Accurate Measurement of the Positron Asymmetry from Oriented ^{58}Co†

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(Received 15 December 1972)

An accurate measurement of the asymmetry of the $2^+\rightarrow 2^+$ 474-keV positron group emitted by oriented ^{58}Co is reported. The ^{58}Co source, which had been ion implanted into an iron foil, was cooled to about 0.013°K by thermal contact with a demagnetized chromium potassium sulphate salt pill. The positrons were detected in a semiconductor detector telescope which reduced corrections for backscattering and γ-ray response. A complete description of the detector system and its operation is given. The accuracy of the result reported here is such that it is the first rigorous independent test of the commonly applied β-γ circular-polarization-correlation technique. The asymmetry parameter was found to be $A_1=0.243\pm0.007$ which corresponds to a Fermi–Gamow-Teller mixing ratio $C_V M_F/C_A M_{GT}=-0.0063\pm0.0056$. This result is discussed in terms of possible second-forbidden corrections and used to derive a value for the Fermi matrix element and to determine the isospin mixing of the analog of the daughter in the parent ground state of ^{58}Co.

1. INTRODUCTION

This paper describes an accurate measurement of the positron asymmetry from oriented ^{58}Co. There have been few β-asymmetry results published since the years immediately following the discovery of parity violation. This paucity of data is largely due to the experimental difficulties involved in doing accurate β-ray measurements in the hostile cryogenic environment required for the production of nuclear orientation. The result reported here derives its improved accuracy from technical advances such as semiconductor β- and γ-ray detectors, ion implantation for sample preparation, and orientation in ferromagnetic metals.

In addition to β-ray asymmetries, parity violation provides another valuable measurement possibility – the β-γ circular polarization correlation. The two techniques give comparable and complementary information in those instances where both methods are applicable. There are many cases, however, where only one or the other can be used and thus it is important to perfect both techniques. The circular-polarization method has been widely used although it is also very difficult to apply. The fact that the efficiencies of γ-ray circular-polarization analyzers are very low, results in small experimental effects, and this, coupled with the necessity of a coincidence measurement, makes it hard to reduce the statistical and systematic errors to an acceptable level. β-asymmetry experiments, in contrast, have a raw effect typically 20 times larger and require only singles counting. The β-asymmetry method thus has intrinsic advantages which make it worthwhile to attempt to overcome its obvious experimental problems.

The data from both types of experiment have been summarized by Schopper.[1] His tabulation reveals the poor consistency and low accuracy which have troubled the field. While the general trends are clear in mixed, allowed β decays, fundamentally new and independent input, such as that presented here, is required for more reliable quantitative conclusions. The situation obtaining for ^{58}Co will be discussed more fully in the final section of this paper. Including this work, it has been the subject of 13 investigations with a large spread in the results. The most recent circular-polarization measurement[2] carries a quoted error comparable with that in this work and the agreement is excellent. However, in view of the history of circular-polarization experiments, it would be difficult to place that degree of confidence in such a result without corroboration by an independent method to similar accuracy.

This paper contains a comprehensive description of the methods we have used to overcome the difficulties of accurate β-ray-asymmetry measurements. This result is the best independent experimental test of circular-polarization techniques to date and thus a detailed discussion of the magnitudes and sources of systematic error is given. The results are interpreted in terms of Fermi–Gamow-Teller mixing in the allowed decay and the isospin impurity deduced from the mixing. The question of second-forbidden corrections is briefly considered.

2. BACKGROUND MATERIAL

The decay scheme of ^{58}Co, based on the most recent *Nuclear Data Sheets*, is shown in Fig. 1. Positron emission, with an end point of 474 keV, accounts for 15.5% of the β decays. This 2^+ to 2^+ β decay is allowed and the zero angular momentum change admits the possibility of Fermi–Gamow-Teller mixing. The Fermi component, however, is strongly suppressed by the approximate isospin selection rule $\Delta T = 0$. A measurement of the Fermi component in the decay gives the degree of isospin mixing of the analog of the daughter in the parent state.[1] The interpretation of the β-ray asymmetry in terms of the Fermi–Gamow-Teller mixing of the β transition would be modified by the presence of significant second-forbidden components. The matter of second-forbidden contributions will be discussed in the final section of this paper.

