V. SUMMARY

In conclusion we have shown that the multiple-scattering series for a scattering problem cannot, in general, converge in some part of the phase space of one channel and diverge in others. Hence, recent fits to p-d inelastic scattering on the quasi-free peak by the first few terms of that series are fortuitous since the series is known to diverge or at best converge very slowly for this reaction in general at the energies studied. We are not saying that the first impulse approximation for the quasi-free scattering is not the dominant reaction mechanism, but rather that it is not a good approximation. We also show that, since the question of convergence of the multiple-scattering series is the same for all reaction channels, attempts to rearrange the series to leave a convergent remainder must be sufficiently sensitive to the physical details of the system to be in some sense a "good approximation" in all channels.

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$^{12}C(^3He,p)^{14}N$ Reaction from 7 to 17 MeV*

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Yield curves of the $^{12}C(^3He,p)^{14}N$ reaction were taken at 11 angles from 30 to 169° in the beam energy interval of 7 to 17 MeV. From these data, angular distributions and the total cross section were deduced at 0.2-MeV intervals for 7 to 11 MeV and at four higher energies. The results are compared with recent measurements of the same reaction leading to the first nine excited states of ^{14}N. Compound-nuclear effects appear to dominate. There is little correlation between excitation functions for the different states. In contradiction to previous suggestions, little evidence is seen for direct processes.

INTRODUCTION

We have measured the yield of the $^{12}C(^3He,p)^{14}N$ ground-state reaction at a number of angles over the range of ^3He energies of 7 to 17 MeV. The aim of the experiment was to look for resonances corresponding to levels in ^{15}O in the region of excitation energy of 17.6 to 25.6 MeV and to determine the relative importance of direct and compound-nucleus reaction mechanisms in this energy range. Our results for the ground state are compared with the results of Haas *et al.*[1] for the total cross sections for the reaction leading to the second through ninth excited states of ^{14}N. We discuss the conclusions drawn by these authors concerning compound-nucleus formation and the importance of direct-reaction mechanisms.

Measurements were made on this reaction some time ago by Hinds and Middleton.[2] They covered the range 5.50 to 10.23 MeV, taking excitation curves at 10 and 90°. Angular distributions were taken for the ground state and first six excited states of ^{14}N at energies of 5.98, 8.83, 9.37, and 10.14 MeV. Excitation curves were measured at 10° for the first six excited states. The angular distributions did not, however, extend to angles greater than 120°. In contrast to other $(^3He,p)$ reactions, direct-reaction patterns were not seen in the angular distributions and the authors suggested the possibility of a compound-nuclear contribution comparable in magnitude to the direct reaction.

EXPERIMENTAL PROCEDURE

Protons were observed with 2- and 3-mm-thick Kevex lithium-drifted silicon detectors. Alumi-

num absorbers were used to stop α particles and collimators limited the spread in reaction angle to 2°. Typical solid angles were in the vicinity of 3.5×10^{-3} sr.

The targets were 150- and 300-μg/cm^2 natural carbon foils. A preliminary measurement of thickness was made by weighing, but differential cross sections were obtained by comparing the yield with those of protons elastically scattered from ^{12}C. The numbers of protons scattered at 85 and 105° lab, at $E_p = 11.5$ MeV were compared (using the same target and detector) with the proton yield from a 9.4-MeV ^3He beam at lab angles of 83.5, 70, and 30°. By using the results of Moss and Haeberli[3] for carbon elastic scattering we calculated the differential cross section for ^{12}C-$(^3He, p)^{14}N$. Target nonuniformity is thought to be the major source of the estimated ±10% uncertainty. The stated uncertainty for the results of the proton elastic scattering is ±4%.

Yield curves for the $^{12}C(^3He, p)^{14}N$ ground-state reaction were measured at 11 angles from 30 to 169° in the ^3He energy range of 7 to 11 MeV. At 30, 83.5, and 169° the measurements were extended to 17 MeV. One detector was set at 83.5° for all the runs. The reproducibility of the points in the yield curve from this detector was ±6%. This is taken as the uncertainty in the relative values of the points on all the yield curves up to 11 MeV. Past 11 MeV the statistics become poorer and uncertainties average ±25%. Yield curves taken at 30, 83.5, and 169° are shown in the top three sections of Fig. 1 as representative results for forward, middle, and backward angles.

