

Two-Hole Multiplets in  $^{88}\text{Y}$  from  $^{89}\text{Y}(d, t)$  and  $^{90}\text{Zr}(d, \alpha)$  Studies\*

W. W. Daehnick and T. S. Bhatia†

Nuclear Physics Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania 15213

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States of  $^{88}\text{Y}$  up to 2 MeV in excitation were studied with 8–12-keV resolution by  $(d, t)$  and  $(d, \alpha)$  experiments, and interpreted in terms of two-hole configurations.  $^{89}\text{Y}(d, t)$  angular distribution and spectroscopic factors were obtained at 14.0- and 15.75-MeV bombarding energy. Six low-lying levels not detected in earlier ( $^3\text{He}, \alpha$ ) experiments were resolved. The summed spectroscopic ( $C^2S$ ) strength obtained for the five strong  $l=1$  transitions was  $\frac{2}{3}$  of the sum-rule limit in contrast to previous ( $^3\text{He}, \alpha$ ) results which exceeded it. Successful distorted-wave Born-approximation (DWBA) predictions for  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  angular distributions permitted the assignment of 15  $L$  values and corresponding  $J^\pi$  limits for the final states, about half of which had not been resolved in previous particle-transfer work. Levels dominated by  $(f_{5/2}g_{9/2})^{-1}$  and  $(p_{3/2}g_{9/2})^{-1}$  configurations were identified by comparison with previous data for  $^{86}\text{Rb}$ . Higher-spin states dominated by the  $g_{9/2}^2$  configuration were not seen, as expected; however, a comparison of measured and “absolute” microscopic DWBA cross sections for known two-hole states showed evidence of strong configuration mixing. Additional evidence for a  $2^-(2^+)$  doublet at 706 keV excitation was found.

NUCLEAR REACTIONS  $^{89}\text{Y}(d, t)^{88}\text{Y}$ ,  $E=14.0, 15.75$  MeV, measured  $\sigma(E_t, \theta)$ ,  $^{88}\text{Y}$  level energies; resolution 8 keV. DWBA analysis, deduced  $l, s$ .  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$ ,  $E=17.0$  MeV, measured  $\sigma(E_\alpha, \theta)$ ; resolution 12 keV. Microscopic DWBA analysis, deduced  $L$  transfers, enhancement factors.

## I. INTRODUCTION

$^{88}\text{Y}_{49}$  is of particular spectroscopic interest because of its close proximity to the “closed shell” nuclei  $^{88}\text{Sr}_{50}$  and  $^{90}\text{Zr}_{50}$ . A number of low-lying  $^{88}\text{Y}$  levels may be interpreted as particle-hole states with respect to a  $^{88}\text{Sr}$  core or as two-hole states with respect to a  $^{90}\text{Zr}$  core. Both models lead to the prediction of a  $(p_{1/2}g_{9/2})_{4-,5-}$  ground-state doublet in  $^{88}\text{Y}$  and a slightly higher-lying  $p_{1/2}^2$  doublet with  $J^\pi = 1^+$  and  $0^+$ . The corresponding states have been known for some time.<sup>1</sup> More recent studies<sup>2-6</sup> of  $^{88}\text{Y}$  have extended our knowledge of this nucleus considerably and have exposed considerable complexity of the  $^{88}\text{Y}$  level structure below 2 MeV. Many of these more recently observed states have been classified as members of  $(g_{9/2}g_{9/2})^{-1}$ ,  $(p_{1/2}p_{3/2})^{-1}$ , and  $(p_{1/2}f_{5/2})^{-1}$  particle-hole multiplets with respect to an inert  $^{88}\text{Sr}$  core.<sup>3</sup> The centroids of these multiplets were estimated to lie between 0.8 and 1.5 MeV<sup>3</sup> in  $^{88}\text{Y}$ , so that the proximity of the various expected  $1^+$ ,  $2^+$ , and  $3^+$  levels makes considerable configuration mixing likely. Nevertheless, the  $3^+ - 9^+$  states of the  $(g_{9/2}g_{9/2})^{-1}$  multiplet appear remarkably pure. These and other data<sup>7</sup> attest to the usefulness of considering  $^{88}\text{Sr}$  as a closed shell core.

It is important to confirm the many new  $J^\pi$  assignments suggested in Refs. 2, 3, and 6, and of considerable interest to investigate the degree of purity of the various multiplets proposed in Ref. 3. The present high resolution study uses the  $^{89}\text{Y}(d, t)$

reaction to ascertain which members of previously unresolved clusters of levels have measurable  $(p_{1/2}p_{1/2})$ ,  $(p_{1/2}p_{3/2})$ ,  $(p_{1/2}g_{9/2})$ , and  $(p_{1/2}f_{5/2})$  components. High resolution is even more important for the  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  study.<sup>2,4</sup> This reaction should establish independent  $J^\pi$  limits for the less well known states, and in conjunction with microscopic distorted-wave Born-approximation (DWBA) predictions provide a sensitive check on the purity of the configurations proposed for various levels.

Previous  $^{90}\text{Zr}(d, ^3\text{He})$  studies<sup>7,8</sup> have indicated approximately 7% filling of the  $1g_{9/2}$  proton shell in  $^{90}\text{Zr}$ , but no appreciable  $1g_{9/2}$  proton strength for the  $^{89}\text{Y}$  and  $^{88}\text{Sr}$  ground states. This result is in accord with theoretical expectations,<sup>9,10</sup> and means that  $^{89}\text{Y}(d, t)$  should only excite proton  $(p_{1/2})$  particle-neutron hole states in  $^{88}\text{Y}$ , whereas  $^{90}\text{Zr}(d, \alpha)$  may excite a much larger variety of states, including (albeit weakly) the  $(g_{9/2}g_{9/2})_{J=\text{odd}}^{-1}$  states. Direct reaction selection rules predict the absence of  $(d, \alpha)$  peaks for the  $4^+$ ,  $6^+$ , and  $8^+$  ( $g_{9/2}^2$ ) states. The absence of these states would indirectly confirm the corresponding assignments of Ref. 3. Microscopic transfer calculations based on the  $^{90}\text{Zr}$  ground-state wave function<sup>8</sup>  $|^{88}\text{Sr}, (0.8 \pi p_{1/2}^2 + 0.6 \pi g_{9/2}^2)\rangle$  predict that pure  $(g_{9/2}^2)_{J=\text{odd}}$  transfers should have  $^{90}\text{Zr}(d, \alpha)$  cross sections of no more than 1–3  $\mu\text{b}/\text{sr}$ , and be virtually invisible. The lack of detailed theoretical wave functions for  $^{90}\text{Zr}$  and  $^{88}\text{Y}$  also restricts us to qualitative tests regarding states with mixed configurations.

For  $^{90}\text{Zr}(d, \alpha)$ ,  $^{88}\text{Y}$  levels are most conveniently discussed as two holes in an inert  $^{90}\text{Zr}$  core. This approximation is too rough (and will be amended) whenever quantitative predictions for ( $p_{1/2}$  proton) + neutron transfers are computed. If the final-state configurations have few terms and are indeed dominated by a single configuration, the cross sections predicted for the transfer of a pure proton-neutron configuration should be very close to the observed ones. A large disagreement would demonstrate the existence and significance of more than the postulated or assumed "dominant" term.

## II. EXPERIMENT

### A. Beam and Targets

The  $^{89}\text{Y}(d, t)$  reaction was studied at 14.0 and 15.75 MeV. These particular energies were chosen in order to keep the elastic deuteron peak above or between the position-sensitive surface-barrier detectors mounted in the focal plane of the split-pole spectrograph. A 17.0-MeV deuteron beam of the Pittsburgh three-stage Van de Graaff was used for the investigation of  $^{90}\text{Zr}(d, \alpha)$ . Energy resolution was 8 and 11–12 keV, respectively, for the ( $d, t$ ) and ( $d, \alpha$ ) reactions, and primarily determined by the kinematic spread

due to the angular divergence of the incident beam and the differential energy loss in the  $^{90}\text{Zr}$  target. The  $^{89}\text{Y}$  target consisted of approximately  $60 \mu\text{g}/\text{cm}^2$  of naturally pure yttrium evaporated onto a  $20\text{-}\mu\text{g}/\text{cm}^2$  carbon backing. A typical  $^{89}\text{Y}(d, t)^{88}\text{Y}$  spectrum is shown in Fig. 1. Absolute cross sections were rechecked at both energies with a self-supporting  $1.01\text{-mg}/\text{cm}^2$   $^{89}\text{Y}$  target.

The Zr target consisted of about 97% pure  $^{90}\text{Zr}$ ,  $30 \mu\text{g}/\text{cm}^2$  thick, supported by a  $20\text{-}\mu\text{g}/\text{cm}^2$  carbon backing. At all angles some very weak but well-focused low-lying  $\alpha$  groups were observed which did not correspond to  $^{88}\text{Y}$  levels seen in other reactions. We surmise that most are due to target impurities with a mass very close to or slightly heavier than that of  $^{90}\text{Zr}$ . Since ( $d, \alpha$ )  $Q$  values for the heavier Zr isotopes are significantly more positive, these doubtful groups (in particular one corresponding to 212 keV excitation in  $^{88}\text{Y}$ ) would lie in the high excitation region in these isotopes, which is not known in detail at present.

