

where σ_{DWBA} is the (d, t) cross section calculated from the code JULIE.⁶ Triton optical-model parameters were obtained from Ref. 12. A real-well radius of $1.16A^{1/3}$ fm was used. The spectroscopic factors were evaluated at the first peak in the an-

gular distribution beyond 12° .

The previously known ground-state Q value for the $^{106}\text{Cd}(d, t)$ reaction was -4.670 ± 0.10 MeV.¹³ From the present work we were able to establish the Q value as -4.661 ± 0.05 MeV.

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Gamma Decay of Excited States in ^{210}Bi and an Interpretation with the Shell Model

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The γ decay of excited states in ^{210}Bi up to 3250-keV excitation energy has been studied with the $^{209}\text{Bi}(d, p\gamma)$ reaction at 8.0- and 10.0-MeV incident deuteron energies, using particle- γ and γ - γ coincidence techniques and a pulsed deuteron beam. For almost all of the states excited in this reaction, γ -decay branching ratios were obtained. The branched decay of states with a dominant configuration $(d_{5/2}^{\nu} \otimes h_{3/2}^{\pi})$ into states with a $(g_{9/2}^{\nu} \otimes h_{3/2}^{\pi})$ configuration was analyzed to determine experimentally the amount of configuration mixing between these two multiplets. The obtained mixing amplitudes are in remarkably good agreement with shell-model calculations. From the observed lifetimes of the $(g_{9/2}^{\nu} \otimes h_{3/2}^{\pi})_{5^{-}, \gamma^{-}}$ states, a reduced matrix element of $\langle g_{9/2}^{\nu} || E2 || g_{9/2}^{\nu} \rangle = -38 \pm 4 e \text{ fm}^2$ has been extracted. This value is in good agreement with an intermediate coupling calculation which yields $-35 e \text{ fm}^2$.

I. INTRODUCTION

The shell-model interpretation of the low-lying excited states of ^{210}Bi as states resulting from the coupling of one proton and one neutron to the inert core of the doubly magic nucleus ^{208}Pb has been the subject of various experimental¹⁻⁶ and theoretical⁷⁻⁹ studies. The nucleus ^{210}Bi is of special theoretical interest since the simple two-particle character of the excited states provides an excellent example for studying the effective nucleon-nucleon interaction. Several shell-model calculations⁷⁻⁹ have been performed to understand the spectrum of the excited states of ^{210}Bi in terms of

two-particle multiplets. All calculations showed the necessity of introducing tensor forces besides the usual central forces. A remarkable result of these calculations is the prediction of generally very pure wave functions of the states with appreciable configuration mixing only in special cases.

The first high-resolution studies on the levels of ^{210}Bi have been made in 1962 by Erskine, Buechner, and Enge¹ who measured with a spectrograph angular distributions for the protons emitted in the $^{209}\text{Bi}(d, p)$ reaction. They showed that the first 10 levels excited in this reaction can be interpreted as the members of the $(g_{9/2}^{\nu} \otimes h_{9/2}^{\pi})_{0^{-}, \dots, 9^{-}}$ multiplet. In recent experi-

ments this interpretation has been confirmed and several other proton-neutron multiplets were identified.

Studies on the γ decay of excited states in ^{210}Bi have been reported,^{2,3} so far, only for the $(g_{9/2}^v \otimes h_{9/2}^\pi)$ ground-state multiplet and for the $(i_{11/2}^v \otimes h_{9/2}^\pi)$ multiplet. The magnetic moments of two isomeric states first reported by Ref. 2 have recently been measured by bombarding ^{208}Pb with a pulsed ^7Li beam.⁵

It was the aim of the present investigations to extend the knowledge on the γ -decay properties of excited states in ^{210}Bi . The levels were populated in the reaction $^{209}\text{Bi}(d, p\gamma)$ at bombarding energies near the Coulomb barrier. With particle- γ and γ - γ coincidence techniques and with a pulsed deuteron beam the γ -decay modes have been established. Several previously suggested spin assignments were confirmed.

An attempt has been made in the present paper to analyze the γ -decay branching ratios of states of the $(d_{5/2}^v \otimes h_{9/2}^\pi)$ multiplet into the ground-state multiplet in terms of mutual mixing. The amount of configuration mixing between these two multiplets has been determined and it is shown that only very small admixture amplitudes are needed in order to fit the measured branching ratios. The determined amplitudes are in remarkably good agreement with the ones obtained by the shell-model calculations.^{8,9}

The half-lives of the two isomeric states of the ground-state multiplet (7^- at 433.1 keV and 5^- at 438.9 keV) have been reinvestigated with a pulsed deuteron beam and found to be 59.0 ± 1.5 nsec and 38.0 ± 1.0 nsec, respectively. The interpretation of the initial and final states as $(g_{9/2}^v \otimes h_{9/2}^\pi)$ states including $(d_{5/2}^v \otimes h_{9/2}^\pi)$ admixture leads to a value of $-38 \pm 4 e \text{ fm}^2$ for the reduced matrix element $\langle 2g_{9/2}^v || E2 || 2g_{9/2}^v \rangle$ from which a quadrupole moment $Q = -28 \pm 3 \text{ fm}^2$ for the ground state of ^{209}Pb can be deduced.

II. EXPERIMENT

The γ decay of the low-lying states in ^{210}Bi has been studied using the $^{209}\text{Bi}(d, p\gamma)$ reaction with a deuteron beam of energies between 8 and 10 MeV. At these low deuteron energies the (d, p) reaction is the strongest observed reaction. Additional γ rays have been observed from the (d, n) and $(d, 2n)$ reactions leading to ^{210}Po and ^{209}Po . The decay scheme of the ^{210}Bi states has been established in a p - γ coincidence measurement. In this experiment a self-supporting ^{209}Bi target of 2-mg/cm² thickness was bombarded by a deuteron beam of 10.0 MeV supplied by the Heidelberg EN tandem accelerator. Backscattered protons have been detected by two surface-barrier silicon detectors at

150° to the beam axis each providing a solid angle of 0.3 sr. The energy signals of both detectors were fed through one preamplifier. Detection was accomplished by conventional fast-slow coincidence electronics described elsewhere.¹⁰ The γ rays have been observed in a 40-cm³ Ge(Li) detector. In order to protect the Ge(Li) counter against neutron radiation damage, the beam defining slits were 6 m upstream from the target in front of the last quadrupole lens and the beam was stopped in a Faraday cup behind a concrete wall. The coincident events (about 100/sec) were stored on magnetic tape, event by event.

In addition to this particle- γ coincidence experiment a γ - γ coincidence measurement with two Ge(Li) detectors using similar coincidence techniques was performed with the MP tandem accelerator of the Beschleunigerlaboratorium der LMU und der TU München. One detector of 40-cm³ active volume was used for the low-energy γ rays from the decay of the ground-state multiplet and the other detector of 60-cm³ active volume for the high-energy γ rays feeding the different states of this multiplet. In this experiment, as well as in another measurement of singles γ spectra to determine the relative yield of each γ transition, the beam was stopped at the target position in a ^{209}Bi layer of 300-mg/cm² thickness. The γ rays have been detected at 55 and 125° relative to the beam axis avoiding corrections for possible anisotropies in the γ -ray angular distributions resulting from A_2 terms. A typical spectrum is shown in Fig. 1.

The energy dependence of the efficiency for both the low-energy and the high-energy Ge(Li) counter was carefully determined with $^{177\text{m}}\text{Lu}$, ^{228}Th , and ^{56}Co sources. After the experiments, the magnetic tapes were scanned off line. The proton spectrum coincident with all γ rays is shown in the lower part of Fig. 2. The use of a thick target and large solid angle caused poor resolution. For comparison, a proton spectrum from Ref. 1 taken with a spectrograph is shown on top of the figure. For each of these proton groups the corresponding γ spectrum was created. Such γ spectra can be seen in Fig. 3. Similarly the γ - γ coincidence run was analyzed. γ - γ coincidence spectra are shown in Fig. 4.

