Investigation of the ^{208}Pb(p,p') Reaction at $E = 54$ MeV*

M. B. Lewis, F. E. Bertrand, and C. B. Fulmer
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 30 August 1972)

Collective excitations of ^{208}Pb were investigated by inelastic proton scattering at 54 MeV. The spectra covered the entire bound-state region for scattering angles between 11 and 58°, and scattered particles were detected in a broad-range spectrograph with energy resolution $\approx$35–40 keV full width at half maximum. The experimental angular distributions were compared to those predicted by distorted-wave Born-approximation calculations, utilizing a collective-model form factor. The location of the low-spin members of a sequence (2^+–8^+) of positive-parity states was confirmed. Likely candidates for $J > 8$ states were found, as well as several collective fragments of 3^- and 4^+ strengths. No appreciable fragmentation of the 2^+ strength was found. Energy-weighted sum-rule strengths were estimated for all multipoles studied and were compared with the measured values.

[NUCLEAR REACTIONS ^{208}Pb(p,p'), $E = 54$ MeV; measured absolute $\sigma(E,\theta)$. ^{208}Pb levels, deduced L, β_L^2. Spectrograph.]

1. INTRODUCTION

The applicability of the nuclear shell model to nuclear level structure probably reaches its highest degree in ^{208}Pb. This fact is an expected consequence of: (1) the double shell closure ($Z = 82$, $N = 126$) property, (2) the statistically large nucleon number, and (3) the effective weakness of the two-nucleon force because of the large nuclear volume (or surface) in which the two-particle wave functions are normalized. It is the object of the present study to examine more closely the collective properties of ^{208}Pb by the method of inelastic scattering.

Although there exist abundant energy level data to 7 MeV excitation, it is apparent from Ref. 1 that the corresponding information on collective transition strengths is scarce. Coulomb excitation[2] has been measured only for the first excited (3^-) state. Inelastic electron data[3] have added to the knowledge of collective excitations up to about 6 MeV. However, except for the first collective 3^-, 5^-, and 2^+ states, the limited resolution of the electron experiments [full width at half maximum (FWHM) $\gtrsim$100 keV] makes level property assignments questionable due to the high level density. Inelastic scattering experiments with α particles[4] have also suffered from the limitation of energy resolution as well as from the problem of ambiguous angular distribution structure. The analysis of inelastic proton data[5] at 24.5 MeV significantly increased our understanding of collectivity in ^{208}Pb by identifying a kind of positive-parity band ($J = 2^+, 4^+, 6^+, 8^+$) between 4.086 and 4.608 MeV. The resolution (FWHM $\approx$25 keV) in this work was a considerable improvement over that of earlier studies, but the low-energy bombardment yields rather characterless angular distributions for high-lying states. For example, in Ref. 5 a 4.690-MeV level is proposed to be 10^+, and we will show below that this assignment should be 3^-.

In the measurements presented below, we extend the information found from the lower energy proton inelastic scattering measurements. Through the use of higher bombarding energy, the angular distribution characteristics are considerably enhanced, allowing more positive l determination, and thus we are able to examine nearly the entire bound-state excitation region.

2. EXPERIMENT

The experiments were performed at the Oak Ridge isochronous cyclotron (ORIC) using a beam of 54-MeV protons. Inelastically scattered protons were detected on photographic plates in a broad-range spectrograph. An energy resolution of $\approx$35–40 keV FWHM was obtained.

Figure 1 shows part of the ^{208}Pb(p,p') spectrum at 45° in the laboratory. Peaks from many strongly excited states are observed in the spectrum, and several of the peaks have strengths comparable to that of the well-known 2^+, 4^+, 6^+ group. Several peaks are seen to be broad (as compared with the 3.198-MeV peak, for example). These broad peaks were carefully analyzed and yielded multiple levels which are discussed in more detail below. Angular distributions between $\theta_L = 11°$ and $\theta_L = 58°$ were obtained for those peaks shown in Fig. 1 which are labeled with an excitation energy.

