This work is part of the research program of the Institute for Nuclear Physics Research (I.K.O.), made possible by financial support from the Foundation for Fundamental Research on Matter (F.O.M) and the Netherlands Organization for the Advancement of Pure Research (Z.W.O.).

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‡Research sponsored by Stichting Fundamenteel Onderzoek der Materie, Netherlands.

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¶Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

[1]T. Yamazaki, H. Ohnuma, Y. Hashimoto, M. Fujioka, E. Takekoshi, A. Hashizume, H. Ikegami, and M. Sakai, J. Phys. Soc. Japan 17, 1223 (1962).

[2]N. G. Zaitseva, B. Kratsik, M. G. Loshchilov, G. Muziol, Chan Tkhan' Min', and H. Strusny, Izv. Akad. Nauk SSSR Ser. Fiz. 33, 1283 (1969) [transl.: Bull. Acad. Sci. USSR, Phys. Ser. 33, 1186 (1969)].

[3]J. Konijn, A. C. Rester, B. van Nooijen, J. J. Pinajian, and N. R. Johnson, Instituut vor Kernphysisch Onderzoek Amsterdam, Progress Report 1970-1971 (unpublished).

[4]T. A. Doron and M. Blann, Nucl. Phys. A161, 12 (1971).

[5]D. G. Alkhazov, D. S. Andreev, V. D. Vasil'ev, Yu. P. Gangrskii, I. Kh. Lemberg, and Yu. I. Udralov, Izv. Akad. Nauk SSSR Ser. Fiz. 27, 1285 (1963) [transl.: Bull. Acad. Sci. USSR, Phys. Ser. 27, 1263 (1963)].

[6]W. G. Davies, J. E. Kitching, W. McLatchie, J. Morton, D. G. Montague, K. Ramavataram, N. S. Chant, T. A. Hodges, F. Uridge, and D. A. Morris, Rutherford High Energy Laboratory Proton Linear Accelerator Progress Report No. RHEL/R170, 1968 (unpublished), p. 49.

[7]T. Inamura, S. Nagamiya, A. Hashizume, Y. Tendow, and T. Katron, Institute of Physical and Chemical Research (Japan) Cyclotron Progress Report No. 4, 1970 (unpublished), p. 67.

[8]J. M. Sauer, Rev. Sci. Instr. 36, 1374 (1965).

[9]P. D. Kunz, University of Colorado, unpublished.

[10]J. S. Forster, L. L. Green, N. W. Henderson, J. L. Hutton, G. D. Jones, J. F. Sharpey-Schafer, A. G. Craig, and G. A. Stephens, Nucl. Phys. A101, 113 (1967).

[11]P. H. Stelson and L. Grodzins, Nucl. Data A1, 21 (1965).

[12]A. V. Ramayya, J. L. C. Ford, Jr., R. L. Robinson, and J. H. Hamilton, Bull. Am. Phys. Soc. 17, 445 (1972).

[13]G. A. P. Engelbertink and G. van Middelkoop, Nucl. Phys. A138, 588 (1969).

[14]R. G. R. Engmann, Ph.D. thesis, University of Utrecht, 1971 (unpublished).

[15]J. B. Ball, J. S. Larsen, J. J. Pinajian, and A. C. Rester, to be published.

PHYSICAL REVIEW C VOLUME 7, NUMBER 1 JANUARY 1973

Nuclear Structure Calculations in ^{21}Ne and ^{23}Na

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(Received 17 August 1972)

The energy levels, static moments, and electromagnetic transition probabilities in ^{21}Ne and ^{23}Na are calculated from the projected multishell Hartree-Fock wave functions obtained with a realistic *NN* interaction. The modified Elliott interaction is employed as the realistic *NN* interaction. The results of these calculations are in good agreement with the experimental data. In particular, the magnetic moment of the ground state and the excitation energy of the first $\frac{1}{2}^+$ state in both the nuclei are predicted accurately. The computed ft values of the β transitions to the states in the two nuclei are in semiquantitative agreement with the experimental data.

