6. ACKNOWLEDGMENTS

I would like to thank Dr. J. V. Noble for suggesting this problem and Dr. R. Amado, Dr. H. Galbraith, and Dr. R. Dreizler for useful discussions. I am grateful for the financial support of the Comissão Nacional de Energia Nuclear (Brazil) and the Universidade Federal de Pernambuco (Brazil).

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†Work based in part on a Ph.D thesis submitted to the Physics Department of the University of Pennsylvania and supported partially by the U. S. Atomic Energy Commission.

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PHYSICAL REVIEW C VOLUME 7, NUMBER 6 JUNE 1973

Off-Shell Effects in Nucleon-Nucleus Scattering*

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(Received 30 January 1973)

It is shown that the usual multiple-scattering treatment of nucleon-nucleus elastic scattering does not provide a complete description of the process. In particular, the replacement of the two-nucleon interaction potential by a *two-body* T matrix (evaluated at a single parametric energy) leads to an inadequate treatment of the off-shell aspects of the multiple-scattering problem. As an example, we discuss the manner in which short-range correlations in the nuclear targets affect the off-shell behavior of the two-body scattering operators used to describe nucleon-nucleus scattering. The influence of these off-shell effects on the calculation of total cross sections is discussed.

I. INTRODUCTION

Recently we have presented a theory of nucleon-nucleus scattering[1,2] which is formulated such that the off-shell properties of the two-body T matrices entering the theory can be made explicit. The choice of the parametric energies in these T matrices is unambiguous and is different depending on the particular physical process (or "diagram") considered. (In this respect our approach is rather close to that used in the formulation of the theory of nuclear matter.)

Before expanding on these matters it is worth reviewing the more conventional treatment of multiple scattering to indicate how off-shell effects may be treated in such theories. This is done in Sec. II, where it is pointed out that the usual approximations made in the application of the Watson multiple-scattering theory are not capable of correctly describing the off-shell aspects of the problem.

In Sec. III, we show that taking into account the short-range correlations in the target leads to two-body T matrices that are off shell. These off-shell effects can be important in that they even affect the *leading term* in the multiple scattering expansion of the optical-model potential at moderate energies.

In Sec. IV, a rough estimate of the influence of these off-shell effects on the calculation of total cross sections is given. The magnitude of these effects appears to be generally comparable to the modifications of the total cross section arising from the exclusion principle, as well as to the uncertainties in the nucleon-nucleon data and those uncertainties introduced through the use of different form factors for the target.

II. MULTIPLE-SCATTERING SERIES

In the theory of the scattering of nucleons by nuclei, where the nuclear force is considered to be singular, it is customary to eliminate the singular two-body potential in favor of a two-body scattering matrix. In the Watson theory,[3] one writes a scattering matrix, T, which satisfies an equation of the form

$$T(E)=\sum_{i=1}^{A} v_{0i}+\left(\sum_{i=1}^{A} v_{0i}\right)\frac{1}{E-h_0-H_A+i\epsilon}\,T(E), \tag{2.1}$$

where

$$H_A=\sum_{i=1}^{A} h_i+\sum_{\substack{i,j=1\\ i\neq j}}^{A} v_{ij}, \tag{2.2}$$

and

$$h_i=-(\hbar^2/2m)\nabla_i^2, \quad (i=0,\,1,\,\ldots,\,A). \tag{2.3}$$

We define

$$T(E)=\sum_{i=1}^{A} T_{0i}(E), \tag{2.4}$$

and then construct an operator $\tau_{0i}(E)$, which satisfies an equation of the form

$$\tau_{0i}(E)=v_{0i}+v_{0i}\,G(E)\tau_{0i}(E). \tag{2.5}$$

Here $G(E)=(E-h_0-H_A+i\epsilon)^{-1}$ is the same propagator as that which appears in Eq. (2.1). If we choose to solve Eq. (2.5) for v_{0i} we obtain

$$v_{0i}=\frac{1}{1+\tau_{0i}(E)G(E)}\,\tau_{0i}(E), \tag{2.6}$$

which when substituted into Eq. (2.4) yields

$$T_{0i}(E)=\tau_{0i}(E)+\tau_{0i}(E)G(E)\sum_{j\neq i}^{A} T_{0j}(E). \tag{2.7}$$

Now Eq. (2.7) presents no advantages whatever over Eq. (2.1), because in order to use this we must solve Eq. (2.5) for $\tau_{0i}(E)$. However, Eq. (2.5) is obviously no easier to solve than is Eq. (2.1). Rather, we seek to reexpress Eq. (2.1) in terms of the two-body operator $t_{0i}(\omega_i)$, where $t_{0i}(\omega_i)$ satisfies the relation

$$t_{0i}(\omega_i)=v_{0i}+v_{0i}\,g_{0i}(\omega_i)t_{0i}(\omega_i). \tag{2.8}$$

Here $g_{0i}(\omega_i)=(\omega_i-h_0-h_i+i\epsilon)^{-1}$ is the free propagator. If we solve Eq. (2.8) for v_{0i} and substitute the result into Eq. (2.5), we readily obtain

$$\tau_{0i}(E)=t_{0i}(\omega_i)+t_{0i}(\omega_i)[G(E)-g_{0i}(\omega_i)]\,\tau_{0i}(E). \tag{2.9}$$

The circumstances under which the second term on the right-hand side of Eq. (2.9) can be taken to be negligible are the circumstances under which we may substitute $t_{0i}(\omega_i)$ for $\tau_{0i}(E)$ in Eq. (2.7). This substitution, of course, greatly simplifies the scattering problem.