Since the ^{208}Pb target used was not of uniform thickness, absolute cross sections were obtained by normalization of the measured 45° elastic scat-

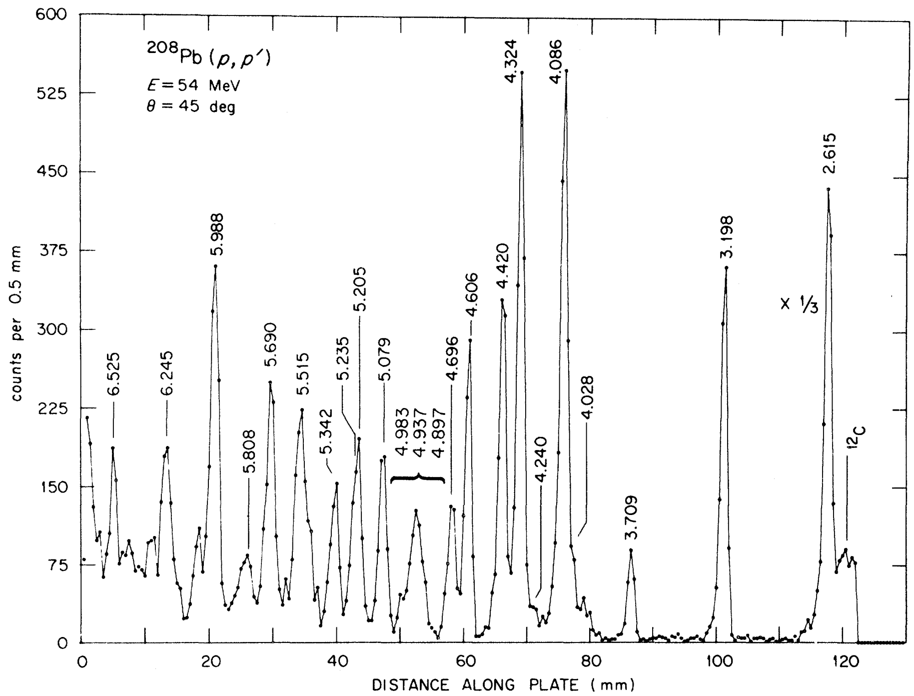

FIG. 1. Spectrum of ^{208}Pb levels in the ^{208}Pb(p,p') reaction at $\theta_{lab}=45°$.

tering to that predicted by the optical model using the parameters shown in Table I. In addition, a second normalization was made utilizing the accurately measured[2] $B(E3)$ for the 2.615-MeV 3^- state. Good agreement was obtained using the two methods and we have assigned a ±30% uncertainty to the absolute cross sections. In some cases the uncertainty may be greater due to factors such as poor separation of close-lying peaks. (The absolute cross sections, if calculated directly from the normal target thickness, were found to be ≈20% higher than those shown here, still well within the assigned 30% uncertainty.)

3. ANALYSIS

A. Point of View

We assume in our analysis that the shape and magnitude of most of the differential cross sections in ^{208}Pb(p,p') at 54 MeV can be described by the usual approach in which the orbital angular momentum L' is absorbed by the target in the excitation of level J, and the final-state parity is $\pi_f=(-1)^{L'}$. Furthermore, spin effects are assumed to be unimportant and we expect that $L'=L$, the angular momentum transferred to the continuum. Finally, the reaction is presumed to be one step, i.e., can be treated by the first-order distorted-wave Born-approximation (DWBA).[6]

B. DWBA Treatment

The theoretical angular distributions shown in Fig. 2 were obtained from first-order distorted-wave analysis using the code DWUCK.[7] This analysis used a collective-model $[\partial/\partial r(U+iW)]$ form factor for the interaction, as has been successfully applied to the (p,p') reaction at lower (≤40-MeV) proton energies[8] as well as higher energies.[9,10]

The optical-model parameters considered in this analysis are those deduced at 62,[9] 55,[11] and 40 MeV,[8] all on ^{208}Pb or ^{209}Bi. The 55-MeV set was somewhat different in that the imaginary volume was more than twice as deep as for the 40- or 60-MeV set. This set accentuated the maxima and minima in the predicted inelastic angular distributions well beyond those which we

TABLE I. Optical-model parameters used in the DWBA treatment of the present data (conventional notation); see text for discussion of the parameters. (Units: MeV, fm.)