1. INTRODUCTION

The extensive experimental investigations[1] carried out recently on the nuclear reactions ^{18}O-$(\alpha, n\gamma)$, ^{23}Na$(\alpha, \alpha'\gamma)$, ^{23}Na(p, p'), and ^{24}Mg$(t, \alpha\gamma)$ have established the sequence of positive-parity nuclear states in ^{21}Ne and ^{23}Na. The energy-level spectra of these two odd-A nuclei are expected to be basically characteristic of the eleventh odd nucleon in the *sd* shell and this expectation is borne out by the striking similarity in the two experimental spectra. The static electromagnetic moments for the $J=\frac{3}{2}^+$ ground state in both the nuclei are accurately measured. The $B(E2)$ and $B(M1)$ values for the electromagnetic transitions and ft values for the β transitions to the states in ^{21}Ne and ^{23}Na are also available. The shell-model (SM) calculations carried out recently by considering

only the "valence nucleons" in the *sd* shell explain[2] the level spectrum of ^{23}Na. In these SM calculations,[2] however, the static moments and transition probabilities of β and γ transitions in ^{23}Na are not evaluated. The SM calculations are also performed in the case of ^{21}Ne but they do not satisfactorily explain[3] the level spectrum of ^{21}Ne in the sense that the excitation energies of the second $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states and particularly of the first $\frac{1}{2}^+$ state are not correctly reproduced. It is stressed in Ref. 3 that no SM calculation has yet given a good account of the experimentally observed $\frac{1}{2}^+$ state at an excitation energy of 2.8 MeV in ^{21}Ne. The magnetic moments of the $\frac{3}{2}^+$ ground states of ^{21}Ne and ^{21}Na predicted by the SM calculations[3] are quite large in magnitude as compared to the experimental values. The large deviation of $0.44\mu_N$ between the computed and experimental values of magnetic moments in $A = 21$ nuclei is attributed[3] to an error in the estimation of the isovector part of the magnetic-dipole operator. In view of this situation in SM calculations[3] for ^{21}Ne and also the accumulated data in both ^{21}Ne and ^{23}Na, it was felt worthwhile to carry out detailed spectroscopic calculations in these two nuclei.

The calculations reported in this paper are performed in the framework of Hartree-Fock (HF) projection formalism.[4] In addition to the good accuracy[5] of the HF projection method, its advantage lies in the fact that all the nucleons in a nucleus can be explicitly treated in a sufficiently large configuration space. In the present calculations we have employed a modified Elliott interaction[6] as the realistic *NN* interaction. The *s*-state matrix elements of Elliott *et al.*[6] are modified so as to get reasonable binding energies and rms radii of the *sd*-shell nuclei. The binding energy is very sensitive to the $^3S_1$ matrix elements which are precisely the matrix elements not accurately determined owing to the approximations involved in the derivation[6] and rather large uncertainties in the experimental $T = 0$ phase shifts. As pointed out by Elliott *et al.*,[6] there is enough uncertainty in the experimental phase-shift data themselves to produce an uncertainty of about 15% in the matrix elements of the $^3S_1$ state. The effect of increasing the overall $^3S_1$ matrix elements of the Elliott interaction was studied in *p*-shell nuclei.[7] It was found that the low-energy spectra of *p*-shell nuclei could be very well explained by increasing $^3S_1$ matrix elements but the rms radii then became too small. The off-diagonal matrix elements, particularly in the *s* state, are quite important in determining the sizes of the nuclei. It has been pointed out[8] that the off-diagonal $^1S_0$ and $^3S_1$ matrix elements of Elliott *et al.*[6] are overestimated by as much as 50% in many cases. Taking a clue from these observations,[7,8] we have modified the Elliott interaction by increasing the diagonal $^3S_1$ matrix elements by 25% and reducing the off-diagonal *s*-state matrix elements by 20 to 40%. The same modified interaction has been used in our earlier work on *sd*-shell nuclei.[9] By employing this modified Elliott interaction, the HF calculations for ^{21}Ne and ^{23}Na are carried out in a configuration space of the first five major oscillator shells. The HF calculations, restricting to axially symmetric deformations, show that the lowest intrinsic state is a prolate state with band quantum number $K = \frac{3}{2}$ in both the nuclei. The excited $K' = \frac{1}{2}$ HF state in both the nuclei also lies energetically quite close, thus necessitating a band-mixing calculation. In the HF calculations, the Coulomb and center-of-mass corrections are neglected. The effect of these corrections on the nuclear properties such as rms radii and mass quadrupole moments of *sd*-shell nuclei has been studied[10] earlier and found to be very small, of the order of 2%. In view of the small effect of these corrections on nuclear properties, we have neglected them here as in earlier calculations.[7,9] The Hartree-Fock-Bogoliubov calculations for odd-*A* nuclei are quite involved and consequently we have performed only the HF calculations, neglecting the effect of pairing corrections. The band-mixing formulation to compute the energy levels, static moments, and transition probabilities for β and γ transitions is outlined in Sec. 2. The results of the computation are given in Sec. 3. The conclusions are presented in the last, Sec. 4.

