = (-1)^L .$$

In order to determine the value of the total orbital angular momentum transfer L, we have compared the shape of angular distribution with distorted-wave-Born-approximation (DWBA) calculations. The program used is a zero-range two-particle transfer code written by Chant.[20] The nonlocal options in this program were not used since they made little difference.

The spectroscopic factor for the 0^+ ground state was calculated using the prescription given by Yoshida,[21] i.e., for $L=0$, ground-state to ground-state transitions,

$$B(o, j, j) = (j + \tfrac{1}{2})^{1/2} U_j V_j' ,$$

where U_j is the emptiness amplitude for the residual nucleus and V_j' is the fullness amplitude for the target nucleus.

The values of V_j^2 taken from the papers of Hjorth and Cohen,[22] and Diehl *et al.*,[23] are given in Table I.

The spectroscopic amplitudes for the ground-state transition then becomes $0.62[(3s_{1/2})^2]$,

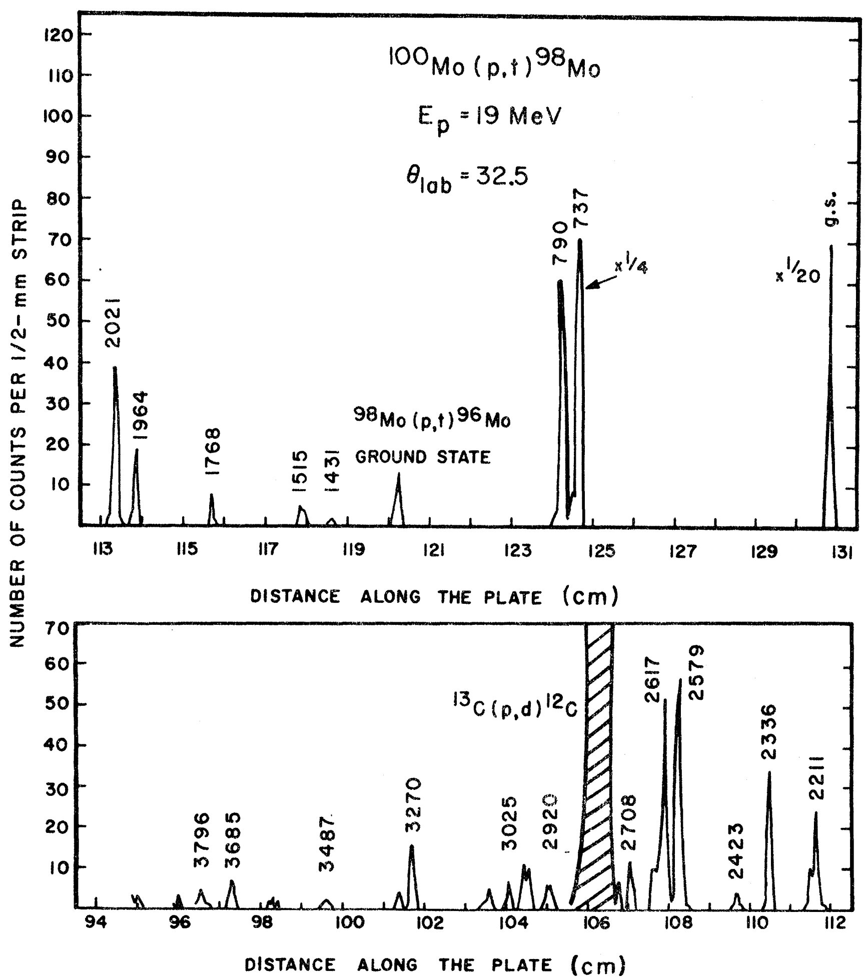

FIG. 1. A triton spectrum from the ^{100}Mo$(p, t)^{98}$Mo at a laboratory angle of 32.5°. The incident proton energy was 19.0 MeV.