### B. Experimental Procedure

( $d, t$ ) and ( $d, \alpha$ ) reaction products were analyzed with our Enge split-pole spectrograph which was used with an acceptance aperture of 1.4 msr. Beam spot size was held to 0.5 mm by 2 mm by a collimating slit 2 cm in front of the target. At

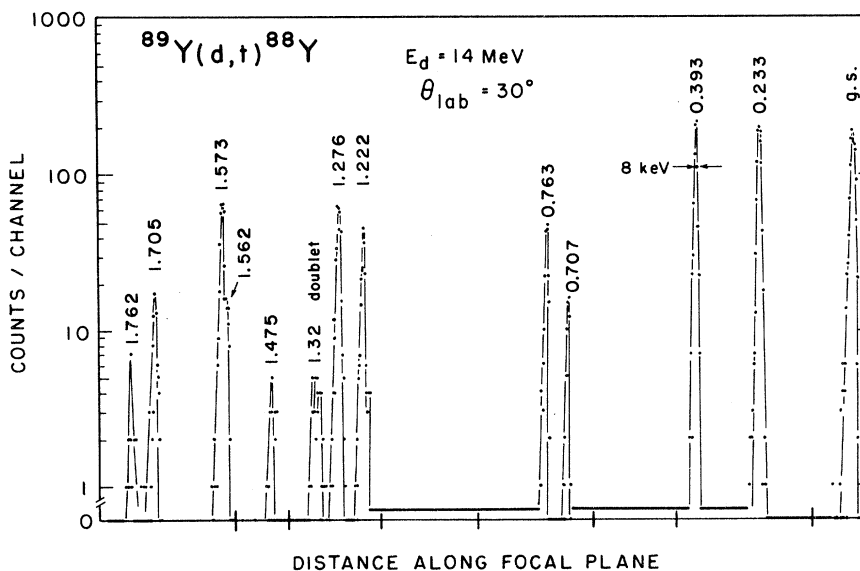


FIG. 1. Composite high-resolution triton spectrum for the reaction  $^{89}\text{Y}(d, t)^{88}\text{Y}$  obtained with position-sensitive surface barrier counters at  $\theta_{\text{LAB}} = 30^\circ$ . Resolution is 8 keV or better and the average background is 0 or less than 1 count per channel. Horizontal bars are averages over separated, statistically insignificant counts (1–3 per channel). Note that because of the strong selectivity of the  $^{89}\text{Y}(d, t)$  reaction of 25 levels known for the excitation range shown only 15 are detected. Only half of these levels would be noticeable on a linear plot. As expected the  $(g_{9/2})^2$  levels at 0.678 MeV ( $8^+$ ), 0.847 MeV ( $5^+$ ), and 0.989 MeV ( $4^+$ ) levels are invisible. The states at 0.707 and 1.475 MeV [both very strong in ( $d, \alpha$ )] are almost certainly *not* identical with  $g_{9/2}^2$  levels reported at 0.712 MeV ( $6^+, 7^+$ ) and 1.478 MeV ( $9^+$ ), respectively (Ref. 3).

angles below  $20^\circ$  a 1-mm by 2-mm collimating slit was used. The geometry and details of the apparatus have been described previously.<sup>11</sup> The deuteron beam was monitored by charge integration and by observation of elastic scattering into a NaI monitor counter placed at  $38^\circ$ . Reaction products were detected by an array of four position-sensitive surface barrier counters in the focal plane of the spectrograph as in previous experiments. Position- and particle-type resolution were good at all angles reported.

The somewhat nonlinear response of the position

counters made it desirable to remeasure the  $^{88}\text{Y}$  level energies with nuclear emulsions. The  $^{89}\text{Y}$ -( $^3\text{He}, \alpha$ ) reaction (at 18 MeV) was used as a substitute for ( $d, t$ ) nuclear emulsion measurements which were not possible at the beam energies available. The deduced level energies for neutron hole states are listed in Table I together with relative  $^{89}\text{Y}$ ( $^3\text{He}, \alpha$ ) cross sections measured at  $\theta_{\text{c.m.}} = 51^\circ$ .  $^{90}\text{Zr}(d, \alpha)$  counter measurements were repeated for  $\theta_L \geq 40^\circ$  with K-1 photographic emulsions as detectors. Reliable relative excitation energies could be extracted; however accurate

TABLE I. Summary of neutron pickup results for  $^{88}\text{Y}$  levels.

$E_x^a$ (MeV)	$\sigma(^3\text{He}, \alpha)$ at $51^\circ$ (rel. units)	14 MeV	15.75 MeV	$l_j$	$\text{C}^2\text{S}$		$\text{C}^2\text{S}$ $^3\text{He}, \alpha$ (Ref. 2)	$J^\pi$ limits based on ( $d, t$ ) only
		$\sigma(d, t)$ $36^\circ$ ( $\mu\text{b}/\text{sr}$ )	$\sigma(d, t)$ ( $\mu\text{b}/\text{sr}$ ) max		14 MeV	15.75 MeV		
0	380	275	484	$g_{9/2}$	$4.0 \pm 0.25$	3.71	4.4	$4^-(5^-)$
0.233	585	300	576	$g_{9/2}$	$5.1 \pm 0.35$	4.88	5.5	$5^-(4^-)$
0.393 (0.678)	90 <2	265 <1	1315 <2	$p_{1/2}$ ...	1.04 ...	0.94 ...	1.5 ...	$1^+(0^+)$ ...
0.707 (0.712)	6 ...	17 ...	40 ...	$\left\{ \begin{array}{l} p_{3/2} \\ f_{5/2} \end{array} \right\}$	$\left\{ \begin{array}{l} (0.04) \\ (0.18) \end{array} \right\}$	...	...	$(0^+ - 2^+)$ ...
0.763 (0.847)	25 <2	57 <1	320 $\leq 3$	$p_{1/2}$ ...	0.31 ...	0.35 ...	0.7 ...	$0^+(1^+)$ ...
(0.881)	<2	<1	<2	...	...	...	...	...
(0.989)	<2	<1	$\leq 2$	...	...	...	...	...
(1.087)	<2	<1	$\leq 2$	...	...	...	...	...
1.127	8	$\leq 4$	7.5	$g_{9/2}$	...	0.10	...	$4^-, 5^-$
1.215	...	(~5)	...	...	...	...	...	...
1.225	85	66	360	$\left\{ \begin{array}{l} p(\frac{3}{2}) \\ \text{or } p(\frac{1}{2}) \end{array} \right\}$	$\left\{ \begin{array}{l} 0.49 \\ 0.56 \end{array} \right\}$	$\left\{ \begin{array}{l} 0.48 \\ 0.55 \end{array} \right\}$	1.0	$0^+ - 2^+$
(1.234)	...	...	...	...	...	...	...	...
(1.262)	...	...	...	...	...	...	...	...
1.276	144	106	600	$\left\{ \begin{array}{l} p(\frac{1}{2}) \\ p(\frac{3}{2}) \end{array} \right\}$	$\left\{ \begin{array}{l} 1.03 \\ (0.90) \end{array} \right\}$	1.00 (0.87)	1.6	$0^+ - 2^+$
(1.285)	...	...	...	...	...	...	...	...
1.315	<2	$\left\{ \begin{array}{l} \leq 4 \\ \leq 4 \end{array} \right\}$	~36	?	...	...	...	...
1.325	<2	...	...	...	...	...	...	...
(1.460)	<2	...	...	...	...	...	...	...
1.475	$\leq 2$	$\approx 8$	$\leq 8$	...	...	...	...	...
1.562	(30)	...	...	...	...	...	...	...
1.573 (1.598)	150 $\leq 2$	88 $\leq 1$	~(450) ...	$p(\frac{3}{2})$ ...	0.90 ...	(0.92) ...	1.9 ...	$0^+ - 2^+$ ...
1.705 (1.732)	110 ...	20 $\leq 4$	(>50) ...	( $f_{5/2}$ ) ...	(1.8) ...	(>1.0) ...	2.7 ...	$(2^+, 3^+)$ ...
1.762 (1.827)	~2 <2	~10 ...	...	...	...	...	...	...
(1.897)	10	...	...	...	...	...	...	...
1.951	20	...	...	...	...	...	...	...
2.056 <sup>b</sup>	32	...	...	...	...	...	...	...

<sup>a</sup> Energy assignments without brackets were obtained from the ( $^3\text{He}, \alpha$ ) and ( $d, t$ ) data of this study only. Energy uncertainties increase from  $\pm 2$  keV for the first excited state to  $\pm 4$  keV at 2 MeV excitation. Bracketed energies were taken from other sources, in particular Refs. 3 and 6 and our ( $d, \alpha$ ) work reported below.

<sup>b</sup>Doublet.

scanning of this set of plates proved difficult. Hence for angles where counter *and* plate data were available the former were weighted twice as heavily. The plate data showed no improvement in resolution which indicates that the inherent limitation in counter resolution did not matter here and was better than 0.25 mm.

### C. Experimental Errors

Previous  $^{89}\text{Y}(^3\text{He}, \alpha)$  studies have shown considerable variation in the spectroscopic factors  $S$  deduced by DWBA.<sup>1,2</sup> Hence some effort was made to measure absolute  $(d, t)$  cross sections carefully. We believe that the absolute cross section scale for the  $(d, t)$  data shown in Figs. 2 and 3 is accurate to better than  $\pm 15\%$ , for the 14.0-MeV as well as the 15.75-MeV data. Yet our spectroscopic factors are considerably lower than those deduced from the most recent  $(^3\text{He}, \alpha)$

data<sup>2</sup> which are shown in Table I for comparison. We do not think that the large differences in the extracted spectroscopic factors for the higher levels could be explained by experimental scale errors in either experiment. Only a moderate effort was made to obtain an accurate absolute scale for the  $(d, \alpha)$  cross sections. We assign a  $\pm 20\%$  scale error to all  $(d, \alpha)$  cross sections given. Scale errors are not shown in the figures. Random errors were frequently but not always determined by statistics, and are shown by error bars. Monitoring uncertainties and errors in resolving close groups are responsible for the error bars for large cross sections, and for unusually large errors in general.