2. METHOD OF CALCULATION

The HF method of solving for self-consistent single-particle wave functions and energies is well known and can be found, for example, in Ref. 10. The projection method for obtaining states of good angular momentum J from instrinsic HF state is described in Ref. 4. In the case of band-mixing calculations, the normalized band-mixed wave function Ψ^J_M with energy ϵ_J can be written in the form

$$\Psi^J_M = N_J(\psi^J_{MK} + \omega_J \psi^J_{MK'}) , \tag{1}$$

where ψ^J_{MK} ($\psi^J_{MK'}$) is the normalized wave function with energy E^J_{KK} ($E^J_{K'K'}$) projected from the HF state Φ_K ($\Phi_{K'}$). The band-mixing coefficient ω_J is given by

$$\omega_J = \left(\frac{E^J_{KK} - \epsilon_J}{E^J_{K'K'} - \epsilon_J}\right)^{1/2} . \tag{2}$$

The normalization factor N_J in Eq. (1) is given by the relation

$$N_J = (1 + \omega_J{}^2 + 2\omega_J q^J_{KK'})^{-1/2} , \tag{3}$$

where $q^J_{KK'}$ is the overlap of the wave functions ψ^J_{MK} and $\psi^J_{MK'}$:

$$q^J_{KK'} = p^J_{KK'}(p^J_{KK}\,p^J_{K'K'})^{-1/2}$$

with

$$p^J_{KK'} = \langle \Phi_K | P^J_{KK'} | \Phi_{K'} \rangle \ .$$

Here $P^J_{KK'}$ is the angular momentum projection operator.[4] The energy ϵ_J can be expressed as

$$\epsilon_J = X_J \pm (X_J{}^2 + Y_J)^{1/2} \tag{4}$$

with

$$X_J = \frac{\frac{1}{2}(E^J_{KK} + E^J_{K'K'}) - q^J_{KK'}E^J_{KK'}}{1-(q^J_{KK'})^2} \ ,$$

$$Y_J = \frac{(E^J_{KK'})^2 - E^J_{KK}E^J_{K'K'}}{1-(q^J_{KK'})^2} \ ,$$

where the band-mixing matrix element $E^J_{KK'}$ of the Hamiltonian H between the states ψ^J_{MK} and $\psi^J_{MK'}$ is given by

$$E^J_{KK'} = h^J_{KK'}(p^J_{KK}\,p^J_{K'K'})^{-1/2}$$

with

$$h^J_{KK'} = \langle \Phi_K | H P^J_{KK'} | \Phi_{K'} \rangle \ .$$

The computation of the static electromagnetic moments and transition probabilities requires the evaluation of the matrix element of a tensor operator T^λ (of rank λ) between the initial and final states Ψ^J_M and $\Psi^{J'}_{M'}$, respectively. It is straightforward to obtain the following expressions:

$$\langle \Psi^J_J | T^\lambda | \Psi^J_J \rangle = N_J{}^2(JJ, \lambda O | JJ)\,(T^{JJ}_{KK} + \omega_J{}^2 T^{JJ}_{K'K'} + \omega_J T^{JJ}_{KK'} + \omega_J T^{JJ}_{K'K}), \tag{5}$$

$$B(\lambda; J \to J') = N_J N_{J'} \frac{2J'+1}{2J+1}\,(T^{JJ'}_{KK} + \omega_J\omega_{J'}T^{JJ'}_{K'K'} + \omega_J T^{JJ'}_{K'K} + \omega_{J'}T^{JJ'}_{KK'})^2 \ . \tag{6}$$

