$E0$ Contributions in 2^+ to 2^+ Transitions in ^{160}Dy, ^{134}Ba, ^{110}Cd, and ^{82}Kr†

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A Ge(Li)-Ge(Li) directional-correlation system has been utilized to obtain directional-correlation functions for the 1–3 sequence, $I_i(L_1, L_1')2(E0, M1, E2)2(L_3, L_3')I_f$, in ^{160}Dy, ^{134}Ba, ^{110}Cd, and ^{82}Kr. The following $E0$ admixtures were determined: $T(E0)/T(E2_\gamma) \leq 0.010$ (^{160}Dy, 879 keV), $T(E0)/T(E2_\gamma) < 0.045$ (^{134}Ba, 563 keV), $T(E0)/T(E2_\gamma) = 0.097^{+0.139}_{-0.097}$ (^{110}Cd, 818 keV), and $T(E0)/T(E2_\gamma) = 0.009^{+0.073}_{-0.009}$ (^{82}Kr, 698 keV). In addition, combinations of NaI(Tl) and Ge(Li) detectors have been utilized to obtain mixing ratios for other transitions in the above nuclei.

INTRODUCTION

In a transition $I \rightarrow I$ with no parity change, the electric-monopole ($E0$) mode of decay can compete with the permissible and usually dominant $M1$ and/or $E2$ modes of decay (for $I=0$, only $E0$ modes exist). The $E0$ rate is expressed in terms of a matrix element ρ which is dependent on the radial coordinates of the protons and the nuclear radius. The monopole operator is a radial operator and cannot connect nuclear states which differ in their angular dependence. The $E0$ rate is therefore a useful addition to other data ($E2$, $M1$ rates, etc.) which are often used as tests of specific nuclear models. For example, the presence of a measurable $E0$ component in the $2^{+\prime} \rightarrow 2^+$ transition for a number of so-called "harmonic" nuclei is evidence for the existence of a deformation (anharmonic vibrational or asymmetric rotational) from the spherical equilibrium shape of the nuclear surface.

Recently Anicin *et al.*[1] have proposed a method to determine the $E0$ admixture from the 1–3 correlation, $I_i(L_1, L_1')I_a(E0, M1, E2)I_a(L_3, L_3')I_f$. The normalized and reduced $U_{kk}(\gamma_2)$ given by Anicin *et al.*[1] requires prior and separate knowledge of the $E2/M1$ mixing ratio for the $2^{+\prime} \rightarrow 2^+$ transition as well as the total $E2$ and $M1$ conversion coefficients $\alpha(E2)$ and $\beta(M1)$. The total conversion coefficients were determined using the tables of Hager and Seltzer.[2] The $E2/M1$ mixing ratios were determined separately in this laboratory from direct-correlation measurements.

ANALYSIS AND EXPERIMENTAL ARRANGEMENTS

For the purpose of the present investigation two directional-correlation systems were used:

System I 7.6-×7.6-cm NaI(Tl) – 29-cm^3 Ge(Li),

System II 32-cm^3 Ge(Li) – 29-cm^3 Ge(Li).

In those correlation measurements for which good resolution was required in only one detector, system I was used; in those cases that required good resolution in both detectors, system II was used. The systems utilized a Nuclear Data computer controlled analyzer. Both systems acquired data automatically at 15° intervals in a double quadrant sequence. Dilute liquid sources were used to obtain all reported correlation results.

Interfering correlations were present in several correlations. In each such case, the correlation of interest was corrected for the interfering correlation following the procedure outlined by Frauenfelder and Steffen.[3]

After corrections, the data were analyzed following the method of Rose.[4] A least-squares fit was made to the function $W(\theta) = A_0' + A_2' P_2(\cos\theta) + A_4' P_4(\cos\theta)$ so that the normalized and geometric corrected correlation coefficients $A_0 = A_0'/A_0'$, $A_2 = A_2'/(A_0' Q_2)$, and $A_4 = A_4'/(A_0' Q_4)$ and their associated uncertainties[5] could be obtained.