Delayed γ rays from $^{209}\text{Bi} + d$ at 8.0-MeV deuteron energy have been measured using the pulsed beam of the Munich MP tandem accelerator. The prompt width of the beam pulse was less than 1.5 nsec, the distance between two pulses was 800 nsec. Also in this experiment the standard electronics mentioned above was used and the data were recorded on magnetic tape. Besides the well-known delayed γ rays from ^{210}Po and ^{209}Po [excited

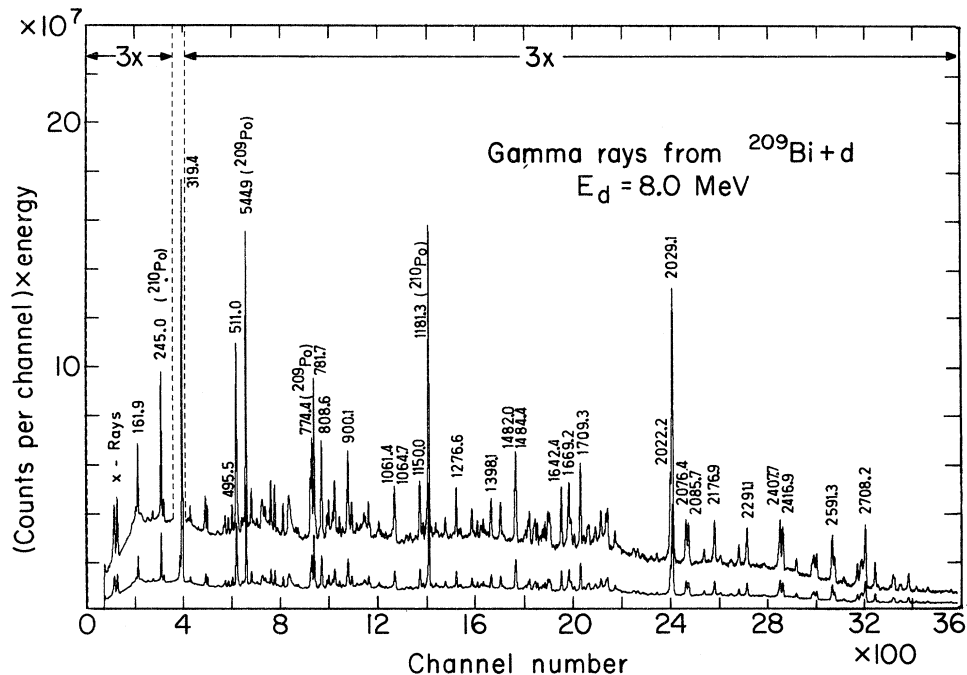


FIG. 1. γ spectrum obtained from the bombardment of a thick-beam stopping target of ^{209}Bi with deuterons of 8.0 MeV. The yield is multiplied with the γ energy in order to correct for the efficiency loss of the Ge(Li) detector at high energies. Most of the transitions originate from the (d, p) reaction. γ rays have been observed also from the (d, n) and $(d, 2n)$ reactions leading to ^{210}Po and ^{209}Po .

in the (d, n) and $(d, 2n)$ reactions, respectively] and the 511.0-keV annihilation radiation only two prominent delayed γ transitions of 162.0 and 319.4 keV have been observed. They have been reported previously in Ref. 2. Time spectra have been taken over several decay periods. The decay curve for the the 319.4-keV transition is shown in Fig. 6.

III. RESULTS

At the low bombarding energies used in the present experiment only a few number of multiplets with states up to 3.5-MeV excitation energy are excited in the (d, p) reaction. Most of the higher-lying states decay into the ground-state multiplet. Some of them also decay into levels which are not excited in the (d, p) reaction directly. No attempt was made to identify these levels. Some of the states reported in Refs. 1, 4, and 5 are so weakly excited in the present experiment that their γ decay could not be observed.

The identification of the decay γ rays of individual levels has mainly been made on the basis of the particle- γ and γ - γ coincidence experiments. Weak γ rays have been accepted as decay branches if the γ -ray energies fit within the experimental uncertainties and if the initial and final spins make a transition probable.

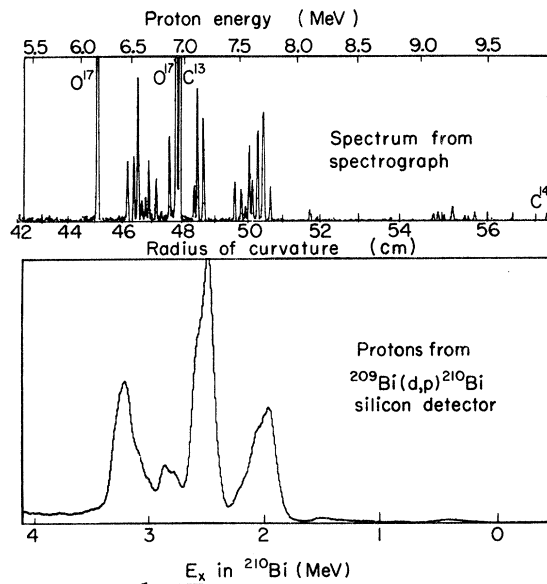


FIG. 2. Proton spectra from the reaction $^{209}\text{Bi}(d, p)$ at deuteron energies near the Coulomb barrier taken with a spectrograph as reported in Ref. 1 (top) and taken with two surface-barrier Si detectors at backward angles during the particle- γ coincidence run (bottom). Target thickness of 2 mg/cm² and solid angle of 0.3 sr caused poor resolution in the later one.

The excitation energies of the states excited in the $^{209}\text{Bi}(d, p)^{210}\text{Bi}$ reaction are given in Table I for the low-lying states, in Table II for the states between 1580 and 2650 keV, and in Table III for all higher states. Spins are also given as reported previously or suggested from the results of this work. Additionally the energies and intensities of all observed γ transitions from the quoted states are listed. The intensities of the γ transitions are obtained from high-resolution singles spectra taken in a special run at 8.0 MeV and 55° (Fig. 1). To get accurate values for the peak areas a peak-fitting computer code was used. All intensities are relative values. For not observed transitions an upper limit for their intensities has been listed in the tables in a number of cases. It was not possible to give all decay branches of a given state, since often a level can decay by low-energy transitions which could not have been detected in our experiment.

We do not want to discuss in detail the γ decay of every state studied in this experiment: It follows for most of them quite in a straightforward

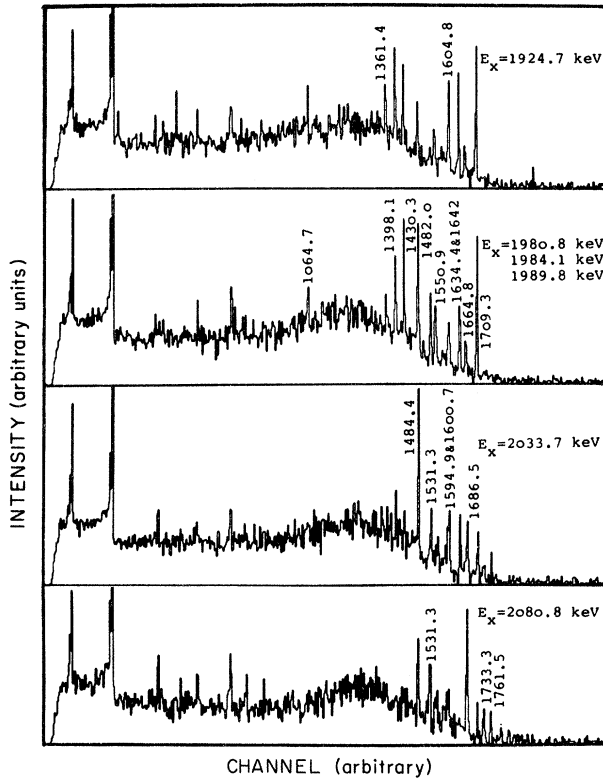


FIG. 3. Some γ spectra in coincidence with various proton groups. The corresponding excitation energies in ^{210}Bi are listed at top right. The γ rays resulting from the decay of a given state are labeled with their energy.

