Shell-Model Study of the Calcium Isotopes ($A = 42$–48) Using a Pairing-Plus-Surface-Tensor Interaction

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The calcium isotopes ^{42}Ca through ^{48}Ca are described within the framework of the conventional shell model using a phenomenological effective two-body interaction. The residual interaction consists of usual j-j pairing plus a surface-tensor force. ^{40}Ca is assumed to be an inert core. Besides all possible configurations arising from $1f_{7/2}$ and $2p_{3/2}$ orbits, configurations of the form $(1f_{7/2})^{n_1}(1f_{5/2})^{n_2}$ where $n_1 + n_2 = n$ (the number of extracore neutrons) and $n_2 = 1, 2$, are also considered in the basis vector spaces. The calculated ground-state energies as well as the excited energy levels are in good agreement with the experimental values. Reproduction of single-neutron spectroscopic factors for stripping and pickup reactions is also good.

1. INTRODUCTION

The calcium isotopes form a good set of nuclei for a conventional shell-model calculation. Following the shell-model picture, the nucleons are filling a well-isolated $1f_{7/2}$ orbit. A large amount of experimental information has been gathered from one- and two-nucleon pickup and stripping reactions. The experimental situation as regards the $1f_{7/2}$ nuclei has recently been reviewed by Ricci and Maurenzig.[1]

In the past, many calculations of the level structures of the calcium isotopes using the empirical effective-interaction method have been reported. The basic idea of this method is well known and has been emphasized many times.[2] We shall briefly report such calculations as an introduction to the investigation described in this paper. In the earliest works,[3-5] only pure $1f_{7/2}$ neutron configurations were included. A reasonable first-order agreement with the experiments was obtained. Some improvements have been achieved in a more realistic approach allowing neutrons also in the $2p_{3/2}$ orbital. In fact, there is experimental evidence for admixtures from higher configurations. For example, the first $\frac{3}{2}^-$ state in ^{43}Ca has a large admixture of $[f_{7/2}^2(J_i = 2)p_{3/2}]$. This is why one scarcely sees the $p_{3/2}$ admixture in the ^{42}Ca(d, p)-^{43}Ca reaction.[6]

England and Osnes[7] considered pure $1f_{7/2}^n$ as well as $(1f_{7/2}^{n-1}2p_{3/2})$ configurations. They obtained good agreement for the odd isotopes. Their results for the even isotopes, however, were not as good as for the odd isotopes. The first excited 0^+ and second 2^+ states in ^{42}Ca and ^{44}Ca were not reproduced. These states, however, are believed to arise from core excitations. In another systematic calculation of the calcium isotopes, Federman and Talmi[8] included all shell-model states arising from the ground configurations $(1f_{7/2}^n$ up to ^{48}Ca, $1f_{7/2}^{\ 8}2p_{3/2}^{\ 1}$ for ^{49}Ca, and $1f_{7/2}^{\ 8}2p_{3/2}^{\ 2}$ for ^{50}Ca) and of those obtained from it by raising one or two $1f_{7/2}$ neutrons into the $2p_{3/2}$ orbit. They also included deformed states in ^{42}Ca. They were able to reproduce the second 0^+ and 2^+ states in ^{44}Ca with the $1f_{7/2}$ and $2p_{3/2}$ active neutrons only. These states were taken in the fitting procedure. Recently, Federman and Pittel[9] have made a detailed study of the calcium isotopes with particular attention on the (t, p) reaction data.[10] It has been observed that the best agreement with the experiment is obtained when the 0_2^+ levels in ^{44}Ca (1.90 MeV) and ^{46}Ca (2.42 MeV) are assumed to arise from core-excited configurations. In their work also, the states were described in terms of an inert ^{40}Ca core plus neutrons in the $1f_{7/2}$ and $2p_{3/2}$ single-particle orbitals. For the isotopes through ^{48}Ca, two neutrons were allowed to occupy the $2p_{3/2}$ orbital. In Table I, we present some of the resulting set of two-body matrix elements determined by the empirical effective-interaction method. We find that even in the same basis-vector spaces (Refs. 8 and 9) the resulting set of two-body matrix elements depends to some extent on the energy levels taken in the fitting procedure. Thus in one case, 0_2^+ and 2_2^+ states in ^{44}Ca were well reproduced, while in the other work these were not obtained. However, the agreement with the experiments for both the cases is of the same order.

In the above mentioned method, the two-body matrix elements are determined directly by the least-squares fit to the experimental data. Generally, there are large numbers of adjustable parameters. As a result, a good agreement with the experimental values is obtained. However,

one has to make a very restrictive assumption regarding the configuration spaces. There is little scope to enlarge the model vector spaces. Then the number of variable two-body matrix elements increases drastically. The experimental information becomes insufficient to determine them with reasonable accuracy. Even in the case where only $1f_{7/2}$ and $2p_{3/2}$ orbitals were taken into considerations, the two-body matrix elements determined by the least-squares fit to the experimental binding energies and level energies had large statistical errors when all of them were allowed to vary. The parameters could undergo rather large changes without seriously affecting the agreement with the experiments. For this reason, Federman and Pittel[9] did not vary the nondiagonal two-particle matrix elements. There are also uncertainties in the configurations necessary to describe the nuclear states under investigation.

In another popular method, the effective two-body matrix elements are calculated from free nucleon-nucleon interactions by reaction-matrix methods. Interaction matrix elements can be calculated between states of many configurations. Thus uncertainties in the configuration spaces can be removed to a large extent. McGrory, Wildenthal, and Halbert[11] studied the calcium isotopes ^{42}Ca through ^{50}Ca within the framework of the conventional shell model. They used a "realistic" effective interaction derived for this mass region by Kuo and Brown[12] from the Hamada-Johnston free nucleon-nucleon potential. The quality of the agreement was not as good as that obtained by the empirical effective-interaction method which included the $2p_{3/2}$ orbit in the model vector space. Ground-state energies were poorly reproduced. They also observed an abrupt change in the quality of the agreement between theoretical and experimental values from ^{46}Ca to ^{47}Ca. This is in contradiction to the results obtained by the empirical effective-interaction method. An improvement was achieved by changing some of the two-body matrix elements. This method of calculating nuclear properties with the realistic potential is promising. Any lack of agreement with experiments directly points to the inadequacy in the treatment of the many-body problem. But there are difficulties. There are uncertainties in the nucleon-nucleon phase shifts, even at low energy, especially in the ^3S-^3D mixing parameter.[13] The uncertainty at high energy is considerable in most channels. Thus a potential is not obtained uniquely by the fit to the observed phase shifts. Secondly, it is well known that the renormalization effects are very important. Renormalizations are done to compensate for the omission of excitations into orbits absent from the active model space. The problem is then how to compensate for the omission of the configurations involving partially occupied orbits for some basis states within the model space. Finally, a variation of 10–15% of the matrix elements is probable due to approximations used in calculating the reaction matrix as pointed out by Kuo and Brown.[12] Our aim of the above discussion is to point out that there are still considerable uncertainties in the effective interaction itself when it is deduced from a realistic potential. Here we like to point out that there is a profound difference between the force used by us and the realistic forces. This point will be clear if one makes a comparative study of the matrix elements obtained by us (see Table I) and those obtained by Kuo and Brown[12] from the Hamada-Johnston free nucleon-nucleon potential. The comparison should be made both for the bare and core-renormalized matrix elements. Let us consider $f_{7/2}$-$p_{3/2}$ two-body matrix elements for $J=2$, 3, 4, and 5. For our interaction [pairing-plus-surface-tensor interaction (PSTI)] these values are, respectively, +0.92, +0.52, −0.03, and −0.72 and the corresponding values with the bare ones are +0.631, +0.213, +0.216, and +0.119. In our notation a positive sign indicates attraction. Thus we get much more repulsion in high J, i.e., $J=5$. Note that a $J=5$ matrix element is also unique in L-S coupling. It is $L=4$ and $S=1$, i.e. pure spin triplet. Since it is also $T=1$ we have the relative

TABLE I. Values of effective-interaction two-particle matrix elements which involve only $1f_{7/2}$ and/or $2p_{3/2}$ orbits. Entries in the column headed PSTI are two-particle matrix elements used in our work.