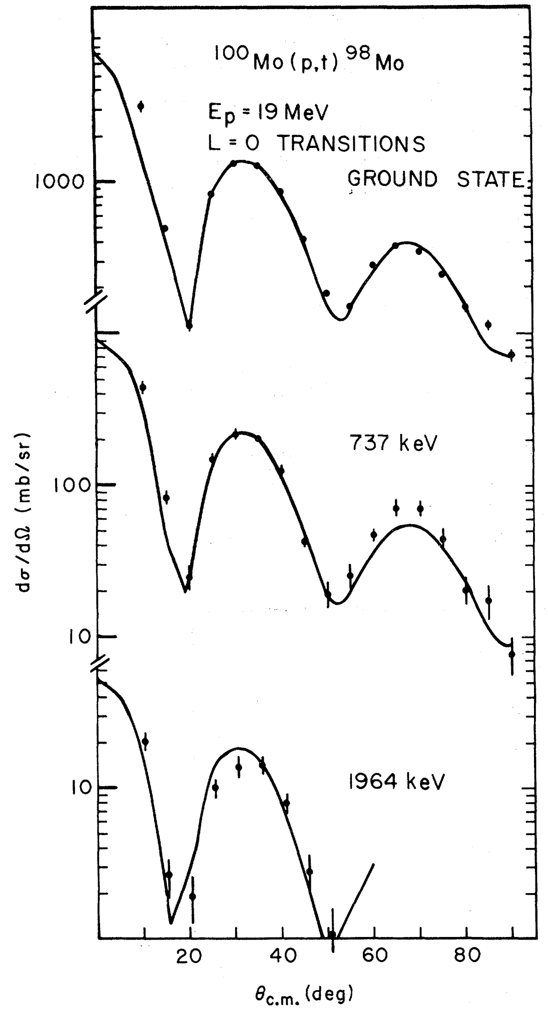

FIG. 2. Measured angular distributions for the ^{100}Mo-$(p, t)^{98}$Mo reaction. The results are for the states populated with $L=0$ angular momentum transfers. The points are the data and the curves are DWBA fits as described in the text The energies listed are excitation energies in keV. Other angular distributions are shown in Figs. 2–7.

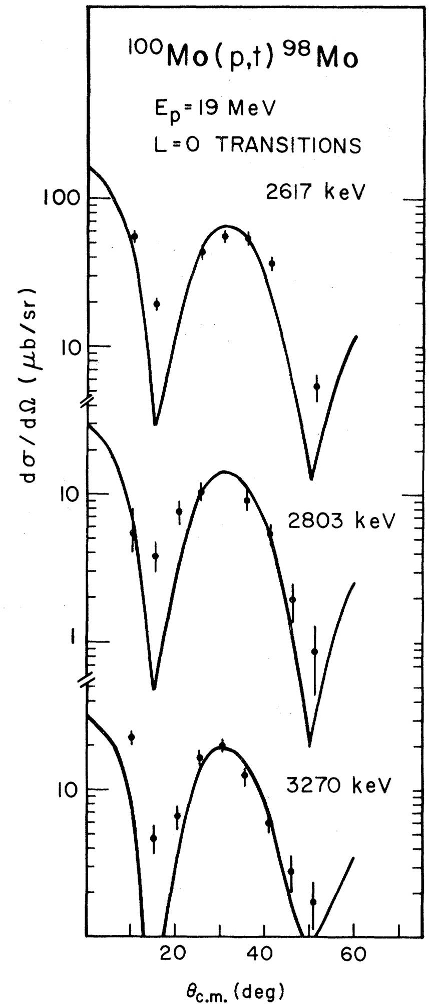

FIG. 3. Angular distributions for states in ^{98}Mo with $L=0$ at excitation energies 2617, 2803, and 3270 keV. See Fig. 2 caption.

$0.87[(2d_{5/2})^2]$, $0.11[(2d_{3/2})^2]$, and $0.24[(1g_{7/2})^2]$.