V	r_0	a	W	r'_0	a'	V_s
−42	1.20	0.74	−8.4	1.39	0.66	7.0

measured. We believe the 40- and 62-MeV sets are consistent although their parameters resulted from slightly different analyses. The 40-MeV set yields similar in phase, but somewhat less structured, inelastic angular distributions than the 62-MeV set. The set adopted for this analysis is an interpolation to 55 MeV between the 40- and 62-MeV sets. This set is shown in Table I in which the parameters correspond to a potential of the Woods-Saxon form given in Ref. 9.

The nonlocality correction ($\beta = 0.85$) incorporated in the DWUCK code has the effect of reducing the predicted cross section (≈15%). Coulomb excitation corrections were also included for all L values but were significant for only $L=1$ and $L=2$ angular distributions. These latter curves were computed with 75 partial waves and an integrated radius of 60 fm. The effects of the Coulomb excitation were important for $\theta \lesssim 20°$. In the $L=2$ case the Coulomb and nuclear fields interfered destructively, and damping of the forward-angle cross section resulted. In the $L=1$ case the Coulomb effects were large enough to enhance the forward-angle predictions beyond the purely nuclear predictions.

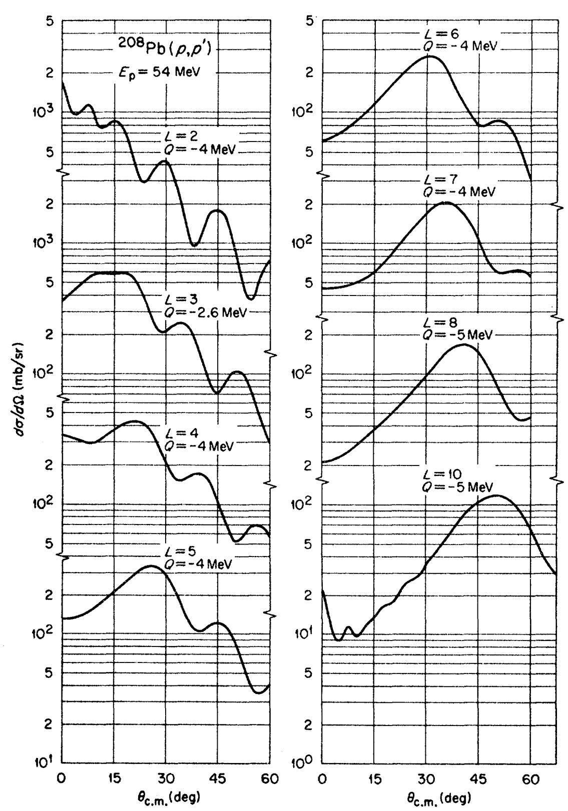

FIG. 2. DWBA predictions for ^{208}Pb(p,p') reaction for cases applicable to the present measurements. See text for discussions of parameters.

The predictions for the angular distributions shown in Fig. 2 illustrate the unique characteristic shape of each L transfer when the bombarding energy is as high as 54 MeV. We noted that the differences in shape between predicted angular distributions for neighboring L values for heavy targets became considerably more pronounced for $E_p \gtrsim 40$ MeV.

4. EXPERIMENTAL RESULTS

The level energies in ^{208}Pb determined in this experiment are shown in the first column of Table II. The uncertainty in the excitation energy for an isolated peak was dominated by the uncertainty in peak location which we estimated to be no more than 15 keV. Peaks which are weak or poorly resolved have greater uncertainty as shown in Table II. Our objective was to study the entire bound-state region of ^{208}Pb, and the levels shown represent all peaks in the spectra (up to ≈7 MeV) which were strong enough to be analyzed. Certainly levels other than those shown are excited in the high-energy region of ^{208}Pb; however, within the statistics and energy resolution available in our experiment, no other peaks were analyzable.