$2j_a$	$2j_b$	$2j_c$	$2j_d$	J	Federman and Talmi [a] (MeV)	Federman and Pittel [b] (MeV)	PSTI (MeV)
7	7	7	7	0	2.64	2.80	2.71
7	7	7	7	2	0.83	1.29	0.94
7	7	7	7	4	0.20	0.17	0.42
7	7	7	7	6	−0.28	−0.34	−0.39
7	3	7	3	2	1.98	0.35	0.92
7	3	7	3	3	−1.98	−0.78	0.52
7	3	7	3	4	0.93	0.60	−0.03
7	3	7	3	5	−0.59	−0.60	−0.72
3	3	3	3	0	1.40	1.35	1.67
3	3	3	3	2	0.00	0.28	0.18
7	7	3	3	0	1.64	0.78	1.10
7	7	3	3	2	1.20	0.27	0
7	7	7	3	2	0.74	0.50	0
7	7	7	3	4	0.35	0.31	0
7	3	3	3	2	0.60	0.33	0

[a] See Ref. 8.

[b] See Ref. 9.

l odd, for example $l=1$. So the only way to get this matrix element repulsive and large is to have triplet repulsion. The trouble with the realistic forces is that, being Serber-like, they have the $l=1$ interaction very weak, and even slightly attractive. We feel that it is very important to have a large and repulsive $\langle f_{7/2}p_{3/2}, J=5|V_{12}|f_{7/2}p_{3/2}, J=5\rangle$ matrix element for getting a good fit with the experimental values. The specific tensor force used by us fulfils this requirement.

The uncertainties mentioned above can be removed to a great extent if we use, in the shell-model study, a phenomenological effective two-body interaction with a few adjustable parameters. The only criterion is that such an interaction should give a good set of two-body matrix elements. A large configuration space can be used and it is also possible to obtain a good agreement with the experiments. The wave functions obtained, in view of greater configurational freedom, are also realistic. Thus calculations in this category are also attractive. Komada[14] used Serber and Rosenfeld spin-exchange forces for the study of ^{42}Ca, ^{43}Ca, and ^{44}Ca with $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ mixing. The calculated $(\frac{11}{2}^-)$ level in ^{43}Ca was found to be lower than the calculated $(\frac{3}{2}^-)$ level for the Rosenfeld mixture. Raz and Soga[15] dealt with the four calcium isotopes ^{42}Ca through ^{45}Ca using a potential with seven adjustable parameters. They considered all possible configurations of neutrons distributed in the $1f_{7/2}$ and $2p_{3/2}$ orbits. The $(\frac{9}{2}^-)$ levels in ^{43}Ca and ^{45}Ca were predicted too low.

In a previous communication,[16] we have proposed a phenomenological effective two-body interaction for a shell-model study of the calcium isotopes. It has been shown that it is possible to get a good set of two-body matrix elements from this interaction. This conclusion was drawn by an attempt to reproduce an empirical set of two-body matrix elements[9] by which the experimental data were well described. In this paper, we present our study of the calcium isotopes ^{42}Ca through ^{48}Ca using our effective two-body interaction. We shall limit ourselves to discussing binding energies, energy levels, and spectroscopic factors for one-neutron transfer reactions. In our calculations we allow active neutrons in the $1f_{7/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbits with the restriction of a maximum two neutrons in the $1f_{5/2}$ orbit. The model vector spaces and different assumptions are described in Sec. 3. We describe the effective two-body interaction in detail in Sec. 2. In Sec. 4 we describe the calculation and in Sec. 5 we discuss the results. Single-particle spectroscopic factors are presented in Sec. 6. A brief summary of our investigation is given in Sec. 7.

2. EFFECTIVE TWO-BODY INTERACTION

In the work of Levinson and Ford[17] on the calcium isotopes, we find that the empirical potential should be shallower and of longer range than that required by the nucleon-nucleon scattering at low energy. Mitler[18] had shown in the work of intermediate shell-model calculation for ^{42}Ca and ^{43}Ca that the effective two-body internucleon potential should be long ranged, with a weak-repulsive (rather than attractive) triplet-odd part and a strong-attractive (rather than weak-repulsive) tensor part.

It is desirable that a phenomenological form of an effective interaction will contain some of the features believed to be present in a realistic nucleon-nucleon potential. Let us consider Hamada-Johnston's[19] nucleon-nucleon potential. Its short-range part has a strong central-force component in the singlet-even state. So there is some justification for treating the short-range part of the internucleon potential by the j-j pairing force.[20] The j-j pairing force acts only in the single-even state. Moreover, the pairing nature of particle-particle interaction is enhanced by renormalization effects due to core excitations. An important feature of ground-state energies of nuclei with fixed proton or neutron number is the pairing effect, but a pairing force alone is a very poor approximation to the effective nuclear interaction. The longe-range part of the realistic force is mainly determined by the one-pion exchange potential which is of tensorial character. The need for a tensor force for shell-model study has been demonstrated in the past.[21-24] There are indications that a part of the residual interaction takes place at the nuclear surface. The long-range part of the "pairing-plus-quadrupole" interaction[25] which has been quite successful in accounting for many systematic features of nuclear levels, is well known to act mainly when nucleons are at the nuclear surface. In view of the above discussions we have considered an effective two-body interaction which consists of the usual j-j pairing force and a long-range part of tensorial character acting at the nuclear surface only. The form of the effective two-body interaction used in our investigation is given below. As before, we shall refer to it as PSTI[16] throughout this paper.

$$V_{12} = -Aq_{12} + B(r^2/R_0^2)\delta(r_1 - R_0)\delta(r_2 - R_0)S_{12}, \tag{1}$$

where

$$S_{12} = \left[\frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \frac{1}{3}(\vec{\sigma}_1 \cdot \vec{\sigma})\right]$$

and the operator q_{12} is defined by its antisymmetrized matrix elements

$$\langle j_a j_b JM | q_{12} | j_c j_d JM\rangle = [(2j_a+1)(2j_c+1)]^{1/2}\delta_{j_a j_b}\delta_{j_c j_d}\delta_{J0}\,.$$

Antisymmetrized two-body matrix elements for the tensor part of the relation (1) in the j-j coupling scheme can be easily obtained by the multipole-expansion method given by Talmi.[26] The result for $T=1$ is given below in $\vec{j}=\vec{s}+\vec{l}$ coupling scheme:

$$\begin{aligned}
&\langle j_a j_b JM | B(r^2/R_0^2)\delta(r_1-R_0)\delta(r_2-R_0)S_{12} | j_c j_d JM\rangle \\
&\quad = -6\sqrt{10}\,B\hat{j}_a\hat{j}_b\hat{j}_c\hat{j}_d\hat{l}_a\hat{l}_b\hat{l}_c\hat{l}_d[(1+\delta_{ab})(1+\delta_{cd})]^{-1/2}(-)^{j_b+j_c+l_a+l_b+J} \\
&\qquad \times\Bigg[\frac{1}{\sqrt{5}}\begin{Bmatrix} j_a & j_b & J \\ j_d & j_c & 1\end{Bmatrix}\Bigg(\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_c & j_c \\ 1 & 2 & 1\end{Bmatrix}\begin{Bmatrix} \frac12 & l_b & j_b \\ \frac12 & l_d & j_d \\ 1 & 0 & 1\end{Bmatrix}\langle l_a 0 l_c 0|20\rangle\langle l_b 0 l_d 0|00\rangle \\
&\qquad\qquad +\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_c & j_c \\ 1 & 0 & 1\end{Bmatrix}\begin{Bmatrix} \frac12 & l_b & j_b \\ \frac12 & l_d & j_d \\ 1 & 2 & 1\end{Bmatrix}\langle l_a 0 l_c 0|00\rangle\langle l_b 0 l_d 0|20\rangle\Bigg) \\
&\qquad +(-)^J\frac{1}{\sqrt{5}}\begin{Bmatrix} j_a & j_b & J \\ j_c & j_d & 1\end{Bmatrix}\Bigg(\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_d & j_d \\ 1 & 2 & 1\end{Bmatrix}\begin{Bmatrix} \frac12 & l_b & j_b \\ \frac12 & l_c & j_c \\ 1 & 0 & 1\end{Bmatrix}\langle l_a 0 l_d 0|20\rangle\langle l_b 0 l_c 0|00\rangle \\
&\qquad\qquad +\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_d & j_d \\ 1 & 0 & 1\end{Bmatrix}\begin{Bmatrix} \frac12 & l_b & j_b \\ \frac12 & l_c & j_c \\ 1 & 2 & 1\end{Bmatrix}\langle l_a 0 l_d 0|00\rangle\langle l_b 0 l_c 0|20\rangle\Bigg) \\
&\qquad +\frac{1}{3}\sqrt{10}\sum_K(2K+1)\begin{Bmatrix} 1 & 1 & K \\ 1 & 1 & 2\end{Bmatrix}\Bigg(\begin{Bmatrix} j_a & j_b & J \\ j_d & j_c & K\end{Bmatrix}\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_c & j_c \\ 1 & 1 & K\end{Bmatrix}\begin{Bmatrix} \frac12 & l_b & j_b \\ \frac12 & l_d & j_d \\ 1 & 1 & K\end{Bmatrix}\langle l_a 0 l_c 0|10\rangle\langle l_b 0 l_d 0|10\rangle \\
&\qquad\qquad +(-1)^J\begin{Bmatrix} j_a & j_b & J \\ j_c & j_d & K\end{Bmatrix}\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_d & j_d \\ 1 & 1 & K\end{Bmatrix}\begin{Bmatrix} \frac12 & l_b & j_b \\ \frac12 & l_c & j_c \\ 1 & 1 & K\end{Bmatrix}\langle l_a 0 l_d 0|10\rangle\langle l_b 0 l_c 0|10\rangle\Bigg)\Bigg]\,.
\end{aligned} \tag{2}$$

In the above relation, $\hat{x}$ signifies $(2x+1)^{1/2}$, $\langle l_a 0 l_c 0|20\rangle$, etc. are Clebsch-Gordan coefficients.

