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Spin-Orbit-Bound Nuclei

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It has been noted recently that a local spin-orbit nucleon-nucleon interaction, if taken seriously in the framework of nonrelativistic quantum mechanics, is generally inconsistent with the asymptotic saturation property of nuclear binding energies. This has motivated the conjecture that nuclear aggregates bound essentially by the spin-orbit interaction exist. The present paper constitutes an attempt to subject this conjecture to scrutiny, to the extent possible without abandoning the firm ground of well-understood, and mathematically sound, many-nucleon nonrelativistic theory, with "realistic" interactions mediated by potentials.

I. INTRODUCTION

Recently the old issue[1] of nuclear saturation has been revived, and rigorous constraints that nuclear forces must satisfy to be consistent with the asymptotic saturation property of nuclear binding energies have been exhibited.[2] These conditions are sufficiently flexible to be applicable, and to yield significant results, also in the case of "realistic" potentials. Remarkably, they imply that several, indeed most, of the currently employed models of the nuclear interaction are questionable as regards their compatibility, in the framework of nonrelativistic quantum theory, with the asymptotic property of saturation of nuclear binding energies. A general discussion of the actual impact of these findings on the present status of the theory of finite nuclei and of nuclear matter can be found elsewhere.[3] Here we limit ourselves to recall that one implication of the necessary conditions for saturation is that local spin-orbit forces are incompatible with saturation, unless the central part of the nuclear interaction contains strongly singular repulsive cores, or repulsive velocity-dependent terms, to prevent the occurrence of collapse.[2(b), 2(c), 2(e)]

This is due to the nonpositive-definiteness of the spin-orbit operator $\vec{L}\cdot\vec{S}$ implying that, irrespective of the sign of the potential $V_{LS}(r)$ the spin-orbit part $V_{LS}(r)\vec{L}\cdot\vec{S}$ of the Hamiltonian can always yield a negative (attractive) contribution to the potential energy; and, moreover, a contribution that can become dominant over all others, for sufficiently large values of the orbital momentum.

This observation has motivated the conjecture, that nuclear aggregates might exist, bound mainly by the spin-orbit part of the nuclear interaction.[4] The purpose and scope of this paper is to subject this conjecture to scrutiny, to the extent possible without abandoning the firm ground of well-understood, and mathematically sound, many-nucleon nonrelativistic theory, with "realistic" interactions mediated by pair potentials.

After the conjecture described above was put forward, a considerable research effort was invested into the attempt to test it by Yu. A. Simonov, E. L. Surkov, and one of the authors of this paper (FC) (to the extent allowed by geography; the first two authors live in Moscow, the last in Rome). The analysis focused on nuclear matter. A number of results were obtained, whose general trend was consistent with those reported in this paper. No result was, however, considered sufficiently conclusive to warrant publication.[5] More recently, by shifting the focus of the analysis from nuclear matter to finite (or semifinite; see below) nuclear configurations, the two authors of this paper were able to obtain more conclusive (or less inconclusive) results. These are the findings that are presented and discussed in this paper.[6]

It should also be mentioned that the conjectured existence of collapsed nuclei has been recently discussed by Bodmer,[7] whose motivation also originated, at least in part, from the above mentioned results concerning saturation.[2] The analysis of Bodmer is very interesting, but it focuses on phenomenological aspects, little attention being instead devoted to the investigation of specific dynamical mechanisms that might be responsible for the actual occurrence of collapsed nuclei. A sound

justification for this point of view is provided by the remark that the internal structure in collapsed configurations is presumably characterized by dynamics about which very little is known, there being no reason to expect that it resembles that governing the internal structure of ordinary nuclei (about which we know a lot, even though we are as yet far from a complete understanding). Of course the main difference is that, while the nonrelativistic many-nucleon problem provides a sufficient basis for at least a gross understanding of the internal structure of ordinary nuclei, collapsed nuclear structures require presumably a fully relativistic treatment. And yet, attempts at a quantitative understanding, based on our present knowledge of nuclear theory, are in order, since only such analyses can hope to provide some information on the likelihood that collapsed nuclear structures exist, and on their possible properties (that might be quite different from those of ordinary nuclear structures; see below). Moreover, such investigations have the potential to yield information on the nuclear interaction itself; a model of it that was reliably shown to imply the existence of collapsed nuclear structures should be questioned unless the predicted structures are actually found in nature, or their absence is adequately explained. Thus the point of view adopted in this paper is to stick to the known framework provided by the nonrelativistic many-nucleon problem with "realistic" pair potentials, employing moreover variational techniques that, because of the extremum property of the Ritz principle for the computation of ground-state energies, can yield unambiguous answers (within the stated theoretical framework). The actual physical significance of these rigorous results requires then a discussion; at this point opinions may differ, but a solid, and easily understood, background will have been provided.

In the following section a general outline and discussion of the approach is given. In Sec. III the results are derived, reported, and discussed.

II. DESCRIPTION OF THE APPROACH EMPLOYED

Our aim is to set up a variational computation for the ground-state energy of a many-nucleon system, under the assumption that the nucleon-nucleon interaction be mediated by local "realistic" "soft-core" potentials. The requirement that the interaction be "realistic" also implies that it contain spin-orbit forces, on which our attention shall be mainly focused. The restriction to "soft-core" potentials is introduced so as to be able to use variational techniques based on uncorrelated single-particle wave functions, this being an essential condition for the feasibility of exact variational computations. Let us immediately emphasize, however, that by "soft-core" potentials we merely indicate potentials that are integrable at short distance (namely, less singular than r^{-3} as $r \to 0$), even though they may contain strongly repulsive cores; indeed, the requirement of "realism," together with the requirement that these potentials be consistent with saturation (apart from the presence of spin-orbit forces; see below), implies that some mechanism yielding sizable repulsion at short range and/or at high internucleon momenta be present.