The determination of L values was accomplished by assuming the excited states were of natural parity (no L mixtures) and by finding the best fit to each experimental curve. Since states with known $L=2$, 3, 4, 5, 6, and 8 are clearly observed, the quality of the calculation could be carefully determined, and the empirical angular distribution could be compared, if needed, to made an L-transfer determination.

For most of the levels excited, a deformation or coupling strength parameter $\beta_L{}^2$ was calculated by the usual relation $\beta_L{}^2 = \sigma_{\rm exp}(\theta)/\sigma_{\rm DW}(\theta)$. The results for L and $\beta_L{}^2$ determinations for each level are given in Table II. Fits to the angular distributions of individual levels are given in Fig. 3 and discussed below.

5. SUMMARY OF THE COLLECTIVE LEVEL PROPERTIES

2^+ Levels

The only quadrupole state found in the present measurements is the well-known 4.086-MeV level. Other 2^+ levels in ^{208}Pb excited by two-nucleon transfer reactions[12] at 4.934, 5.550, 5.629, 5.801, 5.973, and 6.172 MeV appear to have negligible collective strength, although the region near 6.2 MeV, in which 2^+ strength has been reported,[3] contains an unresolved multiplet.

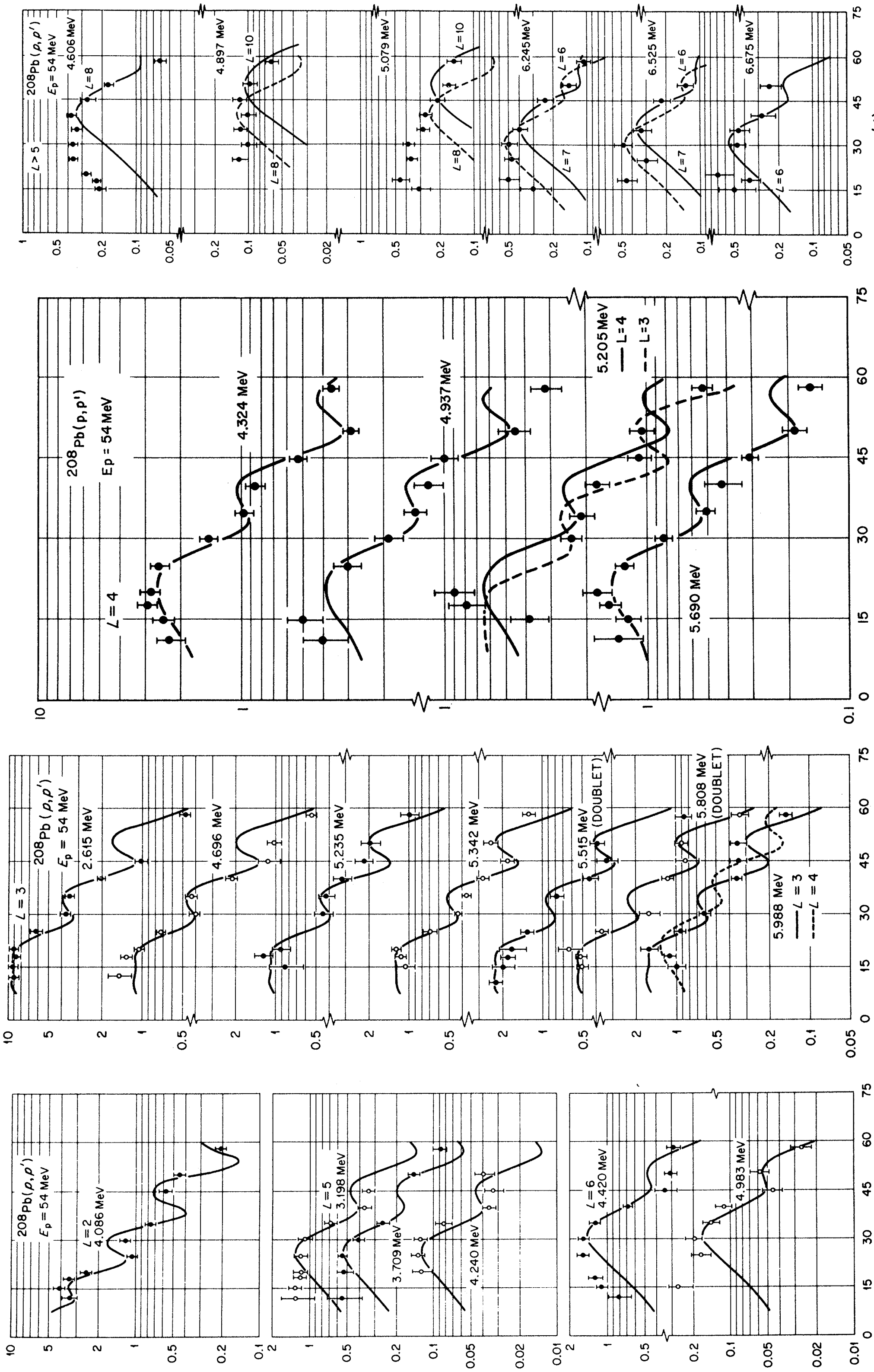

FIG. 3. (a)–(d) Experimental angular distributions and DWBA predictions arranged in groups characterized by value of L transfer.

3^- Levels

As expected from the fact that 3^- states can be formed by a plethora of particle-hole excitations across the 82 and 126 shell closures, we find the inelastic proton spectra are dominated by octupole transitions. As can be seen in Fig. 3(b), several collective fragments of the octupole