It is known from the previous work that the predicted configuration of the angular distributions is not very sensitive to the configuration assumed. Thus, we have used the ground-state $L=0$ shape for the other $L=0$ transitions. For $L=2$ and 4 we have used pure $(2d_{5/2})^2$ transfer. For $L=3$ we have taken $(2d_{5/2})(1h_{11/2})$ for the configuration of the transferred pair.

The optical potential used in the DWBA calculation was

$$V(r) = Vf_v(r) + iWf_w(r) - i\,4W_D\,\frac{df_w(r)}{dr} + V_c\,,$$

where

$$f_i(r) = \left[1 + \exp\left(\frac{r - R_i A^{1/3}}{a_i}\right)\right]^{-1}.$$

Several optical-model parameter sets were tried to find the best fit for the (p, t) ground state. These parameters are given in Table II.

For the neutron bound-state wave function the spin-orbit potential was included and the parame-

TABLE I. Occupation probabilities.

Isotope / orbit		$3s_{1/2}$	$2d_{5/2}$	$2d_{3/2}$	$1g_{7/2}$	$1h_{11/2}$
^{98}Mo	v_j^2	0.29	0.56	0.16	0.17	(0)
^{100}Mo	v_j^2	0.53	0.66	0.34	0.35	0.09

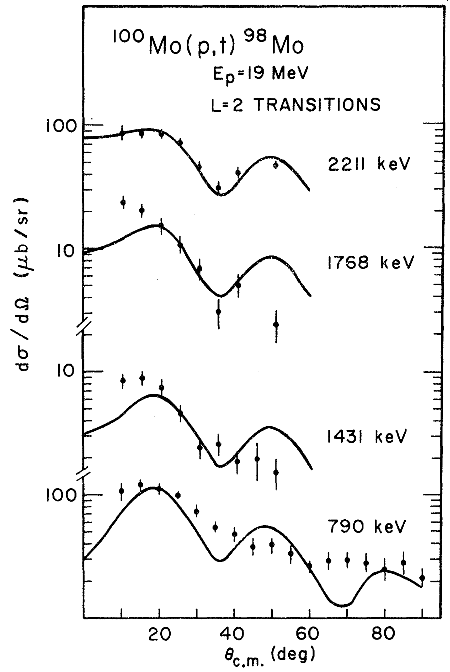

FIG. 4. Angular distributions for states populated with $L=2$. DWBA curves are shown along with data points.

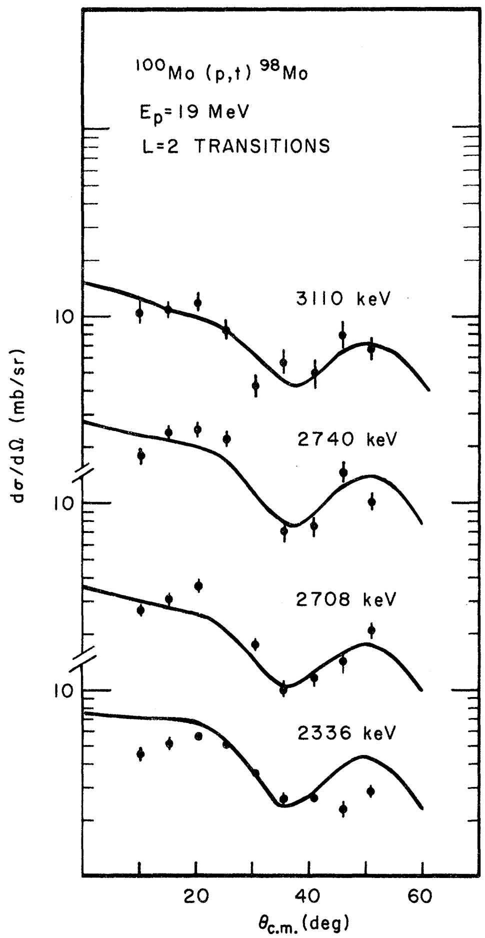

FIG. 5. Angular distributions for states in ^{98}Mo populated with $L=2$. Excitation energies are shown. DWBA curves are shown along with data points.

ters used were $R_v = 1.25$ fm, $a_v = 0.65$ fm, $V_{so} = 6.0$ MeV, $R_{so} = 1.25$ fm, $a_{so} = 0.65$ fm, and $R_c = 1.25$ fm. The transfer program searches on the depth of the real potential to give the correct separation energy for given Q and l, j values. The neutron separation energy was taken as $S_{2n}/2$.