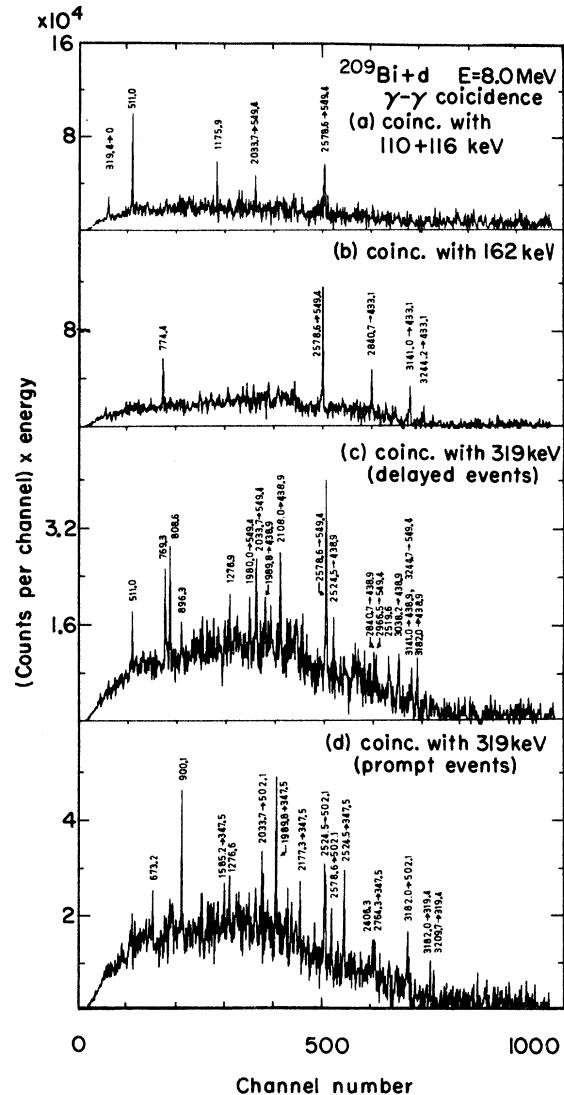


FIG. 4. Several γ - γ coincidence spectra. Identified γ rays are labeled with the excitation energies of the initial and final states; unidentified ones with their energy. Figure 4(a) is the spectrum in coincidence with the 110.3- and 116.2-keV γ rays showing the transitions ending at the 6^- (549.3-keV) state. Figure 4(b) is the spectrum in coincidence with the 161.9-keV γ ray giving the transitions to the 7^- (433.1-keV) state. The time interval for this coincidence spectrum was chosen from $t = 0$ to $t = 300$ nsec, since the half-life of the 7^- state is 59 nsec. Figure 4(c) is the spectrum coincident with the delayed part ($t = 8$ to $t = 300$ nsec) of the 319.4-keV γ ray which gives the transitions to the 5^- (438.9-keV) state. Figure 4(d) shows the γ rays in coincidence with the prompt part of the 319.4-keV transition. They feed the 2^- (319.4-keV) and 3^- (347.5-keV) states.

manner from the particle- γ coincidence spectra. The proposed decay scheme is shown in Fig. 5. 10 levels from zero to 581-keV excitation energy have been interpreted previously¹ as the members of the $(g_{9/2}^{\nu} \otimes h_{9/2}^{\pi})_{0-\dots_9}$ -multiplet. The spin assignments of Ref. 1 to the different states based on the rule that the (d, p) cross section to the multiplet member of spin I is proportional to $(2I + 1)$, are in complete agreement with the γ -decay properties of these states.

In this experiment the excitation energies for

the states of the ground-state multiplet with spin higher than 4^- relative to the states with lower spin had to be determined from the branched decay of higher-lying states, since the excitation energy of the 9^- state was not known accurately, and since the 5^- (438.9-keV) to 3^- (347.5-keV) transition could not be observed because of its high conversion coefficient.

The half-lives of the 7^- state at 433.1 keV and 5^- state at 438.9 keV have been reinvestigated with the pulsed deuteron beam. The time spectrum of

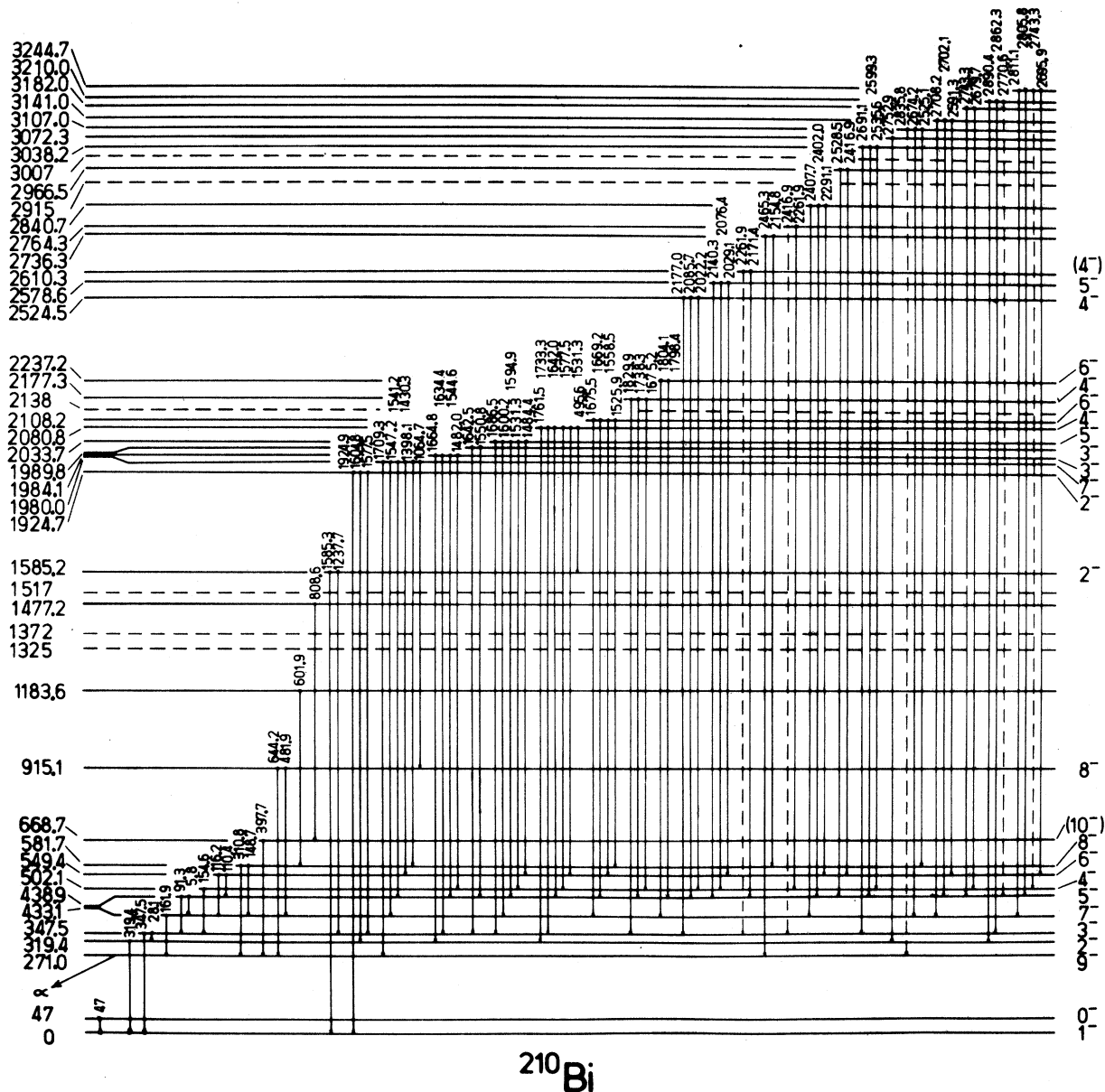


FIG. 5. Proposed decay scheme for the states in ^{210}Bi excited in the $^{208}\text{Bi}(d, p)$ reaction at $E_d = 8.0$ MeV.

the 162.0-keV transition arising from the decay of the 7^- state at 433.1 keV has to be corrected for a delayed feeding of the 5^- state at 438.9 keV. The transition energy is only 5.8 keV, hence the transition could not be seen directly. Its γ strength was calculated on the basis of pure configurations in the way discussed below in Sec. IV and its conversion coefficient was taken from the computer code of Pauli.¹¹ The calculated value for the transition strength is 15%. The γ - γ coincidence run suggests this decay branch; however, an accurate value cannot be deduced because of poor statistics. So we get $t_{1/2} = 59.0 \pm 1.5$ nsec for the half-life of the 7^- state at 433.1 keV.