$\begin{Bmatrix} j_a & j_b & J \\ j_d & j_c & 1\end{Bmatrix}$ etc.

are $6j$ symbols and

$\begin{Bmatrix} \frac12 & l_a & j_a \\ \frac12 & l_c & j_c \\ 1 & 2 & 1\end{Bmatrix}$ etc.

stand for $9j$ coefficients. We assume that the particle radial wave functions all have the same amplitude at the nuclear surface and the state-independent radial integrals thus obtained are absorbed in the parameter B.

3. ASSUMPTIONS AND CHOICE OF CONFIGURATIONS

A. Assumptions

We assume that the basic assumptions of general shell models are valid and the effective nuclear forces are two-body forces. In our calculation we consider ^{40}Ca as an inert core. Thus, total energy of a calcium isotope $^{40+n}$Ca can be written as the sum of two parts: $E=E_0+E_{\text{int.}}$, where E_0 is the binding energy of the ^{40}Ca core and $E_{\text{int.}}$ is the interaction energy of the n extracore neutrons moving in a central field supplied by this core. We take the central field in which the extracore neutrons move to be the same for all nuclei in which these particles can be placed in the same j orbit. This is consistent with our previous findings on the systematics of nuclear separation energies.[27] Accordingly we expect that the interaction parameters determining $E_{\text{int.}}$ remain constant for the isotopes ^{41}Ca through ^{48}Ca. Moreover, there is evidence that ^{48}Ca forms a good closed core.[28, 29]

B. Choice of Configurations

In our investigation we consider $1f_{7/2}$, $2p_{3/2}$, and $1f_{5/2}$ as active orbitals. We form states by distributing extracore neutrons in these orbits. However we allow a maximum of two neutrons in the

$1f_{5/2}$ orbit. All Pauli-allowed configurations of the form $(1f_{7/2})^n$, $[(1f_{7/2})^{n_1}(2p_{3/2})^{n_2}]$ where $n_1+n_2=n$ (the number of extracore neutrons) $[(1f_{7/2})^{n-1}(1f_{5/2})^1]$, and $[(1f_{7/2})^{n-2}(1f_{5/2})^2]$ are considered. The unperturbed $1f_{5/2}$ orbit is rather high. It is plausible to assume that the excitations of three or more neutrons in the $1f_{5/2}$ orbit will not affect the low-lying states appreciably. On the other hand, since our effective two-body interaction has a strong pairing character, we believe that the excitations of two particles in the $1f_{5/2}$ orbit are important.

4. DESCRIPTION OF THE CALCULATION

The calculations are made within the framework of the conventional shell model. The energy matrices are constructed in terms of two-particle matrix elements and the single-particle energies. We then diagonalize the model Hamiltonian in the model vector spaces described before. The eigenvalues are characterized as nuclear energy levels and the eigenvectors as wave functions of nuclear states. We work in the seniority scheme.

Now we discuss the choice of the single-particle energies (spe). In our truncated basis-vector spaces, we need three single-particle energies $\epsilon_{1f_{7/2}}, \epsilon_{2p_{3/2}}, \epsilon_{1f_{5/2}}$. The values we use, are considered from the shell-model interpretation of the ^{41}Ca spectrum. We take the values from the work of Belote, Sperduto, and Buechner[30] on the ^{40}Ca-$(d,p)^{41}$Ca reaction. We find that in ^{41}Ca there is splitting of the single-particle strength of the $2p_{3/2}$ and $1f_{5/2}$ orbits. The position of the $(\frac{7}{2})^-$ ground state of ^{41}Ca has a spectroscopic factor of about 1. Thus the neutron separation energy in ^{41}Ca determines the unperturbed $1f_{7/2}$ single-particle energy. The $2p_{3/2}$ strength was found to be split over two states and the unperturbed level was determined by weighting the two $\frac{3}{2}^-$ levels at 1.95 and 2.47 MeV by their spectroscopic factors. The $1f_{5/2}$ level which was observed to be split over three states, carries only half of the single-particle strength. The spe taken in our investigation are given below

$$\epsilon_{1f_{7/2}} = 8.36 \text{ MeV},$$

$$\epsilon_{2p_{3/2}} = 6.29 \text{ MeV},$$

$$\epsilon_{1f_{5/2}} = 2.86 \text{ MeV}.$$

For the sake of convenience we consider binding energies as positive quantities. We use the convention in which a larger number represents a more tightly bound state. We take the absolute values of spe because we are interested in calculating the ground-state energies as well as the energy spacings.

The splitting of the single-particle strength in ^{41}Ca suggests that ^{40}Ca is not a very inert core. It has been shown that this can be understood with the introduction of core-excited states.[31,32] It is desirable to include core-excited configurations for the shell-model study. For simplicity, we shall not consider this in our investigation. This is, of course, a major imperfection of the work we are discussing. There are uncertainties in the single-particle energies. We expect that the slight changes in the spe can be absorbed by the effective interaction between nucleons without altering the quality of agreement between theory and experiment significantly.

There are two adjustable parameters in the effective two-body interaction which we use in the present study. We determine these parameters by a fit to the experimental values. For this purpose we focus our attention on the experimental ground-state energies. Since we describe the calcium isotopes relative to the ^{40}Ca core, the excess of the binding energies relative to ^{40}Ca is determined by the interaction of the extracore neutrons. A major part of it comes from the one-body part of the interaction. For example, if we assume that the ground state of ^{48}Ca is a pure $(1f_{7/2})^8$ state then the one-body contribution is 66.88 MeV and only 7.06 MeV comes from the two-body part. Thus, considerable discrepancies between calculated and experimental ground-state energies point to a defect of the residual two-body interaction. We think so because we have assumed that a slight change in the spe can be absorbed in the effective two-body interaction. From previous works, is clear that the low-lying states of the calcium isotopes are dominated by pure $(1f_{7/2})^n$ configurations. Thus the four two-body matrix elements $\langle f_{7/2}^2 J | V_{12} | f_{7/2}^2 J \rangle$ with $J=0$, 2, 4, and 6 are very important in determining these states. The results on strengths for ground-state to ground-state transitions suggest that the ground states for $^{42-48}$Ca are almost pure. Moreover, the $\epsilon_{1f_{7/2}}$ is less uncertain. The ground states are affected primarily by the matrix elements $\langle f_{7/2}^2 J | V_{12} | f_{7/2}^2 J \rangle$ and are less sensitive to other two-body matrix elements and to the single-particle energies. So, we hope to get a good set of two-body matrix elements by adjusting the interaction parameters so as to get a reasonable agreement with the experimental binding energies. Finally, it is well known that the core-excited states appear in the calcium isotopes through ^{46}Ca. If the excited energy levels are considered in determining the interaction parameters, there is a chance that some core-excited states are also included in the fitting procedure. We do not want to make any assumption regarding the choice of the core-excited states. Once the parameters are de-

TABLE II. Calculated and experimental binding energies for ^{42}Ca–^{48}Ca relative to ^{40}Ca. The units are MeV. The percentage of ground-state wave function made up of pure $(1f_{7/2})^n$ configuration is also shown.