All four combinations of proton and triton parameters were tried. The best fit for the ground state was obtained using the proton parameters given by Perey[24] and the triton parameters of Becchetti and Greenlees.[25] These parameters were then used for fitting the rest of the states. The DWBA results shown in Figs. 2–7 are individually normalized at forward angles.

IV. RESULTS

The results of the present ^{100}Mo(p, t)^{98}Mo experiment together with spin and parity assignments, are shown in Tables III and IV. Also shown are results from previous experiments on ^{98}Mo. The angular distributions and DWBA fits are shown in Figs. 2–7; the error bars reflect statistical errors only and apply to the relative cross sections. Absolute cross sections are good to ±10–15% as discussed above.

The (p, t) results significantly increase the num-

TABLE II. Optical-model parameters.

	V	R_V	a_v	W	R_W	a_W	W_D	R_C	Reference
Protons	53.3	1.25	0.65	0.0	1.25	0.47	16.1	1.25	24
Protons	55.0	1.17	0.75	0.0	1.33	0.61	8.76	1.30	a
Tritons	165.1	1.16	0.78	15.0	1.51	0.78	0.0	1.25	b
Tritons	161.8	1.20	0.72	25.9	1.40	0.84	0.0	1.30	25

[a] F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).
[b] E. R. Flynn, D. D. Armstrong, J. G. Beery, and A. G. Blair, Phys. Rev. 182, 1113 (1969).

ber of states of known spin and parity. Our results in general are in good agreement with the (p, p') data of Lutz, Heikkinen, and Bartolini,[12] the (n, γ) experiment of Heck *et al.*,[14] and β-decay studies in the determination of energies, spin, and parities.

The agreement between the experimental and theoretical angular distributions is quite good except for low-lying states at 790 keV (2^+), 1431 keV(2^+), 1515 keV($2^+, 4^+$), and 1768 keV(2^+). The 2^+ assignment for these states was made on the basis of their similarities in the angular distribution and by comparison with the spin assignment from other experiments. The state at 1515 keV, assigned 4^+ in other work, shows a rise at small angles, unlike the other $L=4$ shapes. This rise could be due to the presence of an unresolved 2^+ state nearby. However, no evidence of the doublet structure is seen in the spectra of this or other experiments.

A level is seen at 2208 keV by Lutz, Heikkinen, and Bartolini,[12] in (p, p') which they assign $J=4^+$. We see a state at 2211 keV, which is presumably the same state. Our spin assignment is clearly 2^+ which is in agreement with the (p, t) experiment of Taketani[26] [2.22(2^+)].

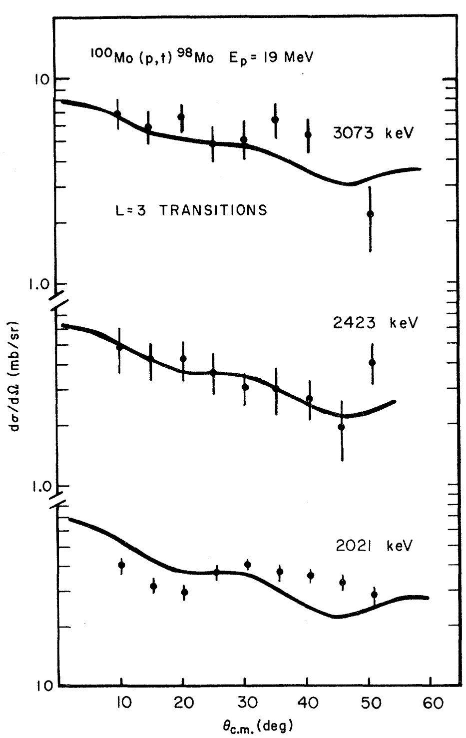

FIG. 6. Angular distributions for states probably populated with $L=3$. Excitation energies are shown. Curves are DWBA calculation.