The lifetime of the 5^- state at 438.9 keV has been determined by the time spectrum of the ground-state transition of the 2^- state at 319.4 keV which is fed by the delayed decay of the 5^- state. Such a time spectrum is shown in Fig. 6. The 91.3-keV transition from the 5^- state to the 3^- (347.5-keV) state is almost completely converted¹¹ ($\alpha = 10.4$), so its time spectrum could not be measured, but from the γ - γ coincidence experiment it follows uniquely (see Fig. 4) that it is indeed the 5^- state at 438.9 keV whose half-life has been determined to 38.0 ± 1.0 nsec.

Above the states of the $(g_{9/2}^v \otimes h_{9/2}^\pi)$ multiplet the states with the dominant configuration $(i_{11/2}^v \otimes h_{9/2}^\pi)$ and $(j_{15/2}^v \otimes h_{9/2}^\pi)$ are expected to be excited in the $^{209}\text{Bi}(d, p)$ reaction. The cross sections to these states, however, are very small because of the high angular momentum of the transferred neutron, so only a few of these states could be studied.

Between about 1600 and 2200 keV the states with dominant $(d_{5/2}^v \otimes h_{9/2}^\pi)$ configuration are expected. Nine proton groups have been observed in previous $^{209}\text{Bi}(d, p)$ studies^{1, 4, 5} in this region of excitation energy. They show typical $l_n = 2$ angular distributions justifying their interpretation as $(d_{5/2}^v \otimes h_{9/2}^\pi)$ states. One of the proton groups at 1981 keV has been identified as a doublet. Spin assignments based on the proportionality of the (d, p) cross sections to $(2I+1)$ are made in Refs. 4 and 5. The assignments suggested in the two references differ slightly. Our spin assignments based on the γ decay of the states are in complete agreement with Ref. 4. We actually observe three states at 1980.0 keV (7^-), 1984.1 keV (3^-), and 1989.8 keV (3^-) which could not be resolved in the previous experiments. Our data are most consistent with the assignment of 5^- for the state at 2033.7 keV and 6^- for the state at 2108.0 keV.

TABLE I. Excitation energies and spins for the states in ^{210}Bi below 1500 keV as excited in the $^{209}\text{Bi}(d, p)$ reaction. Excitation energies and spins of the states where they decay to are given as well as transition energies and relative intensities for the γ transitions.

$E_x(i)$ (keV)	ΔE_x	Spin	$E_x(f)$ (keV)	Spin	E_γ (keV)	ΔE_γ	I_γ (relative units)	ΔI_γ
47 ^a		0^-	0	1^-	47		Transition not observed	
271.0	0.8	9^-	Decays by α emission					
319.4	0.3	2^-	0	1^-	319.4	0.3	2220	50
347.5	0.3	3^-	0	1^-	347.5	0.3	31.5	1.7
			319.4	2^-	28.1		Transition not observed	
433.1	0.5	7^-	271.0	9^-	161.9	0.3	885	20
438.9	0.5	5^-	347.5	3^-	91.3		Transitions not observed	
			433.1	7^-	5.8		observed	
502.1	0.5	4^-	347.5	3^-	154.6	0.3	187	10
549.3	0.5	6^-	433.1	7^-	116.2	0.3	83.5	3
			438.9	5^-	110.3	0.3	73.7	3
581.7	0.8	8^-	271.0	9^-	310.8	0.3	108	13
			433.1	7^-	148.7	0.3	44.2	2.4
668.7	0.8	10^-	271.0	9^-	397.7	0.5	22.6	2.0
915.2	0.8		271.0	9^-	644.2	0.5	60.0	2.0
915.0	0.8		433.1	7^-	481.9	0.5	25.8	1.2
915.1	0.6	8^-	549.4	6^-	365.7		≤ 1.8	
			581.7	8^-	333.4		≤ 1.8	
			581.7	8^-	601.9	0.5	19.9	1.4
1183.6	0.8							
1325 ^a		}	Decay uncertain					
1372 ^a								
1477.3	0.8			668.7	10^-	808.6		$\ll 127$

^a Level reported in Ref. 1.

TABLE II. γ decay of the states between 1500- and 2600-keV excitation energy. See also caption for Table I. For showing that our spin assignments based on the γ decay of the states is in agreement with that deduced from applying the $(2I+1)$ proportionality rule to the (d, p) cross sections, the cross sections measured by Ref. 1 as well as those calculated with the $(2I+1)$ rule are given.

$E_x(i)$ (keV)	ΔE_x	Spin	$\sigma(d, p)$ at $\theta=172.5^\circ$ [(mb/sr) \times 100]	$\sigma(2I+1)$	$E_x(f)$ (keV)	Spin	E_γ (keV)	ΔE_γ	I_γ (relative units)	ΔI_γ
1585.3	0.4				0	1 ⁻	1585.3	0.4	33.5	2.1
					319.4	2 ⁻	1265.9		≤ 1.6	
1585.2	0.4				347.5	3 ⁻	1237.7	0.3	31.3	1.6
					502.1	4 ⁻	1081.2		≤ 1.4	
1585.2	0.3	2 ⁻	6.9							
1924.9	0.5				0	1 ⁻	1924.9	0.5	11.6	1.7
					47.0	0 ⁻	1877.7		≤ 1.9	
1924.2	0.5				319.4	2 ⁻	1604.8	0.3	24.3	5.5
1925.0	0.5				347.5	3 ⁻	1577.5	0.3	13.0	5.3
					502.1	4 ⁻	1422.6		≤ 2.3	
					562.8 ^a	1 ⁻	1361.4	0.3	15.3	1.5
					1585.2	2 ⁻	339.4	0.3	12.9	1.9
1924.7	0.4	2 ⁻	$\frac{21.6}{\sum(2^-)=28.5}$	=29.0						
1980.3	0.5				271.0	9 ⁻	1709.3	0.2	123	3
1980.2	0.8				433.1	7 ⁻	1547.2	0.5	19.7	2.5
					438.9	5 ⁻	1541.2		≤ 2.5	
1979.7	0.5				549.4	6 ⁻	1430.3	0.2	64.8	2.0
1979.8	0.6				581.7	8 ⁻	1398.1	0.2	54.1	1.8
1979.8	0.6				915.1	8 ⁻	1064.7	0.2	35.8	1.9
1980.0	0.4	7 ⁻	86.5							
					0	1 ⁻	1984.1		≤ 2.1	
1984.2	0.5				319.4	2 ⁻	1664.8	0.3	44.1	2.7
					347.5	3 ⁻	1636.6		≤ 3.0	
1983.5	2.0				438.9	5 ⁻	1544.6	2.0	20	5
1984.1	0.7				502.1	4 ⁻	1482.0	0.4	36.2	8.6
1984.1	0.4	3 ⁻	$\sum(3^-)=40.0$							
					0	1 ⁻	1989.8		≤ 1.4	
					319.4	2 ⁻	1670.4		≤ 5.6	
1990.1	0.5				347.5	3 ⁻	1642.6	0.4	96	14
1989.7	0.5				438.9	5 ⁻	1550.8	0.3	44.4	2.6
					502.1	4 ⁻	1487.5		≤ 3.9	
1989.8	0.4	3 ⁻	$\sum(7^-+3^-)=123$	=126.5						
2034.0	0.5				347.5	3 ⁻	1686.5	0.3	25.8	2.1
2033.3	0.5				433.1	7 ⁻	1600.2	0.2	46.8	5.3
2033.8	0.5				438.9	5 ⁻	1594.9	0.2	49.3	2.3
					502.1	4 ⁻	1531.3		≤ 43	
2033.7	0.5				549.4	6 ⁻	1484.3	0.3	124	9
2033.7	0.4	5 ⁻	58.9							
2138 ^b		5 ⁻	$\frac{9.3}{\sum(5^-)=68.2}$	=64						
2080.9	0.4				319.4	2 ⁻	1761.5	0.2	23.5	4.0
2080.8	0.4				347.5	3 ⁻	1733.3	0.2	27.0	2.6
					438.9	5 ⁻	1642.0		≤ 13.7	
					502.1	4 ⁻	1578.7		≤ 5.3	
					549.4	6 ⁻	1531.3		$\ll 43.4$	
2080.8	0.3	4 ⁻	26.3		1585.2	2 ⁻	495.6		≤ 136	