Just as we combined Eqs. (2.1) and (2.5) to obtain Eq. (2.7), so can we combine Eqs. (2.8) and (2.1) to obtain the result that

$$T_{0i}(E)=t_{0i}(\omega_i)+t_{0i}(\omega_i)G(E)\sum_{j\neq i} T_{0j}(E)$$
$$+t_{0i}(\omega_i)[G(E)-g_{0i}(\omega_i)]\,T_{0i}(E). \tag{2.10}$$

Here again, we see that under the circumstances that $\tau_{0i}(E)\approx t_{0i}(\omega_i)$, Eq. (2.10) becomes identical to Eq. (2.7). If we were to define the operator $\hat{T}_{0i}(E,\{\omega\})$ as

$$\hat{T}_{0i}(E,\{\omega\})=t_{0i}(\omega_i)+\sum_{j\neq i}\hat{T}_{0j}(E,\{\omega\})G(E)t_{0i}(\omega_i) \tag{2.11}$$

then solve Eq. (2.11) for $t_{0i}(\omega_i)$ and finally substitute this into Eq. (2.10), we would obtain

$$T_{0i}(E)=\hat{T}_{0i}(E,\{\omega\})+\hat{T}_{0i}(E,\{\omega\})G(E)\sum_{j\neq i} T_{0j}(E)$$
$$-\sum_{j\neq i}\hat{T}_{0j}(E,\{\omega\})G(E)T_{0i}(E)$$
$$+\hat{T}_{0i}(E,\{\omega\})[G(E)-g_{0i}(\omega_i)]\,T_{0i}(E). \tag{2.12}$$

If we then define $\hat{T}(E,\{\omega\})$ to be

$$\hat{T}(E,\{\omega\})=\sum_i \hat{T}_{0i}(E,\{\omega\}), \tag{2.13}$$

we may sum Eq. (2.12) to obtain

$$T(E)=\hat{T}(E,\{\omega\})+\sum_i \hat{T}_{0i}(E,\{\omega\})[G(E)$$
$$-g_{0i}(\omega_i)]\,T_{0i}(E). \tag{2.14}$$

The implication is clear that judicious choices of the ω_i could minimize the effect of the second term on the right-hand side of Eq. (2.14). When the problem is formulated in this way, however, it is not especially clear how this choice might be made, particularly if we wish to take account of the identity of the particles within the nucleus. In the Kerman-McManus-Thaler (KMT) formulation,[4] where the identity of the particles within the nucleus is accounted for at the outset, the possibility of using different single-particle energies, ω_i, is completely lost, since in that formulation all the t_{0i} are identical. It is perhaps instructive to repeat the derivation given above using the KMT formalism.

Thus, following KMT, we write the relation which is equivalent to Eq. (2.1) as

$$T(E)=Av+AvG(E)T(E), \tag{2.15}$$

and then, in analogy with Eq. (2.5), define an operator $\tau(E)$ as

$$\tau(E)=v+vG(E)\tau(E). \tag{2.16}$$

Upon solving Eq. (2.16) for v and substituting this into Eq. (2.15), we obtain

$$T(E)=A\tau(E)+(A-1)\tau(E)G(E)T(E). \tag{2.17}$$

Here again, we define an operator $t(\omega)$ just as in Eq. (2.8) such that

$$t(\omega)=v+vg(\omega)t(\omega), \tag{2.18}$$

Now, $\tau(E)$ and $t(\omega)$ are related, as in Eq. (2.9) by

$$\tau(E)=t(\omega)+t(\omega)[G(E)-g(\omega)]\tau(E). \tag{2.19}$$

The use of the $t(\omega)$ in place of the v in Eq. (2.15) leads in the by now familiar way to the result

$$\begin{aligned} T(E)&=At(\omega)+t(\omega)[AG(E)-g(\omega)]T(E)\\ &=At(\omega)+(A-1)t(\omega)G(E)T(E)\\ &\quad+t(\omega)[G(E)-g(\omega)]T(E). \end{aligned} \tag{2.20}$$

Again in analogy to Eq. (2.11), we define $\hat{T}(E,\omega)$ to satisfy

$$\hat{T}(E,\omega)=At(\omega)+(A-1)t(\omega)G(E)\hat{T}(E,\omega), \tag{2.21}$$

from which we readily obtain, by eliminating $t(\omega)$ between Eqs. (2.20) and (2.21), the relation

$$T(E)=\hat{T}(E,\omega)+A^{-1}\hat{T}(E,\omega)[G(E)-g(\omega)]T(E). \tag{2.22}$$

It is clear that Eqs. (2.22) and (2.14) are the same equation, provided only that we add the constraint that, because of particle identity, we choose all the τ_{0i} to be equal, and also all the t_{0i}. This implies that all the $T_{0i}(E)$ are equal and likewise all the $\hat{T}_{0i}(E,\{\omega\})$, so that we may identify $T_{0i}(E)$ as

$$T_{0i}(E)=A^{-1}T(E), \tag{2.23}$$

and $\hat{T}_{0i}(E,\{\omega\})$ as

$$\hat{T}_{0i}(E,\{\omega\})=A^{-1}\hat{T}(E,\omega). \tag{2.24}$$

These identifications are all that is required to make the KMT formulas identical with the Watson relations.