Mass number of calcium isotopes	Binding energies Experimental [a] (MeV)	Calculated (MeV)	Percentage of pure $(1f_{7/2})^n$ configuration
42	19.84	19.65	95.88
43	27.76	27.66	95.88
44	38.90	38.58	91.80
45	46.32	46.24	92.16
46	56.72	56.80	88.38
47	64.00	64.09	89.87
48	73.94	74.26	86.99

[a] From Ref. 33.

termined we calculate the excited energy levels. It is, therefore, possible to judge which of the experimentally observed states can or cannot be accounted for by our chosen configurations. The two interaction parameters A and B are determined by a fit to the seven experimental ground-state energies. The parameters thus determined are given below:

$$A = -0.194 \text{ MeV} \quad \text{and} \quad B = -2.020 \text{ MeV}.$$

In order to give an idea of the two-body matrix elements which we have used in our calculations, we present some of them in the Table I. The $\langle f_{7/2}{}^2 J|V_{12}|f_{7/2}{}^2 J\rangle$ matrix elements are all quite similar to those obtained in empirical effective-interaction calculations. The average value, defined by $V_{\text{ave}} = \sum_J (2J+1)\langle f_{7/2}p_{3/2}J|V_{12}|f_{7/2}p_{3/2}J\rangle / \sum_J (2J+1)$ of the $f_{7/2}$-$p_{3/2}$ two-body matrix elements is -0.0005. It is slightly repulsive. Federman and Pittel pointed out that it was essential to consider the various 2^+ levels in ^{48}Ca in the fitting procedure to reduce the stastical errors in $f_{7/2}$-$p_{3/2}$ two-body matrix elements.[9] However, the assignment of 2^+ to levels at 6.33 and 6.79 MeV are less certain. We did not consider excited levels to get the interaction parameters.

5. RESULTS

A. Binding Energies

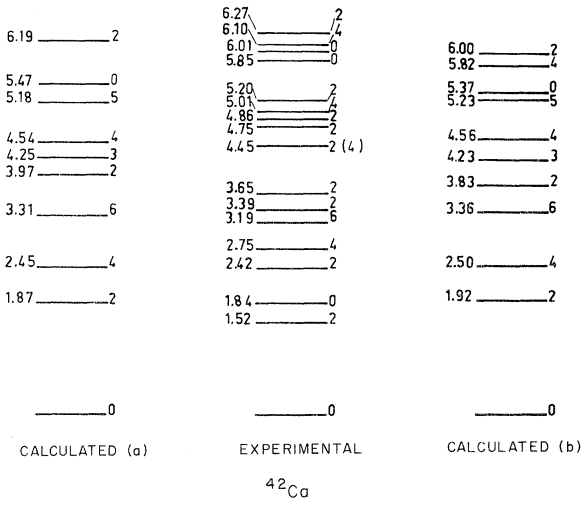
In Table II we show the calculated and experimentally determined binding energies relative to ^{40}Ca. The experimental values are taken from the 1964 mass table.[33] The agreement is good. We also show the percentage of ground-state wave functions made up of pure $(1f_{7/2})^n$ configurations. It indicates that the ground states of these isotopes are dominated by pure $(1f_{7/2})^n$ configurations. This is consistent with the experimental results on strengths for ground-state to ground-state transitions.

B. Energy Spectra for the Even Isotopes

We have calculated only positive-parity states for even-even nuclei. For this reason, we omit parity assignments from the energy spectra for the even calcium isotopes and we also do not present the experimental negative-parity levels.

^{42}Ca

In the case of the ^{42}Ca nucleus, the calculations are done in two basis-vector spaces with the same interaction parameters. For the first model space, extracore neutrons are allowed to occupy only $1f_{7/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbits. For the second model space, all the f-p shell orbits, i.e. $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbits, are active. The single-particle energy taken for the $2p_{1/2}$ orbit is 4.23 MeV. The calculated and observed positions of levels in ^{42}Ca are shown in Fig. 1. "Calculated (a)" in Fig. 1 shows the results for the first model space and



^{42}Ca

FIG. 1. Experimental and calculated energy levels of ^{42}Ca.

"Calculated (b)" shows the spectrum for the second model space. The experimental values are taken from the (t,p) reaction data of Bjerragard *et al.*[10] We have not shown the unassigned experimental levels. The calculated binding energy for the second model space is 19.70 MeV relative to the ^{40}Ca nucleus while for the first model space it is 19.65 MeV. We see that the relative positions of the levels with respect to the ground state are not affected significantly with the inclusion of the $2p_{1/2}$ orbit in the basis-vector space. However, one 4^+ level appears at 5.82 MeV which corresponds to the observed 4^+ level at 6.10 MeV. The dominant strength in this state is $|f_{7/2}p_{1/2}, J=4\rangle$, which has an amplitude of 90%. The 0^+ level observed at 1.84 MeV is not described in both the model spaces. The same result has been obtained in most of the previous calculations described in the introduction. Federman and Talmi[8] described this level as a deformed state which is in agreement with the (d,t) reaction data.[34] Pickup data[34] also show that the 1.52- and 2.42-MeV 2^+ levels equally share the $|f_{7/2}^2, J=2\rangle$ configuration. These two states have been nicely described by mixing deformed states formed by exciting two protons from the s-d shell.[35-37] The resulting wave functions for both these states have been found to contain a strong $|$deformed, $J=2\rangle$ component. In our calculations we have not considered core excitations. Accordingly, we expect that our model predicts one 2^+ level in this region sharing the total $|f_{7/2}^2, J=2\rangle$ strength. We also expect that the position of this unperturbed level is determined by weighting the two 1.52- and 2.42-MeV 2^+ levels by their spectroscopic factors for neutron pickup from the $1f_{7/2}$ orbit. The unperturbed position thus determined $(\sum S_i E_i / \sum S_i)$ is at 1.99 MeV. Spectroscopic factors for $l=3$ pickup are taken from the Ref. 34. For the first model space, the calculated 2_1^+ level is at 1.87 MeV and the $l=3$ spectroscopic factor is 0.40 which is nearly the sum of the observed $l=3$ spectroscopic factors $(\sum S_i = 0.42)$. For the second model space, the calculated 2_1^+ level is at 1.92 MeV. The 4^+ assignment to the level at 4.45 MeV was made by Lippincott and Bernstein.[38] This assignment is based on the inelastic α-particle scattering experiment. In view of our results we feel that this assignment is correct. The calculated second 0^+ state roughly matches with the observed 0^+ state at 5.85 MeV. The agreement between the calculated and experimental spectra appears good. The density of states observed in the ^{40}Ca$(t,p)^{42}$Ca reaction[10] is greater than our calculated density. This indicates the importance of core-excited configurations not included in our basis-vector spaces.

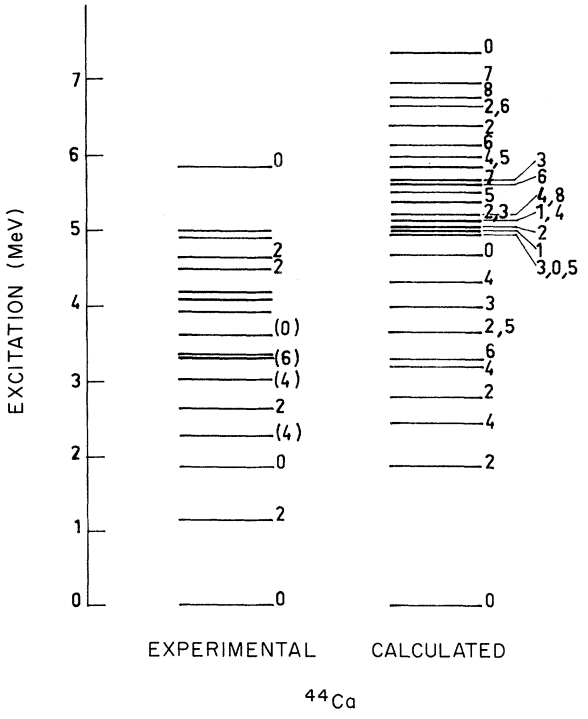


FIG. 2. Experimental and calculated energy levels of ^{44}Ca.