TABLE II (Continued)

$E_x(i)$ (keV)	ΔE_x	Spin	$\sigma(d, p)$ at $\theta = 172.5^\circ$		$E_x(f)$ (keV)	Spin	E_γ (keV)	ΔE_γ	I_γ (relative units)	ΔI_γ
			$\sigma(2I+1)$ [(mb/sr) $\times 100$]							
2177.4	0.4				319.4	2 ⁻	1857.9		≤ 2.5	
2177.2	0.4				347.5	3 ⁻	1829.9	0.2	27.4	2.1
2177.3	0.8				438.9	5 ⁻	1738.3	0.2	29.6	2.6
					502.1	4 ⁻	1675.2	0.8	11	6
					549.4	6 ⁻	1627.9		≤ 2.5	
2177.3	0.3	4 ⁻	$\frac{24.9}{\sum(4^-) = 51.2}$	= 52						
2108.6	0.4				271.0	9 ⁻	1837.0		≤ 1.9	
2108.1	0.5				433.1	7 ⁻	1675.5	0.2	53	6
					438.9	5 ⁻	1669.2	0.2	112	4
					502.1	4 ⁻	1605.9		≤ 2.1	
2107.9	0.5				549.4	6 ⁻	1558.5	0.2	31.0	2.0
2107.7	0.5				581.7	8 ⁻	1525.9	0.2	29.8	1.9
2108.0	0.4	6 ⁻	51.4							
2237.2	0.4				433.1	7 ⁻	1804.1	0.2	64.2	2.5
2237.3	0.4				438.9	5 ⁻	1798.4	0.2	57.9	2.4
					502.1	4 ⁻	1735.1		≤ 2.3	
					549.4	6 ⁻	1687.8		≤ 1.7	
					581.7	8 ⁻	1655.5		≤ 5.5	
2237.2	0.3	6 ⁻	$\frac{23.5}{\sum(6^-) = 74.9}$	= 75						
2524.5	0.4				347.5	3 ⁻	2177.0	0.3	64	8
2524.6	0.4				438.9	5 ⁻	2085.7	0.3	69.4	1.9
2524.3	0.4				502.1	4 ⁻	2022.2	0.3	170	8
2524.5	0.3	4 ⁻	125							
2610.3	0.8	4 ⁻	24.7		438.9	5 ⁻	2171.4	0.4	28.5	7.1
					502.1	4 ⁻	2107.8		≤ 3.0	
					549.4	6 ⁻	2060.5		≤ 1.8	
2610.3	0.8		$\frac{\sum(4^-) = 149.7}{}$	= 149						
2579.2	0.8				438.9	5 ⁻	2140.3	0.5	25.8	1.8
2578.5	0.5				502.1	4 ⁻	2076.4	0.3	84.8	3.0
2578.5	0.6				549.4	6 ⁻	2029.1	0.3	502	8
2578.6	0.5	5 ⁻	181	182						

^a Level reported in Ref. 3.^b Level reported in Ref. 1.

These two spins had been interchanged in Ref. 5. Finally it is seen from our data that the state at 2237.2 keV is a 6⁻ state rather than a 3⁻ state. In order to show that our spin assignments are consistent with the (2I+1) rule for the (d, p) cross sections, we give in Table II the experimental cross sections to the states measured by Ref. 1 together with the cross sections calculated by applying the (2I+1) rule.

The ($d_{5/2}^\nu \otimes h_{9/2}^\pi$) configuration for all except the 7⁻ state is split up into two states. This is predicted by the shell-model calculations^{8,9} which show relatively strong mixing with the ($i_{11/2}^\nu \otimes f_{7/2}^\pi$) configuration.

Above the states discussed so far one observes

the ($s_{1/2}^\nu \otimes h_{9/2}^\pi$)_{4,5}- states. They are strongly excited in the (d, p) reaction. It seems that the strength of the 4⁻ member of this configuration is fragmented into two states at 2524.5- and 2610.3-keV excitation energy.

The higher-lying states are believed to belong to the ($d_{3/2}^\nu \otimes h_{9/2}^\pi$) and ($g_{7/2}^\nu \otimes h_{9/2}^\pi$) multiplets. The angular distributions of the (d, p) cross sections to these states show $l_n = 2$ and $l_n = 4$ mixing^{4,5} indicating strong configuration mixing between these two multiplets. Assignment of spins to these states has not been tried; however, the γ decay of these states may give an estimate of their spins confirming the suggestions made in Ref. 4.

We have studied the γ decay of states up to level

No. 40 of Ref. 1. We get from the transition energies an excitation energy of 3244.7 keV for the level No. 40, whereas Ref. 1 gives a value of 3205 keV. Yet, as can be seen from Fig. 1 of the cited paper, this energy must be about 3240 keV and the excitation energy given there is probably misprinted. In all other cases our energy values agree well with those of Ref. 1 and also with those given in Refs. 2, 3, and 5. A systematic deviation, however, exists to the excitation energies from Ref. 4 where smaller excitation energies are given.

IV. ANALYSIS

A. Transition Probabilities Between Two-Particle States

In this section we want to show that an analysis of the branched γ decay allows one to determine

experimentally the configuration mixing between different states of two-particle multiplets. An application of this method to our experiment is presented for some cases.

For this purpose we write the states of ^{210}Bi as one neutron and one proton outside the ^{208}Pb core:

$$|^{210}\text{Bi}, I\rangle = \sum_{J, j} c_{\alpha J}^I [a_v^+(J) \otimes a_p^+(j)] |^{208}\text{Pb, g.s.}\rangle, \quad (1)$$

where the neutron $a_v^+(J) |^{208}\text{Pb}\rangle$ occupies the single-particle orbits J of ^{209}Pb and the proton $a_p^+(j) |^{208}\text{Pb}\rangle$ those of ^{209}Bi . The coupling between the two particles is defined as

$$[a_v^+(J) \otimes a_p^+(j)]_I = \sum_{m_J m_j} \langle J m_J j m_j | I M \rangle a_v^+(J) \cdot a_p^+(j). \quad (2)$$

TABLE III. γ decay of the states above 2700-keV excitation energy. See also caption of Table I.