Let us now consider the leading term of Eq. (2.20),

$$T(E)\simeq At(\omega). \tag{2.25}$$

We may define (nonantisymmetrized) states containing a plane wave for the projectile and the antisymmetrized ground state of the target, $|\Phi_A\rangle$; these states may be denoted as $|\vec{k},\Phi_A\rangle$. Matrix elements of $At(\omega)$ between such states give the leading term of the optical potential in the KMT approach. Thus, we obtain

$$\langle\vec{k}'|v_{\text{opt}}|\vec{k}\rangle\simeq A\langle\vec{k}',\Phi_A|t(\omega)|\vec{k},\Phi_A\rangle=A\iint d\vec{r}_1 d\vec{r}_1'\langle\vec{k}',\vec{r}_1'|t_{01}(\omega)|\vec{k},\vec{r}_1\rangle\langle\vec{r}_1|\rho|\vec{r}_1'\rangle, \tag{2.26}$$

where, in Eq. (2.26), $\langle\vec{r}_1|\rho|\vec{r}_1'\rangle$ is the density matrix of the target. In terms of the creation and destruction operators for particles, $a^\dagger(\vec{r})$ and $a(\vec{r})$, we may write the density matrix as

$$\langle\vec{r}_1|\rho|\vec{r}_1'\rangle=\langle\Phi_A|a^\dagger(\vec{r}_1')a(\vec{r}_1)|\Phi_A\rangle. \tag{2.27}$$

It is apparent from the form of Eq. (2.26) that one has only limited freedom in describing the off-shell aspects of the individual collisions, since there is only a single parameter, ω. In the next section, we will discuss the leading term of the optical potential as obtained in the formalism of Ref. 2 and contrast that result with that of KMT [Eq. (2.26)].

In summary, it should be stated that without further analysis of the Watson multiple-scattering series, one has no guide as to the optimum choice for the parameter ω. In addition, with only a single parameter at one's disposal it is not possible to obtain a reasonable description of the various thresholds which are important for obtaining an accurate description of the analytic properties of the optical potential at low energy.

It is also clear that if we take $\omega=\omega_i=E$, then at high enough energies $T(E)=\hat{T}(E,E)$. This yields a form of the impulse approximation. However, this remark gives us no indication as to what energy is sufficiently high for this purpose. In a Hartree-Fock description of the target nucleus, one expects that if the incident energy is very much greater than the single-particle energies, then this approximation ($T=\hat{T}$) might be sensible. However, singular interactions require a more detailed description of the target than is possible in a Hartree-Fock treatment. Since v_{0i} is singular (or very strong), it is necessary to replace

v_{0i} by some operator (such as t_{0i}) for which one expects to be able to define a convergent series. This resummation has been carried through for the nuclear matter problem, and except for some question of convergence, the *bound-state* problem is reasonably well understood. It is therefore worth pointing out that it is inconsistent to consider the resummation of the interaction between the projectile and the target particles and neglect the short-range correlations of the target. In the next section we describe some of the features which result if one considers the correlations in the target and their effect on the analysis of nucleon-nucleus scattering.

III. OFF-SHELL EFFECTS DUE TO SHORT-RANGE CORRELATIONS

It is well known that the ground state of a nucleus cannot be entirely represented by a superposition of shell-model states if the forces are singular. Brueckner-Hartree-Fock calculations[5] indicate that each normally occupied nuclear orbital is depleted by about 15%, and high-momentum continuum states become occupied. This is a direct consequence of the strong repulsive nature of the fundamental two-body force at short distances. A detailed dynamical theory establishes the connection between the occupation of these continuum states and the short-range correlation structure of the nucleus. The existence of such high-momentum single-particle states implies that the projectile must be at a higher energy before one may use the impulse approximation in the treatment of the scattering of a nucleon from nucleons occupying such states in the nucleus. The foregoing remarks indicate qualitatively how the short-range correlations can affect even the leading term in the impulse approximation, since the scattering from the continuum states needs to be considered very much further off shell than does the scattering from the normally occupied orbitals.

A diagrammatic representation of the single-scattering contribution to the optical potential is shown in Fig. 1. The first diagram on the right-hand side represents the scattering from a nucleon in a discrete orbital $|\phi_b\rangle$, with single-particle energy $\epsilon_b = -|\epsilon_b|$, and occupation number ρ_b. This leading term contributes to the optical potential as[2]

$$\langle \vec{k}' | v_{\rm opt} | \vec{k}'' \rangle = \sum_b \langle \vec{k}', \phi_b | K_{12}(\epsilon_{\vec{k}} - |\epsilon_b|) | \vec{k}'', \phi_b \rangle_A \rho_b + \cdots , \tag{3.1}$$

where K_{12} is the Bethe-Goldstone reaction matrix evaluated at the "off-shell" energy $E = (\hbar^2 k^2/2m) - |\epsilon_b|$, and the subscript A stands for antisymmetrization of the right-hand ket. The symbol ρ_b is defined by

$$\rho_b = \langle \Phi_A | \eta_b^\dagger \eta_b | \Phi_A \rangle , \tag{3.2}$$

where $|\Phi_A\rangle$ is the *correlated* ground-state ket and η_b is the destruction operator for the orbit $|\phi_b\rangle$. The occupation numbers ρ_b are less than unity. For those forces in current use which are characterized by strong short-range repulsion, it is found that[5]

$$\sum_b \rho_b \sim 0.85A , \tag{3.3}$$

and ~15% of the density is accounted for by the occupation of the continuum orbits.