Energy assignments to peaks recorded on photographic emulsions were computed with code SPECTRE<sup>12</sup> and commonly used parameters for the split-pole spectrograph. The scale uncertainty for  $(^3\text{He}, \alpha)$  energies was  $\pm 0.3\%$  of excitation energy

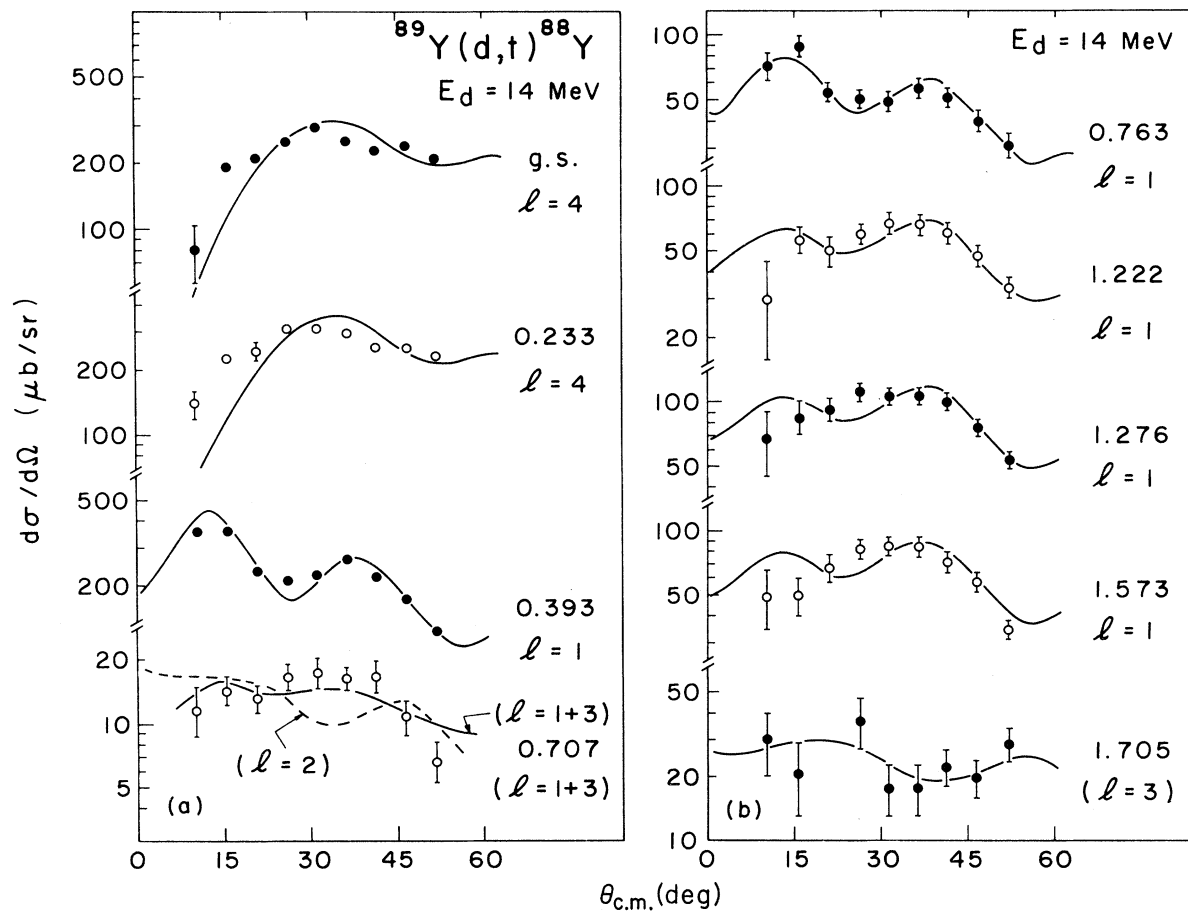


FIG. 2.  $^{89}\text{Y}(d, t)^{88}\text{Y}$  differential cross sections obtained at  $E_d = 14.0$  MeV. Level energies are indicated in MeV. The curves represent DWBA calculations. Error bars include known and estimated random errors. The absolute scale error is  $\pm 15\%$ .

$E^*$ . For  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  calibrations were performed at 12-MeV bombarding energy and at 17 MeV, with the result that scale errors in excess of  $\pm 0.2\%$  may be excluded. An additional (random) uncertainty of  $\pm 2$  keV results from the error in locating the centroids of the peaks. Deduced ( $^3\text{He}, \alpha$ ) and ( $d, \alpha$ ) level energies agree to within better than  $0.003E^*$ . The ( $d, \alpha$ ) values also show excellent agreement with recent ( $p, n\gamma$ ) energy assignments.<sup>6</sup> Differences are never in excess of the larger of 2 keV or  $0.002E^*$ . Uncertainties in the scattering angle  $\theta$  are smaller than  $\pm 0.2^\circ$ .

### III. DWBA ANALYSIS

#### A. $^{89}\text{Y}(d, t)^{88}\text{Y}$

DWBA calculations for ( $d, t$ ) reactions were carried out with code JULIE<sup>13</sup> and code DWUCK.<sup>14</sup>

For local, zero-range calculations both computer codes gave identical results, and led to angular distributions which were not sensitive to the optical-model parameters used. As there is good evidence that nonlocality and finite-range corrections improve the reliability of DWBA calculations for transfer reactions,<sup>15,16</sup> the final calculations were performed with code DWUCK, with the commonly used nonlocality parameters<sup>14</sup>  $\beta_d = 0.54$ ,  $\beta_t = 0.25$ , and the finite-range parameter  $R(d, t) = 0.845$ . The optical-model parameters used are shown in Table II. Nonlocality corrections were not used for the bound-neutron wave function.

A comparison of data and calculations in Figs. 2 and 3 shows good but not outstanding agreement. We note particularly that the DWBA predictions fall off too fast at small angles for  $l=4$  transfers,

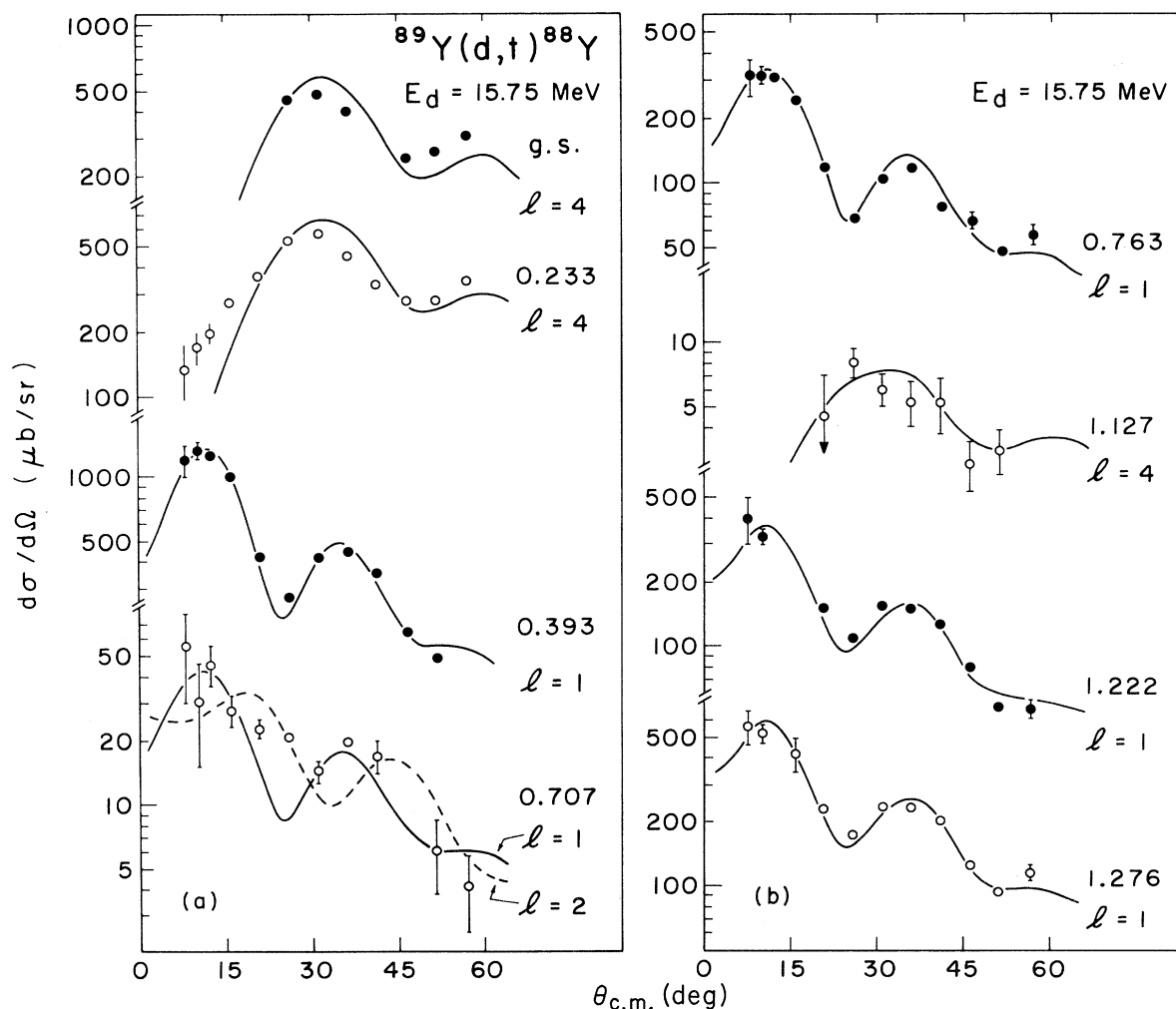


FIG. 3.  $^{89}\text{Y}(d, t)^{88}\text{Y}$  differential cross sections obtained at  $E_d = 15.75$  MeV. Level energies are indicated in MeV. The curves represent DWBA calculations. Error bars include known and estimated random errors. The absolute scale error is  $\pm 15\%$ .

and too slowly for some  $l=1$  transfers at the lower (14-MeV) bombarding energy. Nevertheless, the  $l$  assignments are unique if the 14 and 15.75-MeV data are considered together; and generally there is little uncertainty about the normalization factors needed (i.e., the spectroscopic factors to be extracted). A similar deficiency in  $l=4$  ( $d, t$ ) fits in this mass and energy region has been experienced before.<sup>17,18</sup> This does not seem to be associated with the particular choice of deuteron and triton parameters from sets of equivalent potentials, since Ref. 17, for instance, finds the same effect using a 10-MeV deeper deuteron well and an 8-MeV shallower triton well than shown in Table II. But it may very well be caused by the fact that the triton parameters were extracted from scattering of 20-MeV particles,<sup>19</sup> whereas they are used here for tritons of about 10 MeV. We may also see a  $j$  dependence for  $g_{9/2}$  which is not reproduced by DWBA calculations. Whatever the reason, less than perfect agreement calls for some caution in the interpretation of the spectroscopic factors extracted.