$E_x(i)$ (keV)	ΔE_x (keV)	$E_x(f)$ (keV)	Spin	E_γ (keV)	ΔE_γ (keV)	I_γ (relative units)	ΔI_γ (relative units)	$E_x(i)$ (keV)	ΔE_x (keV)	$E_x(f)$ (keV)	Spin	E_γ (keV)	ΔE_γ (keV)	I_γ (relative units)	ΔI_γ (relative units)
2736.3	0.7	271.0	9 ⁻	2465.3	0.3	29.6	2.3				1 ⁻	3072.3		≤ 1.6	
		433.1	7 ⁻	2303.3		≤ 2.1		3072.3	1.0	319.4	2 ⁻	2752.9	1.0	7.3	3.7
<u>2763.5</u>	<u>1.2</u>	581.7	8 ⁻	2154.8	1.0	4	2			347.5	3 ⁻	2724.8		≤ 1.4	
2736.3	0.7									271.0	9 ⁻	2836.0		≤ 7.3	
		319.4	2 ⁻	2444.9		≤ 3.7		3107.3	0.8	433.1	7 ⁻	2674.2	0.5	18.5	2.1
2764.5	0.7	347.5	3 ⁻	2416.9	0.7	≤ 90				549.4	6 ⁻	2557.6		≤ 1.6	
		438.9	5 ⁻	2325.4		≤ 2.1		<u>3106.8</u>	<u>0.8</u>	581.7	8 ⁻	2525.1	0.5	39.7	5.2
<u>2764.0</u>	<u>0.7</u>	502.1	4 ⁻	2261.9	0.7	35	} +2 -10	<u>3107.0</u>	<u>0.6</u>						
2764.3	0.5								3141.3	0.6	433.1	7 ⁻	2708.2	0.4	144
2840.8	0.5	433.1	7 ⁻	2407.7	0.5	126	3	3141.0	1.0	438.9	5 ⁻	2702.1	1.0	64	14
2840.9	0.5	438.9	5 ⁻	2402.0	0.5	35.3	2.2			502.1	4 ⁻	2638.9		≤ 1.7	
		502.1	4 ⁻	2338.4		≤ 2.2		3140.7	0.6	549.4	6 ⁻	2591.3	0.4	96.3	2.4
2840.5	0.5	549.4	6 ⁻	2291.1	0.5	75.6	2.4			581.7	8 ⁻	2559.3		≤ 1.9	
		581.7	8 ⁻	2259.0		≤ 2.1		<u>3141.0</u>	<u>0.5</u>						
2840.7	0.4									319.4	2 ⁻	2862.5		≤ 40	
2915 ^a		Decay uncertain						3182.0	1.0	347.5	3 ⁻	2834.5	1.0	22.5	1.6
		433.1	7 ⁻	2533.2		≤ 2.3				433.1	7 ⁻	2748.9		≤ 1.4	
2967.4	1.2	438.9	5 ⁻	2528.5	0.9	17	5			438.9	5 ⁻	2743.3		≤ 63	
2966.3	0.8	549.4	6 ⁻	2416.9	0.7	≤ 90		3181.8	0.7	502.1	4 ⁻	2679.7	0.5	67.7	2.4
		581.7	8 ⁻	2384.5		≤ 2.2				549.4	6 ⁻	2632.6		≤ 1.8	
<u>2966.5</u>	<u>0.8</u>							<u>3181.9</u>	<u>0.7</u>						
3007 ^a		Decay uncertain								0	1 ⁻	3209.8		≤ 4.2	
		319.4	2 ⁻	2718.8		≤ 1.7		3209.8	0.8	319.4	2 ⁻	2890.4	0.8	11.8	1.6
3038.6	0.8	347.5	3 ⁻	2691.1	0.5	69.1	3.5			347.5	3 ⁻	2862.3	0.8	≤ 40	
		433.1	7 ⁻	2605.1		≤ 1.7				502.1	4 ⁻	2707.7		≤ 7	
3038.2	0.8	438.9	5 ⁻	2599.3	0.5	49	2	<u>3209.8</u>	<u>0.7</u>						
3037.8	0.8	502.1	4 ⁻	2535.6	0.5	53	2			347.5	3 ⁻	2897.2		≤ 1.5	
		549.4	6 ⁻	2488.8		≤ 1.9		3244.2	0.8	433.1	7 ⁻	2811.1	0.7	20.2	2.9
<u>3038.2</u>	<u>0.6</u>							3244.7	0.7	438.9	5 ⁻	2805.8	0.6	33.3	3.0
										502.1	4 ⁻	2743.3		≤ 63	
								3245.3	0.6	549.4	6 ⁻	2695.9	0.5	64.0	2.5
										581.7	8 ⁻	2663.0		≤ 1.7	
								<u>3244.7</u>	<u>0.6</u>						

^a Level reported in Ref. 1.

Every state in ^{210}Bi is written as a sum over all possible configurations and the coefficients $c_{\alpha J}^I$ give the amplitudes of the different configurations. The restriction on configurations based on the ground state of ^{208}Pb is valid as long as we con-

sider low-lying multiplets.

In this framework we get the following expression for the reduced transition matrix element for transitions between two-particle states in ^{210}Bi :

$$\begin{aligned} \langle I_2 \| M(\lambda) \| I_1 \rangle = & [(2I_1 + 1)(2I_2 + 1)]^{1/2} \\ & \times \sum_{J_1 j_1 J_2 j_2} c_{J_2 j_2}^{I_2} c_{J_1 j_1}^{I_1} \left((-)^{J_2 + j_2 + I_1 + \lambda} \begin{Bmatrix} J_1 & j_1 & I_1 \\ I_2 & \lambda & J_2 \end{Bmatrix} \langle J_2 \| M(\lambda) \| J_1 \rangle \delta_{j_1 j_2} \right. \\ & \left. + (-)^{J_1 + j_1 + I_2 + \lambda} \begin{Bmatrix} J_1 & j_1 & I_1 \\ \lambda & I_2 & j_2 \end{Bmatrix} \langle j_2 \| M(\lambda) \| j_1 \rangle \delta_{J_1 J_2} \right). \end{aligned} \quad (3)$$

$M(\lambda)$ is the electromagnetic multipole operator of the multipolarity λ ; I_1, I_2 are the spins of the initial and final state, respectively, in ^{210}Bi ; J_1, J_2, j_1, j_2 are the spins of the neutron and proton, respectively, making the transition from state I_1 to state I_2 ; and $\langle J_2 \| M(\lambda) \| J_1 \rangle$ and $\langle j_2 \| M(\lambda) \| j_1 \rangle$ are the reduced matrix elements for the single-particle transition moments taken from the neighboring nuclei ^{209}Pb and ^{209}Bi , respectively. The units are $e \text{ fm}^2$ in the case of $E2$ and $eh/2Mc$ in the case of $M1$ transitions. The values of the matrix elements are listed in Table IV. $c_{J_1 j_1}^{I_1}, c_{J_2 j_2}^{I_2}$ are the amplitudes of the different components of the wave functions of the initial state I_1 and final state I_2 . The reduced transition matrix elements are connected to the experimentally observed transition

probabilities by standard formulas, see for example Ref. 12. To calculate lifetimes and branching ratios, an incoherent sum over all possible multipolarities has to be taken.

In this approach we make use of the concept of the additivity of effective g factors and effective charges which has been proved to be valid for the nuclei of the lead region in many experimental and theoretical studies.¹³⁻¹⁶

B. Configuration Mixing

In formula (3), in principle, all values are known except the coefficients $c_{J_1 j_1}^{I_1}$ and $c_{J_2 j_2}^{I_2}$; so one can get from the experiment, roughly speaking, as many coefficients $c_{J_i j_i}^{I_i}$ as one measures transition probabilities from state I_1 to state I_2 . In the case

TABLE IV. Reduced matrix elements used in the analysis described in Sec. IV.

Nucleus	Matrix element ^a	Electromagnetic moment	Reference
^{209}Bi	$\langle \frac{3}{2}^- \ M1 \ \frac{3}{2}^- \rangle = 6.97$	$\mu = 4.08$	b
	$\langle \frac{3}{2}^- \ E2 \ \frac{3}{2}^- \rangle = -51 \pm 2$	$Q = -37.9 \pm 1.5$	c
	$\langle \frac{3}{2}^- \ M1 \ \frac{7}{2}^- \rangle = 0.11 \pm 0.03$	$\delta(E2/M1) = 1.0 \pm 0.25$	d, e
	$\langle \frac{3}{2}^- \ E2 \ \frac{7}{2}^- \rangle = 15.5 \pm 0.8$	$B(E2, \frac{7}{2}^- \rightarrow \frac{3}{2}^-) = 30 \pm 3$	d, e
	$\langle \frac{3}{2}^- \ E2 \ \frac{5}{2}^- \rangle = -33$		f
^{209}Pb	$\langle \frac{3}{2}^+ \ M1 \ \frac{3}{2}^+ \rangle = -2.27 \pm 0.10$	$\mu = -1.33 \pm 0.06$	6
	$\langle \frac{3}{2}^+ \ E2 \ \frac{3}{2}^+ \rangle = -38 \pm 4$	$Q = -28 \pm 3$	Present work
	$\langle \frac{3}{2}^+ \ E2 \ \frac{5}{2}^+ \rangle = -33 \pm 5$	$B(E2, \frac{5}{2}^+ \rightarrow \frac{3}{2}^+) = 185 \pm 50$	d
	$\langle \frac{5}{2}^+ \ M1 \ \frac{5}{2}^+ \rangle = -1.95$	$\mu = -1.38$	Migdal theory (g)
	$\langle \frac{5}{2}^+ \ E2 \ \frac{5}{2}^+ \rangle = -21$	$Q = -16$	Migdal theory (g)
	$\langle \frac{5}{2}^+ \ E2 \ \frac{7}{2}^+ \rangle = -17.5 \pm 0.5$	$B(E2, \frac{7}{2}^+ \rightarrow \frac{5}{2}^+) = 154 \pm 8$	20
	$\langle \frac{3}{2}^+ \ M1 \ \frac{7}{2}^+ \rangle = 2.42$	$B(M1, \frac{7}{2}^+ \rightarrow \frac{3}{2}^+) = 0.73$	Migdal theory (g)

^a $M1$ matrix elements are given in units of $e\hbar/2Mc$; $E2$ matrix elements are given in units of $e \text{ fm}^2$.