A number of new 0^+ states have been found at 1964, 2617, 2803, and 3270 keV. None of these states have been reported in the earlier literature, except possibly a state at 2630 keV seen by Heck *et al.*[14] in an (n, γ) experiment. They have assigned $J=2^+$ for this level.

V. DISCUSSION

From the results of the present investigation, it seems highly unlikely that the 737-keV 0^+ excited state has a rotational band built upon it, at least one with the properties observed for the band built on the 1.256-MeV 0^+ state in ^{150}Sm. The only

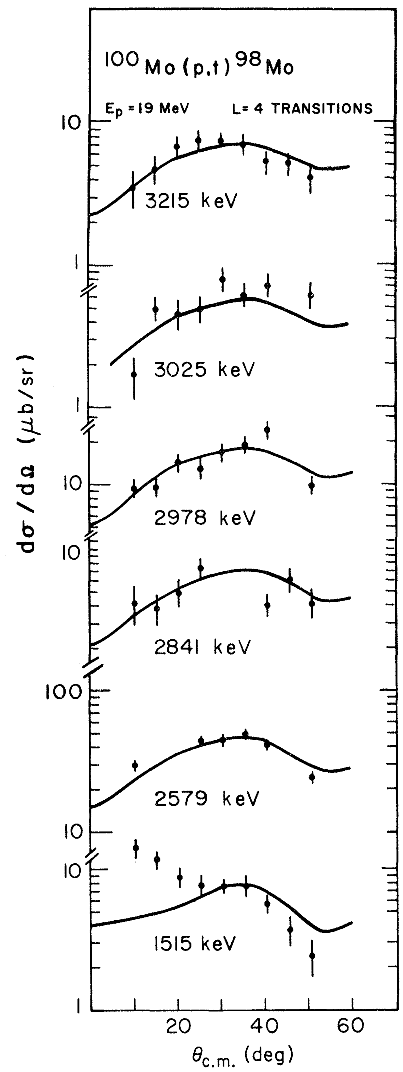

FIG. 7. Angular distributions for states probably populated with $L=4$. Excitation energies are shown. Curves are the DWBA calculations.

2^+ candidate with suitable excitation energy is the state at 1431 keV. However, the integrated strength of this state is 0.027 relative to the 737-keV 0^+ member. This is nearly an order of magnitude too low for it to qualify as a 2^+ rotational state when compared with a 2^+ strength of 0.20 ±0.04 in ^{150}Sm (0^+ member at 1.256 MeV) and 0.272±0.015 in ^{152}Sm for the ground band. The 2^+ state at 790 keV is too low in energy for a rotational excitation of the 737-keV 0^+ state.

A recent calculation by Gneuss and Greiner[27] shows that a collective potential surface with two minima (one at $\beta=\gamma=0$) and one axially asymmetric, ($\beta\neq 0$) gives rise to a low-lying spectrum

TABLE III. Summary of ^{98}Mo energy levels. Energies in keV.