Here $(J_1M_1, J_2M_2 | J_3M_3)$ is the vector-coupling coefficient and $T^{JJ'}_{KK'}$, the reduced matrix element of the tensor operator

$$T^{JJ'}_{KK'} = \sum_\nu (JK' - \nu, \lambda\nu | J'K') \langle \Phi_K | T^\lambda_\nu P^J_{K'-\nu,K} | \Phi_K \rangle, \tag{7}$$

can be computed by evaluating

$$\langle \Phi_{K'} | T^\lambda_\nu e^{-i\theta J_y} | \Phi_K \rangle = \sum_{i=1}^{A}\sum_{k=1}^{A} (-)^{i+k} D_{i,k}(\theta) \sum_{\alpha,\beta} \langle \alpha | T^\lambda_\nu | \beta \rangle C^i_\alpha(K') C^k_\beta(K) d^{j_\beta}_{m_\beta m_k}(\theta) \ . \tag{8}$$

Here $C^i_\alpha(K)$ is the expansion coefficient of the ith deformed single-particle wave function in the HF state Φ_K where $\alpha \equiv (\eta_\alpha, l_\alpha, j_\alpha, m_\alpha)$ symbolizes the basis state in HF calculation and $D_{i,k}(\theta)$ is the overlap function $\langle \Phi_{K'} | e^{-i\theta J_y} | \Phi_K \rangle$ evaluated after eliminating the ith and kth single-particle states from the determinantal HF states $\Phi_{K'}$ and Φ_K, respectively. In evaluating the reduced transition probability [in Eq. (6)] of the allowed β transitions from the state J of the parent nucleus to the state J' of the daughter nucleus, the relation (8) is slightly modified. For a β^- transition,

$$\langle \Phi_{K'}(Z+1, N-1) | T^\lambda_\nu e^{-i\theta J_y} | \Phi_K(Z,N) \rangle = \sum_{p=1}^{Z+1}\sum_{n=1}^{N} (-)^{p+n} D_p(\theta) D_n(\theta) \sum_{\alpha,\beta} \langle \alpha | T^\lambda_\nu | \beta \rangle C^p_\alpha(K') C^n_\beta(K) d^{j_\beta}_{m_\beta m_n}(\theta), \tag{9}$$

where $Z(N)$ is the number of protons (neutrons) in the parent nucleus, $D_p(\theta)$ is the proton overlap function evaluated after eliminating the pth proton state from the HF state $\Phi_{K'}$ of the daughter nucleus and $D_n(\theta)$ is the neutron overlap function evaluated after eliminating the nth neutron state from the HF state Φ_K of the parent nucleus. For a β^+ transition, one obtains

$$\langle \Phi_{K'}(Z-1, N+1) | T^\lambda_\nu e^{-i\theta J_y} | \Phi_K(Z,N) \rangle = \sum_{n=1}^{N+1}\sum_{p=1}^{Z} (-)^{p+n} D_n(\theta) D_p(\theta) \sum_{\alpha,\beta} \langle \alpha | T^\lambda_\nu | \beta \rangle C^n_\alpha(K') C^p_\beta(K) d^{j_\beta}_{m_\beta m_p}(\theta) \ . \tag{10}$$

3. CALCULATIONS AND RESULTS

The variational wave functions obtained from the intrinsic HF state by projection methods are employed in the calculations reported here. The modified Elliott interaction has been used as the realistic *NN* interaction and all the nucleons are explicitly taken into account in carrying out HF and HF projection calculations are performed in a configuration space of the first five major oscillator shells. In the nuclei

^{21}Ne and ^{23}Na under consideration here, the lowest HF states are found to be axially symmetric prolate states with band quantum number $K=\frac{3}{2}$. The energies in the lowest $K=\frac{3}{2}$ HF states are -166.1 and -188.7 MeV in ^{21}Ne and ^{23}Na, respectively. The corresponding mass rms radii are 2.87 and 2.88 fm, respectively. It is also found in both the nuclei that the excited $K'=\frac{1}{2}$ HF band lies energetically quite close to the $K=\frac{3}{2}$ band.