The recent results on saturation mentioned above[2] imply that the presence of a spin-orbit local interaction is inconsistent with saturation (assuming no velocity-dependent force to be present; see below); a collapsed configuration, with large internucleon angular momenta, becomes energetically favored, due to the dominating contribution to the binding energy originating from the spin-orbit part of the interaction. In such a configuration, the binding energy per nucleon grows monotonically with the number of nucleons and the mean nuclear density. The main question to be asked in this connection is[2(e)]: At what nucleon number, and/or at what nuclear density, does the collapsing regime take over? And, more specifically, does this phenomenon initiate before, or after, the borderline density beyond which a nonrelativistic treatment of the nucleons becomes altogether inappropriate?

It is clearly the answer to this question that indicates whether the actual existence of nuclear configurations bound mainly by the spin-orbit part of the nucleon-nucleon interaction should be considered likely or unlikely. Thus the task is to try and compute variationally the binding energy per nucleon in a configuration appropriate to display the collapse mechanism just described. As mentioned above, the first attempts to do this for nuclear matter met with considerable difficulties; it is not easy to devise a wave function describing such a system, that is sufficiently simple for computation, and yet that does yield a nonvanishing spin-orbit contribution, indeed one that has the correct dominating behavior at high density. In fact such a configuration not only must be, in some sense, "unbalanced" in spin, but it must also be characterized by density fluctuations, with a size commensurate with the range of the spin-orbit potentials; otherwise the expectation value of the spin-orbit interaction, that is the quantity entering the Ritz principle, averages to zero (this need not be true for the exchange contribution, that is, however, damped at high density, and is therefore unable to sustain the collapse mechanism).

The results reported in this paper are instead

based on the consideration of a finite (or semi-finite; see below) nuclear aggregate. In this case a very simple approach is able to display explicitly the collapse mechanism, and can therefore provide the basis for the estimate that is our goal. To be sure, the wave function that we employ is certainly far from having the flexibility and complication that would be necessary to perform an accurate computation of the ground-state energy of the system. It is, however, sufficiently reasonable to yield more than just a proof of the occurrence of collapse, that requires only the identification of the different exponents of the nucleon number A characterizing, in a hypothetical collapsed configuration, the asymptotic behaviors of the contributions to the binding energy originating from the different terms in the many-nucleon Hamiltonian: kinetic energy, central and tensor interactions, spin-orbit interaction.[2(b), 2(e)] Indeed this wave function makes it possible to obtain an estimate of the relative magnitude of the different contributions to the binding energy in a collapsed aggregate of A nucleons, due to the various terms of the Hamiltonian that have just been enumerated.

To describe our approach, let us begin by a qualitative analysis based on a very simple model. Consider a "nucleus" composed of A nucleons and having a size d of the order of the range of nuclear forces; for definiteness, let us assume it can be described by a Slater determinant of single-particle wave functions, using for convenience cylindrical coordinates ρ, z, and φ, with the z axis parallel to the direction of spin quantization. Then, indicating by n'_ρ, n'_z, and n'_φ the corresponding quantum numbers, and assuming that n_ρ, n_z, and n_φ are the maximal values of the moduli of these quantum numbers that enter in the Slater determinant, we have

$$A = c\, n_\rho n_z n_\varphi , \tag{2.1}$$

and

$$\overline{T} = \frac{\hbar^2 A}{2md^2}(C_1 n_\rho{}^2 + C_2 n_z{}^2 + C_3 n_\varphi{}^2) . \tag{2.2}$$

Here A is the total number of nucleons, $\overline{T}$ the expectation value of the kinetic energy of the nucleus, and the C's used here and below indicate some numerical constants.

Evaluating the mean value of the potential energy associated with local potentials one also gets

$$\overline{E}_{C,T} = C'_{C,T} n_\rho{}^2 n_z{}^2 n_\varphi{}^2 = C_{C,T} A^2 , \tag{2.3}$$

and

$$\overline{E}_{LS} = C'_{LS} n_\rho{}^2 n_z{}^2 n_\varphi{}^3 = C_{LS} A^2 n_\varphi . \tag{2.4}$$

Here clearly $\overline{E}_{C,T}$ and $\overline{E}_{LS}$ are, respectively, the contributions to the expectation value of the total energy of the system of the central plus tensor, and of the spin-orbit, parts of the local interaction; and we clearly assume that the potentials are integrable, so that the constants $C_{C,T}$ and C_{LS} are finite. These formulas refer of course only to the leading terms as the quantum numbers n_ρ, n_z, n_φ, and therefore the total number of nucleons A, becomes large. The constant C_{LS} can be different from zero only if the filling of the Slater determinant is asymmetrical in the quantum number n'_φ; it is also clear that it can be made negative by correlating appropriately the filling in the quantum number n'_φ with the spin orientation of the nucleons.