With the conventional DWBA normalization for ( $d, t$ ) reactions<sup>20</sup> the sum of all  $1g_{9/2}$  spectroscopic factors given in Table I is  $\sum_i C^2 S^i(g_{9/2}) = 8.7$  for the 15.75-MeV data, or 9.2 for the 14-MeV data. These numbers appear quite reasonable, in fact more so than the value of  $\approx 10$  usually given in ( $^3\text{He}, \alpha$ ) analyses, since it must be expected that some  $g_{9/2}$  strength is found in higher-lying levels. For instance, for  $^{87}\text{Rb}_{50}$  a total  $g_{9/2}$  strength of 1.1 was found for levels between 1.2 and 2 MeV.<sup>18</sup> In ruling out any justification for scale renormalization for our data or DWBA calculations we are confronted with the very small values [compared

to ( $^3\text{He}, \alpha$ ) results<sup>2</sup>] obtained for  $l=1$  pickup. We will give arguments below why we consider the ( $d, t$ ) values more realistic.

### B. $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$

Microscopic DWBA calculations for ( $n+p$ ) transfer were made with code DWUCKII,<sup>14</sup> which performs the computation of the microscopic form factor by the method of Bayman.<sup>21</sup> A slight modification in code DWUCKII allows us to choose the proper size parameter for the larger projectile. Here  $r=1.4$  fm was used for the  $\alpha$  size.

As in previous work<sup>22-24</sup> nonlocality and finite-range corrections were made in the local-energy approximation (LEA) with the parameters  $\beta_d=0.54$  for the entrance channel and  $\beta_\alpha=0.2$  for the exit channel. The finite-range parameter was  $R=0.4$ . The conditions under which LEA finite-range corrections for ( $d, \alpha$ ) are meaningful were discussed in detail in Ref. 22. Their use has been continued although it was shown that for well-matched optical-model parameters finite-range effects in the LEA become small and the resulting DWBA predictions are almost identical to zero-range calculations.<sup>22</sup> The difference of both sets of calculations lies primarily in the over-all normalization needed. The optical-model parameters used for ( $d, \alpha$ ) calculations are shown in the lower half of Table II.

The two sets of form factor geometries shown in Table II may need a word of explanation. Experience with ( $d, \alpha$ ) reactions has shown that, surprisingly, cluster predictions are almost always in somewhat better agreement with experiment than microscopic calculations which use the same form-factor well geometry.<sup>23-25</sup> This effect

TABLE II. Optical model parameters used in the DWBA calculations for  $^{89}\text{Y}(d, t)^{88}\text{Y}$  and  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$ .

Channel	$V$ (MeV)	Real well			$W$ (MeV)	Imaginary well			$\lambda_{\text{so}}$
		$r_0$ (fm)	$r_c$ (fm)	$a$ (fm)		$4W_D$ (MeV)	$r_I$ (fm)	$a_I$ (fm)	
$^{89}\text{Y} + d$ <sup>a</sup>	90.3	1.20	1.3	0.75	...	49.2	1.33	0.72	
$^{88}\text{Y} + t$ <sup>b</sup>	166.6	1.16	1.3	0.75	22.9	...	1.50	0.82	
Bound neutron for ( $d, t$ )	c	1.17	1.3	0.75	...	...	...	...	25
$^{90}\text{Zr} + d$ <sup>d</sup>	90.0	1.20	1.3	0.75	...	66.3	1.30	0.70	
$^{88}\text{Y} + \alpha$ <sup>d</sup>	181.3	1.20	1.3	0.75	15.0	...	1.70	0.60	
Bound deuteron cluster	c	1.20	1.3	0.75	...	...	...	...	0
Bound nucleons for ( $d, \alpha$ )	e	1.25	1.3	0.75	...	...	...	...	25

<sup>a</sup> J. Childs, W. W. Daehnick, and M. Spisak, Bull. Am. Phys. Soc. **17**, 446 (1972); and to be published.

<sup>b</sup> Reference 19.

<sup>c</sup> Adjusted by code to yield correct separation energy.

<sup>d</sup> Well-matched parameters for  $^{89}\text{Y}$  taken from Ref. 22.

<sup>e</sup> Adjusted by code to give the binding energy  $E = \{Q(\gamma, d) - 2.225 \text{ MeV}\} / 2$  for a core of  $A=89$ .

seems closely connected to the need for well matching of the potentials of all three channels.<sup>22</sup> In a *single-nucleon- or cluster-transfer calculation*, the potentials  $V_i(r)$  entering into the distorted-wave formalism have the very geometry given by the optical-model or bound-state well parameters. However, if the bound-state wave function is expressed as the product of two independent single-nucleon wave functions,<sup>24, 26</sup> the c.m. motion of the resulting dinucleon overlaps well with a mass-2 cluster wave function only if the latter is generated in a well with a smaller radius.<sup>23, 25</sup> Or conversely, for good well matching<sup>22</sup> the well radius for two independent nucleons is to be chosen slightly larger than that for a mass-2 cluster, i.e., somewhat larger than the optical-model radii in the entrance and exit channels.

The form-factor search code MIFF written by Drisko and Rybicki<sup>26</sup> permits the determination of the cluster radius that leads to the best overlap with a microscopic form factor; hence a microscopic form factor well that leads to a proper  $V_x(r)$ ,<sup>22</sup> so that  $\text{Re}[V_d(r) + V_x(r) - V_\alpha(r)] = 0$ , can be deduced. Although the potential radii involved differ only by a few tenths of a Fermi, the effect on the DWBA predictions can be significant. The increase from  $r_0 = 1.2$  fm for clusters to  $r_0 = 1.25$  fm for noninteracting dinucleons has been found sufficient. Further details regarding our  $(d, \alpha)$  calculations can be found in Ref. 22.

It has been pointed out previously that the absolute normalization for zero-range  $(d, \alpha)$  calculations depends sensitively on various computational parameters.<sup>23, 22</sup> However, once these parameters are chosen and fixed, a single normalization constant  $N$  may be used for all DWBA calculations of this kind for a given nucleus.<sup>27, 28</sup> This constant  $N$  can be derived from transitions to states of known configuration. For  $^{88}\text{Y}$  we assume that the  $[f_{5/2} g_{9/2}]^{-1} 2^-, 7^-$  states are essentially pure and use as our (DWUCK) normalization for  $^{90}\text{Zr}(d, \alpha)$  the weighted average  $N(d, \alpha) = 1900(2S + 1)/2$ . Since at present we have no detailed wave functions for  $^{88}\text{Y}$ , each state is characterized by a single dominant term, usually the one proposed in Ref. 3, and the transfer of a pure configuration with the spectroscopic amplitude

$$\beta_{\gamma LSJ}(J, 0) = \left\{ \frac{n_\pi n_\nu (2J+1)}{(2j_\pi+1)(2j_\nu+1)} \right\}^{1/2} \times \begin{bmatrix} l_\pi & \frac{1}{2} & j_\pi \\ l_\nu & \frac{1}{2} & j_\nu \\ L & S & J \end{bmatrix}$$

is computed. [The factor  $(2J+1)^{1/2} [LS-JJ]$  is part of the DWUCK II output for microscopic form factors.]  $n_\pi$  and  $n_\nu$  are the numbers of proton and neutrons in the  $j_\pi$  and  $j_\nu$  orbits, respectively, and the square bracket is the  $LS-JJ$  transforma-

tion coefficient. We measure the degree of success of this approximation in terms of an "enhancement factor"  $\mathcal{E}$ , and define it as  $\mathcal{E} = \sigma_{\text{exp}}/\sigma_{\text{DW}}$ , where  $\sigma_{\text{exp}}$  is the measured cross section and  $\sigma_{\text{DW}}$  is our "absolute" microscopic DWBA calculation for the pure configuration indicated in column 5 of Table III. For constructive interference of several amplitudes we generally expect  $\mathcal{E} > 1$  and for destructive interference  $\mathcal{E} < 1$ . However, occasionally we may obtain  $\mathcal{E} \neq 1$  solely because the assumed "dominant" configuration predicts  $(d, \alpha)$  cross sections that are very small compared to those for the neglected terms of the wave functions, or vice versa. [The most and least favorable allowed terms often lead to  $(d, \alpha)$  cross sections differing by an order of magnitude.]

#### IV. RESULTS AND DISCUSSION

##### A. $^{89}\text{Y}(d, t)^{88}\text{Y}$

The assignment of  $l$  transfers to the  $^{89}\text{Y}(d, t)$  data shown in Figs. 2 and 3 is unique for all strong transitions and in agreement with published  $^{89}\text{Y}-(^3\text{He}, \alpha)$  neutron-transfer interpretations.<sup>1, 2</sup> (See Table I.) The new information primarily concerns the magnitude of the spectroscopic factors and the "level" seen at 0.707 MeV. This level is excited with 20  $\mu\text{b}/\text{sr}$ , i.e., an order of magnitude weaker than typical states (compare Fig. 1), but still strong enough for us to expect that it should be reproduced by single-step DWBA calculations. Nevertheless, it is quite clear from Figs. 2(a) and 3(a) that no single  $l$  value will fit the data. An acceptable, if not perfect fit is found for both energies by a combination of  $l=1$  and  $l=3$ , which would limit the level spin to  $2^+$ ; and this is our tentative assignment. Any admixture of  $l=2$  expected from the  $2^-$  assignment to a level at 0.706 MeV in Ref. 6 would be very small. The nonobservation of  $l=2$  in  $(d, t)$  does not contradict the existence of a  $2^-$  level near 0.706 MeV, for such a level in  $^{88}\text{Y}$  would have  $(\pi f_{5/2}^{-1} \nu g_{9/2}^{-1})$  as its dominant hole configuration which could not be reached by neutron pickup from the  $(p_{1/2}^{-1})$  ground state of  $^{89}\text{Y}$ . Similarly the  $6^+, 7^+$  levels postulated by Ref. 3 for this excitation energy cannot be populated by single-step neutron pickup from  $^{89}\text{Y}$ . Since good resolution and low background rule out significant data errors, the existence of a third or fourth level with low spin but positive parity (probably  $2^+$ ) seems to be established. This is in agreement with some early suggestions based on  $(p, d)$  data.<sup>1</sup> If all recent studies of  $^{88}\text{Y}$  are correct in their conclusions (there are no known deficiencies in these analyses) there must be four levels between 0.70 and 0.71 MeV in  $^{88}\text{Y}$  (i.e.,  $1^+$  or  $2^+, 2^-, 6^+$ , and  $7^+$ ). The *simultaneous* obser-