This work [a]		(n,γ) (Ref. 14)		(d,p) (Ref. 17)	(p,p') (Ref. 12)		β decay (Ref. 16)	
737	0^+	734.9 ±0.3	0^+	736±20	736	0^+	734.9	0^+
790	(2^+)	787.42±0.1	2^+	787±15	788	2^+	787.5	2^+
1431	2^+	1432.32±0.1	2^+	1435±15	1433	2^+	1432.3	2^+
1515	$4^+(+2^+)$	1510.13±0.1	4^+	1513±15	1510	4^+	1510.1	4^+
1768	2^+	1785.5 ±0.2	2^+	1761±15	1760	2^+	1758.8	2^+
		1880.9 ±0.3						
1964	0^+						1985.1	
2021	3^-	2017.61±0.1	3^-	2025±15	2024	3^-	2018.0	
		2104.9 ±0.2	$1,2^+$	2110±30				
2211	2^+	2206.9 ±0.2	1, 2	2216±15	2208	4^+	2207.2	
2227		2224.0 ±0.2	$3,4^+$				2223.8	
2336	2^+	2333.4 ±0.2	$3,4^+$				2333.6	
		2343.7 ±0.2		2340±20	2343		2343.7	
2423	3^-	2419.8 ±0.2	$3^\pm,4^+$	2430±15	2450	4^+	2419.9	
		2485.4 ±0.2	$2^+,3,4^+$					
2504	3^-	2506.3 ±0.2		2530±25	2500	3^-	2506.2	
		2562.3 ±0.2	(1)					
2579	4^+	2572.9 ±0.2		2585±15				
2617	0^+	2620.9 ±0.2	2^+	2630±15			2608.5	
2646								
2708	2^+	2700.5 ±0.4					2679.0	
2740	2^+	2767.7 ±0.4	$3^\pm,4^+$				2767.9	
2803	0^+	2795.6 ±0.3						
2840	4^+			2829±20				
2920				2925±20				
		2962.4 ±0.4	$3^-,4^-$					
2978	4^+	2977.1 ±0.4		2980±30				
3025	4^+	3045.9 ±0.4					3022.2	
3073	3^-			3066±15				
		3103.1 ±0.5						
3110	2^+	3108.8 ±0.3		3124±20				
		3155.5 ±0.4		3168±20				
		3195.5 ±0.5						
3215	4^+	3210.7 ±0.4					3212.0	
3270	0^+			3270±30				
3302				3340±20				
							3395.2	
				3430±20			3455.2	
3487				3512±20			3502.8	
				3570±15				
3634				3636±20				
3685				3695±20				
				3740±30				
3796				3790±30				
3851								
3951								
4169								
4253								
4356								

[a] The energies are good to ±10 keV.

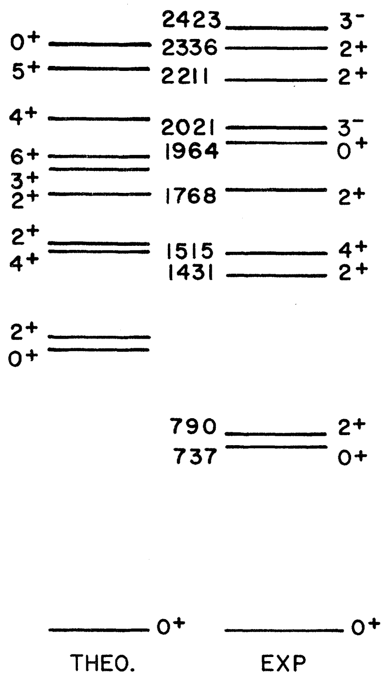

FIG. 8. Comparison of some low-lying states in ^{98}Mo with Gneuss and Greiner (Ref. 27) calculation. The energy scale of the theoretical prediction has been arbitrarily doubled.

which is very close to that observed for ^{98}Mo. A comparison of the theoretical and experimental states is shown in Fig. 8. The energy scale of the theoretical prediction has been arbitrarily doubled. The potential surface is shown in Fig. 9. Following tentative spin assignments by Heck *et al.*,[14] and Hubenthal, Monnand, and Moussa,[16] Sakai,[28] in a survey on quasibands in even-even nuclei, locates the 6^+ and 3^+ levels, predicted by the theory, at 2344 and 2333 keV, respectively. We see a 2^+ level at 2336 keV. With the usual selection rules, a transfer to a 3^+ level is forbidden in a (p, t) reaction on a nucleus with a 0^+ ground-state spin. We observe a (p, t) strength of 4.4% to this level making a 3^+ assignment very unlikely. Concerning the 6^+ level, it is well known that $L=6$ transfers are very weak in (p, t) reactions and therefore, its existence cannot be deduced from this experiment. Further measurements to confirm the existence of these levels and their spins would be very helpful to test the model.