The energy levels of these nuclei are obtained from the band-mixing calculations described in Sec. 2. These calculations reveal that the band mixing is small in the case of ^{21}Ne where $\omega_J \approx 0.15$ is obtained and the mixing is comparatively large in the case of ^{23}Na where $\omega_J \approx 0.30$ is obtained for $J=\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$ states. The experimental and computed energy spectrum of ^{21}Ne is shown in Fig. 1. The agreement between the calculated and experimental spectrum is quite good. The first $\frac{1}{2}^+$ and the second $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states are predicted very correctly. It should be mentioned here that by using the best-fit phenomenological effective interaction in the *sd*-shell SM calcalation, Arima, Sakakura, and Sebe[3] obtain a good agreement with the experimental spectrum for the first $J=\frac{3}{2}^+$ to $J=\frac{13}{2}^+$ states. However, they are unable to account for the second $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states and particularly the first $\frac{1}{2}^+$ state which, in their calculation,[3] appears at a very low excitation energy. It has been stressed[3] that no SM calculation has yet reproduced the first $J=\frac{1}{2}^+$ state in ^{21}Ne. The failure of the SM calculation[3] in reproducing the states corresponding to the $K'=\frac{1}{2}$ band may be due to the shortcomings in the employed effective interaction or due to the inadequacy of the configuration space employed. It is possible that the *sd*-shell-model space, though adequate to reproduce the states from the ground band, is inadequate to account for

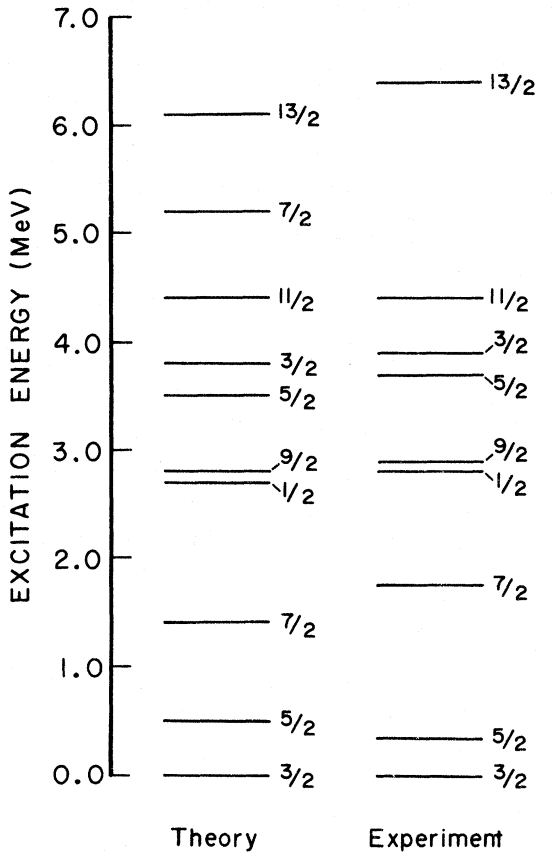

FIG. 1. The energy spectrum of ^{21}Ne.

TABLE I. The experimental (expt) and calculated (calc) values of the magnetic moment μ (in units of $\mu_0 = e\hbar/2Mc$) and the electric quadrupole moment Q (in units of e fm^2) of the ground state (spin J) of $A=21$ and $A=23$ nuclei are tabulated. The values predicted from the shell-model calculations (SM) are also tabulated. The experimental values are taken from Refs. 3 and 11.

		μ			Q		
Nucleus	J	Expt	Calc	SM	Expt	Calc	SM
^{21}Ne	$\frac{3}{2}^+$	-0.66	-0.72	-1.10	9.3 ± 1.0	9.07	10.07
^{21}Na	$\frac{3}{2}^+$	2.39	2.44	2.83	···	9.84	11.77
^{23}Na	$\frac{3}{2}^+$	2.22	2.30	···	11.0	10.1	···
^{23}Mg	$\frac{3}{2}^+$	···	-0.56	···	···	10.4	···
^{23}Ne	$\frac{5}{2}^+$	···	-1.61	···	···	15.0	···

TABLE II. The $B(E2)$ and $B(M1)$ values of the electromagnetic transitions from the initial state (spin J_i) to the final state (spin J_f) in ^{21}Ne are tabulated. The $B(E2)$ values are in units of e^2 fm^4 and $B(M1)$ values in units of $\mu_0{}^2$. The experimental values are taken from Ref. 3.