^b G. H. Fuller and V. W. Cohen, Nucl. Data **A5**, 433 (1969).

^c G. Eisele, I. Koniordos, G. Müller, and R. Winkler, Phys. Letters **28B**, 256 (1968).

^d O. Häusser, F. C. Khanna, and D. Ward, to be published.

^e W. Kratschmer, V. Klapdor, and E. Grosse, to be published.

^f Calculated with $e_{\text{eff}} = 1.5$.

^g R. Bauer, P. Ring, J. Speth, E. Werner, and T. Yamazaki, to be published.

of ^{210}Bi an application of this method to members of the $(g_{9/2}^{\nu} \otimes h_{9/2}^{\pi})$ and $(d_{5/2}^{\nu} \otimes h_{9/2}^{\pi})$ multiplets is given in the following. Since we did not measure the lifetimes of all the states, we used their branching ratios for the analysis. The transition probabilities between the members of these two multiplets are governed only by the electromagnetic properties of the $2g_{9/2}^{\nu}$, $3d_{5/2}^{\nu}$, and $1h_{9/2}^{\pi}$ single-particle states. Admixtures from other configurations as $(i_{11/2}^{\nu} \otimes h_{9/2}^{\pi})$ or $(g_{9/2}^{\nu} \otimes f_{7/2}^{\pi})$ and $(i_{11/2}^{\nu} \otimes f_{7/2}^{\pi})$ to the main configurations of the considered states give negligible contributions to the transition probabilities because of the l forbiddenness of the $i_{11/2}^{\nu} \rightarrow g_{9/2}^{\nu}$ and $f_{7/2}^{\pi} \rightarrow h_{9/2}^{\pi}$ $M1$ transitions or because of the two-particle excitation character of the admixed configuration. Therefore $M1$ transition strength between the states with main configuration $(d_{5/2}^{\nu} \otimes h_{9/2}^{\pi})$ and $(g_{9/2}^{\nu} \otimes h_{9/2}^{\pi})$ is caused only by the configuration mixing between these two multiplets.

To get the $M1$ transition strength from the experimentally observed branching ratios, the $E2$ part of the transition probability has to be subtracted out. This can be done when the initial state decays via a $\Delta I = 2$ transition to a member of the $(g_{9/2}^{\nu} \otimes h_{9/2}^{\pi})$ multiplet. This is obviously an $E2$ transition and the relative intensities of all other $E2$ transitions from this level to states of the lower multiplet are governed then only by the statistical and geometrical factors of formula (3). Additional observed strength in the transition probabilities is then attributed to $M1$ strength resulting from configuration mixing and therefore allows the determination of the amount of the mixing. Since, in general, for two admixed amplitudes, three branching ratios are available, the system is over-

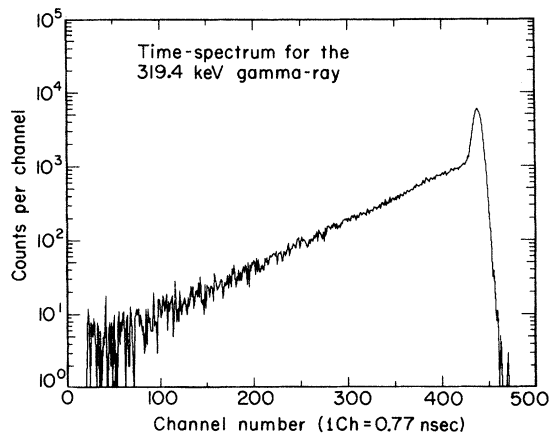


FIG. 6. Decay curve for the 319.4-keV γ ray of the 2^- (319.4-keV) to 1^- (g.s.) transition as observed with a pulsed deuteron beam. The 2^- state is fed delayed by the decay of the 5^- (438.9-keV) state whose half-life is 38.0 ± 1.0 nsec.

determined and admixtures from $(g_{7/2}^{\nu} \otimes h_{9/2}^{\pi})$ into the $(d_{5/2}^{\nu} \otimes h_{9/2}^{\pi})$ states can be estimated. In some cases the (d, p) spectroscopic factors from Ref. 1 have been used to determine the strength of the main configuration.

The amplitudes of the configurations calculated in the described way are listed in Table V. The signs of the amplitudes could not be determined giving rise to nonunique values in some cases. The obtained values are compared with those from shell-model calculations of Ref. 8 (KR) and of Ref. 9 (KH). A remarkably good agreement is found. However, since our experimental uncertainties are rather large, it is not possible to distinguish between the wave functions calculated by Ref. 8 (KR) and those from Ref. 9 (KH) where slightly larger configuration mixing is predicted.

The decay of the 4^- and 2^- states could not be analyzed in the described manner because their $\Delta I = 2$ branch could not be determined accurately enough. Also the two 3^- states could not be analyzed since the strengths of their main configurations are not known.

C. Reduced Matrix Element $\langle g_{9/2}^{\nu} \| E2 \| g_{9/2}^{\nu} \rangle$

As shown above in Sec. III, two $E2$ transition probabilities between states belonging to the ground-state multiplet of ^{210}Bi have been measured absolutely. According to formula (3) several single-particle transition amplitudes contribute to the transition strengths. All these amplitudes are known or do not contribute to the transition strengths except the one related to the matrix element $\langle g_{9/2}^{\nu} \| E2 \| g_{9/2}^{\nu} \rangle$, which can therefore be deduced from our data. The situation is shown in Table VI where the different contributions to the two transitions as obtained from the wave functions of Ref. 9 are listed. It is apparent that the contributions from the unknown matrix elements $\langle g_{9/2}^{\nu} \| E2 \| i_{11/2}^{\nu} \rangle$ and $\langle g_{9/2}^{\nu} \| E2 \| g_{7/2}^{\nu} \rangle$ can be neglected for reasonable values of these matrix elements. From the difference between all known contributions and the experimental values, the unknown matrix $\langle g_{9/2}^{\nu} \| E2 \| g_{9/2}^{\nu} \rangle$ element is derived. It should be pointed out that the admixtures of $(d_{5/2}^{\nu} \otimes h_{9/2}^{\pi})$ into the initial and final states give the only important contributions besides the main configurations. One can also see that the two values for the matrix element deduced from the two lifetimes agree only for the negative sign within their errors so the sign has been determined too. If one assumes additivity, that means that this matrix element is the same in ^{210}Bi and ^{209}Pb ; then one gets

$$\begin{aligned} \langle g_{9/2}^{\nu} \| E2 \| g_{9/2}^{\nu} \rangle \\ = \langle ^{209}\text{Pb, g.s.} \| E2 \| ^{209}\text{Pb, g.s.} \rangle = -38 \pm 4 e \text{ fm}^2. \end{aligned}$$

TABLE V. Excitation energies and configuration amplitudes for a number of ^{210}Bi states as determined from this work. The label Main is given if the strength of the dominant configuration is only slightly less than unity. No sign for any component of the wave functions could be determined giving rise to some nonunique values, which are listed below each other in one line. The experimental numbers are compared with those determined in the shell-model calculations from Ref. 8 (KR) and from Ref. 9 (KH).

Excitation energy (keV)				Amplitudes of the configurations								
Exp.	KH	KR	Spin	$(g_{9/2}^{\nu} \otimes h_{9/2}^{\pi})$			$(d_{5/2}^{\nu} \otimes h_{9/2}^{\pi})$			$(g_{7/2}^{\nu} \otimes h_{9/2}^{\pi})$		
				Exp.	KH	KR	Exp.	KH	KR	Exp.	KH	KR
433.1	470	376	7^{-}	Main	-0.987	-0.996	0.14 ± 0.03	-0.127	-0.083			
1980.0	2030	1996	7^{-}	0.10 ± 0.05	0.125	-0.085	Main	-0.967	0.992	≤ 0.1	0.017	0.021
549.3	600	510	6^{-}	Main	-0.994	-0.995	0.02 ± 0.04	-0.058	-0.021			
2108.0	2180	2077	6^{-}	0.09 ± 0.07	0.085	-0.035	0.828^a	-0.903	0.938	0.2 ± 0.2	-0.120	0.074
438.9	470	392	5^{-}	Main	-0.988	-0.997	0.11 ± 0.04	-0.095	-0.056			
2033.7	2100	2015	5^{-}	0.09 ± 0.04	-0.112	0.071	0.928^a	0.929	0.942	≤ 0.1	-0.011	0.003
502.1	520	459	4^{-}	Main	0.991	$0.994 \leq \begin{cases} 0.20 \\ 0.05 \end{cases}$		0.086	0.041			

^a This amplitude has been deduced from the (d, p) cross section as given by Ref. 1.