The geometric corrections Q_2 and Q_4 were obtained using the expressions given by Camp and Van Lehn.[6]

For the NaI(Tl) detector the ratio J_k/J_0 and the efficiencies $\epsilon(\gamma)$ used to determine Q_k were obtained from the calculated values given by Yates.[7] The ratio J_k/J_0 for the Ge(Li) detectors were obtained from the calculated values of Camp and Van Lehn.[6] The photopeak efficiencies for the Ge(Li) detectors were calculated by means of the Monte Carlo program of Aubin *et al.*[8] Raeside, Wiedenbeck, and Ludington have shown that the calculations of Refs. 6 and 8 give results which are in good agreement with the experimental results.[9,10]

RESULTS AND DISCUSSION

^{160}Dy

High-purity Tb_2O_3 powder was irradiated in the University of Michigan Ford nuclear reactor.

TABLE I. Directional-correlation results for ^{160}Dy.

Cascade (E in keV)	Sequence	A_2 [a] / A_4	$\delta(\gamma)$
(299, 966)	$2(D,Q)2(Q)0$	0.265 ± 0.005 0.004 ± 0.007	$\delta(299) = 0.021 \pm 0.007$
(299, 879)	$2(D,Q)2(D,Q)2$	-0.088 ± 0.005 -0.004 ± 0.006	$\delta(879) = 16.9^{+3.3}_{-2.4}$ [b]
(879, 87)	$2(D,Q)2(Q)0$	-0.030 ± 0.004 0.322 ± 0.017	$\delta(879) = 16.1^{+1.5}_{-1.3}$ [b]
(1178, 87)	$2(D,Q)2(Q)0$	0.215 ± 0.016 0.014 ± 0.037	$\delta(1178) = -0.047 \pm 0.021$
(1272, 87)	$2(D,Q)2(Q)0$	0.252 ± 0.019 0.024 ± 0.045	$\delta(1272) = 0.003 \pm 0.026$
(197, 87)	$4(Q)2(Q)0$	0.102 ± 0.006 0.014 ± 0.012	
(299, 879, 87)	$2(D,Q)2(E0,M1,E2)2(Q)0$	-0.058 ± 0.005 0.012 ± 0.010	

[a] After all corrections, including the attenuation across the 2.6-ns, 87-keV state.
[b] Used to compute the weighted average $\delta(879) = 16.3 \pm 1.3$.

The radioactive powder was dissolved in HCl, and the resultant $TbCl_3$ solution was diluted with distilled water.

System II was used to measure the ^{160}Dy (299-879), (299-966), (879-87), (1178-87), (197-87), (299-879-87), and (1272-87) correlations. The results of the correlation measurements are presented in Table I. The spins of the levels, as well as the multipolarity of the 87-, 197-, and 966-keV transitions in ^{160}Dy, have been well established.[11-14]

Because the first excited state of ^{160}Dy has a lifetime of approximately 2.6 ns, it is necessary to obtain the attenuation factors G_{kk} for the correlations across the 87-keV state before these correlations can be analyzed. The attenuation factors were obtained by comparing the measured value of the correlation coefficients to the corresponding theoretical values for certain selected cascades. The A_2 and A_4 coefficients of the (197-87) and (879-87) correlations were used to determine G_{22} and G_{44}, respectively. The mixing ratio of the 879-keV transition used in the analysis to determine G_{22} and G_{44} was obtained from the (299-879) correlation where $\delta(299) = 0.021 \pm 0.007$ was determined from the (299-966) correlation. Taking a weighted average of the separate values one obtains

$$G_{22} = 0.92 \pm 0.04,$$

$$G_{44} = 0.49 \pm 0.02.$$

Other values for G_{kk}, as well as references to other measurements, are given in Refs. 13 and 15.

TABLE II. Directional-correlation results for ^{134}Ba.