TABLE III. Comparison of previous assignments for  $^{88}\text{Y}$  with independent results of the present study. The assumed configurations are taken from Ref. 3 unless marked by an asterisk. The latter present our own choices and are often obtained from a comparison with corresponding  $^{86}\text{Rb}$  levels, since they are not reached by single-particle transfer to  $^{88}\text{Y}$ . As pointed out in the text there is much evidence that most of these assumed configurations have significant admixtures. They are listed primarily to specify the meaning of the enhancement factor  $\mathcal{E}$ . Entries under "present work" represent weighted averages for  $E^*$  [from  $^{90}\text{Zr}(d, \alpha)$ ,  $^{89}\text{Y}(^3\text{He}, \alpha)$ , and  $^{89}\text{Y}(d, t)$ ] and for  $C^2S$  (for 14.0- and 15.75-MeV data). Uncertainties in  $E^*$  increase linearly from  $\pm 1$  keV at 233 keV to  $\pm 4$  keV at 2 MeV and are primarily due to scale errors. Energies in brackets have uncertainties of  $\pm 6$  keV. The bracketed  $J^\pi$  values in the last column are permitted by strong selection rules, and sometimes are only slightly less acceptable than the preferred assignments.

$E^*$ (MeV) (Refs. 6 and 3)	Previous work				Configuration assumed	$E^*$ (MeV)	Present work			$\mathcal{E}$	$J^\pi$
	$^{87}\text{Sr}(^3\text{He}, d)$ $l$ (Ref. 3)	$C^2S$ (Ref. 3)	$J^\pi$ (Refs. 1-3, 6)	$^{89}\text{Y}(d, t)$ $l$ $C^2S$			$^{90}\text{Zr}(d, \alpha)$ $L$	$\sigma_{\text{max}}$ ( $\mu\text{b/sr}$ )			
0.000	1	0.82	$4^-$	$(p_{1/2} g_{9/2})$	0.000	4	3.85	3	195	2	$4^-$
0.232	1	0.83	$5^-$	$(p_{1/2} g_{9/2})$	0.233	4	5.0	5	45	0.8	$5^-$
0.393	...	n.o.	$1^+$	$(p_{1/2})^2$	0.393	1	0.99	0+2	307	5	$1^+$
0.678 <sup>a</sup>	4	1.18	$8^+$	$(g_{9/2})^2$	n.o.	...	...	...	...	$\sqrt{\quad}$	...
0.706	...	...	$2^-$	$(f_{5/2} g_{9/2})^*$	0.706	...	n.o.	1	138	{(40)}	$2^-$
...	...	...	...	$(p_{3/2} p_{1/2})^*$	0.707	(1+3)	(0.04, 0.19)	2	{(98)}	0.5	$(2^+)(1^+)$
0.712 <sup>a</sup>	4	1.04	$6^+ + 7^+$	$(g_{9/2})^2$	n.o.	...	...	...	...	$\sqrt{\quad}$	...
0.766	...	n.o.	$0^+$	$(p_{1/2})^2$	0.763	1	0.33	...	<6	$\sqrt{\quad}$	$0^+(2^+)$
0.842	4	1.12	$5^+$	$(g_{9/2})^2$	0.844	...	n.o.	...	(16)	(17)	...
0.983	4	1.06	$4^+$	$(g_{9/2})^2$	n.o.	...	...	...	<3	$\sqrt{\quad}$	...
1.089	(1)	...	$(5^-, 6^-)$	...	1.087	...	n.o.	...	(8)	...	...
1.134 <sup>a</sup>	1	(0.08)	$(5)^-$	$(g_{9/2} p_{1/2})$	1.127	4	0.10	...	(5)	$\sqrt{\quad}$	$5^-(4^-)$
1.220	[4	0.86	$(2)^+(1)^+$	$(g_{9/2}^2 + ?)$	1.215	...	Weak	...	...	...	...
1.225 <sup>a</sup>	...	...	$(4)^-$	$(p_{1/2} p_{3/2})$	1.225	1	0.49	...	<10	<0.08	$2^+(1^+, 0^+)$
1.234	...	...	...	...	...	...	...	...	...	...	...
1.262	[(1)	...	$(3^-)(4^-)$	$(f_{5/2} g_{9/2})^*$	(1.264)	...	...	(3)	{(30)}	2	$(3^-)$
1.275	[+]	...	$(1^+)$	$(p_{1/2} p_{3/2})^*$	1.274	1	1.0	2+0	93	{(70)}	$1^+$
1.285	[4	1.15	$(3^+)$	$(g_{9/2})^2$	...	...	...	...	...	...	...
1.323 <sup>a</sup>	1	0.2	$(5, 6)^-$	$(p_{3/2} g_{9/2})^*$	{(1.315)	...	Weak	5	174	0.7	$6^-(5^-, 4^-)$
...	...	...	...	$(f_{5/2} g_{9/2})^*$	{(1.325)	...	Weak	...	...	...	...
...	...	...	...	$(p_{3/2} g_{9/2})^*$	1.460	...	n.o.	(7)	21	0.7	$(7^-)$
1.478 <sup>a</sup>	4	1.0	$9^+$	$(g_{9/2})^2$	1.475	...	Weak	3	50	0.5	$3^-(4^-, 2^-)$
...	...	...	...	$(p_{3/2} p_{1/2})^*$	1.562	...	Weak	...	...	...	...
1.570	...	Weak	$(2^+)$	$(p_{1/2} p_{3/2})$	1.573	1	0.90	0+2	111	$\sim 1$	$1^+$
1.595	1	...	$(4^-)$	$(f_{5/2} g_{9/2})^*$	1.598	...	n.o.	3	35	2.3	$4^-(3^-, 2^-)$
1.701	...	...	$3^+$	$(p_{1/2} f_{5/2})$	1.702	(3)	(1.8)	4	64	1.2	$3^+, (4^+)$
1.732	(1)	...	...	...	1.735	...	n.o.	...	(10)	...	...
1.761	...	...	$(3^-)(5-8)$	$(p_{3/2} g_{9/2})^*$	1.762	...	Weak	5	30	2	$5^-(4^-, 6^-)$
1.832	1	...	$(4^-)$	$(p_{3/2} g_{9/2})^*$	1.827 <sup>b</sup>	...	n.o.	3	122	1.0	$4^-(3^-, 2^-)$
1.881 <sup>a</sup>	(1)	...	...	...	1.900	...	Weak	...	(20)	...	...
1.913 <sup>a</sup>	1	...	...	...	...	...	...	...	...	...	...
1.952 <sup>a</sup>	(4)	(0.39)	$(2^+)$	$(p_{1/2} f_{5/2})^*$	1.948	...	Seen	2	50	2.4	$2^+(3^+, 1^+)$
1.971 <sup>a</sup>	...	Seen	...	...	(1.971)	...	...	...	(15)	...	...
2.056 <sup>a</sup>	4	0.37	$2^+$	$(p_{1/2} f_{5/2})^*$	2.056 <sup>b</sup>	...	Seen	...	(10)	(0.5)	$(2^+)?$
...	...	...	...	$\{(f_{5/2} p_{1/2})^*\}$	2.127 <sup>b</sup>	...	...	(4)	67	(0.7)	$3^+(4^+, 5^+)$
2.136 <sup>a</sup>	(1)	...	$(5-8)$	{ ? }	[	...	...	...	...	...	...

<sup>a</sup> Energies assigned by Ref. 3. These energies tend to be systematically by 0.2% higher than the energies of Ref. 6 and the present work.

<sup>b</sup> Broadened peaks or partially resolved doublet.



vation and resolution of four levels in this narrow energy range at present seem a rather formidable task. [ $^{85}\text{Rb}(\alpha, n)$  can populate all four states, but the distinction of the  $\gamma$  decay of an unresolved  $2^-$ ,  $2^+$  doublet may be difficult since both states would preferentially decay to the  $1^+$  level at 0.393 MeV.]

The partial resolution at all angles in the  $(d, t)$  data of multiplets at (1.225), (1.279), (1.315, 1.325), and (1.562, 1.573) MeV (see Fig. 1) confirms the indirect assignment of multiplets to these energies by previous<sup>3,6</sup> studies.

The most interesting result of the  $(d, t)$  experiment appears to be the significant lack of  $p_{1/2}$  and  $p_{3/2}$  spectroscopic strength in the lowest four  $l=1$  transitions. Generally<sup>1-3</sup> the 0.393- ( $1^+$ ) and 0.763-MeV ( $0^+$ ) states are considered as rather pure  $(p_{1/2})^2$  states. The neutron  $p_{1/2}$  orbit is full in  $^{89}\text{Y}_{50}$ , hence spectroscopic factors of 1.5 and 0.5 would be expected, but we see only  $\frac{2}{3}$  of this strength in the lowest states. We are inclined to think that some of the missing  $1p_{1/2}$  strength is found in the 1.276-MeV ( $1^+$ ) level, and that an appreciable portion of the  $2p_{3/2}$  strength is fractionated and lies above the excitation energy range investigated. This interpretation would be in line with the results for  $^{89}\text{Rb}_{49}$ , where the  $l=1$  neutron strength is spread over at least twice the expected number (6) of final states.<sup>18</sup> Had our  $^{89}\text{Y}(d, t)$  data been obtained at higher energies and shown

more perfect agreement with DWBA, they could have been considered sufficient to refute the idea of a pure  $(p_{1/2})^2_{0^+, 1^+}$  doublet. In the situation at hand we cannot rule out the possibility that our DWBA predictions systematically overestimate the  $l=1$  cross sections while giving correct  $l=4$  cross sections, although we consider this explanation as unlikely.