The theoretical description of nuclear orientation is covered in the review articles of De Groot, Tolhoek, and Huiskamp[3] and of Blin-Stoyle and Grace.[4] We shall present here only the results which are relevant to the interpretation of this experiment. The angular distribution of the 811-keV $E2$ γ ray is given by

$$W^\gamma(\theta) = W_0^\gamma[1 - 0.2965 Q_2 B_2 P_2(\cos\theta) + 0.7067 Q_4 B_4 P_4(\cos\theta)]. \quad (1)$$

Here $W^\gamma(\theta)$ is the number of γ rays emitted at an angle θ with respect to the axis of orientation when the nuclei are oriented and W_0^γ is the corresponding number when there is no orientation. B_2 and B_4 are the orientation parameters of the initial β-decaying state. The numerical factors in Eq. (1) take into account the multipolarity of the γ ray and the decrease in the orientation due to the preceding β radiation. Since the Fermi contribution is known to be small, it was assumed in Eq. (1) that the β rays are pure Gamow-Teller. Q_2 and Q_4 are attenuation factors depending on the solid angle subtended by the detectors. The experimentally measured γ-ray anisotropy is defined by

$$\epsilon^\gamma(\theta) = [W^\gamma(\theta)/W_0^\gamma] - 1. \quad (2)$$

The corresponding β-ray anisotropy is defined in a similar fashion. In the allowed approximation the β-ray angular distribution is given by

$$W^\beta(\theta, E) = W_0^\beta(E)[1 + (v/c) Q_1 A_1 B_1 \cos\theta], \quad (3)$$

where E is the energy of the β ray. The factor Q_1 corrects for the finite solid angle of the detector and the attenuation due to multiple scattering in the source. For a $\Delta J = 0$, $J = 2$ positron transition, the quantity of interest, A_1, is given by

$$A_1 = (0.2357 - 1.1547x)/(1 + x^2), \quad (4)$$

where x is the Fermi–Gamow-Teller mixing ratio. This expression assumes the allowed approximation, the $V-A$ form of the interaction and time reversal invariance with maximal violation of parity. The expression for the result of the equivalent β-γ circular-polarization correlation experiment is

$$\tilde{A}_1 = -\frac{1}{\sqrt{2}}(0.2357 + 1.1547x)/(1 + x^2). \quad (5)$$

We shall ultimately convert our values of A_1 to the corresponding value of $\tilde{A}_1$ for comparison with the previous measurements.

In this experiment the nuclei were oriented by a variant of the "brute-force" thermal-equilibrium method. The ^{58}Co nuclei were implanted as dilute impurities in an iron foil. This source was cooled to roughly 0.013°K by thermal contact with a paramagnetic salt which had been cooled by adiabatic

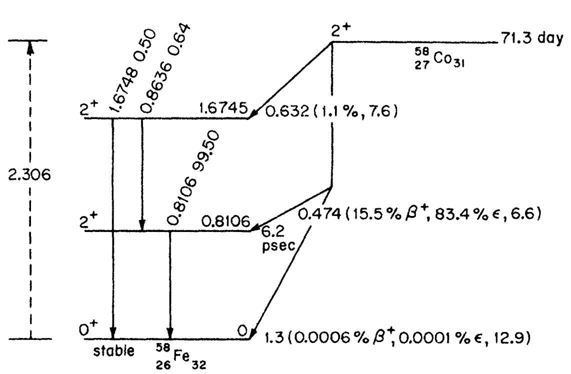

FIG. 1. Decay scheme of ^{58}Co.

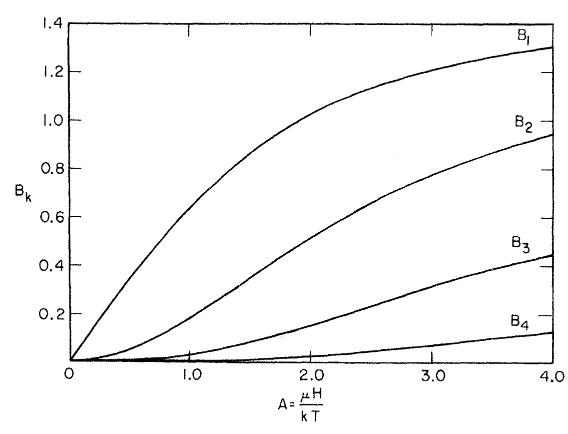

FIG. 2. Orientation parameters for a spin-2 nucleus.

demagnetization from 1°K. The hyperfine field at a Co nucleus in an iron lattice has been measured to be −288 kG.[5] The negative sign signifies that the internal field is directed opposite to the magnetization of the iron. The combination of the large hyperfine field and the low temperature produces the nuclear orientation.

The populations of the magnetic sublevels which appear in the definition of the orientation parameters are given by a Boltzmann distribution:

$$a_m = \exp\left(\frac{m\mu H}{JkT}\right) \Big/ \sum_{m'=-J}^{J} \exp\left(\frac{m'\mu H}{JkT}\right). \qquad (6)$$

Here H is the hyperfine field of Co in Fe and μ is the magnetic moment of ^{58}Co [3.996(11) nuclear magnetons[5]]. The four nonvanishing orientation parameters are shown in Fig. 2 as a function of $A = \mu H/kT$. Most of the data for this experiment were taken with A about 3. Since B_k are monotonic functions of $\mu H/kT$, any one can, in principle, be used to determine the values of the others.