The consequence of these estimates, that could be backed by quantitative computations in the variational framework [see Ref. 2(e) and below], is, of course, that the presence of local spin-orbit forces implies collapse, since clearly A and n_φ can grow so that the (negative!) term $\overline{E}_{LS}$ dominate over all the others, implying unlimited growth with A of the binding energy per nucleon. The behavior at large A of the different contributions to the binding energy per particle implied by these formulas is summarized in Table I.

Clearly the first two cases are the more likely candidates to describe the collapse caused by spin-orbit forces; and roughly speaking, the behavior in the first line is more appropriate to a situation where the main task of the spin-orbit force is to overcome the kinetic-energy contribution, whereas the behavior in the second line is more appropriate to a situation where the main task of the spin-orbit force is to overcome the contribution of the static part of the interaction (central and tensor), that is presumably repulsive in the collapsed state, due to the mainly repulsive character at short range of the central part of the nuclear force.

All the results reported in this paper have been obtained with a trial many-nucleon wave function of the type outlined above, and filling the Slater determinant consistently with the behavior of the second line of Table I. The principal motivation for this is simplicity. For the same reason, for the z variable we have used plane waves, with the assumption that in the z direction the aggregate under consideration have an infinite extension

TABLE I. Behavior at large A of the different contributions to the binding energy per particle.

n_ρ	n_z	n_φ	$\overline{T}/A$	$\overline{E}_{C,T}/A$	$\overline{E}_{LS}/A$
$A^{1/3}$	$A^{1/3}$	$A^{1/3}$	$A^{2/3}$	A	$A^{4/3}$
1	$A^{1/2}$	$A^{1/2}$	A	A	$A^{3/2}$
1	1	A	A^2	A	A^2

(with one qualification, see below); then of course the role of n_z is taken over by the Fermi momentum in the z direction, hereafter denoted by p. The single-nucleon wave functions employed are

$$\psi(r) = L^{-1/2}\exp(ikz)\,\theta(L-z)\,\theta(z)(2\pi)^{-1/2}\exp(in'_\varphi\varphi) \times (2^{1/2}/d^2)\rho\exp[-\rho^2/(2d^2)]\,\alpha_\lambda\,. \tag{2.5}$$

Here α_λ indicates the spin-isospin part of the wave function (α_1 corresponds to spin-up proton, α_2 to spin-down proton, α_3 to spin-up neutron, α_4 to spin-down neutron), $\kappa = 2\pi n'_z/L$, and the limit of large L is understood. The Slater determinant is filled choosing wave functions with $0<|\kappa|<p$ and with $n'_\varphi = 0,\ s_\lambda,\ 2s_\lambda,\ \ldots,\ (M-1)s_\lambda$, where $s_\lambda = +1$ or $s_\lambda = -1$; so that M characterizes the number of angular momentum states and $M-1$ is the maximum value of the angular momentum. Note that only nonnegative, or nonpositive, values of n'_φ are included, depending on the spin-isospin state (even M could depend on it, but for simplicity we take it independent of λ). The length d characterizes the radial size of the "nucleus," in the plane orthogonal to z. The presence of the factor ρ implies the vanishing of the density for $\rho = 0$; this factor is introduced, again for simplicity, in place of the factor $\rho^{n'_\varphi}$ that would be appropriate to oscillator wave functions (omission altogether of ρ would yield a logarithmically divergent kinetic energy contribution).

In conclusion, the many-nucleon wave function is constructed as a Slater determinant of single-nucleon wave functions, separable in cylindrical coordinates. The filling of the determinant is accomplished using in the radial coordinate, only one wave function; in the z coordinate, a continuum of plane waves; and in the φ coordinate, the first $M-1$ eigenfunctions of L_z, all corresponding to rotation in one and the same sense. This same filling is realized for each spin-isospin state (so as to yield balanced nuclear matter), but with the possibility that the sense of rotation be different for particles in different spin-isospin states, so as to introduce in the many-nucleon wave function the correlation between spin and orbital momentum that is required to yield a nonvanishing expectation value for the spin-orbit interaction. The possibility is, moreover, kept open to include, in the single-particle states that fill the Slater determinant, only some of the four spin-isospin nucleon states, as described in detail below. In any case, because the z and φ parts of the nucleon wave functions have constant moduli, the density variation is only associated with the radial wave functions, and since there is always only one of these, all the "nuclei" that we consider have one and the same simple shape, that of a long tube with cylindrical symmetry around the z direction, the density profile in the radial plane being proportional to $\rho^2\exp(-\rho^2/d^2)$. Clearly this choice, motivated by the sake of simplicity, is certainly very far from being the optimal one. It is, nonetheless, suitable, in the variational framework, to yield estimates that are reliable, indeed rigorous, in the sense of allowing conclusions valid *a fortiori* (of course, within the nonrelativistic approach we are using).

III. RESULTS

The results that we now report have been obtained by a variational computation, using as trial wave function that described in the previous section. The computational technique is standard, and needs no detailed reporting. It should, however, be emphasized that the results reported below have been obtained neglecting the exchange contribution to the expectation value of the potential energy. The motivation for doing this is the quest for maximal simplicity. A valid justification is provided by the remark that, at the high densities that we shall consider, the exchange contribution amounts only to a small correction, and therefore it certainly cannot affect the results qualitatively; indeed even its quantitative impact is presumably quite marginal.