The second term of Fig. 1 (together with its various exchange diagrams) may be written as

$$\langle \vec{k}' | V_2(\epsilon_{\vec{k}}) | \vec{k}'' \rangle = \sum_{b b'} \int d\vec{k}_1 d\vec{k}_1' d\vec{k}_2 \langle \vec{k}', \vec{k}_1' | K_{12}(\epsilon_{\vec{k}} - \epsilon_{\vec{k}_2} - |\epsilon_b| - |\epsilon_{b'}|) | \vec{k}'', \vec{k}_1 \rangle_A \langle k_1 | \rho(\epsilon_b, \epsilon_{b'}, \vec{k}_2) | \vec{k}_1' \rangle \rho_b \rho_{b'} , \tag{3.4}$$

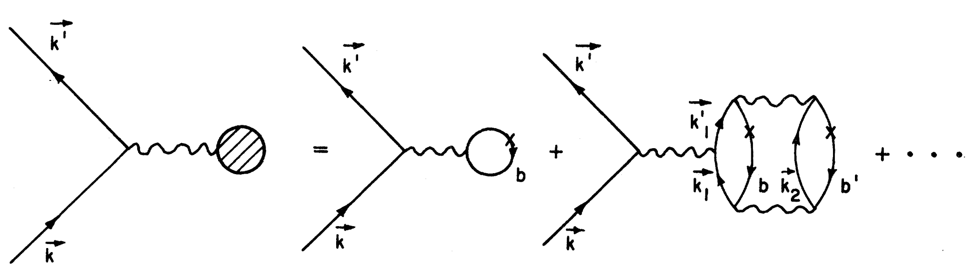

FIG. 1. A schematic representation of the leading term in the optical potential (direct terms only are indicated). Here the wavy line represents a nuclear reaction matrix which is the solution of a Bethe-Goldstone equation (Ref. 2). The cross-hatched portion of the left-hand diagram depends on the detailed structure of the target and is expanded on the right into a bound-state term and a continuum term. The small crosses on the down-going (hole) lines indicate that these diagrams are calculated including occupation probabilities [see diagrams Eqs. (3.1) and (3.4)].

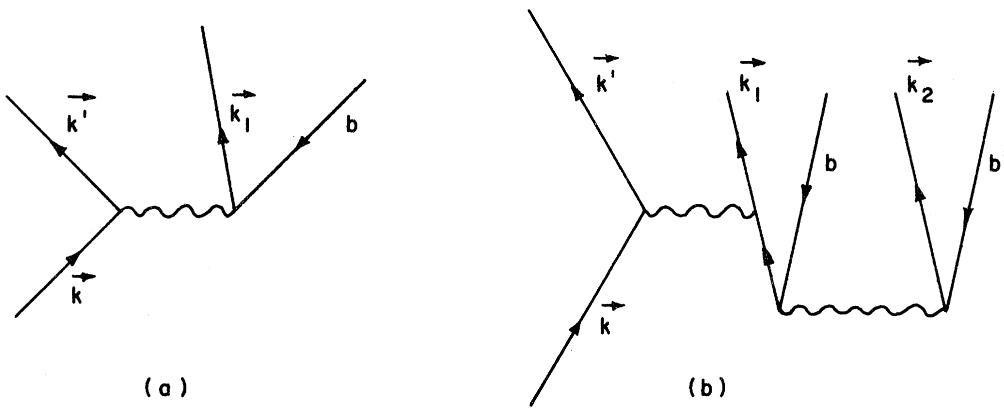

FIG. 2. Inelastic processes associated with the imaginary portions of the optical potential arising from the two diagrams on the right of Fig. 1. The threshold for process (a) is $|\epsilon_b|$ and that for (b) is $|\epsilon_b|+|\epsilon_{b'}|+(k_2^2/2m)$, consistent with the energy arguments of the reaction matrices, $K_{12}(\epsilon_{\vec{k}}-|\epsilon_b|)$ and $K_{12}(\epsilon_{\vec{k}}-|\epsilon_b|-|\epsilon_{b'}|-k_2^2/2m)$, in Eqs. (3.1) and (3.4), respectively.

where we have separated the reaction matrix involving the projectile from the entire analytic expression for the diagram. This separation is useful if we wish to consider the high-energy limit $\epsilon_{\vec{k}}\rightarrow\infty$. It may be worth remarking upon the very different off-shell behavior of the two terms in Fig. 1. The origin of this difference in behavior may be seen from inspection of Fig. 2. While the inelasticity associated with the first term of Fig. 1 is one-particle knockout [Fig. 2(a)] with threshold at $|\epsilon_b|$, the inelasticity associated with the second term of Fig. 1 [see Fig. 2(b)] is the generation of two-particle, two-hole states whose threshold is (for fixed $\vec{k}_2$), $\epsilon_{\vec{k}_2}+|\epsilon_b|+|\epsilon_{b'}|$.