TABLE VI. Evaluation of the reduced matrix element $\langle g_{9/2}^{\nu} || E2 || g_{9/2}^{\nu} \rangle$ from the lifetimes of the 7^{-} (433.1-keV) and 5^{-} (438.9-keV) states. The contributions from admixed configurations have been calculated using the wave functions of Ref. 9. The difference between the sum of all known contributions and the experimental value is attributed to the $2g_{9/2}^{\nu}$ $E2$ moment as given in the last row. $c^I(\text{stat})\{6J\}$ is the amplitude of the wave function (taken from Ref. 9) multiplied by the statistical and geometrical factors of formula (3).

Contribution from	$c^I(\text{stat})\{6J\}$		Matrix element ^a
	$5^{-} \rightarrow 3^{-}$	$7^{-} \rightarrow 9^{-}$	
$h_{9/2}^{\pi} \rightarrow h_{9/2}^{\pi}$	-0.458	-0.210	-51.2 ± 2.0
$f_{7/2}^{\pi} \rightarrow h_{9/2}^{\pi}$	0.0	-0.004	15.5 ± 0.8
$f_{5/2}^{\pi} \rightarrow h_{9/2}^{\pi}$	-0.017	-0.030	-33
$d_{5/2}^{\nu} \rightarrow g_{9/2}^{\nu}$	-0.054	-0.174	-33 ± 5
$i_{11/2}^{\nu} \rightarrow g_{9/2}^{\nu}$	-0.015	-0.023	$\langle g_{9/2}^{\nu} E2 i_{11/2}^{\nu} \rangle^b$
$g_{7/2}^{\nu} \rightarrow g_{9/2}^{\nu}$	-0.015	-0.006	$\langle g_{9/2}^{\nu} E2 g_{7/2}^{\nu} \rangle^b$
$\sum c^I(\text{stat})\{6J\}$ ME	$+25.8 \pm 1.2$	$+17.5 \pm 1.3$	
Experimental ME	$\pm 43.6 \pm 0.6$	$\pm 25.3 \pm 0.3$	
Difference = (stat) $\{6J\} \times$ $\langle g_{9/2}^{\nu} E2 g_{9/2}^{\nu} \rangle$	-17.8 ± 1.8 or 69.4 ± 1.8	-7.8 ± 1.6 or 42.8 ± 1.6	
Deduced ME ^c $\langle g_{9/2}^{\nu} E2 g_{9/2}^{\nu} \rangle$	-39 ± 4	-37 ± 8	
Mean value		-38 ± 4	

^a All matrix elements (ME) are given in $e\text{fm}^2$.

^b Matrix element is not known; however, the contribution is small for reasonable values so it can be neglected.

^c Only the negative sign gives an unique value for the matrix element.

The error in this number is mainly due to the experimental uncertainties in the $E2$ matrix elements involved. From this matrix element one can deduce the quadrupole moment of the ground state of ^{209}Pb to be

$$Q(^{209}\text{Pb, g.s.}) = -28 \pm 3 \text{ fm}^2.$$

This value can be parametrized by an effective charge

$$e_{\text{eff}}(^{209}\text{Pb, g.s.}) = (-0.87 \pm 0.09) e.$$

As has been shown previously¹⁷ $E2$ matrix elements between neutron single-hole states in ^{207}Pb can be explained quantitatively in the intermediate coupling model by taking the coupling to the collective 2^+ state of ^{208}Pb at 4.08 MeV into account. Higher-lying quadrupole strength, which is expected from sum-rule considerations¹⁸ and recent electron scattering data,¹⁹ was included in these calculations phenomenologically by increasing the core $E2$ matrix element from the experimental value by about 50%. This "renormalization" was adjusted to fit the very accurately measured²⁰ $\langle s_{1/2}^v || E2 || d_{5/2}^v \rangle$ matrix element in ^{209}Pb . A calculation of the $\langle g_{9/2}^v || E2 || g_{9/2}^v \rangle$ matrix element in ^{209}Pb was performed in the same way as given in Ref. 17 where calculational details may be found, resulting in a value which is too low by 20% compared to our experiment. As suggested in Ref. 21 the rather large quadrupole moment²² of the 3^- state in ^{208}Pb may contribute to the $E2$ matrix element. Whereas in ^{209}Bi the possible contributions were negligible²¹ this is not true in the case of ^{209}Pb because of the strong admixture of the $|3^- \otimes j_{15/2}\rangle$ configuration into the ^{209}Pb ground state. By including core excitation to the 3^- state into our calculation and using the experimentally observed core strength¹⁷ the following wave function for the ground state of ^{209}Pb was found:

$$|(^{209}\text{Pb, g.s.})_{9/2^+}\rangle = 0.95 |g_{9/2}^v\rangle + 0.13 |2^+ \otimes g_{9/2}^v\rangle + 0.24 |3^- \otimes j_{15/2}\rangle + \dots$$

The first term gives no contribution to our $E2$ matrix element, since we are dealing with neutrons. The core-polarization effect is treated by the second term which contributes $-31 e \text{ fm}^2$ to $\langle g_{9/2}^v || E2 || g_{9/2}^v \rangle$. The contribution of the third term is $-4 e \text{ fm}^2$ summing up to a total calculated value of $-35 \pm 3 e \text{ fm}^2$, which agrees with experiment within the errors. The calculated value has errors, too, due to core-parameter errors.

CONCLUSION

The investigation of the γ decay of excited states in ^{210}Bi has led to a more complete experimental

knowledge on the properties of states resulting from the coupling of one neutron and one proton to the inert doubly magic core of ^{208}Pb . Previous interpretations of the main configurations of several two-particle multiplets have been supported although some slight corrections in the earlier spin assignments had to be made.

From the analysis of the branched γ decay of states of the $(d_{5/2}^v \otimes h_{9/2}^\pi)$ multiplet into the ground-state multiplet the amount of configuration mixing between these two multiplets could be determined for a number of states. The obtained amplitudes of the admixed configurations are very small and in good agreement with those predicted by shell-model calculations. At least partially the extreme purity of the two-particle states in ^{210}Bi is proved as well as the reliability of the existing shell-model calculations. The accuracy of the analysis however is limited by the relatively large error in the $3d_{5/2}^v - 2g_{9/2}^v$ single-particle transition probability and by the errors of some branching ratios for the decay of states with the main configuration $(d_{5/2}^v \otimes h_{9/2}^\pi)$. An experimental determination of the $E2/M1$ mixing ratios for the decay of these states would improve the present arguments. It would be nice to extend the analysis to the states with the main configuration $(s_{1/2}^v \otimes h_{9/2}^\pi)$ by comparing their decay into the ground-state multiplet with that into the $(d_{5/2}^v \otimes h_{9/2}^\pi)$ multiplet. That is not possible at present since the branching ratios for the second decay mode are very weak. A measurement of a Compton suppressed γ spectrum would certainly help in this case. The analysis of the lifetimes of two states of the ground-state multiplet in ^{210}Bi has led to a value of $-38 \pm 4 e \text{ fm}^2$ for the matrix element $\langle 2g_{9/2}^v || E2 || 2g_{9/2}^v \rangle$. Since it is difficult to measure the quadrupole moment of the ground state of ^{209}Pb directly, it would be nice to compare our value with values extracted from the lifetimes of the $(g_{9/2}^v)_{6^+, 8^+}^+$ states in ^{210}Pb which should be measurable. One would then get an extra check of the additivity of effective charges in the lead region for neutron states which has been proved for proton states in several cases.¹⁴

The good agreement between the experimental $\langle g_{9/2}^v || E2 || g_{9/2}^v \rangle$ matrix element and the one calculated in the unified model is another confirmation of the assumption that the electromagnetic properties of single-particle states around ^{208}Pb are determined by the collective parameters of the core alone.

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