		$B(E2)$			$B(M1)$		
J_i	J_f	Expt	Calc	SM	Expt	Calc	SM
$\frac{5}{2}^+$	$\frac{3}{2}^+$	63 ± 13	68.8	69.1	0.06 ± 0.02	0.16	0.15
$\frac{7}{2}^+$	$\frac{3}{2}^+$	16 ± 6	27.9	34.8			
$\frac{7}{2}^+$	$\frac{5}{2}^+$	24 ± 10	42.5	60.4	0.13 ± 0.04	0.20	0.17
$\frac{9}{2}^+$	$\frac{5}{2}^+$	22 ± 10	38.9	50.3			
$\frac{9}{2}^+$	$\frac{7}{2}^+$	21 ± 14	25.6	33.4	0.22 ± 0.06	0.39	0.53
$\frac{11}{2}^+$	$\frac{7}{2}^+$	43 ± 19	38.2	54.7			
$\frac{11}{2}^+$	$\frac{9}{2}^+$	13 ± 10	15.2	28.3	0.14 ± 0.05	0.53	0.53
$\frac{13}{2}^+$	$\frac{9}{2}^+$	···	25.2	48.7			
$\frac{13}{2}^+$	$\frac{11}{2}^+$	>0.3	7.2	14.9	<0.03	0.21	0.70

the states from the excited band in ^{21}Ne. The experimental and computed energy spectrum of ^{23}Na is displayed in Fig. 2. The agreement between the calculated and experimental spectrum is quite good except for $J=\frac{11}{2}^+$ and $\frac{13}{2}^+$ states. The computed energies for these two states are lower than the corresponding experimental energies. It should, however, be noted that the assignment of spins $\frac{11}{2}^+$ and $\frac{13}{2}^+$ to the levels at 5.5 and 6.2 MeV is not uniquely determined. It may be mentioned here that the SM calculations[2] also predict the excitation energies of the $\frac{11}{2}^+$ and particularly of the $\frac{13}{2}^+$ state much lower than the experimental energies of 5.5 and 6.2 MeV, respectively. As in the case of ^{21}Ne, our calculations reproduce the experimentally observed first $\frac{1}{2}^+$ and second $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states in ^{23}Na quite well. The SM calculations[2] also reproduce these states quite well except that the energy separation between the second $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states is predicted to be very small.

The evaluation of the static moments and the transition probabilities of β and γ transitions provides a good criterion to test the accuracy of the model wave functions. The results of our calculations for the magnetic-dipole and electric-quadrupole moments of a few *sd*-shell nuclei are displayed in Table I. The computed values agree quite well with the experimental values wherever available. It should be pointed out here that the SM calculations[3] do not yield correct values for the magnetic moments of $A=21$ nuclei. The values predicted by SM calculations[3] for the $J=\frac{3}{2}^+$ ground-state magnetic moments of both ^{21}Ne and ^{21}Na are larger in magnitude by $0.44\mu_N$ than the corresponding experimental values. This large deviation in the computed magnetic moments is attributed[3] to an error in the estimation of the isovector part of the magnetic-dipole operator. The present calculations, however, reproduce the experimental magnetic moments of $A=21$ nuclei very well by assigning the free-nucleon magnetic moments to all the nucleons. Consequently, there is no need to introduce the suggested[3] quenching in evaluating the contribution from the isovector part of the magnetic-dipole operator. In evaluating electric-quadrupole moments and $B(E2)$ values, the SM calculations[3] employ effective charges $e_p=1.5e$ and $e_n=0.5e$. Since our model space and active nucleons are quite large as compared to those in SM calculations,[3] we expect effective charges to be quite small. The results reported in this paper are obtained with effective charges $e_p=1.1e$ and $e_n=0.1e$. The computed quadrupole moments of the ground states of ^{21}Ne and ^{23}Na are in good agreement with the experimental data.