Now it is clear that the off-shell character of the expression of Eq. (3.1) is quite different from that of Eq. (3.4). If $\epsilon_{\vec{k}}\gg|\epsilon_b|$, the standard approximation for Eq. (3.1) yields (with $\vec{q}=\vec{k}'-\vec{k}''$)

$$\langle\vec{k}'|v_{\rm opt}|\vec{k}''\rangle\rightarrow K_{12}(\epsilon_{\vec{k}},\vec{q})\sum_b\rho_b(\vec{q})\,. \tag{3.5}$$

Here $\rho_b(\vec{q})$ is the Fourier transform of that portion of the density arising from discrete states alone, $\rho_b(\vec{r})$, where

$$\sum_b\int\rho_b(\vec{r})\,d\vec{r}\sim 0.85A\,. \tag{3.6}$$

However, when one considers Eq. (3.4), one sees that the neglect of the off-shell features of this expression requires $\epsilon_{\vec{k}}\gg\epsilon_{\vec{k}_2}+|\epsilon_b|+|\epsilon_{b'}|$. Due to the strong repulsive interaction, the mean value of $\epsilon_{\vec{k}_2}$ may be several hundred MeV, while the mean value of $|\epsilon_b|$ may be about 40 MeV. In the very high energy limit the expression given in Eq. (3.4) may be written

$$\begin{aligned}\langle\vec{k}'|V_2(\epsilon_{\vec{k}})|\vec{k}''\rangle&\simeq\int\langle\vec{k}',\vec{k}_1'|K_{12}(\epsilon_{\vec{k}})|\vec{k}'',\vec{k}_1\rangle_A\Big[\sum_{bb'}\int d\vec{k}_2\langle\vec{k}_1|\rho(\epsilon_b,\epsilon_{b'},\vec{k}_2)|\vec{k}_1'\rangle\rho_b\rho_{b'}\Big]d\vec{k}_1'\,d\vec{k}_1\\&=\int\langle\vec{k}',\vec{k}_1'|K_{12}(\epsilon_{\vec{k}})|\vec{k}'',\vec{k}_1\rangle_A\langle\vec{k}_1|\rho|\vec{k}_1'\rangle\,d\vec{k}_1d\vec{k}_1'\,,\end{aligned} \tag{3.7}$$

where $\langle\vec{k}_1|\rho|\vec{k}_1'\rangle$ is now the *continuum portion* of the density matrix of the target. We may assume that the relevant parameter in Eq. (3.5) is the momentum transfer, $\vec{q}$, so that

$$\langle\vec{k}'|V_2(\epsilon_{\vec{k}})|\vec{k}''\rangle\rightarrow K_{12}(\epsilon_{\vec{k}},\vec{q})\rho_c(\vec{q})\,, \tag{3.8}$$

where ρ_c is the portion of the *density* arising from the occupation of continuum orbits. Note that the total density, $\rho(\vec{r})=\sum_b\rho_b(\vec{r})+\rho_c(\vec{r})$, satisfies $\int\rho(\vec{r})d\vec{r}=A$.

Combining Eq. (3.5) and (3.8), we have the conventional result

$$\langle\vec{k}'|v_{\rm opt}|\vec{k}''\rangle\rightarrow K(\epsilon_{\vec{k}},\vec{q})\rho(\vec{q})+\cdots\,. \tag{3.9}$$

The point we wish to stress, however, is that, at energies below a few hundred MeV, the off-shell nature of V_2 will have important consequences which have not been fully explored. In particular, let us consider the calculation of the imaginary part of V_{opt}, from Eqs. (3.5) and (3.8),

$$\operatorname{Im}\langle \vec{k}' | v_{\text{opt}} | \vec{k}''\rangle \simeq \operatorname{Im}K_{12}(\epsilon_{\vec{k}}, \vec{q}) \sum_b \rho_b(\vec{q}) + \operatorname{Im}\langle \vec{k}' | V_2(\epsilon_{\vec{k}}) | \vec{k}''\rangle . \qquad (3.10)$$

The first term of Eq. (3.10) is about 0.85 of what one would calculate from Eq. (3.9). The second term in Eq. (3.10) may be very small, since if $\epsilon_{\vec{k}} \sim 100$ MeV, the K matrix in Eq. (3.4) will be evaluated mainly at *negative energies*, where it is *real*.

Corrections to the imaginary part of V_{opt} of the order of 15% may prove to be important in the interpretation of total cross-section data. Thus the study of cross sections may provide a source of information concerning the correlation structure of nuclei. This suggestion is briefly discussed in the next section.

IV. INVESTIGATION OF OFF-SHELL EFFECTS ON THE TOTAL CROSS SECTION FOR NUCLEONS ON ^{16}O

In this section we will discuss the influence of the effects described above on the calculation of the total cross section for the scattering of nucleons from ^{16}O. We write the spin-independent part of the momentum-space optical potential in the form ($A=16$)

$$U_E(q) = -\frac{1}{2\pi^2}\frac{\hbar^2}{m} A F(q) f_{NN}(q, E), \qquad (4.1)$$

where q is the magnitude of the momentum transfer and E is the incident energy. The quantity $f_{NN}(q, E)$ is given by

$$f_{NN}(q, E) = \tfrac{3}{4} A_1(q, E) + \tfrac{1}{4} A_0(q, E), \qquad (4.2)$$

where $A_1(q, E)$ and $A_0(q, E)$ are the isospin $T=1$ and $T=0$ components of the spin-independent part of the nucleon-nucleon (N-N) scattering amplitude, respectively. The nuclear form factor, $F(q)$, is given by

$$F(q) = \int \rho(r) e^{-i\vec{q}\cdot\vec{r}} d\vec{r}, \qquad (4.3)$$

where $\rho(r)$ is the nuclear mass density distribution, normalized to unit volume integral, $\int \rho(r) d\vec{r} = 1$.