The formulas yielded by the variational computation are

$$\overline{E}/A = \overline{T}/A + \overline{E}_{\rm pot}/A\,, \tag{3.1}$$

with

$$\frac{\overline{T}}{A} = \frac{\hbar^2}{2m}\left[\tfrac{1}{12}p^2 + (\tfrac{1}{3}M^2 - \tfrac{1}{2}M + \tfrac{7}{6})/d^2\right], \tag{3.2}$$

$$\frac{\overline{E}_{\rm pot}}{A} = \frac{1}{A}(\overline{E}_C + \overline{E}_T + \overline{E}_{\rm Coul} + \overline{E}_{LS}) \tag{3.3a}$$

$$= \tfrac{1}{2}Mpd[W + (M-1)W_{LS}]\,, \tag{3.3b}$$

$$W = W_C + W_T + W_{\rm Coul}\,. \tag{3.4}$$

Here the symbols either have already been defined or are such that their significance is self-explanatory. The expressions W_C, W_T, $W_{\rm Coul}$, and W_{LS}, related, respectively, to the contributions of central, tensor, Coulomb, and spin-orbit forces, depend on the spin-isospin composition of the "nucleus"; that of W_{LS} depends also on the signs s_λ. Four representative cases have been considered: the balanced case [(B) equal number of nucleons in the four spin-isospin states]; the case with equal numbers of protons and neutrons, all with spin, say, up (NP); the case of neutrons, one half of them with spin up and one half with spin down (NN); and the case of neutrons all with spin, say, up (N). It turns out that in all these cases there is a unique choice of the signs s_λ that maximizes the spin-orbit contribution (apart from

the over-all sign, that can always be chosen so as to yield an attractive contribution); this choice is also indicated in the following formulas:

B:

$$W_C = \tfrac{1}{2}(9W_C^{33} + 3W_C^{31} + 3W_C^{13} + W_C^{11}), \tag{3.5a}$$

$$W_T = 0, \tag{3.5b}$$

$$W_{\text{Coul}} = 2W_Q, \tag{3.5c}$$

$$W_{LS} = 3W_{LS}^{33} + W_{LS}^{31}, \tag{3.5d}$$

$$s_1 = s_3 = -s_2 = -s_4; \tag{3.5e}$$

NP:

$$W_C = 3W_C^{33} + W_C^{31}, \tag{3.6a}$$

$$W_T = 3W_T^{33} + W_T^{31}, \tag{3.6b}$$

$$W_{\text{Coul}} = W_Q, \tag{3.6c}$$

$$W_{LS} = 3W_{LS}^{33} + W_{LS}^{31}, \tag{3.6d}$$

$$s_1 = s_3; \tag{3.6e}$$

NN:

$$W_C = 3W_C^{33} + W_C^{11}, \tag{3.7a}$$

$$W_T = W_{\text{Coul}} = 0, \tag{3.7b, c}$$

$$W_{LS} = 2W_{LS}^{33}, \tag{3.7d}$$

$$s_3 = -s_4; \tag{3.7e}$$

N:

$$W_C = 2W_C^{33}, \tag{3.8a}$$

$$W_T = 2W_T^{33}, \tag{3.8b}$$

$$W_{\text{Coul}} = 0, \tag{3.8c}$$

$$W_{LS} = 2W_{LS}^{33}. \tag{3.8d}$$

Here of course the superscripts refer to the spin and isospin of the nucleon pair (triplet or singlet), and the quantities $W_{C,T,LS}^{\sigma\tau}$ depend only on d (if the exchange contribution is neglected) and consist of simple integrals over the corresponding potentials:

$$W_C^{\sigma\tau} = (16\pi)^{-1}\int_0^\infty dt\, t^2 V_C^{\sigma\tau}(td)\int_0^1 dx\,[2 + \tfrac{1}{4}t^4(1-x^2)^2]\exp[-\tfrac{1}{2}t^2(1-x^2)], \tag{3.9}$$

$$W_T^{3\tau} = (16\pi)^{-1}\int_0^\infty dt\, t^2 V_T^{3\tau}(td)\int_0^1 dx(3x^2-1)\,[2 + \tfrac{1}{4}t^4(1-x^2)^2]\exp[-\tfrac{1}{2}t^2(1-x^2)], \tag{3.10}$$

$$W_{LS}^{3\tau} = (16\pi)^{-1}\int_0^\infty dt\, t^2 V_{LS}^{3\tau}(td)\int_0^1 dx\,\tfrac{1}{4}t^4(1-x^2)^2\exp[-\tfrac{1}{2}t^2(1-x^2)]. \tag{3.11}$$

As for the Coulomb potential, its contribution diverges logarithmically with L, since (for finite L) it reads

$$W_Q = \frac{e^2}{2\pi d}\left[\ln\left(\frac{L}{d}\right) - 0.74\right]; \tag{3.12}$$

this is of course due to the long range of the Coulomb potential and to the shape of the "nucleus" being considered. The results reported below are, however, essentially insensitive to the Coulomb contribution for any large, but finite, value of the ratio L/d. The figures given below have been obtained setting $W_Q = e^2/(2\pi d)$, corresponding to $L = 5.7d$.

Except for this approximation in the evaluation of W_Q and for the systematic neglect of exchange contributions, the expression for $\overline{E}/A$ given above provides a rigorous, if presumably not very stringent, upper bound to the ground-state energy of the system under consideration; p, d, and M can be regarded as variational parameters. Of course unrestricted increase of M and p makes the (negative) attractive contribution of spin-orbit forces dominant, displaying the collapse. The quantitative answer that we now pursue is: When does the collapsing regime take over?