^{44}Ca

The calculated and observed spectra of ^{44}Ca are shown in Fig. 2 and tabulated in Table III. The experimental values are taken from (d,p) and (t,p) reaction data.[10,39] The experimental spin assignments shown in the table are taken from different experiments.[10,38-40] Only a few levels are drawn in the figure. It appears that the agreement between the calculated and experimental spectra is good. The 0^+ state at 1.89 MeV is not reproduced. We believe that it is a core-excited level. In the ^{43}Ca$(d,p)^{44}$Ca reaction, both the 2^+ states at 1.16 and 2.67 MeV are strongly excited.[39] We think that the 2^+ level at 2.67 MeV is also a core-excited level and this state is strongly mixed with the shell-model state. Accordingly, we expect that the unperturbed position of the shell-model 2_1^+ state is determined by $\sum S_i E_i / \sum S_i$, where S_i denotes $l=3$ spectroscopic factors for these two 2^+ levels. The position of 2^+ level thus determined is at 2.00 MeV. Our calculated 2_1^+ level is at 1.86 MeV and its $l=3$ spectroscopic factor is 1.32 which is very close to the sum of the $l=3$ spectroscopic factors (1.30) of the 1.16- and 2.66-MeV 2^+ levels.[39] The calculated spectrum exhibits one 2^+ state at 2.77 MeV which is close to the observed 2^+ state at 2.67 MeV. However, the wave function of this state is strongly admixed

TABLE III. Calculated and experimental levels in ^{44}Ca.

	Exp.		Cal.			Exp.		Cal.	
	E		E			E		E	
No.	(MeV)	J	(MeV)	J	No.	(MeV)	J	(MeV)	J
0	0	0	0	0	25	5.02	(4)	5.99	4
1	1.16	2	1.86	2	26	5.14		5.99	5
2	1.89	0	2.45	4	27	5.17		6.11	6
3	2.29	(4)	2.77	2	28	5.24		6.39	2
4	2.67	2	3.19	4	29	5.30		6.65	2
5	3.05	4	3.27	6	30	5.35		6.66	6
6	3.30	(6)	3.66	2	31	5.39		6.75	8
7	3.37	(2)	3.66	5	32	5.41		6.96	7
8	3.59	(0)	4.01	3	33	5.47		7.31	2
9	3.66	(5, 6)	4.33	4	34	5.56		7.35	0
10	3.68		4.92	3	35	5.74		7.48	1
11	3.73		4.92	5	36	5.78		7.57	4
12	3.79		4.94	0	37	5.83		7.72	3
13	3.88		4.99	1	38	5.87	(0)	7.76	2
14	4.03		5.04	2	39	5.98		7.84	5
15	4.10		5.10	4	40	6.05		7.97	6
16	4.21		5.12	1	41	6.16			
17	4.36		5.24	4	42	6.44			
18	4.49	(2)	5.24	8	43	6.58			
19	4.60		5.35	3	44	6.74			
20	4.62		5.37	2	45	6.78			
21	4.66	(2)	5.51	5	46	6.91			
22	4.70		5.59	6	47	7.00			
23	4.83		5.67	3	48	7.84			
24	4.99		5.85	7					

and the calculated spectroscopic factor (see Table IX) indicates that this level would be populated very weakly in the (d, p) reaction. Thus we believe that this 2^+ state does not match with the observed 2^+ state at 2.67 MeV. In view of our results, we predict such a 2^+ state in the 3-MeV region.

Based on our results we also feel that the 4^+ assignments to the states at 2.29, 3.05, and 5.02 MeV (see the table) and 6^+ assignment to the level at 3.30 MeV are correct. Of the two possible spins (5^+ and 6^+) for the level at 3.66 MeV, our results justify 5^+ assignment to this level. The calculated second 0^+ level, however, does not correspond to the 0^+ assigned 3.59-MeV level. For comparison between other theoretical and experimental levels, we refer to Table III. From the table it is clear that the calculated density of levels between 3 and 5 MeV is less than the observed density. This indicates the importance of the extension of our basis-vector spaces and the inclusion of core-excited configurations. Another point which we like to mention here is that the calculated density of levels between 5 and 7 MeV agrees well with the observed density in this region.

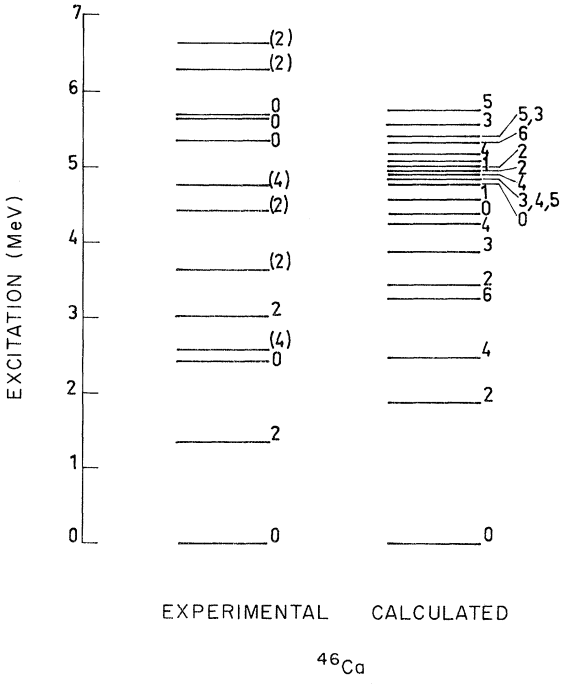


FIG. 3. Experimental and calculated energy levels of ^{46}Ca.

TABLE IV. Calculated and experimental levels in ^{46}Ca.

	Exp.		Cal.			Exp.		Cal.	
No.	E (MeV)	J	E (MeV)	J	No.	E (MeV)	J	E (MeV)	J
0	0	0	0	0	25	6.27	(2)	6.02	6
1	1.35	2	1.87	2	26	6.38		6.13	2
2	2.42	0	2.46	4	27	6.56		6.53	6
3	2.58	(4)	3.25	6	28	6.63	(2)	6.64	4
4	2.98		3.42	2	29	6.75		6.88	2
5	3.02	2	3.87	3	30	6.84		6.88	8
6	3.64	(2)	4.21	4	31	6.97		6.89	2
7	3.78		4.36	0	32	7.03		6.91	0
8	3.86		4.55	1	33	7.11		6.93	7
9	4.23		4.76	0	34	7.18		7.05	1
10	4.28		4.81	5	35	7.24		7.14	4
11	4.43	(2)	4.82	3	36	7.28		7.29	3
12	4.75	(4)	4.87	4	37	7.32		7.33	2
13	5.00		4.94	2	38	7.39		7.41	4
14	5.05		5.02	2	39	7.45		7.44	6
15	5.32	0	5.06	1	40	7.50		7.66	2
16	5.40		5.15	4	41	7.51		7.69	0
17	5.54		5.29	6	42	7.68		7.74	3
18	5.61	0	5.38	5	43	7.75		7.75	3
19	5.64	0	5.39	3	44	7.92		7.77	5
20	5.69		5.55	3	45	8.39		7.82	5
21	5.78		5.74	5	46			7.83	1
22	5.86		5.80	4	47			7.84	9
23	5.96		5.96	2	48			7.84	4
24	6.06		5.99	7	49			7.91	1

^{46}Ca

The calculated and observed spectra of ^{46}Ca are shown in Fig. 3 and tabulated in Table IV. The experimental values are taken from the *Nuclear Data Sheets*.[41] In the figure, only the spin-assigned experimental levels and the calculated levels up to 5.3 MeV are drawn. Table IV gives a detailed comparison between the calculated and the observed spectra. The agreement between the two spectra is, on the whole, good. It appears that some of the states, in particular the 0^+ state at 2.42 MeV and 2^+ state at 3.02 MeV are to be interpreted as core-excited states. We reproduce only one 2^+ level for the two observed at 1.35 and 3.02 MeV. The calculated 2_1^+ level is at 1.87 MeV. This is consistent with the assumption that these two 2^+ levels are strongly mixed as in the case of ^{42}Ca and ^{44}Ca nuclei. The calculated 2_2^+ level matches with the observed 2^+ level at 3.64 MeV. The (t,p) reaction data[10] favor 2^+ assignment to the 4.43 MeV state. Based on the inelastic scattering ^{46}Ca(d,d') results Belote *et al.*[42] assigned 3^- to this level. Our calculated 2_3^+ level does not match with this level. The calculated second and third 0^+ levels seem to be low. The agreement between the observed and calculated density of levels, in particular between 5 and 8 MeV, is very good (see the table). Such agreement is also observed in the case of ^{44}Ca. We have mentioned before that the core excitations also play a role in ^{46}Ca. Inclusion of core-excited configurations in the basis-vector spaces will certainly increase the calculated density. Here we like to mention that in the case of ^{46}Ca, the experimental levels are mainly available from the (t,p) reaction data. In the case of ^{44}Ca, a large number of levels below 5 MeV are observed in the (d,p) reaction which are not seen in the (t,p) experiment. Thus we hope that more levels will be seen in the case of ^{46}Ca also in other experiments. Similar arguments have been given by McGrory, Wildenthal, and Halbert.[11]