In this experiment the measured anisotropy of the 811-keV γ ray was used to determine the orientation parameters B_2 and B_4. The theoretical relationship between the orientation parameters was then used to calculate B_1. Figure 2 shows that because of its larger value and slower variation with A it is possible to obtain a value of B_1 which has only $\frac{1}{3}$ the relative error of the B_2 which is used to deduce it. The calculated B_1 was combined with the measured β-ray asymmetry to obtain a value for A_1 and the Fermi–Gamow-Teller mixing ratio.

3. CRYOGENIC SYSTEM

Most of the low-temperature equipment used in this experiment has been described previously.[6] The modifications made in the inner Dewar for the observation of the β rays are shown in Fig. 3.

The β detectors were operated at roughly 77°K to ensure reliable performance. This was accomplished by placing them in the 4 to 1°K vacuum space, mounting them on a Teflon support, and using a carbon resistor to heat up the detector assembly. The heat input required was roughly 10 mW. The temperature of the system was monitored by measuring the value of the heater resistor.

The β rays reached the detectors through a 3-μm permalloy window in the bottom of the 1°K tail. The collimator was provided by a 3-mm-thick Cu plate which formed the top surface of the detector assembly. The collimator hole had a 3-μm permalloy covering to protect the detector surface from debris and to keep thermal radiation and hot-gas atoms from striking the thin window in the bottom of the 1°K tail. With this system it was possible to maintain the β detectors at 77°K with no perceptible change in the behavior of the 1°K bath or the degree or duration of nuclear orientation.

The salt-pill assembly is shown in Fig. 4. The chief change since Ref. 6 was the addition of another guard salt between the main chromium potassium sulphate pill and the 0.1°K guard salt. The new salt was a mixture of ferric ammonium sulphate and Octoil-S housed in a copper can which held the middle of the nylon support near 0.05°K following demagnetization. The use of this pill

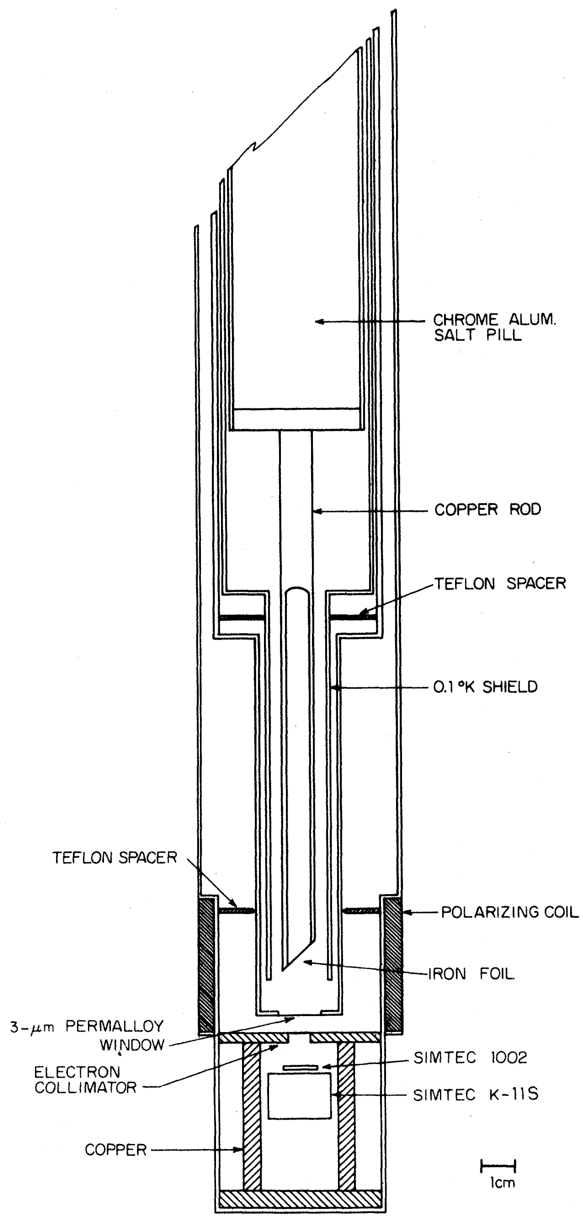

FIG. 3. Drawing of the lower part of the inner Dewar showing the relative positions of the source and the detector.

resulted in an improvement in warm up time by a factor of 2 to 3 so that little loss of orientation was observed during the 10-h counting period following each demagnetization. The slow variation in temperature and the small curvature in the orientation parameters made it possible to average the data over several hours without appreciable error.