The structure of Eqs. (3.1)–(3.3) is so simple, to allow analytic minimization over p. The value of p corresponding to the minimum is denoted by p_0:

$$p_0 = 3\,\frac{M}{d}\,\frac{-(M-1)W_{LS} - W}{\hbar^2/(2md^2)}. \tag{3.13}$$

We consider hereafter only values of M that are sufficiently large to make p_0 positive (recall that W_{LS} is negative). We then get for the potential and kinetic energies the expressions

$$\frac{\overline{E}_{\text{pot}}}{A} = -\tfrac{3}{2}M^2\,\frac{[W + (M-1)W_{LS}]^2}{\hbar^2/(2md^2)}, \tag{3.14}$$

$$\frac{\overline{T}}{A} = \frac{\hbar^2}{(2md^2)}\left(\tfrac{1}{3}M^2 - \tfrac{1}{2}M + \tfrac{7}{6}\right) - \frac{1}{2}\frac{\overline{E}_{\text{pot}}}{A}. \tag{3.15}$$

A "realistic" local nuclear interaction containing static and spin-orbit components must now be sought, to be inserted in these equations. It should should not contain hard cores, to prevent the divergence of the integrals of Eqs. (3.9)–(3.11); and yet its central part should contain enough short-range repulsion to prevent the collapse from occurring independently of spin-orbit forces. Not

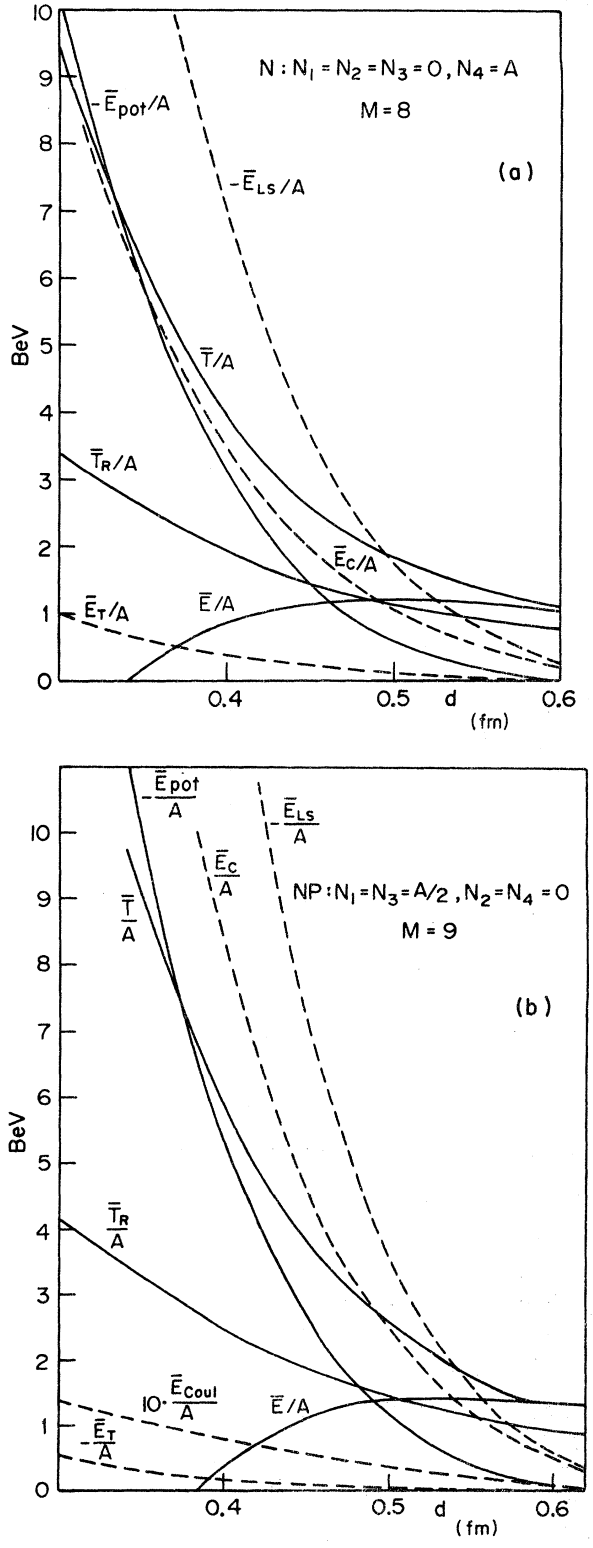

FIG. 1. Various contributions [see Eqs. (3.1)–(3.13)] to the variational computation of the binding energy per particle in a "nucleus" of the kind described in the text, for the Eikemeier-Hackenbroich potential of Ref. 8.

many such potentials are available.[2] One of these is the more recent Eikemeier-Hackenbroich model,[8] that has moreover the advantage that, because the shape of all potential is Gaussian, both of the integrations in Eqs. (3.9)–(3.11) can be performed analytically, yielding explicit, and very simple, expressions. Rather than displaying these formulas here, we report directly the final results for the different contributions to the expectation value of the many-nucleon Hamiltonian, obtained choosing for p the value p_0 of Eq. (3.13). These results are plotted, as a function of d, in Figs. 1(a) and 1(b), for two representative choices of the spin-isospin configuration (N and NP), and for the minimal values of M (8 and 9, respectively) that are sufficiently large to display the possible dominance of the spin-orbit contribution over all others. Although we have obtained results for other cases, it appears that these two examples are sufficient to characterize the sort of results one gets, by this approach, for the Eikemeier-Hackenbroich potential. Note that in each figure, besides the contribution to the total energy per particle arising out of each term of the many-nucleon Hamiltonian, we have also graphed the function