strength were found in levels at 4.696, 5.235, 5.342, 5.515, 5.808, and 5.988 MeV. The 4.696-MeV level had previously been assigned 10^+ (Ref. 5) but appears to have properties[1] of the $(2d_{5/2}3p_{1/2}{}^{-1})$ neutron and $(1h_{9/2}2d_{3/2}{}^{-1})$ proton configurations. The 5.342- and 5.515-MeV levels may correspond to levels excited but not resolved in ^{209}Bi(d, ^{3}He) measurements[13] which excite proton particle-hole states. The L-value determination for the 5.988-MeV level is uncertain and an $L = 4$ assignment cannot be rejected.

4^+ Levels

In addition to the well-known hexadecapole state at 4.324 MeV, three other fragments of the strength were found at 4.937, 5.205, and 5.690 MeV and the angular distributions are shown in Fig. 3(c). The 4.937-MeV level has not been previously reported. Levels at 5.205 and 5.690 MeV may again correspond to unresolved particle-hole levels[13] seen in ^{209}Bi(d, ^{3}He). A possible $L = 3$ assignment cannot be rejected for the 5.205-MeV level [see Fig. 3(c)].

TABLE II. Level energies, L transfers, and strengths observed in ^{208}Pb(p, p'), $E_p = 54$ MeV.

E^*(MeV) ±20 keV [a]	L	$\beta_L{}^2$ (W.u.)	E^*(MeV) ±20 keV	L	(W.u.)
(2.615) [b]	3	(32) [c]	4.847	(1) [d]	≤0.2
(3.198) [b]	5	9.3	4.897	≈10	≈2
(3.475) [b]	(3) [d]	<0.2	4.937	(4)	2.0
(3.709) [b]	5	3.8	4.983	(6)	1.6
3.922?			5.079	≈10	≈4
3.959	(3) [d]	≤0.6	5.110		
4.028	≥6		5.205	4, 3	3.0, 2.2
(4.086) [b]	2	8.9	5.235	(3)	3.4
4.150			5.342	3	4.2
4.240 20	5	0.86	5.515 [e]	(3)	8.2 [e]
4.324 12	4	13.7	5.690	4	7.5
4.420 12	6	13.5	5.808 [e]	(3)	1.8 [e]
4.50			5.988	3, 4	5.2, 6.6
4.606 15	(8)	5.8	6.245 [e]	≈7	≈5 [e]
4.696 15	3	3.8	6.525	≈7	≈4
4.749			6.675	≥6	

[a] Unless noted.