^{48}Ca

The calculated and the experimental energy levels are listed in the Table V and shown in Fig. 4. The experimental values are mainly taken from the ^{48}Ca$(p,p'\gamma)^{48}$Ca reaction.[43] In this experiment mostly all the levels, observed by other experiments, were recognized and in addition many new levels were established. We have also included in the table two more levels observed at 6.48 and

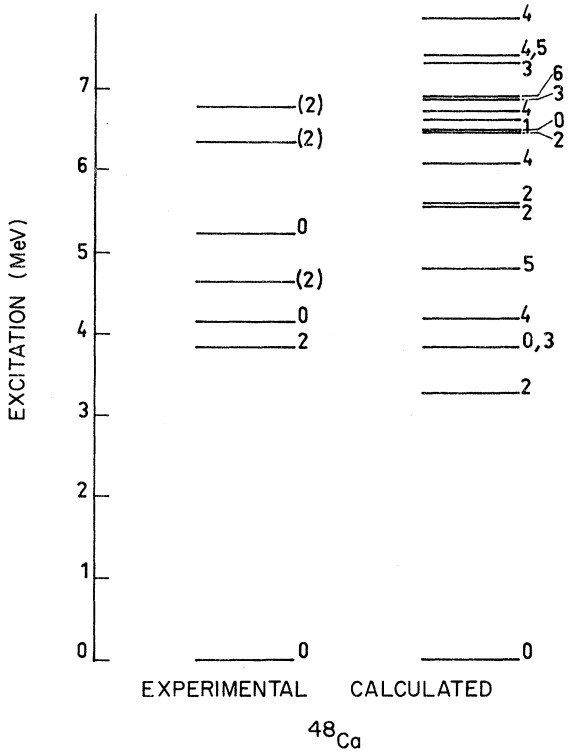


FIG. 4. Experimental and calculated energy levels of ^{48}Ca.

6.79 MeV in inelastic α scattering[44] (6.48 MeV) and (t,p) experiments.[10] Levels which are assigned with only negative-parity spins are omitted. Experimental spin values are taken from different works.[10, 38, 43, 44] The calculated first 2^+ level is a little low. The first excited 0^+ level is well reproduced. This indicates that this is not a core-excited state. This is supported by the (t,p) experiment.[10] The (t,p) transition to the first excited 0^+ state is 60% of the ground-state transition. Configuration of the type $(f_{7/2})^{10}$ $(d_{3/2})^{-2}$ for this 0^+ state cannot explain such strong transition.[45] The calculated second excited 0^+ level is, however, 1 MeV high. Our results do not agree with the 2^+ assignment to the level at 4.62 MeV. The over-all agreement between the calculated and the experimental spectra is good. The striking feature is that the calculated density of states agrees very well with the observed density.

C. Energy Spectra for the Odd Isotopes

Now we describe our results for the odd calcium isotopes. In our model, only negative-parity states for odd-A nuclei are calculated. So we omit parity assignments from all the theoretical and experimental spectra. For the sake of convenience, we show $2J$ value for each level. Multi-

TABLE V. Calculated and experimental levels in ^{48}Ca.

	Exp.		Cal.			Exp.		Cal.	
	E		E			E		E	
No.	(MeV)	J	(MeV)	J	No.	(MeV)	J	(MeV)	J
0	0	0	0	0	23	7.67		8.03	3
1	3.84	2	3.27	2	24	7.96		8.11	5
2	4.29	0	3.82	0	25	8.04		8.23	6
3	4.51	$(3^-,3)$	3.82	3	26	8.07		8.57	4
4	4.62	(2)	4.20	4	27	8.08		8.63	1
5	5.15	$(3^-,5)$	4.79	5	28	8.25		8.65	0
6	5.27		5.55	2	29	8.28		8.70	6
7	5.32		5.59	2	30	8.38		8.79	1
8	5.46	0	6.10	4	31	8.44		8.81	0
9	5.74	$(2,3,5^-)$	6.47	2	32	8.49		8.81	3
10	6.11	(2)	6.50	0	33	8.53		8.91	2
11	6.35	$(4,2,1^-)$	6.64	1	34	8.59		8.93	6
12	6.48		6.73	4	35	8.60		9.01	4
13	6.62		6.88	3	36	8.67		9.07	2
14	6.65	(4)	6.91	6	37	8.79		9.19	7
15	6.79	(2)	6.92	2	38	8.81		9.21	5
16	6.82		7.32	3	39	8.83		9.32	3
17	6.90		7.41	5	40	8.89		9.36	4
18	7.31		7.43	4	41	8.92		9.48	3
19	7.40		7.62	2	42	8.96			
20	7.44		7.86	6	43	9.13			
21	7.59		7.87	4	44	9.21			
22	7.65		7.95	1					

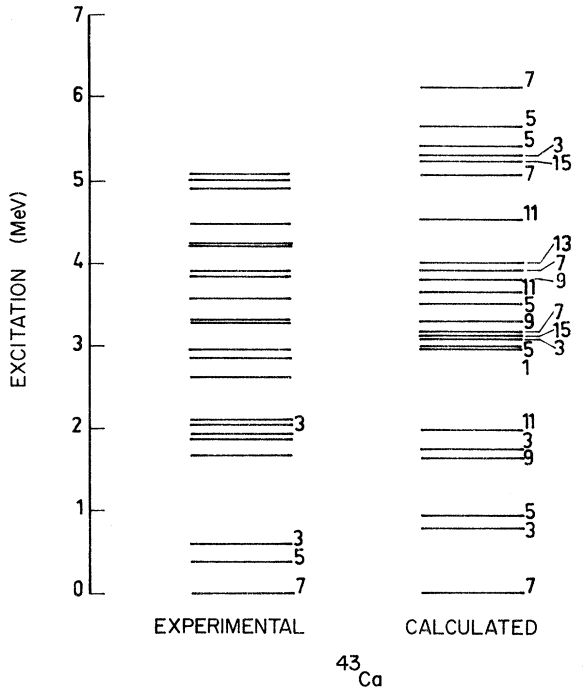


FIG. 5. Experimental and calculated energy levels of ^{43}Ca.

ple spin assignments to one calculated level mean that there is a level for each indicated spin at the same energy. Energy levels which are tentatively assigned with positive-parity spins are omitted from the discussion.

^{43}Ca

Figure 5 and Table VI show the comparison between the calculated and the observed spectra of ^{43}Ca. The experimental values are taken from the compilation of Endt and Van Der Leun[46] and (d,p) reaction data.[47] Since our calculated density of states is much less than the observed density, we have preferred to list only the negative-parity-assigned experimental levels. However, we have included the 2.75-MeV level which has three possible $2J$ values (3^+, 15^-, 1^+). The experimental spins shown in the table are obtained from different sources.[42, 47-51] In order to increase the calculated density of states it is essential to consider core-excited configurations in the basis-vector spaces and also to increase the model space by the inclusion of other orbitals. The position of the first calculated $\frac{5}{2}^-$ level is somewhat too high. Our calculations more or less support the assignments of $\frac{9}{2}^-$, $\frac{11}{2}^-$, and $\frac{15}{2}^-$ to the 2.10-, 1.68-, and 2.75-MeV levels, respectively.[42, 51] The calculated first and second $\frac{3}{2}^-$ levels agree well with the established 0.59- and 2.05-MeV $\frac{3}{2}^-$ levels. The tentative assignments of $\frac{7}{2}^-$ to the levels at 3.28 and 3.81 MeV appear to be valid in view of our results.[49]

^{45}Ca

Figure 6 compares the experimental and calculated energy spectra of ^{45}Ca. The experimental values are taken from *Nuclear Data Sheets*.[41] Only negative-parity spin-assigned experimental levels (except two levels at 1.56 and 1.58 MeV) and the calculated levels up to 5.3 MeV are drawn in the figure. The bracketed experimental spins indicate a tentative assignment. There are 90 levels below 6.31 MeV listed in the *Nuclear Data Sheets* and we have calculated only 50 levels up to 6.5 MeV. It indicates the importance of core ex-

TABLE VI. Calculated and observed energy levels of ^{43}Ca.