$$\frac{\overline{T}_R}{A} = \left[m^2 + 2m\frac{\overline{T}}{A} \right]^{1/2} - m , \qquad (3.16)$$

where m is the nucleon mass (in MeV). This is clearly the relativistic kinetic energy of a nucleon, whose momentum q (in MeV) is related to the nonrelativistic kinetic energy $\overline{T}/A$ by the (nonrelativistic) formula $\overline{T}/A = q^2/(2m)$. Therefore the difference of $\overline{T}_R/A$ from $\overline{T}/A$ may serve as an indicator of the extent by which the nonrelativistic framework is inapplicable; and a comparison of $\overline{T}_R/A$, rather than $\overline{T}/A$, with the potential energy contributions, might provide some hint of how the situation would be modified in a relativistic treatment (although this would of course also affect the potential energy contribution).

Before summarizing the remarks suggested by these graphs, it should be emphasized that these results have been obtained by a variational computation with the many-body trial wave function described above, that has no provision for two-body correlations. Thus the repulsive cores of the central potentials produce large positive contributions, that constitute certainly an overestimate relative to what would obtain in a more realistic, although much less simple, computation, in which two-body correlations were taken into some account. To be sure, in such more refined computations the contribution of the spin-orbit potential might also be reduced, since this also originates largely at short range; but the reduction would presumably be much less significant than for the

repulsive part of the central potential, since the contribution of the spin-orbit force is already damped at short range even in the computation without correlations, as it is apparent comparing Eq. (3.11) with Eq. (3.9). Moreover, as indicated above, even in the absence of two-body correlations there certainly are several improvements that might be introduced in the many-nucleon wave function, and these would certainly imply the possibility to obtain somewhat more binding for the same kinetic energy. It is not expected, however, that such improvements modify the picture drastically.

It should also be emphasized that, in the cases NP and N, the trial wave function that we consider has all the particles rotating in the same sense [for the case N, this is obviously implied by the way we construct the many-nucleon wave function, since in this case there is only one type of nucleons; for the case NP, it follows from Eq. (3.6e)]. This fact implies that the relative orbital momenta of no nucleon pair can exceed the maximal value $M-1$ of the orbital momentum of each nucleon. One might moreover wonder whether, for such a configuration, the kinetic energy associated with the rotational degrees of freedom should not be partly attributed to an over-all rotation of the system. We have not tried to subtract such a contribution; this may result in an additional overestimate of the (internal) ground-state energy of the system.

The data of Figs. 1(a) and 1(b) are clearly only indicative, for the reason indicated above. They point, however, rather clearly towards two considerations: (i) The values of the maximal angular momenta that must be involved in order for the collapsing regime to take over must be very large, although not exceedingly so; (ii) the collapsing regime takes over only at densities that are very much higher than those of ordinary nuclei, and in particular such that the mean kinetic energy per nucleon is way into the relativistic region.

The second remark indicates that the results obtained must be viewed with considerable skepticism; the more so, since also the potential energy contribution arises largely from parts of the interaction at such short ranges to put into question the very applicability of nonrelativistic concepts such as that of two-body local potentials. Nevertheless, were one to take the Eikemeier-Hackenbroich potential very seriously as a reliable model of the nucleon-nucleon interaction, these results would perhaps justify a more detailed analysis, especially since the requirement we are putting on the spin-orbit part of the interaction – to overcompensate for both the kinetic and the potential energy in a variational computation with no provision to suppress the large (and rather unphysical) contribution coming from the repulsive cores – is really an overly strong condition (also in view of the "upper bound" property of the computation, and of the very simple choice made for the many-nucleon trial wave function). Anyway, this would be largely a matter of opinion, the data being there for everybody to exercise their own judgment.

But we have considered a matter of higher priority to try the same approach with other potentials, and in particular with one-boson-exchange potentials (OBEP's). In fact, a basic assumption underlying all these considerations is, that the extrapolation of the validity of a given model of the nuclear interaction to a different regime, involving large angular momenta and very large linear momenta, can provide the basis for a semiquantitative assessment of the likelihood that such a different regime be actually realizable. And there might be some justification in considering such an extrapolation less arbitrary with a model of the nuclear interaction, such as the OBEP, that has some theoretical justification, rather than with purely phenomenological models. Moreover a closer look at the way the OBEP is derived from field theory might provide some hint as to how it should be modified in order to represent less inadequately situations involving large momenta, that are already in the relativistic domain.

The OBEP's that we have considered are some of those originated from Green's school.[9] An important feature of these OBEP's is the presence, in the central part of the interaction, of velocity-dependent contributions, that can be written in the form $\frac{1}{2}[p^2V(r)+V(r)p^2]$, where p indicates the relative momentum of the nucleon pair. These contributions originate from the exchange of vector and scalar bosons, namely, the same bosons that contribute to the spin-orbit part of the interaction; moreover, these interactions should be viewed much on the same footing as the spin-orbit forces since both originate as relativistic, or recoil, corrections, to the static part of the interaction (indeed both disappear in the limit of infinite nucleon mass). A crucial question, to which we shall return below, is whether contributions of this type, that clearly grow with the momenta, should be taken seriously even at large, relativistic, momenta. The answer is clearly negative. But for the time being we shall do just this.