[b] Adopted from Ref. 1 in order to normalize energy scale.

[c] Adopted from Ref. 1 in order to normalize DWBA.

[d] Assumed from other work in order to assign a limit for collective strength.

[e] Unresolved doublet; $\beta_L{}^2$ value refers to sum of both members assuming same L.

5^- Levels

Angular distributions for 5^- levels are shown in Fig. 3(a). In addition to the well-known 5^- levels at 3.198 and 3.709 MeV, we report an $L = 5$ angular distribution for a level at 4.240 MeV which is weakly excited and could be of unnatural parity. This level may correspond to one at 4.258 MeV in Ref. 1.

$\geq 6^+$ Levels

At the bottom of Fig. 3(a) and on Fig. 3(d) are shown measured angular distributions for the relatively high angular momentum transfers. Levels of $J = 6^+$ at 4.420 and $J = 8^+$ at 4.606 MeV were first reported in the 24-MeV (p, p') study.[5] Our angular distributions corroborate the earlier assignments but display an inconsistency with the forward-angle predictions of DWBA. (The forward-angle discrepancy becomes noticeable for the $L = 5$ angular distribution comparisons.) The underestimate of the theory for $\theta \lesssim 30°$ becomes more severe with increasing L transfer. Levels reported here at $E = 4.897$ and 5.079 MeV show little angular distribution structure and correspond to expected high-spin states[13] seen in ^{209}Bi(d, ^{3}He) spectra. These levels probably correspond to the $(1h_{9/2}1h_{11/2}{}^{-1})$ proton particle-hole states in which J may be as high as 10^+. Levels at 6.245, 6.525, and 6.675 MeV have angular distributions indicative of L transfers $\approx 6-8$, but these assignments should be considered tentative.

6. CONCLUSIONS

We have measured angular distributions for collective states in ^{208}Pb in inelastic proton scattering. The results indicate that the collective 3^- and 4^+ strengths are considerably fragmented but 2^+ strengths are much less so. Shell-model calculations for ^{208}Pb include those of Gillet, Green, and Sanderson[14] and the recent work of True, Ma, and Pinkston.[15] Both calculations utilize the full $1\hbar\omega$ configuration space for negative-parity states. The former calculation includes ground-state correlations [random-phase-approximation (RPA) corrections] but does not introduce effective charges, while the latter calculations does not correct for ground-state correlations but makes use of phenomenological nuclear forces and effective charges. Since only the calculations of Ref. 14 include positive- as well as negative-parity states, we show in Fig. 4 the comparison between

Ref. 14 and our experimental results. While there appears to be fair agreement between the predicted[14,15] and measured energy levels, the predicted strengths[14,15] for many of the 3^- fragments are nearly an order of magnitude smaller than those measured. This discrepancy emphasizes the importance of the effective charge for neutrons in the inelastic scattering form factor. There appears to be little if any correspondence between the measured and calculated positive-parity states. For example, the predicted low-lying 2^+ state at 4.46 MeV absorbs only a small fraction of the total quadrupole strength predicted in the bound-state spectra.

An energy-weighted sum of each multipole strength in this work is given in Table III. When compared to the isoscalar sum rule,[16,17] we find that only the octupole vibration contributes a large part of its sum-rule strength in the bound-state spectra of ^{208}Pb. This is reasonable since most