The optical potential for nucleons on ^{16}O also has a spin-orbit term in addition to that given by Eq. (4.1). The effect of the spin-orbit potential on total cross sections should, however, be negligible and so will be ignored in the present calculations.

The potential $U_E(q)$ is completely determined when $\rho(r)$ and $f_{NN}(q, E)$ are specified. In these calculations it proved convenient to use two different models for the density $\rho(r)$. One of these was the Fermi distribution

$$\rho(r) = \frac{\rho_F}{1 + e^{(r-c)/a}} \qquad (4.4)$$

with the parameters, determined from electron scattering, given by $\rho_F = 0.0109$ fm^{-3}, $c = 2.6$ fm, and $a = 0.409$ fm. In all the calculations with this distribution we made the additional approximation of assuming that the variation with q of $f_{NN}(q, E)$ could be ignored in comparison with that of $F(q)$. This is not an especially good approximation for ^{16}O, but it simplifies considerably the task of obtaining the potential U_E in coordinate space. A Fourier inversion of $F(q)$ leads to

$$U_E(r) = \int U_E(q) e^{i\vec{q}\cdot\vec{r}} d\vec{q} = -4\pi \frac{\hbar^2}{m} f_{NN}(E, q=0) A \rho(r). \qquad (4.5)$$

This can be rewritten as

$$U_E(r) = \frac{V + iW}{1 + e^{(r-c)/a}} . \qquad (4.6)$$

The real coefficients V and W can be determined immediately from Eqs. (4.4) and (4.5).

To determine the effect of the variation of $f_{NN}(E, q)$ with q it was convenient to use, in place of Eq. (4.4) a harmonic-oscillator (HO) density distribution for the target nucleus. The HO density distribution appropriate to ^{16}O, and normalized to unity as before, is

$$\rho(r) = \frac{\nu\sqrt{\nu}}{4\pi\sqrt{\pi}} (1 + 2\nu r^2) e^{-\nu r^2}, \qquad (4.7)$$

where we choose $\nu = 0.323$ fm^{-2}, consistent with electron scattering and microscopic calculations. The form factor, Eq. (4.3), corresponding to the density distribution of Eq. (4.7) is easily seen to be

$$F(q) = \left(1 - \frac{q^2}{8\nu}\right) e^{-q^2/4\nu}. \qquad (4.8)$$

For the q dependence of the average NN scattering amplitude we use a phenomonological fit to the data due to Schwaller *et al.*,[6]

$$f_{NN}(q, E) = f_{NN}(q=0, E) e^{-\gamma^2(E) q^2/2}, \qquad (4.9)$$

where $\gamma^2(E)$ is given in Fig. 3. An alternative way of obtaining the relevant nucleon-nucleon amplitude is to use the phase shifts of MacGregor, Arndt, and Wright (MAW).[7] We in fact used the MAW phase shifts to obtain the potential given in Eq. (4.6), where only the amplitude for $q=0$ is needed. The imaginary part of this amplitude is related directly to the total nucleon-nucleon cross

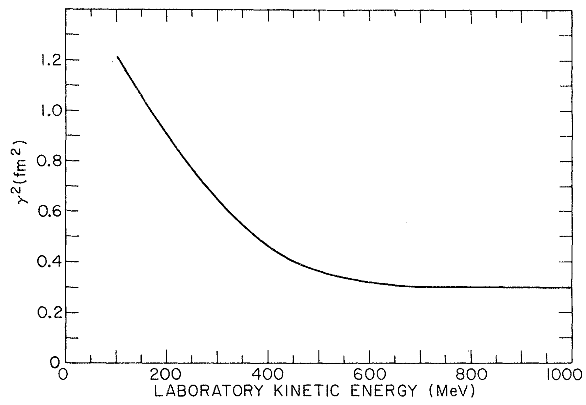

FIG. 3. The parameter $\gamma^2(E)$ occurring in Eq. (4.9). The curve was obtained from Fig. 23 of Ref. 6.