Angular distributions at various energies were calculated by using an angular distribution measured at a ^3He energy of 9.4 MeV to tie together the yield curves taken at various angles. The angular distributions thus obtained are shown in Fig. 2. Again the uncertainties are estimated as ±6%. The energy increment from 7.0 to 10.8 MeV is 200 keV. Figure 2 also shows angular distributions taken at 12.2, 13.4, 14.6, and 16.0 MeV. Values from the angular distributions taken by Hinds and Middleton[2] at 8.83, 9.37, and 10.14 MeV are shown by triangles on our own distributions taken at 8.80, 9.40, and 10.20 MeV. Values from their 10° excitation curve are shown as triangles on our distributions at other energies. Good agreement is seen but it is of special interest to note how the backward-angle points added by the present work change the apparent character of the distributions at 8.8 and 9.0 MeV from forward peaked to nearly symmetric. By the same token when the few points taken by Haas *et al.*[1] (represented by circles) are added to the distributions at 7.0, 8.6, and 9.2 MeV the apparent "pronounced backward peak" is seen to be part of a very symmetric curve. The solid lines are fits using Legendre polynomials of up to sixth order from 7.0 to 10.2 MeV. At ^3He energies above 10.2 MeV only polynomials of up to fifth order are used.

The asymmetry and degree of anisotropy change with beam energy and the most symmetric distributions are those for energies between 8.2 and 9.4 MeV. There is a suggestion of backward peaking at some energies but the degree of anisotropy fluctuates with beam energy.

The yield curves shown in Fig. 1 for observation angles of 30, 83.5, and 169° show fluctuations in the proton yield to the ground state of ^{14}N and

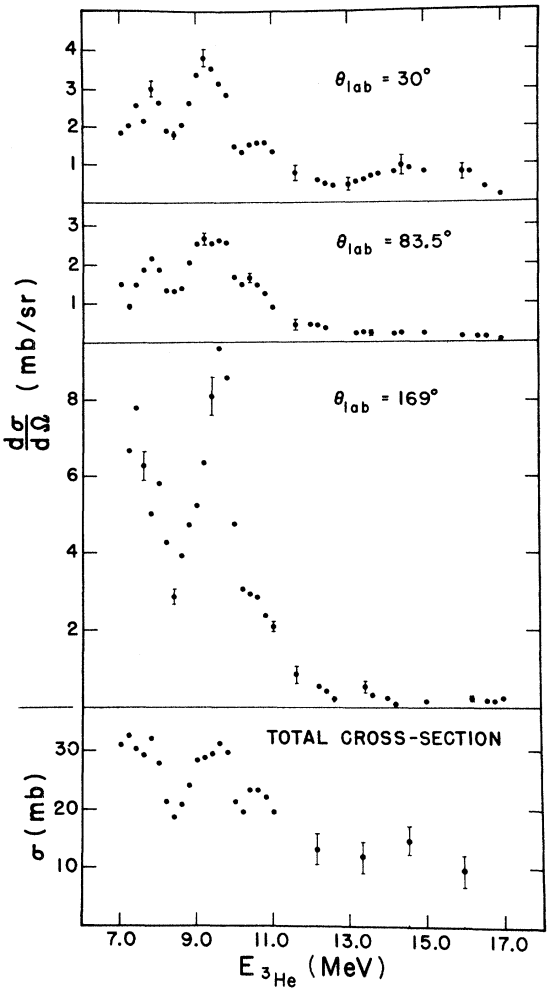

FIG. 1. The top sections show differential cross sections of the $^{12}C(^3He, p)^{14}N$ reaction measured at three angles as a function of energy. The lower section shows the total cross section derived from angular-distribution measurements. A few representative error bars show relative uncertainties between points.