Cascade (E in keV)	Sequence	A_2, A_4	$\delta(\gamma)$
(802, 563)	$4(Q)2(D,Q)2$	-0.046 ± 0.006 -0.012 ± 0.009	$\delta(563) = 8.1^{+2.9}_{-1.8}$
(475, 1168)	$3(D,Q)2(Q)0$	-0.280 ± 0.023 -0.029 ± 0.031	$\delta(475) = 10.0^{+4.6}_{-2.4}$
(475, 563)	$3(D,Q)2(D,Q)2$	0.123 ± 0.013 -0.032 ± 0.017	$\delta(475) = 11.0^{+8.1}_{-3.3}$ [a]
(475, 563, 605)	$3(D,Q)2(E0,M1,E2)2(Q)0$	0.070 ± 0.028 -0.083 ± 0.037	

[a] $\delta(563) = 8.1 \pm 2.4$ used in analysis.

The method of Anicin *et al.*[1] can now be used to determine the relative $E0$ admixture for the 879-keV transition. The mixing ratio $\delta(879) = 16.3 \pm 1.3$ [a weighted average of the values obtained from the (299-879) and (879-87) correlations] together with the total conversion coefficients $\beta(M1) = 6.9 \times 10^{-3}$ and $\alpha(E2) = 4.21 \times 10^{-3}$ were used to determine

$$0 \leqslant \frac{T(E0)}{T(E2_\gamma)} \leqslant 0.010 .$$

The 879-keV transition is therefore primarily $E2$ with an admixture of approximately 0.5% $M1$ and (0–1.0)% $E0$. These small $M1$ and $E0$ admixtures could be the result of mixing of the γ-vibrational and ground-state rotational bands.

In addition to the $E0$ calculations an attempt was made to resolve some of the ambiguities of the K assignments for the negative-parity states in ^{160}Dy. The 299-keV transition ($\leqslant 0.08\%$ $M2$), the 1178-keV transition ($\leqslant 0.4\%$ $M2$), and the 1272-keV transition ($<0.09\%$ $M2$) have been determined to be primarily $E1$, thus providing evidence for rejecting the previous $K = 2$ assignment for the 1265-keV state. The predominately $E1$ character of these transitions from the negative-parity states supports the assumption of K mixing for the negative-parity bands made by Gunther, Ryde, and Krien.[16]

^{134}Ba

The level scheme of ^{134}Ba from the decay of 2.1-yr ^{134}Cs has been well established.[17,18] The spins and parities of the levels, as well as the multipolarities of most of the transitions in ^{134}Ba, have also been determined.[17,19] System I was utilized for the (802-563) and (475-1168) correlations. The (475-563) and (475-563-605) correlations required the use of system II. The results of the correlation measurements are presented in Table II. The mixing ratio for the 563-keV transition, $\delta(563) = 8.1 \pm 2.4$, was obtained from the (802-563) correlation. The mixing ratio, $\delta(475) = 10.3 \pm 3.0$, is the weighted average value of the two values obtained from the (475-1168) and (475-563) correlations in which $\delta(1168) = 0$ and $\delta(563) = 8.1 \pm 2.4$. These mixing ratios together with the total conversion coefficients $\alpha(E2) = 7.73 \times 10^{-3}$ and $\beta(M1) = 9.04 \times 10^{-3}$ were used to determine

$$\frac{T(E0)}{T(E2_\gamma)} < 0.045$$

for the 563-keV $2^{+\prime} \rightarrow 2^+$ transition.

One can see that the 563-keV transition is mainly an $E2\gamma$ transition with at most a 4.5% $E0$ electron contribution. This small $E0$ admixture may be due to an anharmonic term in the nuclear po-

TABLE III. Directional-correlation results for ^{110}Cd.