	Exp.		Cal.			Exp.		Cal.	
No.	E (MeV)	$2J$	E (MeV)	$2J$	No.	E (MeV)	$2J$	E (MeV)	$2J$
0	0	7	0	7	14	3.29	(3, 1)	3.80	9
1	0.37	5	0.80	3	15	3.31	(3, 1)	3.90	7
2	0.59	3	0.94	5	16	3.56	(3, 1)	4.01	13
3	1.68	(11)	1.62	9	17	3.60	(3, 1)	4.53	11
4	2.05	3	1.74	3	18	3.81	(7)	5.08	7
5	2.07	(15)	1.98	11	19	3.86	(1, 3)	5.27	15
6	2.10	(9, 3, 1)	2.96	1	20	4.20	(1)	5.31	3
7	2.27	(9)	3.00	5	21	4.24	(1, 3)	5.42	5
8	2.61	(1, 3)	3.07	3	22	4.46	(5)	5.65	5
9	2.67	(5)	3.12	15	23	4.90	(1, 3)	6.13	7
10	2.75	(3^+, 1^+, 15)	3.18	7	24	4.98	(1, 3)		
11	2.88	(1, 3)	3.30	9	25	5.03	(1, 3)		
12	2.95	(3, 1)	3.52	5					
13	3.28	(7, 5, 3, 1)	3.65	11					

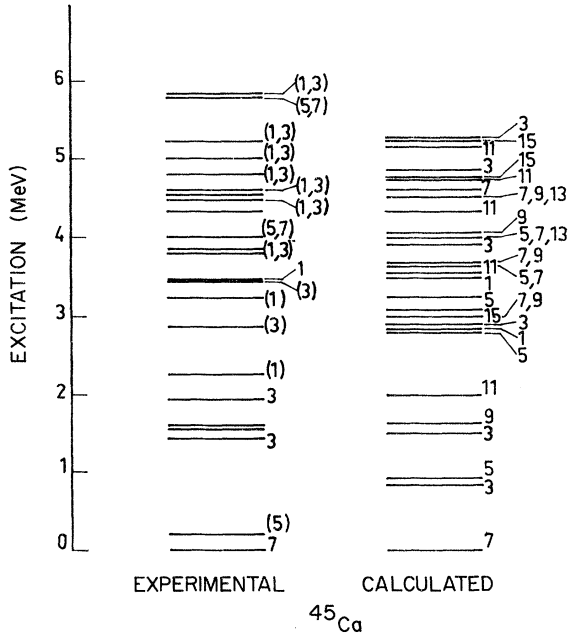


FIG. 6. Experimental and calculated energy levels of ^{45}Ca.

citations in this nucleus also. Since the number of experimental levels is too much we have preferred to list only the calculated energy levels in the Table VII. The calculated first $\frac{5}{2}^-$ level is too high and the first and second $\frac{3}{2}^-$ levels are a little low. We predict a third $\frac{3}{2}^-$ level at 2.87 MeV. This level nicely agrees with the observed level at 2.84 MeV which is tentatively assigned with $\frac{3}{2}^-$. We confirm this assignment. The $\frac{1}{2}^-$ assignment to the level at 2.25 MeV does not agree with our results. The calculated $\frac{1}{2}^-$ level at 3.46 MeV is in very good agreement with the observed $\frac{1}{2}^-$level at 3.46 MeV. It is mentioned in the *Nuclear Data Sheets*[41] that the 1.56- and 1.58-MeV level pair may be the $\frac{9}{2}^-$ and $\frac{11}{2}^-$ components of the $(f_{7/2})^{-3}$ configuration. Our result support these assignments. Many of the spins which are tentatively assigned agree with our results. On the whole the agreement between the calculated and observed spectra is good.

^{47}Ca

The calculated and observed spectra of ^{47}Ca are shown in Fig. 7 and they are tabulated in Table VIII. The experimental values are taken from *Nuclear Data Sheets*.[41] In this case we have also shown those positive-parity spin-assigned experimental levels whose l_n transfers are uncertain. In the figure, however, only negative-parity spin-assigned levels are drawn and the spins of those levels for which authors cannot distinguish between $l_n=2$ and $l_n=3$ transfer, are not indicated (see Ref. 41). For a critical comparison between the two spectra, we refer to Table VIII. The experimental information on the spins of the energy levels in ^{47}Ca is not sufficient. Most of the spins are tentatively assigned. It is difficult to draw any conclusion about the agreement between the two spectra. However, some of the spin assignments agree with our results. The calculated

TABLE VII. Calculated energy levels of ^{45}Ca.

No.	Cal. E (MeV)	$2J$	No.	Cal. E (MeV)	$2J$	No.	Cal. E (MeV)	$2J$
0	0	0	17	3.64	9	34	5.20	15
1	0.83	3	18	3.67	7	35	5.24	3
2	0.91	5	19	3.90	3	36	5.42	5
3	1.50	3	20	3.98	13	37	5.44	5
4	1.64	9	21	3.99	7	38	5.48	13
5	2.00	11	22	4.00	5	39	5.67	7
6	2.75	5	23	4.05	9	40	5.84	17
7	2.81	1	24	4.30	11	41	6.01	9
8	2.87	3	25	4.43	13	42	6.03	1
9	2.97	7	26	4.46	9	43	6.19	1
10	3.00	9	27	4.48	7	44	6.20	7
11	3.06	15	28	4.58	7	45	6.29	9
12	3.22	5	29	4.69	11	46	6.24	3
13	3.46	1	30	4.76	15	47	6.40	3
14	3.53	11	31	4.85	3	48	6.42	11
15	3.61	7	32	5.14	11	49	6.50	7
16	3.62	5	33	5.19	5			

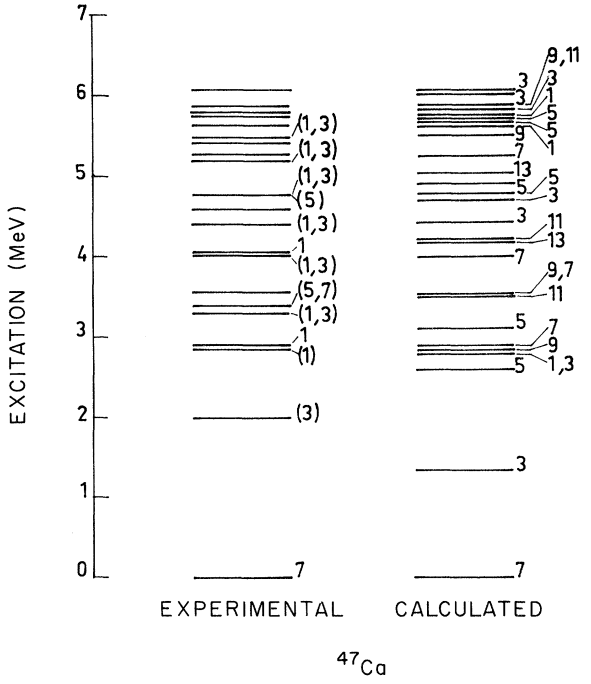


FIG. 7. Experimental and calculated energy levels of ^{47}Ca.

first excited level ($\frac{3}{2}^-$) is too low and the $\frac{1}{2}^-$ assignment to the level at 4.06 MeV does not correspond to our results. The remarkable feature which we like to point out here, is the agreement between the calculated and observed density of states. There are 35 levels (including those whose l_n transfers are uncertain) reported below 6.6 MeV and the calculated number of levels below 6.6 MeV is 36.

6. SINGLE-NUCLEON SPECTROSCOPIC FACTORS

To the extent that the present theory of direct reaction is applicable, stripping and pickup reactions give structural information about the wavefunctions of nuclear energy levels through their spectroscopic factors (S). In this section we compare the experimental spectroscopic factors for these two types of reactions with those calculated using our wave functions.

Table IX shows the comparison between the calculated and experimental spectroscopic factors for stripping reactions. The agreement is good, particularly, for those cases where levels are strongly populated in the transfer reactions. However, our calculated values are reversed for 4^+ levels at 2.29 and 3.04 MeV in ^{44}Ca. The dominant strength in the calculated 4^+ level at 2.45 MeV is $|(f_{7/2})^4, v=2, J=4\rangle$ which has an amplitude of about 96%.

In Table X we compare the spectroscopic factors for pickup reactions with our calculated values. Again, the agreement is good. The calculated spectroscopic factors are large for those levels which are strongly excited in the pickup reactions. This is in accordance with the experiments.

TABLE VIII. Calculated and observed energy levels of ^{47}Ca.