Computations have been performed using exactly the same approach described above. The additional presence of velocity-dependent forces implies, however, a modification of Eqs. (3.3),

that now read

$$\frac{\bar{E}_{\rm pot}}{A}=\frac{1}{A}(\bar{E}_C+\bar{E}_T+\bar{E}_{\rm Coul}+\bar{E}_{LS}+\bar{E}_V) \qquad (3.17a)$$

$$=\tfrac{1}{2}Mpd[W+(M-1)W_{LS}+(M-1)(2M-1)W_{M1}+(M-1)^2W_{M2}+p^2d^2W_P], \qquad (3.17b)$$

where again all the quantities W depend only on d (and on the spin-isospin configuration of the "nucleus" under consideration).

The terms W_{M1}, W_{M2}, and W_P originate, of course, from the velocity-dependent part of the interaction and are positive (otherwise not only would collapse occur, but the Hamiltonian would have no ground state, even for finite A).[10] Their presence guarantees that no collapse can occur,[2(e)] since for large M and p these positive contributions to the total energy become the leading ones. The question we must ask is whether, for some range of values of M and p, the negative contribution of the spin-orbit potential can play a dominant role, in spite of the presence of these additional positive contributions. The answer is rather unambiguously a negative one.

The OBEP's of Ref. 9 always yield positive results, in a computation of the type we are considering. We are not interested, however, in these absolute figures, but rather in a comparison of the attractive contribution of spin-orbit forces relative to the repulsive contribution of the kinetic energy and of velocity-dependent forces; or rather, relative to the extra repulsive contribution from these sources, associated with the asymmetric type of filling in n'_φ that is required in order to have a nonvanishing spin-orbit contribution.

A few representative results corresponding to some typical choices of the spin-isospin filling and of the values of M and d are reported in Table II. In each case we report first the results yielded by the present approach (again analytic minimization in p is possible and p_0 is the value of p corresponding to the minimum) and in the second line the results yielded, for the same value of p_0, by a symmetrical filling in angular momentum, i.e., the results that obtain if the Slater determinant is constructed using the M single-particle wave functions with $n'_\varphi=-(M-1)/2$, $-(M-1)/2+1$, $\cdots(M-1)/2$ (M odd), instead of those corresponding to $n'_\varphi=0, s_\lambda, 2s_\lambda, \cdots (M-1)s_\lambda$,

TABLE II. Various contributions to the variational computation of the binding energy per particle in a "nucleus" of the kind considered in this paper. For an explanation of the significance of all the figures reported, see the text.

Potential	Spin-isospin configuration	M	d (fm)	P_0 (fm^{-1})	$\bar{E}_c/A$ (MeV)	$\bar{E}_T/A$ (MeV)	$\bar{E}_{LS}/A$ (MeV)	$\bar{E}_v/A$ (MeV)	$\bar{E}_{\rm Coul}/A$ (MeV)	$\bar{E}_{\rm pot}/A$ (MeV)	$\bar{T}/A$ (MeV)	$\bar{E}/A$ (MeV)
UG1	B	3	0.9	1.18	−13.5	0	−0.7	7.3	1.6	−5.3	70.7	65.3
				1.18	−13.5	0	0	6.2	1.6	−5.7	45.1	39.4
				1.25	−14.3	0	0	6.6	1.7	−6.0	45.4	39.4
UG1	NP	3	0.9	0.81	−5.2	0.3	−0.5	2.4	0.6	−2.4	69.4	67.0
				0.81	−5.2	0.3	0	2.0	0.6	−2.2	43.8	41.6
				0.78	−4.9	0.3	0	2.0	0.5	−2.1	43.7	41.5
UG1	NN	3	0.9	1.79	−17.9	0	−0.8	6.7	0	−12.1	73.8	61.7
				1.79	−17.9	0	0	5.8	0	−12.2	48.2	36.0
				1.80	−18.1	0	0	5.8	0	−12.2	48.2	36.0
UG1	N	3	0.9	1.66	−11.3	−0.9	−0.8	3.1	0	−9.9	73.0	63.1
				1.66	−11.3	−0.9	0	2.7	0	−9.6	47.4	37.8
				1.61	−11.0	−0.9	0	2.5	0	−9.3	47.1	37.8
UG2	NP	5	0.9	1.88	−35.2	1.1	−3.9	21.7	2.2	−14.1	185.3	171.2
				1.88	−35.2	1.1	0	18.6	2.2	−16.2	82.9	66.7
				2.11	−39.3	1.3	0	17.7	2.4	−17.9	84.5	66.5
UG2	NP	7	0.9	0.14	−3.6	0.1	−0.6	3.8	0.2	−0.1	358.3	358.2
				0.14	−3.6	0.1	0	2.4	0.2	−0.9	128.0	127.1
				1.46	−38.1	1.2	0	26.0	2.3	−8.6	131.7	123.1
UG2	NP	3	0.9	1.90	−21.3	0.7	−1.2	6.9	1.3	−13.6	74.5	60.9
				1.90	−21.3	0.7	0	6.0	1.3	−13.3	48.9	35.6
				1.87	−20.9	0.7	0	5.7	1.3	−13.1	48.7	35.6
UG2	NP	3	0.6	2.45	−59.6	1.5	−8.8	39.5	1.7	−25.6	164.0	138.4
				2.45	−59.6	1.5	0	31.7	1.7	−24.6	106.3	81.7
				2.39	−58.0	1.5	0	30.7	1.6	−24.1	105.8	81.7

with $s_\lambda = +1$ or -1. This yields a nucleus having exactly the same shape, and therefore with exactly the same contribution from central and tensor forces, whereas the contribution of spin-orbit forces is now missing, and the contributions of the kinetic energy and of velocity-dependent forces are somewhat reduced {specifically, the kinetic energy is reduced by the amount $\frac{1}{4}[\hbar^2/(2md^2)] \times (M-1)^2$ relative to that of Eq. (3.2), and the potential energy has the same expression (3.17b) except for the vanishing of W_{M2} and the replacement of the coefficient $(2M-1)$ of W_{M1} by $\frac{1}{2}(M-1)$}.