There seems to be general agreement in the literature that most of the  $1f_{5/2}$  strength lies above 1.8 MeV. This is supported by our present results which suggest that only  $\frac{1}{3}$  of the  $1f_{5/2}$  strength is found below 1.8 MeV.

## B. $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$ Results

### 1. General Comments

A semilog plot of a typical  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  spectrum is shown in Fig. 4. There is practically no continuous background, but there are more peaks, especially weak ones, than can be correlated with known  $^{88}\text{Y}$  levels. Of the sharp peaks weaker than approximately 5% of the strong  $^{88}\text{Y}$  peaks many may stem from heavy target impurities, e.g., other Zr isotopes. We have not tried to extract angular distributions for weak  $(d, \alpha)$  groups but in Table III have listed approximate maximum cross sections for all peaks at energies that correspond to known  $^{88}\text{Y}$  levels. Values of  $10 \mu\text{b/sr}$

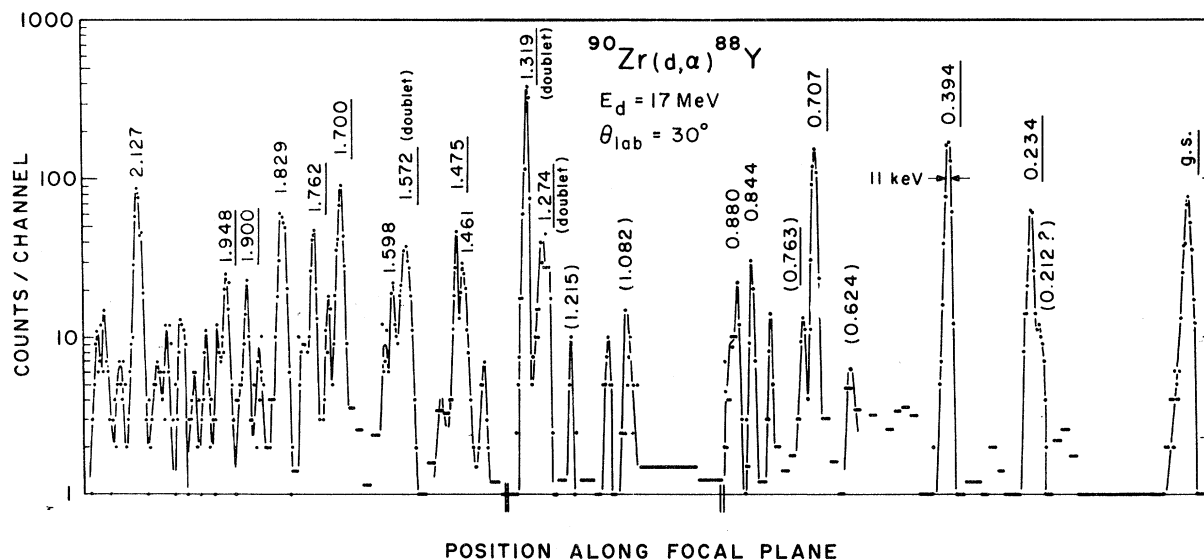


FIG. 4. Composite  $\alpha$  spectrum for  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  at  $\theta_{\text{LAB}}=30^\circ$  obtained with (three) position sensitive surface barrier counters. Peaks seen at most angles are labeled with their measured excitation energy (in MeV). Levels which were also seen in  $^{89}\text{Y}(d, t)^{88}\text{Y}$  are underlined. In order to contract the horizontal scale for this figure, counts in (two) adjacent channels were summed with a slight apparent loss in resolution compared to the raw data. Note that the majority of groups above 1.2 MeV are partially resolved doublets or multiplets. Statistically insignificant counts were averaged over multiples of four channels and are plotted on the 1.0 line for all averages between 0 and 1.

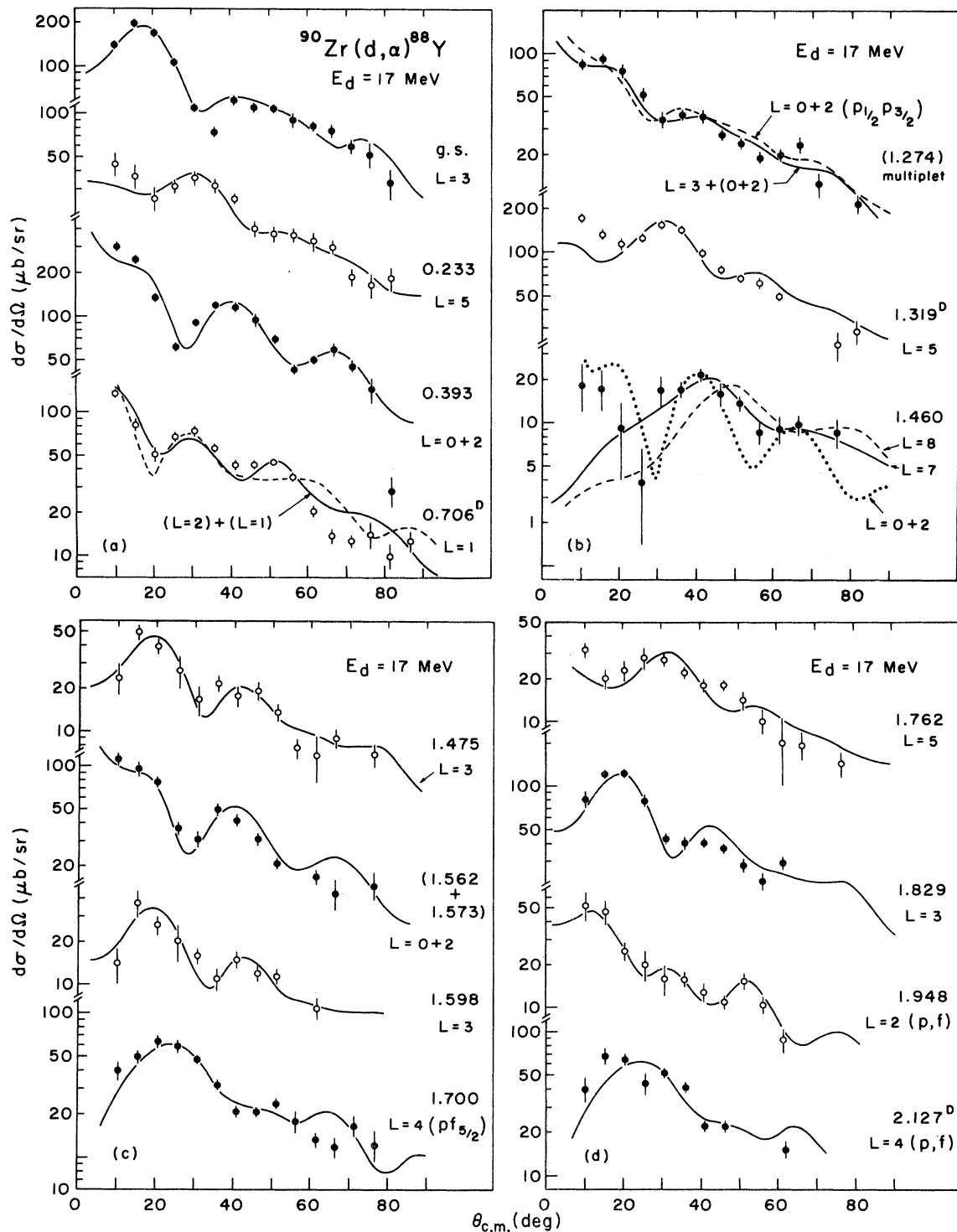


FIG. 5. Experimental angular distributions for  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  at  $E_d = 17$  MeV compared with microscopic DWBA calculation. The solid curves for pure  $L$  transfers correspond to the assumed configurations and the enhancement factors listed in Table III.  $L=0+2$  mixtures are empirical ratios. All known random errors are shown if they exceed the size of the data points. Note that for  $L=4, 5,$  and  $7$  forward-angle DWBA predictions appears systematically low for all angular distributions.

or less should be considered merely upper limits for the excitation of corresponding  $^{88}\text{Y}$  states, since it is difficult to eliminate the possibility that nearby impurity peaks enhance the apparent cross sections by this amount. The largest unexplained group was measured at  $0.212 \pm 0.003$  MeV with  $\sigma_{\text{max}} \approx 20 \mu\text{b/sr}$ . Its angular distribution agrees well with  $L=3$ . Its closeness to the 0.234-MeV level could have made it hard to detect in other work, but it should have been seen in the  $(p, n\gamma)$  work of Ref. 6. The absence of any 212-keV  $\gamma$  line argues strongly for the conclusion that this peak, too, is an impurity.

Angular distributions for all groups that can be assigned to  $^{88}\text{Y}$  states (or clusters of  $^{88}\text{Y}$  levels) are given in Figs. 5(a)–(d). Angular distributions for  $L=0$  (+2),  $L=2$ ,  $L=3$ , and  $L=4$  are well structured and are generally so well predicted by DWBA that  $L$  assignments could be made even without corroborating evidence from  $(d, t)$  or other single-nucleon transfer data. Transitions with  $L \geq 5$  are less structured and show a rise at small angles which is only imperfectly predicted by DWBA, although the main stripping peaks are well fitted. Uncertainties arise for unresolved multiplets for which the angular distributions become flat and uncharacteristic of the contributing  $L$  values. In these cases previous information<sup>1–6</sup> has been used in arriving at the proposed mixture of  $L$  values. It is perhaps interesting to note that for  $^{90}\text{Zr}(d, \alpha)$  odd- $L$  predictions are unique and do not depend on details of the wave functions assumed. On the other hand, in addition to the strong  $L=0, 2$ , and 4 transfers which result from pickup of  $2p$  of  $1f$  nucleons, some weak  $L=0-8$  (even) transfers may result from  $1g_{9/2}^2$  or  $1g2d$  pickup. The latter have predicted angular distributions which differ significantly from  $pf$  transfer.