this is typical of the yield curves at other angles. Between 7.0- and 8.0-MeV ^{3}He input energy, fluctuations in the yield of the ^{14}N ground state are observed at 7.2, 7.4, and 7.8 MeV with only the 7.8-MeV peak showing up in any consistent fashion. Between 9- and 10-MeV ^{3}He energy, two peaks appear in the yield. The peak at 9.2 MeV is most prominent at the forward angles whereas the peak at 9.6 appears strongest at backward angles. A possible structure can be seen at all angles at a

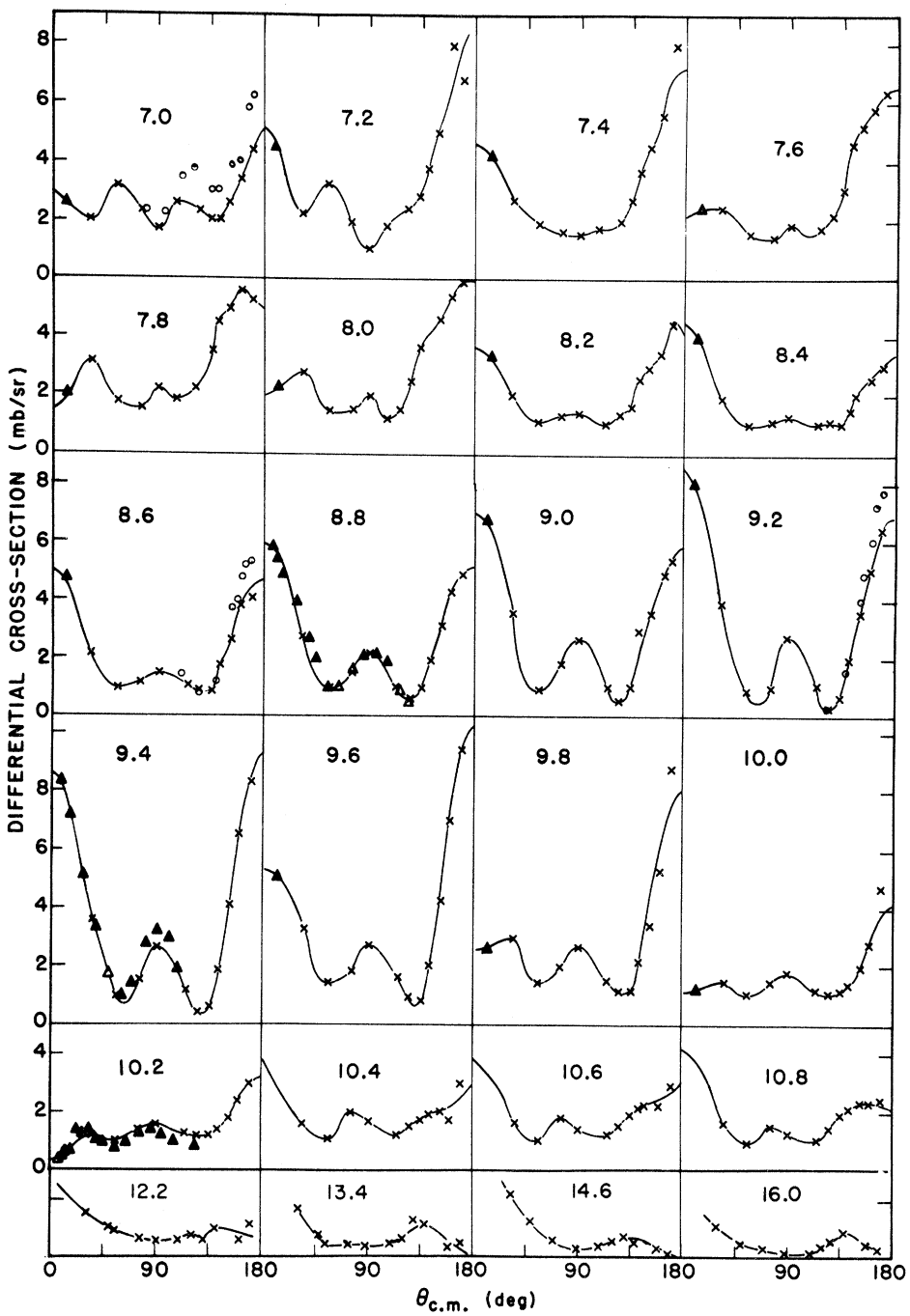

FIG. 2. Angular distributions measured at 24 energies for the ^{12}C(^{3}He, p)^{14}N reaction. The bombarding energy in MeV is shown on each plot. Crosses are the present results, circles are the results of Haas *et al.* (Ref. 1), and triangles are the results of Hinds and Middleton (Ref. 2).