Since the barrier between the two potential minima is small compared to the ground-state zero-point energy of 2.4 MeV, the wave function of the excited state will spread over a large region in β-γ space and so destroy their rotational character. The soft-potential calculations of

TABLE IV. Strengths in ^{100}Mo$(p, t)^{98}$Mo.

E (keV)	Relative strength at 30°	Relative integrated cross section $\theta_L=10°$ to $\theta_L=50°$	Absolute $\frac{d\sigma}{d\Omega}$ at 30°(μb/sr)
0	100	100	1296 ± 31
737	16.9	15.46	219.1 ± 12.9
790	5.67	8.4	73.5 ± 7.5
1431	0.18	0.42	2.4 ± 0.4
1515	0.70	0.82	7.6 ± 0.9
1768	0.53	0.95	6.9 ± 1.2
1964	1.05	0.94	13.7 ± 1.2
2021	3.09	4.45	40.1 ± 2.0
2211	3.56	6.58	46.2 ± 3.2
2336	2.74	4.41	35.6 ± 1.6
2423	0.28	0.13	3.1 ± 0.5
2504			
2579	3.46		44.9 ± 3.1
2617	4.34		56.3 ± 3.5
2708	1.29	2.42	16.8 ± 1.9
2740		1.5	
2803	0.72	0.74	
2840			
2978	1.32	1.8	17.2 ± 1.9
3025	0.61	0.71	7.9 ± 1.3
3073	0.40	0.57	5.1 ± 1.1
3110	0.33	0.91	4.3 ± 0.5
3215	0.57	0.74	7.3 ± 0.9
3270	1.99	1.13	19.9 ± 1.4

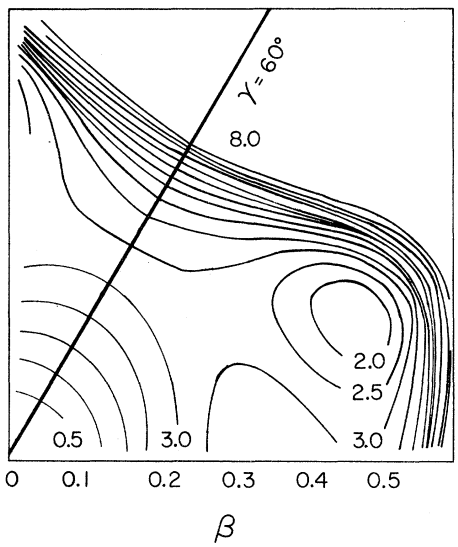

FIG. 9. Polar plot of the potential surface for ^{98}Mo (Fig. 19 of Ref. 27). The radial direction represents β and the azimuthal, γ. Equipotential contours are labeled in MeV.

Gneuss and Greiner[27] need to be extended to predict $B(E2)$ and two-particle transfer spectroscopic factors so that a more complete comparison can be made with the experimental data.

A ^{96}Mo$(t, p)^{98}$Mo experiment would shed more light on the structure of first excited 0^+ states because it would give the overlap with the ^{96}Mo ground state which is presumably more nearly spherical.

The fit of the angular distribution shapes are fairly good except for a few of the low-lying states. The possible reasons for the poor fits to the low-lying states may be:

(i) The wave functions used are too simple. We have used the same configurations for all excited states with the same J^π. This seems unlikely, however, since the shapes are not very sensitive to the configuration assumed.