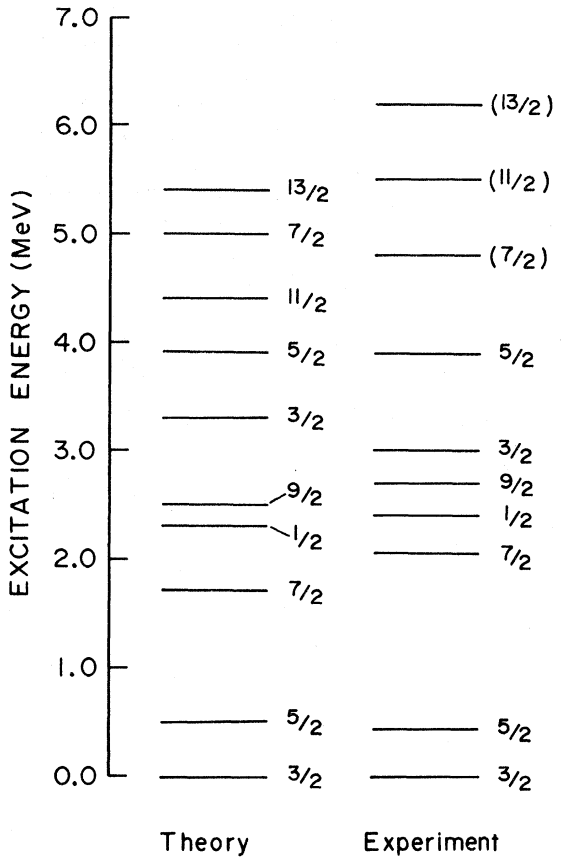

FIG. 2. The energy spectrum of ^{23}Na.

The computed $B(E2)$ and $B(M1)$ values in ^{21}Ne are displayed in Table II and those in ^{23}Na are displayed in Table III. The agreement in the calculated and experimental $B(E2)$ values in ^{21}Ne are quite good. It should be noted that the SM calculations[3] yield much larger $B(E2)$ values as compared

TABLE III. The $B(E2)$ and $B(M1)$ values of the electromagnetic transitions in ^{23}Na are tabulated. The notation is same as in Table II. The experimental values are taken from Ref. 1.

		$B(E2)$		$B(M1)$	
J_i	J_f	Expt	Calc	Expt	Calc
$\frac{5}{2}^+$	$\frac{3}{2}^+$	85 ± 9	82.0	0.43 ± 0.04	0.51
$\frac{7}{2}^+$	$\frac{3}{2}^+$	> 6	32.7		
$\frac{7}{2}^+$	$\frac{5}{2}^+$	>12	54.7	0.59 ± 0.08	0.75
$\frac{9}{2}^+$	$\frac{5}{2}^+$	...	45.0		
$\frac{9}{2}^+$	$\frac{7}{2}^+$	...	32.6	0.86 ± 0.14	0.80
$\frac{11}{2}^+$	$\frac{7}{2}^+$	...	48.6		
$\frac{11}{2}^+$	$\frac{9}{2}^+$	...	25.0	...	0.98
$\frac{13}{2}^+$	$\frac{9}{2}^+$	...	34.8		
$\frac{13}{2}^+$	$\frac{11}{2}^+$	...	11.1	...	0.70

TABLE IV. The calculated and experimental logft values for the β transitions in some sd-shell nuclei are tabulated. The experimental values are taken from Ref. 11.

Parent nucleus	Daughter nucleus	J_i	J_f	logft Expt	logft Calc
^{21}Na	^{21}Ne	$\frac{3}{2}^+$	$\frac{3}{2}^+$	3.6	3 80
		$\frac{3}{2}^+$	$\frac{5}{2}^+$	5.0	4.71
^{23}Ne	^{23}Na	$\frac{5}{2}^+$	$\frac{3}{2}^+$	5.3	4.92
		$\frac{5}{2}^+$	$\frac{5}{2}^+$	5.4	5.26
		$\frac{5}{2}^+$	$\frac{7}{2}^+$	6.3	5.96
^{23}Mg	^{23}Na	$\frac{3}{2}^+$	$\frac{3}{2}^+$	3.7	3.92
		$\frac{3}{2}^+$	$\frac{5}{2}^+$	4.4	4.15

to the experimental data. The experimental $B(M1)$ values in ^{21}Ne are rather small and cannot be well explained by our calculations. The experimental $B(M1)$ values in ^{23}Na, however, are well explained by the present calculations (Table III). Only a few $B(E2)$ values in ^{23}Na are measured and the computed values agree well with the available data as seen from Table III.