^{3}He energy of 10.5 MeV. The peaks at 7.2 and 7.8 MeV were seen by Weller and VonRinsvelt[4] in the $^{12}C(^3He, \alpha)^{11}C$ yield.

The total cross section was obtained from the A_0 term of the Legendre polynomial fits. The result is shown in the lower section of Fig. 1. The estimated error in determining the absolute cross section is ±15% and there is a 10% error in normalizing to the ^{12}C proton elastic scattering data as was stated. The average total cross section for the proton yield to the ground state of ^{14}N between 7- and 11-MeV ^{3}He energies was found to be 23.4 mb.

The total cross sections for 9 of the first 10 states of ^{14}N are shown in Fig. 3. The data for the third to ninth excited state were taken from the paper of Haas *et al.*[1] These authors used different scales for the various excited states of ^{14}N and this must be kept in mind when examining the figure. There appears little in the way of consistent structure above 7.0-MeV ^{3}He energy. The resonance reported by Haas *et al.* at 10.0 MeV is not seen in the ground-state data at any of the angles.

Several comments can be made concerning the reaction mechanism for $^{12}C(^3He, p)^{14}N$. First, it is evident from the fluctuations in the ground-state proton yield as the beam energy is changed and from the changing shape of the angular distributions with changing energy that there is a strong compound-nucleus contribution. This supports the conclusion of Haas *et al.* that compound-nucleus formation is important and agrees with the conclusions of Hinds and Middleton.

In regard to the importance of direct-reaction mechanisms in the $^{12}C(^3He, p)^{14}N$ reaction two comments should be made. The first comment concerns the danger of attributing significance to certain features of the angular distributions. For example in Fig. 19 of the paper by Haas *et al.* a partial angular distribution for the $^{12}C(^3He, p)^{14}N$ reaction leading to the ground state is shown and the text states that "at higher energies in our range, a pronounced backward peak is evident." Looking at the complete angular distributions shown in Fig. 1, one finds the "pronounced backward peak" seen by Haas *et al.* at 8.10, 8.60, 8.92, and 9.20 MeV accompanied by peaking at forward angles to produce a symmetrical distribution. We see no evidence in the ground-state reaction for any significant direct-reaction mechanism. The second comment which should be made is that even a backward-peaked angular distribution does not necessarily imply a direct reaction. This is especially true if the character of the angular distribution changes with bombarding energy. It is well known that asymmetries in the angular distributions can be caused by overlapping resonances in the compound nucleus. The degree of forward and/or backward peaking depends on the