In the third line the results obtained for symmetrical filling, but now with p_0 chosen so as to minimize the total energy in this case, are reported. They do not differ significantly from the results of the second line. The results reported in Table II are only a small subset of those that were obtained, but they constitute a representative sample, the main features being preserved also in the results not reported here.

The most important of these features is the smallness of the spin-orbit contribution, as compared to the increase in the kinetic energy. At larger values of M this imbalance would disappear, but then the spin-orbit contribution would be overcome by the increase of the contribution $\overline{E}_v/A$ of the velocity-dependent part of the interaction (that, at the values of M considered here, already balances the spin-orbit contribution).

As in the case discussed above, these results have been obtained neglecting exchange terms; this is of course less justified for values of p_0, and of the density (or equivalently, the kinetic energy per particle), that are now not much larger than the standard value appropriate to ordinary nuclear matter ($k_F \sim 1.4$ fm^{-1}, $\overline{T}/A \sim 24$ MeV). The very rough nature of the trial wave function employed should also be recalled. Nevertheless, if one believes that, for the purposes of our present discussion, these OBEP's provide a more significant model of the nuclear interaction than a purely phenomenological potential like the Eikemeier-Hackenbroich one, then the results reported here should be taken as an indication pointing towards the unlikelihood of the existence of spin-orbit-bound nuclei. The situation would of course change if, at large interparticle momenta, the velocity-dependent potentials were damped (a likely possibility), but the spin-orbit potential was not damped (an unlikely possibility); and even then quite large values of M (implying ultrarelativistic values of the kinetic energy) would be required in order that the spin-orbit contribution dominate the increase in kinetic energy due to the unsymmetrical filling of the Slater determinant.

A more detailed analysis of this question would require an analysis of the derivation of the OBEP's from field theory, that lies beyond our present scope. Needless to say, such an investigation could only be indicative, since at best it could only point towards the need to formulate a fully relativistic theory of the nuclear interaction, and of the many-nucleon problem. The results reported in this section, preliminary and incomplete as they are, do in fact lead to the conclusion that a detailed understanding of the mechanism of spin-orbit nuclear binding (including a final assessment of its realizability) can be achieved only in the framework of a fully relativistic theory. They do, however, also motivate enough skepticism regarding the likelihood that the mechanism of spin-orbit binding be phenomenologically important, to discourage the immediate undertaking of an aggressive research effort in this direction.

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[5]In the course of that work advantage was also taken of a mathematical trick due to the other author of this paper (FP). Three additional researchers in Rome (E. Olivieri, O. Ragnisco, and M. Scalia) were also involved in this collaboration, although their contribution to this particular problem was quite marginal.

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Electroexcitation of ^{9}Be Levels in the 14–18-MeV Region*

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The form factors for electroexcitation of the lowest $T=\frac{3}{2}$ levels in ^{9}Be (14.39 MeV, $\frac{3}{2}^-$; 16.97 MeV, $\frac{1}{2}^-$) have been measured for momentum transfers between 0.5 and 1.1 fm^{-1}. Results are also presented for levels of unknown TJ^π at 16.63 and 17.48 MeV excitation. Radiative widths have been extracted. The form factor of the 16.63-MeV state is compared with simple-spherical and deformed shell-model form factors. The ground-state rms radius and quadrupole moment have been deduced from the elastic scattering data.

I. INTRODUCTION

An interesting feature of ^{9}Be and a few other light nuclei is the existence of several narrow states ($\Gamma \lesssim 100$ keV) near 16 MeV excitation. For example, in ^{9}Be there are $T=\frac{3}{2}$ states at 14.39 ($\frac{3}{2}^-$) and 16.97 MeV ($\frac{1}{2}^-$), and states of unknown TJ^π at 16.67, 17.28, and 17.48 MeV. These levels lie well within the neutron continuum, but are either bound or slightly unbound to proton emission. The $T=\frac{3}{2}$ levels have very narrow widths, less than 1 keV, which one expects since the isospin-conserving particle-decay channels are energetically unfavored. The spins, isospins, and parities of the other three levels are not known, so it is not clear what mechanism is suppressing the neutron partial widths for decay to the ground and excited states of ^{8}Be.

Few calculations have been made on the high-energy states of ^{9}Be. The most extensive work of which we are aware is the intermediate-coupling calculation of Barker[1] for the states based on the $1s^4 1p^5$ configuration. The interaction parameters were chosen to fit the excitation energies of a number of levels in ^{9}Be, including the $TJ^\pi = \frac{1}{2}\frac{5}{2}^-$ (2.43-MeV), $\frac{1}{2}\frac{7}{2}^-$ (6.66-MeV), $\frac{3}{2}\frac{3}{2}^-$ (14.39-MeV),