Generally, previous  $J^\pi$  suggestions and assignments<sup>1–6</sup> for  $^{88}\text{Y}$  are supported by the present study, as is evident for the comparisons in Table III. Where conflicts seem to exist there is almost always persuasive evidence for two or more close levels, and a level by level discussion of  $^{88}\text{Y}$  does not seem warranted. However, two-nucleon transfer cross sections are extremely sensitive to configuration mixing and we find that few transitions lead to states that can be interpreted satisfactorily in terms of a pure configuration. In Table III we have listed the enhancement factor  $\mathcal{E}$  defined in Sec. III B. Only  $\mathcal{E} \approx 1$  would support (although it does not prove) the dominance of the “pure” configuration assumed. It is seen that  $\mathcal{E}$  varies from 0.1 to 5, but it is rarely very close to 1.  $\mathcal{E} \neq 1$  indicates that other configurations contribute significantly to the two-nucleon transfer studied, so that coherence effects enhance or decrease the

actual cross sections. Some configurations (e.g.  $p^2$  or  $pg_{9/2}$ ) lead to much larger  $(d, \alpha)$  cross sections than others ( $f_{5/2}g_{9/2}$ , for instance), and  $\mathcal{E} \neq 1$  may occasionally indicate an incorrect choice for the dominant term. However, the dominant terms are generally chosen on the basis of single-nucleon transfer data so that they are an established component of the wave function of the state, and  $\mathcal{E} \neq 1$  means that there are others that must be considered in two-nucleon transfer. For real nuclei it is not obvious when a state should be called pure. It seems reasonable to call a state essentially pure if its dominant term has the amplitude  $\geq 0.98$ . Thus a second term could have the amplitude  $\pm 0.2$ , and on the average the acceptable range for  $\mathcal{E}$  would be  $0.6 \leq \mathcal{E} \leq 1.4$ .

## 2. $^{88}\text{Y}$ States Dominated by the $(g_{9/2})^2$ Configuration

A discussion of these states is most convenient if  $^{88}\text{Sr}_{50}$  is used as the inert core so that the configuration is written  $(\pi g_{9/2} \nu g_{9/2}^{-1})$ . Using the same core  $^{90}\text{Zr}$  may be written as  $\pi[(1 - \alpha^2)^{1/2}(p_{1/2})^2 + \alpha^2(g_{9/2})^2]$ , where  $\alpha^2 \approx 0.65$ . Hence in  $^{90}\text{Zr}(d, \alpha)$  pickup of  $(g_{9/2})^2$  is allowed for all  $J^+ = 1^+, 3^+, 5^+, 7^+, 9^+$  states but predicted to be very weak ( $\approx 1-3 \mu\text{b/sr}$ ); it is forbidden for all pure  $(g_{9/2}g_{9/2}^{-1})_{J=\text{even}}$  states. Using the excitation energies suggested by Ref. 3 for pure  $g_{9/2}^2$  states (0.678, 0.712, 0.847, 0.989, 1.282, and 1.478 MeV) we find that only three states [0.678 ( $8^+$ ), 0.847 ( $5^+$ ), 0.989 ( $4^+$ )] are sufficiently well separated from strong  $(d, \alpha)$  groups to check this prediction. No trace is seen of either the 0.678- or the 0.989-MeV level in good agreement with expectation. The 0.847 ( $5^+$ ) level which should have a peak cross section of  $1 \mu\text{b/sr}$  is seen with about  $16 \mu\text{b/sr}$ . A possible but not very attractive explanation for this “discrepancy” is that the  $5^+$  peak may happen to fall on top of an unusually large impurity peak. Configuration mixing is not a likely cause, and in any case would be much more plausible for the  $4^+$  state, where a relatively small ( $\sim 25\%$ )  $p_{3/2}f_{5/2}$  admixture could cause the observed cross section.

Unmixed  $(g_{9/2}^2)_{1^+, 2^+}$  states have not been postulated.<sup>3</sup> It would seem likely that  $3^+$  and  $4^+$  states also have  $(p, f)$  admixtures, whereas the high-spin states ( $5^+ - 9^+$ ) should have the purest  $(g_{9/2}^2)$  configurations.

## 3. Levels Dominated by $(p_{1/2})^2$ , $(p_{1/2}p_{3/2})$ , and $(p_{1/2}f_{5/2})$ Configurations

The centroids of these configurations which can couple from  $0^+$  to  $3^+$  bracket the  $(g_{9/2}^2)$  centroid and fall within a range of 1 MeV. Allowing for the analog  $(p_{3/2}p_{1/2})$  and  $(f_{5/2}p_{1/2})$  configurations we

expect to find in the ( $\approx 2$ -MeV) region investigated as many as two  $0^+$  states, four  $1^+$ , five  $2^+$ , and three  $3^+$  states. Given this large base of terms of like  $J^\pi$ , configuration mixing should be extensive and very difficult to unravel experimentally. Assumed "principal configurations" for this set of states are taken from Ref. 3. It can be seen from Table III that the computed enhancement factors range from 0.1 to 5.0. The highest enhancement (5.0) is found for the lowest-lying  $1^+$  state at 0.393 MeV. The shortcomings of a pure  $(p_{1/2})^2_{1^+}$  interpretation of this state are seen in several ways: (a) The predicted  $(p_{1/2})^2$  pickup cross section is too small by a factor of 5; (b)  $(p_{1/2})^2_{1^+}$  transfer would proceed essentially (90%) by  $L=2$ , whereas in the fit shown in Fig. 5(a)  $L=0$  makes the larger contribution; and finally (c) the  $(d, t)$  spectroscopic factor ( $S=1.0$ ) has only  $\frac{2}{3}$  of the strength expected for a pure  $(p_{1/2})^2_{1^+}$  state.

At the other extreme we have the  $2^+(1^+)$  state<sup>6</sup> at 1.225 MeV for which the principal configuration  $(p_{1/2}p_{3/2})$  was proposed.<sup>1-3</sup> In  $(d, \alpha)$  the level is barely seen, with a peak cross of  $\approx 10 \mu\text{b/sr}$ , whereas pure  $(p_{1/2}p_{3/2})_{2^+}$  pickup would be expected at  $90 \mu\text{b/sr}$  (after taking into consideration the incompletely filled  $\nu p_{1/2}$  orbit in  $^{90}\text{Zr}$ ). Since the level is strong in  $(d, t)$  considerable cancellation in the  $(d, \alpha)$  transition amplitude must take place. Again our  $(d, \alpha)$  data seem to bear out the validity of the small spectroscopic factor ( $S=0.5$ ) given for  $\nu p_{3/2}$  pickup in  $(d, t)$  in Table I, which implies strong mixing.

Although most other enhancement factors for this group of states lie closer to 1 we reach the same conclusion as did the  $^{86}\text{Rb}$  studies:<sup>18,24</sup> the assumption of nearly pure positive-parity states of low spin near the  $^{88}\text{Sr}$  core is not justified.

#### 4. The $(p_{1/2}g_{9/2})_{4^-5^-}$ Ground-State Doublet

$g_{9/2}$  pickup in  $^{89}\text{Y}(d, t)$  shows only two strong transitions, those to the ground and first excited state of  $^{88}\text{Y}$ . If the  $g_{9/2}$  strength is not fractionated the spectroscopic ratio should be  $S_{5^-}/S_{4^-} = \frac{1}{2} = 1.22$ . The observed ratio is 1.3, i.e., very close. Does this mean that the ground-state doublet is pure enough to be treated as a single configuration in two-nucleon transfer work? Apparently not, for the  $(d, \alpha)$  enhancement factors are 2 for the  $4^-$  ground state and 0.8 for the  $5^-$  state (if account is taken of the 65% filling of the  $\pi p_{1/2}$ -orbit in  $^{90}\text{Zr}$ ). This means that at least the  $4^-$  ground state experiences significant constructive enhancement due to admixtures from several other  $4^-$  terms. Similar impurity effects have been seen previously<sup>26,27,24</sup> for the two-hole ground states in  $^{206}\text{Tl}$ ,  $^{56}\text{Co}$ , and  $^{86}\text{Rb}$ .

#### 5. States Dominated by the $(f_{5/2}g_{9/2})$ and $(p_{3/2}g_{9/2})$ Configurations

The  $(f_{5/2}g_{9/2})_{2^-3^-}$  and  $(p_{3/2}g_{9/2})_{3^-6^-}$  configurations in  $^{88}\text{Y}$  can be viewed as two holes in  $^{90}\text{Zr}$ , or as two holes in the  $^{88}\text{Sr}_{50}$  core with two extra protons in the  $2p_{1/2}$  (and  $1g_{9/2}$ ) orbit as passive spectators for all but the  $4^-$  and  $5^-$  states. The splitting of the  $2^-$ ,  $3^-$ ,  $6^-$ , and  $7^-$  levels of these mixed configurations in  $^{88}\text{Y}$  should be very similar to that found in  $^{86}\text{Rb}$ . Hence we may use a comparison with  $^{88}\text{Sr}(d, \alpha)$  to identify these states, which are not readily excited in other transfer experiments. It is easy to see that the  $(f_{5/2}g_{9/2})_{2^-,3^-}$  states are the only candidates for pure configurations strongly excited in  $^{90}\text{Zr}(d, \alpha)$ . The  $2^-$  term dominates

TABLE IV. Comparison of  $^{88}\text{Sr}(d, \alpha)$  and  $^{90}\text{Zr}(d, \alpha)$  transitions believed to excite two-hole configurations of the type  $(f_{5/2}g_{9/2})^{-1}$  and  $(p_{3/2}g_{9/2})^{-1}$ . According to Ref. 24, in  $^{86}\text{Rb}$  these configurations are strongly mixed for the  $3^-$ - $5^-$  states. In  $^{88}\text{Y}$  additional mixing with  $(p_{1/2}g_{9/2})^{-1}$  is possible for the  $4^-$  and  $5^-$  states.