Cascade (E in keV)	Sequence	A_2, A_4	$\delta(\gamma)$
(764, 687)	$5(Q)3(D, Q)2$	-0.228 ± 0.006 -0.027 ± 0.008	$\delta(687) = 0.79 \pm 0.13$
(818, 658)	$2(D, Q)2(Q)0$	0.417 ± 0.012 0.173 ± 0.016	$\delta(818) = 1.20 \pm 0.07$
(687, 818, 658)	$3(D, Q)2(E0, M1, E2)2(Q)0$	-0.051 ± 0.021 [a] -0.033 ± 0.028	
(744-818-658)	$4(Q)2(E0, M1, E2)2(Q)0$	0.017 ± 0.009 [a] 0.006 ± 0.013	
(1505, 658)	$3(D, Q)2(Q)0$	-0.512 ± 0.019 -0.003 ± 0.027	$\delta(1505) = 0.84^{+0.19}_{-0.10}$ [b]
(764, 1505)	$5(Q)3(D,Q)2$	-0.203 ± 0.018 0.017 ± 0.027	$\delta(1505) = 0.46^{+0.18}_{-0.10}$ [b]
(678, 885)	$4(D,Q)4(Q)2$	0.248 ± 0.005 0.002 ± 0.007	$\delta(678) = 0.245^{+0.054}_{-0.034}$ [c]
(678, 885, 658)	$4(D,Q)4(Q)2(Q)0$	0.252 ± 0.009 -0.004 ± 0.012	$\delta(678) = 0.29^{+0.11}_{-0.08}$ [c]

[a] The weighted average value of these two values is $T(E0)/T(E2_\gamma) = 0.097^{+0.139}_{-0.097}$.
[b] The weighted average value of these two values is $\delta(1505) = 0.64 \pm 0.10$.
[c] The weighted average value of these two values is $\delta(678) = 0.27 \pm 0.05$.

tential since the anharmonic-oscillator model allows $E0$, $2^{+\prime} \rightarrow 2^+$ transition.

^{110}Cd

System II was utilized to measure the ^{110}Cd (764-687), (818-658), (687-818-658), and (744-818-658) correlations. The results are presented in Table III. The spins of the levels as well as the multipolarities of the transitions in ^{110}Cd have been well established.[19-22]

The mixing ratio for the 818-keV transition, $\delta(818) = 1.20 \pm 0.07$, was obtained from the (818-658) correlation. The mixing ratio, $\delta(687) = 0.79 \pm 0.13$, was obtained from the (764-687) correlation. These mixing ratios together with the total conversion coefficients $\alpha(E2) = 1.85 \times 10^{-3}$ and $\beta(M1) = 2.15 \times 10^{-3}$ were used to determine

$$\frac{T(E0)}{T(E2_\gamma)} = 0.044\ ^{+0.179}_{-0.044}$$

for the 818-keV $2^{+\prime} \rightarrow 2^+$ transition.

The Compton edge of the 885-keV γ ray falls at the position of the 687-keV γ ray. Consequently the analysis of the (687-818-658) correlation is very difficult and results have a larger uncertainty than is normal.

An attempt was made to circumvent this situation by utilizing the (744-818-658) correlation. Even though the 744-keV γ transition is less intense than the 687-keV transition, the Compton distribution beneath it can be more readily taken into account. Using the results of the (744-818-658) correlation one obtains

$$\frac{T(E0)}{T(E2_\gamma)} = 0.176\ ^{+0.219}_{-0.176}$$

for the 818-keV $2^{+\prime} \rightarrow 2^+$ transition.

It can now be shown using the weighted average

$$\frac{T(E0)}{T(E2_\gamma)} = 0.097\ ^{+0.139}_{-0.097}$$

that the 818-keV transition is

$$(5.4^{+7.8}_{-5.4})\%\, E0 + (38.8 \pm 5.7)\%\, M1 + (55.8 \pm 5.0)\%\, E2 .$$

The anharmonic-oscillator model allows some $E0$ contribution for the $2^{+\prime} \rightarrow 2^+$ transition; however, no calculations have as yet been made concerning the relative magnitude of this contribution. In addition the anharmonic-oscillator model fails to account for the large $M1$ admixtures to many of the transitions in ^{110}Cd. For example, the 1505-keV transition is $M1 + (29.1 \pm 6.4)\%\, E2$, the 687-keV transition is $M1 + (38 \pm 8)\%\, E2$ and the 678-keV transition is $M1 + (6 \pm 2)\%\, E2$.