	Exp.		Cal.			Exp.		Cal.	
No.	E (MeV)	$2J$	E (MeV)	$2J$	No.	E (MeV)	$2J$	E (MeV)	$2J$
0	0	(7)	0	7	18	5.19	(1, 3)	5.03	15
1	2.02	(3)	1.35	3	19	5.22		5.26	7
2	2.85	(1)	2.60	5	20	5.25		5.49	9
3	2.85	(5, 7)	2.76	3	21	5.31	(5, 7)	5.62	1
4	2.88	(1)	2.79	1	22	5.43	(5, 7)	5.66	5
5	3.30	(1, 3)	2.84	9	23	5.46	(5, 7)	5.71	5
6	3.43	(5, 7)	2.89	7	24	5.49	(1, 3)	5.78	1
7	3.57	(1, 3)	3.11	5	25	5.64	(5, 7)	5.79	7
8	3.88		3.52	11	26	5.76	(1, 3)	5.83	3
9	3.97	$(3^+, 5^+)$	3.56	7	27	5.81	(5, 7)	5.88	9
10	4.02	(1, 3)	3.58	9	28	5.84		5.90	11
11	4.06	(1)	4.01	7	29	5.87	(1, 3)	5.99	3
12	4.10		4.18	13	30	6.06	(5, 7)	6.05	3
13	4.22	$(3^+, 5^+)$	4.22	11	31	6.19		6.14	7
14	4.40	(1, 3)	4.44	3	32	6.27		6.17	5
15	4.60	(5, 7)	4.73	3	33	6.37		6.27	11
16	4.78	(5)	4.78	5	34	6.56		6.33	5
17	4.81	(1, 3)	4.90	5	35			6.40	9

TABLE IX. Experimental and calculated spectroscopic factors from stripping reactions.

Final nucleus	Experimental E (MeV)	Experimental $2J$ (odd A) or J (even A)	Experimental S $l=1$	Experimental S $l=3$	Calculated S $l=3$	Calculated S $l=1$	Calculated $2J$ (odd A) or J (even A)	Calculated E (MeV)
^{43}Ca [a]	0.0	7		0.69	0.75		7	0.0
	0.59	3	0.05			0.008	3	0.80
	2.05	3	0.75			0.96	3	1.74
	3.28	3	0.05			0.008	3	3.07
^{44}Ca [b]	0.0	0		2.88	3.83		0	0.0
	1.16	2	0.09	0.58	1.32	0.003	2	1.86
	2.67	2	0.01	0.72	0.001	0.004	2	2.77
	2.29	4		0.20	1.33		4	2.45
	3.04	4		1.32	0.002	0.002	4	3.19
	3.30	6		1.48	1.33		6	3.27
^{45}Ca [c]	0.0	7		0.42	0.51		7	0.0
	1.43	3	0.12			0.011	3	0.83
	1.90	3	0.64			0.92	3	1.50
	2.85	3	0.21			0.011	3	2.87
^{47}Ca [d]	0.0	7		0.27	0.26		7	0.0
	2.01	3	0.82			0.90	3	1.35

[a] Reference 47.
[b] Reference 39.
[c] J. Rapaport, W. E. Dorenbusch, and T. A. Belote, Phys. Rev. 156, 1255 (1967).
[d] J. H. Bjerregaard, O. Hansen, and G. Sidenius, Phys. Rev. 138, B1097 (1965).

TABLE X. Experimental and calculated spectroscopic factors from pickup reactions.

Final nucleus	Experimental E (MeV)	Experimental $2J$ (odd A) or J (even A)	Experimental S $l=1$	Experimental S $l=3$	Calculated S $l=3$	Calculated S $l=1$	Calculated $2J$ (odd A) or J (even A)	Calculated E (MeV)
^{42}Ca [a]	0.0	0		0.75	0.75		0	0.0
	1.53	2	0.03	0.21				
					0.40	0.0002	2	1.87
	2.44	2	0.04	0.21				
	2.76	4	0.10	0.75	0.72	0.0006	4	2.45
	3.20	6		1.08	1.04		6	3.31
^{43}Ca [b]	0.0	7		3.8	3.83		7	0.0
	0.59	3	0.10			0.003	3	0.80
	2.05	3	0.18			0.14	3	1.74
^{45}Ca [c]	0.0	7		6.5	5.76		7	0.0
	1.90	3	<0.04			0.17	3	1.50
^{47}Ca [d]	0.0	7		6.48	7.72		7	0.0
	2.02	3	0.13			0.14	3	1.35

[a] Reference 34.
[b] Reference 49.
[c] J. H. Bjerregaard, O. Hansen, and G. R. Satchler, Phys. Rev. 160, 889 (1967).
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7. SUMMARY

In this paper we have described a conventional shell-model study of the calcium isotopes ($A = 42-48$) using a phenomenological effective two-body interaction. The effective interaction consists of usual j-j pairing force plus a tensor force which acts at the nuclear surface only (PSTI). PSTI has two adjustable parameters. These parameters have been determined by a fit to the experimental ground-state energies only. The reasons for taking ground-state energies to obtain the interaction parameters are discussed in the text. ^{40}Ca is assumed to be an inert core. Besides all possible configurations arising from $1f_{7/2}$ and $2p_{3/2}$ orbits, we have also considered a configuration of the form $(1f_{7/2})^{n_1}(1f_{5/2})^{n_2}$ where $n_1+n_2=n$, the number of extracore neutrons and $n_2=1$, 2 in the basis-vector spaces. Since the $1f_{5/2}$ orbit is rather high we think that the excitations of three or more neutrons in that orbit will not alter our findings appreciably. To have an idea of the effect of increasing the basis-vector spaces, calculations for ^{42}Ca are also done in another model space where all the f-p single-particle orbits are active. In view of our results, we remark that it is necessary to consider excitation of particles out of the ^{40}Ca core to understand the spectroscopy of the calcium isotopes ^{42}Ca through ^{46}Ca in a more accurate way. Some of the excited levels, in particular, the second 0^+ and 2^+ levels of ^{42}Ca, ^{44}Ca, ^{46}Ca, are to be interpreted as core-excited levels. The agreement between the calculated and experimental ground-state energies and excited energy levels is good. The striking feature is the remarkable agreement between the calculated and observed density of energy levels of ^{47}Ca and ^{48}Ca. For these two nuclei, we believe that the core excitations do not play any significant role in determining the energy levels. We have also compared the experimental spectroscopic factors for stripping and pickup reactions with those calculated using our wave functions. The agreement between the two is also good.

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Interaction of 3- and 29-GeV Protons with Vanadium*

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Yields for 40 nuclides have been determined in the interaction of 3- and 29-GeV protons with V. Most of the radionuclides were measured by direct γ-ray spectroscopy of the irradiated targets; stable Ne and Ar isotopes were measured by mass spectrometry; radicargon isotopes were determined both mass spectrometrically and by radioactivity measurements. Charge dispersion curves were derived from independent and cumulative yield measurements for the mass regions 22–29, 36–41, and 42–48. The curves for 29 GeV are almost identical with the corresponding ones for 3 GeV. Total isobaric cross sections were determined at many mass numbers by estimating the unmeasured cross sections, from the charge dispersion curves, and adding them to the directly measured ones. Thus mass-yield distributions were obtained. At 3 GeV the total isobaric cross section is 30 mb at mass 47 and it decreases to a minimum of 8 mb around mass 24. At 29 GeV the cross sections are slightly lower than at 3 GeV for the heavier products, down to about mass 30, and somewhat higher for the lighter products, below mass 30. The mass-yield distributions from V are similar to those determined from Cu and Ag. It is shown that the experimental results (including those from Cu) do not agree as well with the Monte Carlo cascade calculations of Metropolis *et al.*, as has been generally supposed.

INTRODUCTION

Several detailed high-energy proton spallation studies of the light- and medium-weight elements aluminum,[1,2] copper,[3–7] and silver[8] have been made. Most of these studies were based on radiochemical separations and were usually limited to product nuclides with half-lives $\geq$30 min. To study charge-dispersion and mass-yield relationships it is necessary to measure the yields of many nuclides on both sides of β stability. Such data help us understand the nuclear reaction mechanisms.

With the availability of high-resolution Ge(Li) detectors and fast magnetic tape read-out systems it is now possible to identify and measure without chemical separation the yields of nearly all γ-emitting radionuclides produced from a target with $Z \leq 30$ irradiated with GeV protons.[7–10] For targets with $Z > 30$ the problem of unresolved γ-ray lines becomes increasingly troublesome[8] and the proportion of the radionuclides that can be determined becomes more and more limited. Mass spectrometry, though in general less sensitive than radioactivity measurement, has also been used in nuclear reaction studies.[11–14] By combining these two methods we have measured the cross sections for 40 nuclides with half-lives greater than 2 min produced in the bombardment of vanadium with 3- and 29-GeV protons. From