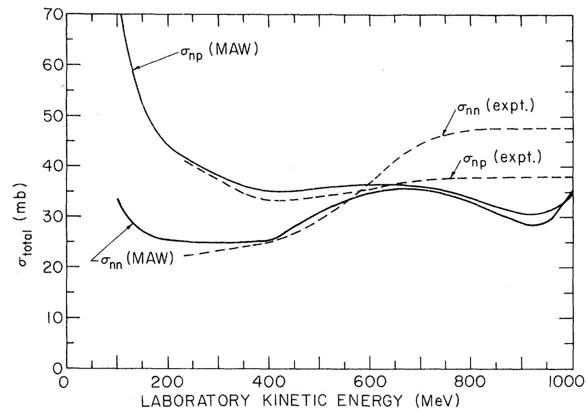

FIG. 4. Nucleon-nucleon total cross sections. The solid curves represent the *np* and *nn* cross sections predicted by the phase shifts of MAW extrapolated to 1 GeV. The dashed lines represent the *np* and *nn* experimental cross sections given in Ref. 8.

section by the optical theorem. In Fig. 4 we compare the experimental total *nucleon-nucleon* cross sections upon which Eq. (4.9) is based with those obtained from the phase shifts of MAW, extrapolated to 1000 MeV. We see that above 600 MeV there is a considerable discrepancy between the two. It is especially marked in the case of the proton-proton total cross sections. The consequences of this discrepancy on the total *nucleon-nucleus* cross sections will be seen shortly.

The coordinate space representation of the optical potential, represented in momentum space by Eqs. (4.1), (4.8), and (4.9) is given by

$$\begin{aligned} U_E(r) &= -\frac{1}{2\pi^2}\frac{\hbar^2}{m} A f_{NN}(E, q=0) \\ &\quad\times \frac{\pi\sqrt{\pi}}{2\alpha\sqrt{2\alpha}}\left(1-\frac{3}{32\nu\alpha}+\frac{r^2}{128\nu\alpha^2}\right)e^{-r^2/8\alpha} \\ &= (V+iW)\,\frac{1.9687}{\alpha\sqrt{\alpha}} \\ &\quad\times\left(1-\frac{0.290\,25}{\alpha}+\frac{0.024\,187}{\alpha^2}\,r^2\right)e^{-r^2/8\alpha}, \end{aligned} \tag{4.10}$$

where

$$\alpha=\frac{1}{8\nu}+\tfrac{1}{4}\gamma^2=0.387\ \mathrm{fm}^2+\tfrac{1}{4}\gamma^2 \tag{4.11}$$

and

$$\begin{aligned} V &= -\frac{8}{\pi^2}\frac{\hbar^2}{m}\,\mathrm{Re} f_{NN}(E, q=0), \\ W &= -\frac{8}{\pi^2}\frac{\hbar^2}{m}\,\mathrm{Im} f_{NN}(E, q=0). \end{aligned} \tag{4.12}$$

The results of the calculation of the total nucleon-^{16}O cross section are shown in Fig. 5.

The curves marked (a) and (b) in Fig. 5 are the total cross sections arising from the potential given in Eq. (4.6) and that given in Eq. (4.10) (with $\gamma^2=0$ in the latter), respectively. In both cases $f_{NN}(E, q=0)$ was obtained from the MAW phase shifts. The difference between these two is then due only to the different density distributions used. The HO distribution gives cross sections which are consistently larger than those predicted by the Fermi distribution, the difference between them becoming larger as the energy increases.

The curve labeled (c) in Fig. 5 was calculated using the potential given in Eq. (4.10) with γ^2 given by **Fig.** 3. When compared to curve (b) it gives a measure of the effect of the momentum transfer dependence of the *N-N* amplitude on the total cross section. We see that the effect of the γ^2 dependence is to *increase* the total cross section. This results from a reduction of the elastic cross section together with an increase in the total inelastic cross section which more than compensates for it. This behavior is reasonable: The increase in the inelastic cross section can be expected because the Gaussian factor in q^2, i.e., $e^{-\gamma^2q^2/2}$, leads to an increase in the mean square radius of the potential by an amount $3\gamma^2$. Since the total inelastic cross section is largely a size effect, we would expect it to increase. The other effect of the factor $e^{-\gamma^2q^2/2}$ is to reduce the strength of the potential. It is easy to see that this is sufficient to overcome the increase in the range of the potential resulting in a decrease in the quantity: (strength of potential)$\times$(square of its range). Since the elastic scattering is largely determined by this quantity the effect would be expected to reduce the elastic cross section, as we have observed is the case.