(ii) As is pointed out in the Gneuss and Greiner[27] paper, the lowest states may have large inelastic collectivity and so two-step mechanism may be important.

*Work supported in part by the U. S. Atomic Energy Commission. This is report number COO-1265-133.

†On leave from Punjabi University, Patiala, India.

‡Present address: C. R. N. Laboratoire des Basses Energies, 67037, Strasbourg-3, France.

[1]K. Yagi, K. Sato, Y. Aoki, T. Udagawa, and T. Tamura Phys. Rev. Letters 29, 1334 (1972).

[2]J. B. Ball, R. L. Auble, J. Rapaport, and C. B. Fulmer, Phys. Letters 30B, 533 (1969).

[3]R. Chapman, W. McLatche, and J. E. Kitching, Nucl. Phys. A186, 603 (1972).

[4]Th. W. Elze, J. S. Boyno, and J. R. Huizenga, Nucl. Phys. A187, 473 (1972).

[5]D. G. Fleming, C. Gunther, B. G. Hagemann, and B. Herskind, Phys. Rev. Letters 27, 1235 (1971).

[6]G. Hagemann, private communication.

[7]P. Debenham and N. M. Hintz, Phys. Rev. Letters 25, 44 (1970).

[8]P. Debenham and N. M. Hintz, Nucl. Phys. A195, 385 (1972).

[9]H. Taketani, M. Adachi, M. Ogawa, K. Ashibe, and T. Hittori, Phys. Rev. Letters 27, 520 (1971).

[10]J. Barrette, M. Barrette, A. Boutard, R. Haroutunian, G. Lamoureux, and S. Monaro, Phys. Rev. C 6, 1339 (1972).

[11]Y. S. Kim and B. L. Cohen, Phys. Rev. 142, 788 (1966).

[12]H. F. Lutz, D. W. Heikkinen, and W. Bartolini, Phys. Rev. C 4, 934 (1971).

[13]Y. Awaya, K. Matsuda, T. Wada, N. Nakanishi, S. Takeda, and S. Yamaji, J. Phys. Soc. Japan 33, 881 (1972).

[14]D. Heck, V. Fanger, W. Michaelis, H. Ottmar, and H. Schmidt, Nucl. Phys. A165, 327 (1971).

[15]E. Cheifetz, R. C. Jared, S. G. Thompson, and J. B. Wilhelmy, Phys. Rev. Letters 25, 38 (1970).

[16]K. Hubenthal, E. Monnand, and A. Moussa, Nucl. Phys. A128, 577 (1969).

[17]K. R. Evans, F. Ajzenberg-Selove, and B. Rosener, Phys. Rev. 165, 1327 (1968).

[18]R. F. Casten, E. R. Flynn, O. Hansen, and T. J. Mulligan, Nucl. Phys. A184, 357 (1972).

[19]D. A. Arseniev, A. Sobieszewski, and V. G. Soloviev, Nucl. Phys. A139, 269 (1969).

[20]N. S. Chant, University of Minnesota, John H. Williams Laboratory Annual Report No. COO-1265-116, 1971 (unpublished).

[21]S. Yoshida, Nucl. Phys. 33, 685 (1964).

[22]S. A. Hjorth and B. L. Cohen, Phys. Rev. 135, B920 (1964).

[23]R. C. Diehl, B. L. Cohen, R. A. Moyer, and L. H. Goldman, Phys. Rev. C 1, 2132 (1970).

[24]F. G. Perey, Phys. Rev. 131, 745 (1965).

[25]F. D. Becchetti, Jr., and G. W. Greenlees, in *Proceedings of the Third International Symposium, Madison, Wisconsin, 1970,* edited by H. H. Barschall and W. Haeberli (Univ. Wisconsin Press, Madison, Wisconsin, 1971.

[26]H. Taketani, private communication.

[27]G. Gneuss and W. Greiner, Nucl. Phys. A171, 449 (1971).

[28]M. Sakai, Nucl. Data A10, 511 (1972).