^{82}Kr

The level structure of ^{82}Kr following the β decay of ^{82}Br($T_{\frac{1}{2}} = 35.4$ h) has been repeatedly investigated.[23-27] The decay scheme of ^{82}Br to levels in ^{82}Kr has been well established.[25,26] The spins and parities of most of the levels as well as the multipolarities of many of the transitions in ^{82}Kr have been determined.[19,24,25,27] System I was used for the (619-1475), (554-619), (619-698), and (698-776) correlations. System II was used for the (619-698-776) correlation. The results of the correlation measurements are presented in Table IV.

The mixing ratio, $\delta(698) = 2.55 \pm 0.32$, was obtained from the (698-776) correlation. The mixing ratio, $\delta(619) = -1.93 \pm 0.07$, is the weighted average value of the three values obtained from (619-1475), (554-619), and (619-698) correlations

TABLE IV. Directional-correlation results for ^{82}Kr.

Cascade (E in keV)	Sequence	A_2, A_4	$\delta(\gamma)$
(619-1475)	$3(D,Q)2(Q)0$	0.144 ± 0.008 -0.077 ± 0.013	$\delta(619) = -1.93 \pm 0.07$ [a]
(554-619)	$4(D)3(D,Q)2$	-0.115 ± 0.014 -0.004 ± 0.021	$\delta(619) = -1.95^{+0.41}_{-0.64}$ [a]
(619-698)	$3(D,Q)2(D,Q)2$ [b]	0.088 ± 0.006 0.028 ± 0.009	$\delta(619) = -1.91^{+0.24}_{-0.27}$ [a]
(698-776)	$2(D,Q)2(Q)0$	-0.282 ± 0.013 0.273 ± 0.019	$\delta(698) = 2.55^{+0.32}_{-0.31}$
(619-698-776)	$3(D,Q)2(E0, M1, E2)2(Q)0$	-0.016 ± 0.008 0.037 ± 0.010	

[a] The weighted average mixing ratio is $\delta(619) = -1.93 \pm 0.07$.
[b] The mixing ratio $\delta(698) = -2.55 \pm 0.32$ was used in the analysis.

[$\delta(1475)=0$, $\delta(554)=0$, and $\delta(698)=-2.55\pm0.32$ were used in the analysis]. The total conversion coefficients $\alpha(E2)=1.22\times10^{-3}$ and $\beta(M1)=1.02\times10^{-3}$ were used with the above mixing ratios to determine

$$\frac{T(E0)}{T(E2_\gamma)}=0.009\,^{+0.073}_{-0.009}$$

for the 698-keV $2^{+\prime}\rightarrow2^+$ transition.

The 698-keV transition is $(1\,^{+6}_{-1})\%\,E0+(13\pm3)\%\,M1+(86\pm3)\%\,E2$. As in ^{110}Cd the anharmonic-oscillator model may account for the $E0$ admixture to the $2^{+\prime}\rightarrow2^+$ transition, however, it is unlikely that it can account for the appreciable $M1$ admixtures of the 698-keV transition [$(13\pm3)\%\,M1$] and the 619-keV transition [$(21.2\pm1.2)\%\,M1$].

SUMMARY

The $E0$ contribution for $2^{+\prime}\rightarrow2^+$ transitions in the medium-mass "vibrational" nuclei are most likely due to an anharmonic term in the nuclear potential. However, they may be due to the addition of a slightly deformed rotational motion of the nuclear surface to the vibrational motion.

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