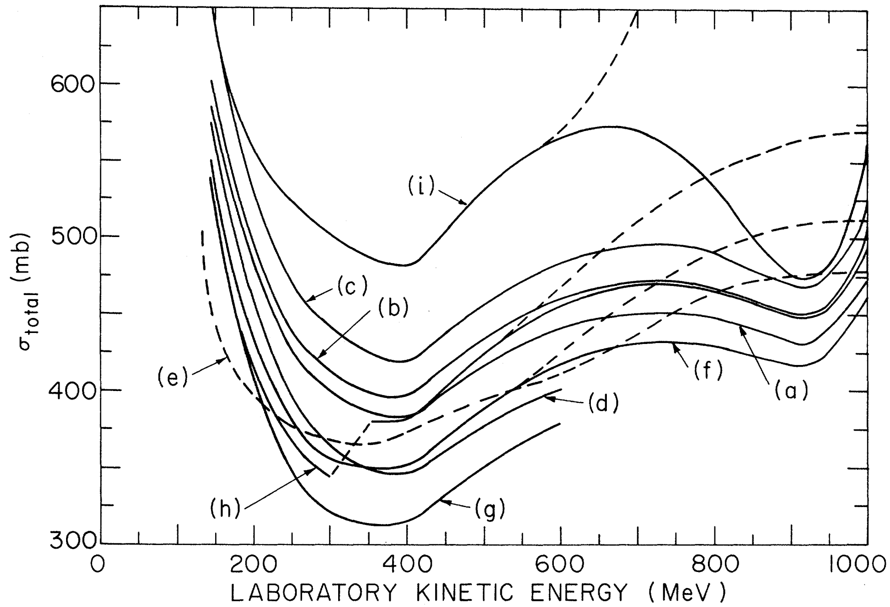

FIG. 5. The total cross sections obtained from: (a) Woods-Saxon potential (W-S), Eq. (4.6). (b) Harmonic-oscillator potential, (HO), Eq. (4.10), with $\gamma^2=0$. (c) Same as (b), but with γ^2 given by Fig. 3. (d) Same as (a), but with imaginary part of potential reduced by 15% (depletion effect). (e) Experimental nucleon-^{16}O total cross section from Ref. 6. (f) Same as (a), but modified for the Pauli effect (Ref. 9). (g) Same as (a), but modified for the Pauli effect and the depletion effect. (h) Same as (c) but modified for the Pauli effect and the depletion effect. (i) Total cross section calculated in the Born approximation. The three dashed lines occurring at high energies in the figure were calculated using the experimental N-N data of Ref. 8 above 700 MeV, instead of the phase shifts of MAW; they were then joined smoothly to the appropriate curves at lower energy.

We next modified the Fermi potential to simulate the depletion effect of the hard core. As described in Sec. III, at moderate energy this effect results in a reduction of the imaginary part of the optical potential. To simulate this effect we have reduced the imaginary part of the optical potential, Eq. (4.6), by 15%. This represents a rough estimate of the depletion effect based on Brueckner calculations. The results of the calculation using this potential are given by the curve labeled (d).

The experimental results for the nucleon-^{16}O total cross sections are indicated by curve (e). The agreement with the results described above is qualitative, at best. The dip appearing in all of the calculated cross sections between 750–950 MeV should be ignored. It arises from the corresponding dip in the N-N cross sections calculated from the MAW phase shifts and we have already noted that these results are in disagreement with experiment above 600 MeV. The experimental N-N data give a much better qualitative fit to the nucleon-^{16}O total cross sections in this region as is indicated by the dashed curves in the figure. The minima in the calculated cross sections also fall at least 50 MeV too high in energy to agree with that in the experimental data. Finally the calculated cross sections are rather close together at the lowest energies investigated and they are all much higher than the experimentally measured cross sections.

The potential was also modified for the Pauli effect. This refers to the diminution of the N-N cross section in the presence of spectator nucleons. Some of the dynamically possible final states are inaccessible, because they are occupied by other nucleons. An estimate of this effect on the total cross section for scattering of two nucleons in nuclear matter was made by Clementel and Villi,[9] who arrived at the result

$$\frac{\langle\sigma\rangle}{\sigma}=1-\frac{7}{5}\frac{\epsilon_F}{E}, \quad E>2\epsilon_F.$$

In this equation $\langle\sigma\rangle$ is the total N-N cross section in nuclear matter, while σ is the free-space N-N cross section; ϵ_F is the Fermi energy, $\epsilon_F\approx 38.3$ MeV. The imaginary part of the forward N-N scattering amplitude, and hence W, is directly proportional to σ. Thus we assume that the net result of the Pauli effect is to reduce the imaginary part of the optical potential by the factor $\langle\sigma\rangle/\sigma$. We assume the real part of the potential is unaffected.

The curve labeled (f) in Fig. 5 gives the results obtained when the potential of Eq. (4.6) is modified for the Pauli effect, while that labeled (g) gives the results obtained when the same potential is modified for both the depletion and the Pauli effects. The curve labeled (h) incorporates all the modifications we have made. It was calculated using the potential of Eq. (4.10) with γ^2 as given in Fig. 3. Up to 300 MeV, W was modified as described above for both the Pauli and depletion effects. (Above 300 MeV it was modified only for the Pauli effect).

The curve labeled (i) gives the total nucleon-^{16}O cross section in the Born approximation.

V. SUMMARY AND CONCLUSIONS

In brief then, we have shown that none of the various forms of the Watson multiple-scattering theory[3,4] of elastic scattering are capable of treating "off-shell" effects. A principal advantage of the formalism suggested by the present authors[1,2] is that "off-shell" effects are inherent to the treatment, and indeed, their consideration is important in achieving an expansion which may have reasonable convergence properties. Also, we have indicated that the short-range correlations arising from the singular part of the nuclear interaction lead to two-body T matrices which are far off shell, even in the region often considered to be properly treated by the impulse approximation ($E > 40$ MeV). Further we have noted that such "off-shell" effects must be considered even in the lowest perturbative order of the multiple-scattering theory.

We have made some crude numerical estimates of these off-shell effects in the lowest order of multiple-scattering theory. It appears that these effects are comparable in size to finite range effects and form factor ambiguities. For this reason we conclude that only a very careful approach to the problem can sort out the different uncertainties. In particular, it would appear premature to investigate two-body correlations effects in the higher-order terms of the multiple-scattering series (using on-shell T matrices) except at very high energies. Since the T matrices are only poorly known at very high energy, we can see that the study of correlations, as they affect multiple scattering, is indeed difficult. Of course, this situation may be improved via knowledge expected to be gained from the new experiments at intermediate and high energies, which are now being planned.

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*Work supported in part by the National Science Foundation.

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