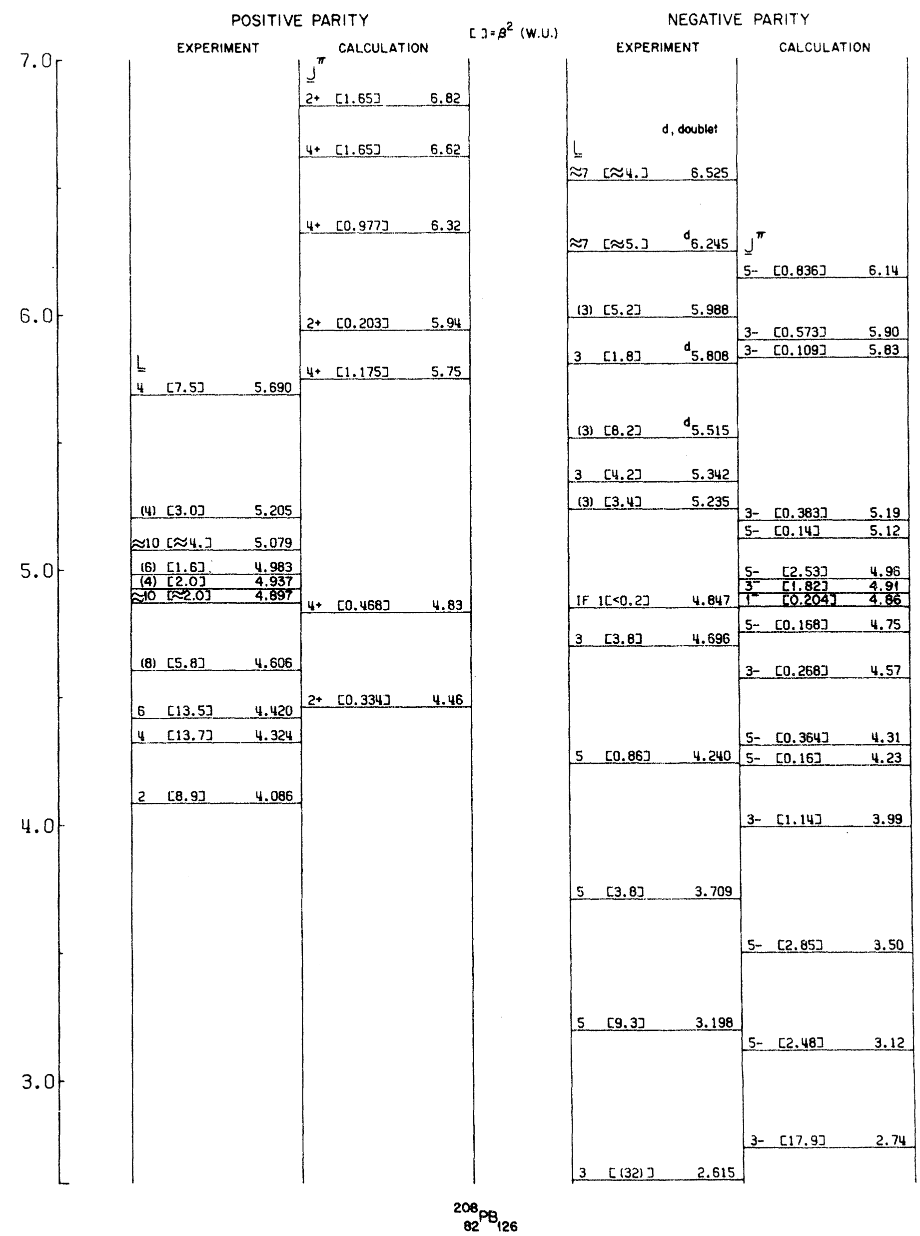

FIG. 4. Comparison of measured levels and corresponding collective strengths to those predicted by RPA shell-model calculations. The spectra are separated into positive- and negative-parity levels.

TABLE III. A summary of the total transition strength to bound-state levels in ^{208}Pb as measured in the ^{208}Pb-(p,p') reaction. Column 1 is L transfer; column 2 the energy-weighted strength for each L transfer; column 3 the theoretical isoscalar (or uniform distribution) energy-weighted sum rule; and column 4 is the percent of column 3 as given in column 2 for each L value.