The ft values for β transitions are calculated from the relation

$$ft = C[(1-x)B_F^2 + xB_{GT}^2]^{-1}$$

with $C = 2600$ and $x = 0.5$. The reduced transition probabilities B_F and B_{GT} for the Fermi and Gamow-Teller transitions are evaluated as indicated in Sec. 2. The computed and experimental[11] logft values for β transitions to the states in ^{21}Ne and ^{23}Na are tabulated in Table IV. The difference of 0.2 to 0.3 between the computed and experimental logft values shows that the theoretical predictions are not quite good. This can be due to the fact that the HF states employed for the odd-A nuclei under consideration are not invariant under time reversal. Moreover the presence of Coulomb force would also affect the β-decay matrix elements. In the absence of Coulomb force and the correct time-reversal properties, we obtain only a semiquantitative agreement between the computed and the experimental logft values.

4. CONCLUSIONS

The energy levels, static moments, and electromagnetic transition probabilities in ^{21}Ne and ^{23}Na are calculated. The HF calculations are performed by considering all the nucleons in the nucleus in a configuration space of the first five major oscillator shells using the modified Elliott interaction. The computed energy spectrum in ^{21}Ne is in good agreement with the experimental spectrum. The energy spectrum of ^{23}Na is also predicted quite well. The computed magnetic-dipole and electric-quadrupole moments are in good agreement with the experimental data. The good agreement in the magnetic moment obtained by assigning free-nucleon magnetic moments to the nucleons in $A = 21$ mirror nuclei does not support the suggestion[3] to introduce "quenching" in evaluating the contribution from the isovector part of $M1$ operator. The $B(E2)$ values in both the nuclei are explained quite well. The computed $B(M1)$ values in ^{23}Na are in good agreement with the experimental values but those in ^{21}Ne are rather large as compared to the experimental values. A semiquantitative agreement for the ft values of the β transitions to the states in ^{21}Ne and ^{23}Na is obtained.

[1] J. G. Pronko, C. Rolfs, and H. J. Maier, Phys. Rev. 186, 1174 (1969); H. J. Maier, J. G. Pronko, and C. Rolfs, Nucl. Phys. A146, 99 (1970); A. R. Poletti, J. A. Becker, and R. E. McDonald, Phys. Rev. C 2, 964 (1970); R. A. Lindgren, R. G. Hirko, J. G. Pronko, A. J. Howard, M. W. Sachs, and D. A. Bromley, Nucl. Phys. A180, 1 (1971).

[2] J. B. McGrory and B. H. Wildenthal, Phys. Letters 34B, 373 (1971).

[3] A. Arima, M. Sakakura, and T. Sebe, Nucl. Phys. A170, 273 (1971).

[4] C. S. Warke and M. R. Gunye, Phys. Rev. 155, 1084 (1967); I. Kelson, Nucl. Phys. 89, 387 (1966).

[5] M. R. Gunye, Phys. Letters 27B, 136 (1968).

[6] J. P. Elliott, A. D. Jackson, H. A. Mavromatis, E. A. Sanderson, and B. Singh, Nucl. Phys. A121, 241 (1968).

[7] M. R. Gunye, J. Law, and R. K. Bhaduri, Nucl. Phys. A132, 225 (1969); M. R. Gunye, Phys. Letters 30B, 64 (1969); Nucl. Phys. A139, 686 (1969).

[8] M. K. Srivastava, A. M. Jopko, and D. W. L. Sprung, Can. J. Phys. 47, 2459 (1969).

[9] M. R. Gunye and C. S. Warke, Phys. Rev. C 5, 1860 (1972); M. R. Gunye and S. B. Khadkikar, Phys. Rev. Letters 24, 910 (1970).

[10] M. R. Gunye, Nucl. Phys. A118, 174 (1968).

[11] C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (Wiley, New York, 1967).