$J^\pi$	$^{86}\text{Rb}$	$^{88}\text{Sr}(d, \alpha)$		$E_x$ (Rb)	$^{88}\text{Y}$	$^{90}\text{Zr}(d, \alpha)$		Dominant configuration	$\Delta E$ (keV)
	$E_x$ (MeV)	$L$	$\sigma_{\text{max}}$ ( $\mu\text{b/sr}$ )	+0.680 MeV $E_{\text{expected}}$	$E_x$ (MeV)	$L$	$\sigma_{\text{max}}$ ( $\mu\text{b/sr}$ )		
$2^-$	0.0	1	105	0.680	0.706	1	(40)	$(f_{5/2}g_{9/2})$	-26
$3^-$	0.555	3	225 (80)	1.235	1.262	(3)	(30)		-27
$6^-$	0.555	5	225 (185)	1.235	1.319 <sup>a</sup>	5	174	$(p_{3/2}g_{9/2})$	-84
$(7^-)$	0.779	(7)	(24)	1.459	1.460	(7)	21	$(f_{5/2}g_{9/2})$	-1
$(3^-)$	0.779	(3)	(30) ?	1.459	1.475	3	50		-16
	or 0.872	3	12	1.552					
$(4^-)$	0.978	3	49	1.658	1.598	3	35		+60
$(5^-)$	1.091	5	$\leq 16$	1.771	1.762	5	30		+9
$(4^-)$	1.195	3+(5)	55	1.875	1.827	3	122		+48

<sup>a</sup> Doublet.

the ground state of  $^{86}\text{Rb}$ . In  $^{88}\text{Y}$  it should also be the lowest state of these two strongly mixed multiplets, and be excited by a practically pure  $L=1$  transition. The only candidate for this level is found among the cluster of states near 0.706 MeV (see Fig. 5).

Table IV compares energies,  $L$  values, and cross sections for members of these multiplets in  $^{86}\text{Rb}$  and  $^{88}\text{Y}$ . The correspondence is surprisingly good. Whereas the multiplets are split by over 1 MeV,<sup>24</sup> corresponding members can generally be found within 80 keV or less. Some uncertainties remain because several states are unresolved in both experiments and because the  $2^-$  levels are excited much stronger than expected. Because of the special interest of the  $7^-$  state the best alternate fits are also shown in Fig. 5(b).  $L=7$  has by far the best  $\chi^2$  value; nevertheless an independent confirmation of the  $L=7$ ,  $J^\pi=7^-$  assignment for this state would be most desirable.

#### 6. $2^-$ State and Other Levels Near 0.706 MeV

The present  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$  experiment was not able to shed much additional light on the surprisingly strong population in  $^{88}\text{Sr}(d, \alpha)$  of the  $^{86}\text{Rb}$   $2^-$  ground state, because here again the 0.706-MeV  $2^-$  state seems unexpectedly strongly populated ( $\mathcal{E} \approx 2$ ). In addition it is unresolved from a strong positive-parity level of low spin ( $2^+$ ). Two close-lying ( $g_{9/2}^2$ )<sub>6<sup>+</sup>7<sup>+</sup></sub> states reported at 0.712 MeV in the ( $^3\text{He}, d$ ) and  $^{88}\text{Sr}(^3\text{He}, t)^{88}\text{Y}$  experiments<sup>3</sup> are almost certainly not excited by  $(d, \alpha)$ , but their existence complicates the experimental problem. If the  $2^-$  level has the pure configuration  $(f_{5/2}g_{9/2})$  it would not be seen in ( $^3\text{He}, t$ ), but an admixture of  $(g_{9/2}f_{5/2})_{2^-}$  should make it visible. The positive-parity ( $2^+$ ) state may have a dominant  $(p_{3/2}p_{1/2})$  term and may also escape detection in ( $^3\text{He}, t$ ) and  $^{87}\text{Sr}(^3\text{He}, d)$ . However, both states should have been seen in the  $^{88}\text{Sr}(p, n\gamma)$  experiment by Gabbard, Chenevert, and Sekkaran<sup>6</sup> who seem to confirm only a  $2^-$  state. Hence further work seems to be required to explain the exceptionally strong peak seen at 0.706 MeV by  $^{90}\text{Zr}(d, \alpha)^{88}\text{Y}$ .

## V. SUMMARY AND CONCLUSIONS

The present high resolution study of  $^{88}\text{Y}$  has detected about 30 low-lying levels excited by pickup, many of them members of close-lying multiplets. Their resolution and accurate energy measurements helped to resolve some divergent assignments in the literature. Independent  $L$  values, spectroscopic strengths, and  $J^\pi$  assignments or close  $J^\pi$  limits were found for about 20 of these states below 2 MeV excitation (Table III). In almost all cases where recent  $J^\pi$  assignments existed these were confirmed or supported. Levels assigned by Comfort and Schiffer<sup>3</sup> to the  $J^\pi \geq 3^+$  members of the  $g_{9/2}^2$  multiplet were not seen or resolved in  $(d, \alpha)$  in good agreement with expectations. Additional evidence for a  $2^-$  level at 0.706 MeV<sup>6</sup> was given and a number of other states dominated by the  $(f_{5/2}g_{9/2})^{-1}$  and  $(p_{3/2}g_{9/2})^{-1}$  configurations were found and discussed.

$^{90}\text{Zr}(d, \alpha)$  angular distributions showed well structured, characteristic shapes and were well fitted by straightforward DWBA calculations.<sup>22</sup> A comparison of measured and "absolute" microscopic DWBA cross sections for  $^{90}\text{Zr}(d, \alpha)$  gave strong evidence of configuration mixing. Most low-lying (two-hole) levels of  $^{88}\text{Y}$  are not well described as multiplets of pure configurations, although often a moderately dominant term can be identified.

$^{88}\text{Y}$  continues to pose a number of unanswered questions caused in part by some very close clusters of levels, the lowest not fully explained group lying near 706 keV. Spectroscopic factors for  $l=1$  pickup derived from the  $^{88}\text{Y}(d, t)^{88}\text{Y}$  experiments were found to be systematically lower than the corresponding values obtained from ( $^3\text{He}, \alpha$ ) studies.<sup>1,2</sup> This new result is consistent with the extensive configuration mixing suggested for the  $(p^2)$  states by the  $^{90}\text{Zr}(d, \alpha)$  analysis.

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<sup>1</sup>Present address: Physics Department, Panjab University, Chandigarh-14, India

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## Properties of $\gamma$ -Ray Transitions in $^{56}\text{Co}$ from $^{56}\text{Ni}$ Decay and $^{56}\text{Fe}(p, n\gamma)^{56}\text{Co}$

L. E. Samuelson, W. H. Kelly, and R. R. Todd\*

*Cyclotron Laboratory,† Department of Physics, Michigan State University, East Lansing, Michigan 48823*

R. A. Warner, Wm. C. McHarris,‡ and F. M. Bernthal

*Department of Chemistry§ and Cyclotron Laboratory,† Department of Physics, Michigan State University, East Lansing, Michigan 48823*

E. M. Bernstein and R. Shamu

*Department of Physics,¶ Western Michigan University, Kalamazoo, Michigan 49001*

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The  $^{56}\text{Ni}$   $\epsilon$  decay and the  $^{56}\text{Fe}(p, n\gamma)^{56}\text{Co}$  reaction with beam energies between 5.5 and 8.4 MeV have been used with Ge(Li) spectrometers to study the properties of  $\gamma$  rays from states of  $^{56}\text{Co}$  below 2.86 MeV excitation. From  $^{56}\text{Ni}$   $\epsilon$  decay both the  $\gamma$ -ray spectrum and  $\gamma$ - $\gamma$  coincidences were studied.  $\gamma$ - $\gamma$  coincidences,  $\gamma$ -ray excitation functions,  $\gamma$ -ray angular distributions, and absolute cross sections were measured for the  $^{56}\text{Fe}(p, n\gamma)^{56}\text{Co}$  reaction. An  $\epsilon$  decay scheme for  $^{56}\text{Ni}$ , which includes six  $\gamma$  rays, and an energy-level diagram for  $^{56}\text{Co}$ , which includes 35  $\gamma$  rays (14 of which are reported for the first time) from 20 excited states, are presented. Comparison of the data from  $^{56}\text{Fe}(p, n\gamma)^{56}\text{Co}$  with predictions of the statistical compound-nuclear model have resulted in spin assignments (in parentheses) for the following states (energies in keV) of  $^{56}\text{Co}$ : 158.4(3), 576.6(5), 829.7(4), 970.3(2), 1009.2(5), 1114.6(3), 1450.8(0), and 1720.3(1). Branching ratios are presented for 14  $\gamma$  rays from these eight states and multipole mixing ratios are given for 12 of these  $\gamma$  rays (10 are predominantly M1). The data are consistent with a spin-4 assignment to the ground state. Contrary to previous suggestions, evidence from all experiments indicates that only one state (believed to be the antianalog of the  $^{56}\text{Fe}$  ground state) exists in  $^{56}\text{Co}$  in the neighborhood of 1451 keV excitation. The level energies,  $\gamma$ -ray multipole mixing ratios, and  $\gamma$ -ray branching ratios agree, in general, with shell-model predictions of McGrory.

### I. INTRODUCTION

The earliest investigations<sup>1-4</sup> of the low-lying excited states of  $^{56}\text{Co}$  began with the  $\epsilon$  decay of  $^{56}\text{Ni}$ . These studies, which included measurements of the  $^{56}\text{Ni}$  half-life,<sup>1</sup> the  $\gamma$ -ray spectrum,<sup>1,3,4</sup>  $\gamma$ - $\gamma$  angular correlations,<sup>1,3</sup> the internal-conversion electron spectrum,<sup>2</sup> and lifetimes of some

$^{56}\text{Co}$  states,<sup>1</sup> produced valuable information. However, only selected states below 2.1 MeV could be populated and unambiguous spin assignments for these states could not be made.

More recently, experiments involving the two-particle transfer reactions,  $^{54}\text{Fe}(^3\text{He}, p)^{56}\text{Co}$ ,<sup>5-9</sup>  $^{54}\text{Fe}(\alpha, d)^{56}\text{Co}$ ,<sup>10</sup>  $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ ,<sup>11</sup> and  $^{58}\text{Ni}(d, \alpha)^{56}\text{Co}$ ,<sup>6,7,9,12,13</sup> and the charge-exchange reactions,