L	$\sum E_i\beta_i^2$ measured (MeV W.u.)	$\sum E_i\beta_L^2(T=0)$ sum rule (MeV W.u.)	Percent of $(T=0)$ sum rule measured
2	36	224	16%
3	229	487	47%
4	127	882	14%
5	48	1425	3.3%
6	68	2160	3.1%
7	≈57	3128	≈1.8%
8	≈55 ($L \geq 8$)	4380	≈1.3%

of the configuration space ($1\hbar\omega$ particle-hole) is associated with energies of the order 3–6 MeV and can form $J^\pi = 3^-$ states. Positive-parity states require at least $2\hbar\omega$ particle-hole excitations to utilize large configuration spaces and this requirement implies excitation energies in the continuum region. Evidence that the giant isoscalar-quadrupole (exhausting the remaining sum rule) can be identified in the continuum ($E^* = 11.0 \pm 0.5$ MeV) near the excitation energy predicted[18] by Mottelson has recently been reported.[19]

Finally, we note that the conventional direct-reaction inelastic scattering analysis appears to be inadequate for high L transfers in (p,p'). Two explanations which present themselves are: (a) Two-step processes become competitive with weakly excited high L transfers (especially at very forward angles) and coupled-channel analysis becomes imperative; (b) the large L transfers sample the extremity form factor tail, the magnitude of which is sensitive to various nuclear potential interactions and nuclear distributions.

ACKNOWLEDGMENTS

We wish to thank the operations staff of the ORIC for efficient operation and helpful assistance rendered in this work. We are also grateful to Vestel Jones and Ruby Shelton for their accurate reading of the photographic plates.

*Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

[1]M. R. Schmorak and R. L. Auble, Nucl. Data B5(No. 3), 207 (1971); M. B. Lewis, Nucl. Data B5(No. 3), 243 (1971); M. J. Martin, Nucl. Data B5(No. 3) 287 (1971).

[2]A. R. Barnett and W. R. Phillips, Phys. Rev. 186, 1205 (1969).

[3]J. F. Ziegler and G. A. Peterson, Phys. Rev. 165, 1337 (1968).

[4]J. Alster, Phys. Rev. 141, 1138 (1966); Phys. Letters 25B, 459 (1967).

[5]J. Saudinos, G. Vallois, O. Beer, M. Gendrot, and P. Lopato, Phys. Letters 22, 492 (1966); J. Saudinos, G. Vallois, and O. Beer, Nucl. Sci. Appl. 3, No. 2, 22 (1967); G. Vallois, Centre à l'Energie Atomique Report No. CEA-R-3500 (unpublished).

[6]R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240 (unpublished); see entire reference list in Ref. 9.

[7]P. D. Kunz, private communication.

[8]M. P. Fricke and G. R. Satchler, Phys. Rev. 139, B567 (1965).

[9]F. E. Bertrand and M. B. Lewis, Nucl. Phys. A168, 259 (1971).

[10]F. E. Bertrand, J. K. Dickens, and T. A. Love, Phys. Letters 24B, 653 (1967).

[11]H. Kamotsubo, H. Ohnuma, K. Ono, A. Nchida, M. Imaizumi, S. Kobayashi, and M. Sekiguchi, J. Phys. Soc. Japan 22, 19 (1967).

[12]G. Igo, P. D. Barnes, and E. R. Flynn, Ann. Phys. (N.Y.) 66, 60 (1971).

[13]E. A. McClatchie, C. Glashausser, and D. L. Hendrie, Phys. Rev. C 1, 1828 (1960).

[14]V. Gillet, A. M. Green, and E. A. Sanderson, Nucl. Phys. 88, 321 (1966).

[15]W. W. True, Chin W. Ma, and W. T. Pinkston, Phys. Rev. C 3, 2421 (1971).

[16]O. Nathan and S. G. Nilsson, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland, Amsterdam, 1965), Vol. 1, pp. 640-641.

[17]G. R. Satchler, Nucl. Phys. A195, 1 (1972).

[18]B. R. Mottelson, in Proceedings of the Solvay Conference in Physics, Brussels, Belgium 1970.

[19]M. B. Lewis and F. E. Bertrand, Nucl. Phys. A196, 337 (1972).