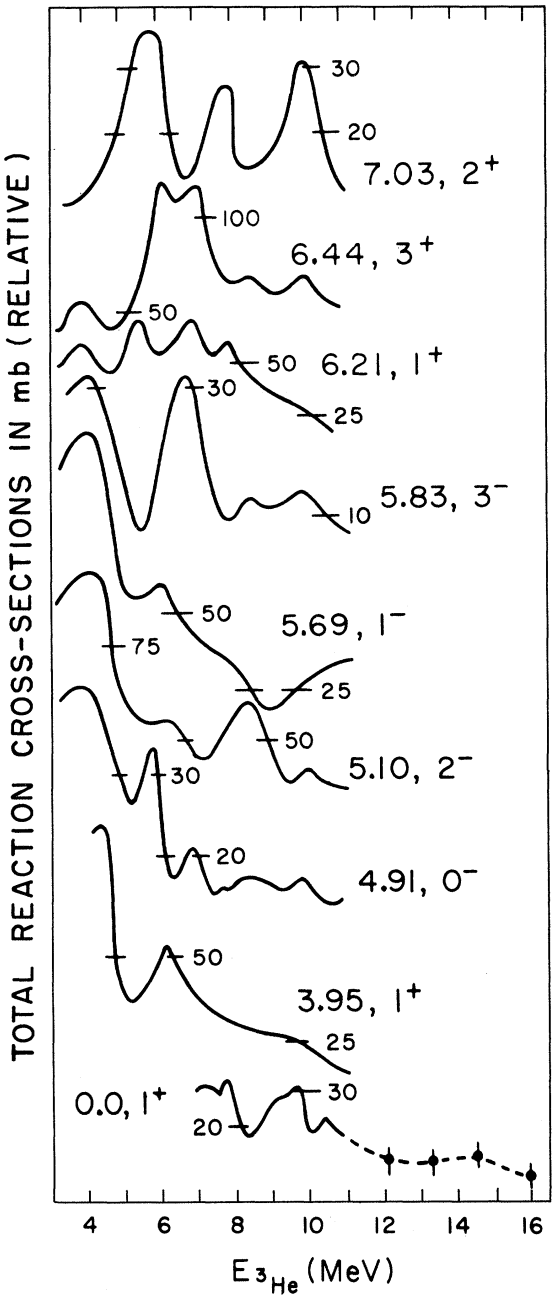

FIG. 3. Total cross section of the $^{12}C(^3He, p)^{14}N$ reaction for 9 of the first 10 states of ^{14}N. The vertical scale varies from one curve to the next but the horizontal dashes are labeled with the cross section in mb. Curves are labeled with the excitation energy and J^π of the final state. The data for the third to the ninth excited state are taken from Fig. 21 of Ref. 1. The ground-state curve shows the present results.

angular momenta involved in the reaction. Since the available angular momenta increase with increasing energy, the degree of forward and/or backward peaking can also increase. Therefore, one should exercise caution in interpreting these features of the angular distributions as evidence for a direct-reaction mechanism.

In summary, we find that up to at least 12 MeV the $^{12}C(^3He, p)^{14}N$ reaction proceeds mainly by compound-nucleus formation. We find no evidence for significant direct-reaction contributions and no consistent structure in yield curves for the various final states.

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Maximum-Overlap Orbitals*

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It is shown that Kobe has not proven the absence of rearrangement terms in the Euler-Lagrange equation for maximum-overlap orbitals for realistic (approximate) reaction-operator calculations.

In a recent paper, Kobe[1] claimed that Löwdin's[2] exact self-consistent field (ESCF) theory provides a reference-spectrum type calculus for wave functions which maximize the overlap with the true wave function. Since the ESCF theory omits rearrangement terms from the single-particle Hamiltonian, it was concluded that the optimal self-consistent potential also does so for an *approximate* reaction operator. We show here that the proof employing the ESCF theory is not valid. Another paper[3] describes how the best wave function is obtained by including the rearrangement terms.

In the ESCF theory, the *exact* reaction operator t is defined by the equation

$$t\Phi_0 \equiv V\Psi , \tag{1}$$

where

$$H\Psi = (H_0 + V)\Psi = E\Psi , \tag{2}$$

and Φ_0 is a model Slater-determinant wave function constructed from single-particle functions

$$H_0\varphi_\nu = \epsilon_\nu\varphi_\nu \tag{3}$$

and normalized by the condition

$$\langle\Psi\,|\,\Phi_0\rangle = 1 . \tag{4}$$

Then it is easy to show that

$$E = \langle\Phi_0|H_0 + t\,|\,\Phi_0\rangle . \tag{5}$$

The Φ_0, the H_0, and t are all functional operators depending on the set $\{\varphi\}$ in such a way that E, and consequently the right-hand side of Eq. (5), is independent of the $\{\varphi\}$. No variational problem arises since E equals the true ground-state energy for any choice of $\{\varphi\}$.

In order to arrive at a variational problem, Löwdin[2] defined the functional $\mathcal{E}$ which involved functional operators φ and χ

$$\mathcal{E}[\varphi, \chi] \equiv \langle\Phi_0[\varphi]\,|H_0[\chi] + t[\chi]\,|\,\Phi_0[\varphi]\rangle \tag{6}$$