The sample, in the form of a 1.25-cm-diam, 25- to 75-μm-thick iron disk was soldered to the end of the copper rod which was in thermal contact with the main chrome-alum pill. The preparation of this pill is described in Ref. 6. The end of the copper rod was milled to hold the sample foil at 45° to the vertical. The 0.95-cm-diam high-purity copper rod was drilled out with a 0.635-cm hole for most of its length and several slits were milled along it to reduce the effects of eddy current heating. The hollowing of the rod served the double purpose of reducing its mass and distributing the heat load from stopped β rays. The iron foil was magnetized with a small superconducting polarizing coil wound around the outside of the 4°K tail. It was found that 1000 G was sufficient to saturate the observed effects; 1200 G was used in the orientation runs reported here.

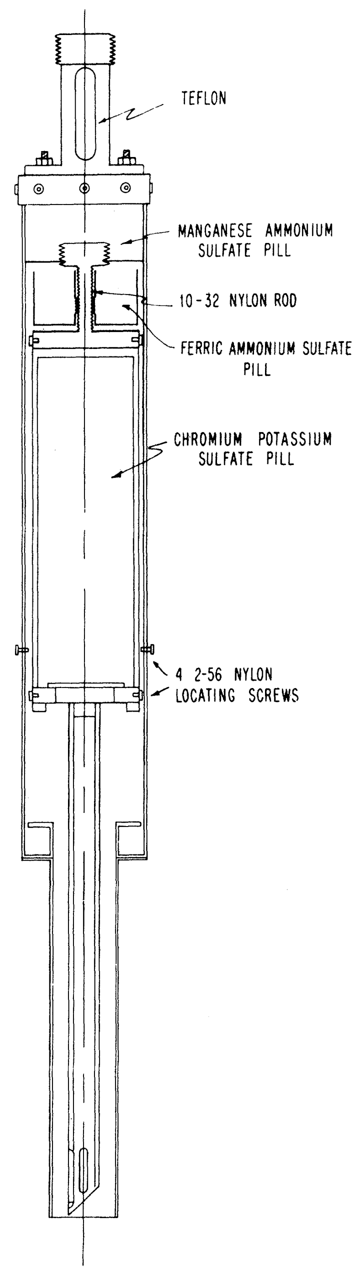

FIG. 4. A drawing showing the salt-pill assembly.

4. SAMPLE PREPARATION AND HANDLING

The choice of a sample-preparation technique depends on the type of measurements to be made and on the chemistry of the elements to be studied. For γ-ray measurements most sources can be prepared in any convenient thickness by alloying or diffusion. For β-ray measurements source thickness is an important consideration because the multiple scattering of the β rays as they leave the source can seriously attenuate the angular distribution. This is illustrated in Fig. 5 where we have plotted the attenuation coefficient Q_1 of the $k=1$ term of the β-ray angular distribution as a function of source thickness and energy. These curves, which were calculated from the multiple scattering theory of Goudsmit and Saunderson,[7] assume a uniform distribution of activity throughout the foil. A 2.5-μm iron foil oriented at 45° has an effective thickness of 2.8 mg/cm^2. Even in this case there is serious attenuation below a few hundred keV in β-ray energy. Not only are such thin foils difficult to handle but only carrier-free isotopes can be used if the impurity level is to be kept low. Similar considerations apply to shallow diffusion into thicker foils with the added problem that the source depth is uncertain so that accurate multiple scattering corrections cannot be calculated.

The use of ion implantation solves both the problems of source thickness and chemistry. High specific activity ^{58}Co obtained from Amersham Searle, Des Plaines, Illinois, was used as charge material for the electromagnetic isotope separator. Ions of ^{58}Co were implanted at an energy of 40 keV into iron disks (median range 18 μg/cm^2). During ion implantation no ion current was detected at the mass-59 beam position confirming the high purity of the carrier-free ^{58}Co charge material. The disadvantage of using ion-implanted sources is that the foil surfaces must be treated with extreme care, since the activity lies so close to the surface. There is also the uncertainty of radiation damage caused by the ion-implantation process which may affect the internal field.

We have found that the following procedures worked well with implanted sources. Before shipment to the isotope separator the foils were cleaned, etched, and encapsulated under vacuum. After implantation, the foils were again stored in evacuated capsules until they were placed in the orientation apparatus. The soldering of the sample foils to the copper rod of the salt-pill assembly was the most risky part of the procedure. The implanted surface was painted with Octoil-S and the foil was soldered to the sample holder with ordinary soft solder. The soldering was carried out in a helium atmosphere. For long-term protection in the cryostat, the foil surface was painted with a 10% solution of Octoil-S in petroleum ether. This provided a coating which prevented corrosion of the surface but which at the same time was thin enough to produce negligible scattering of the β rays. Our ^{58}Co source was treated in this manner and we were able to cycle it between room temperature and helium temperatures several times without any perceptible change in its orientation properties.

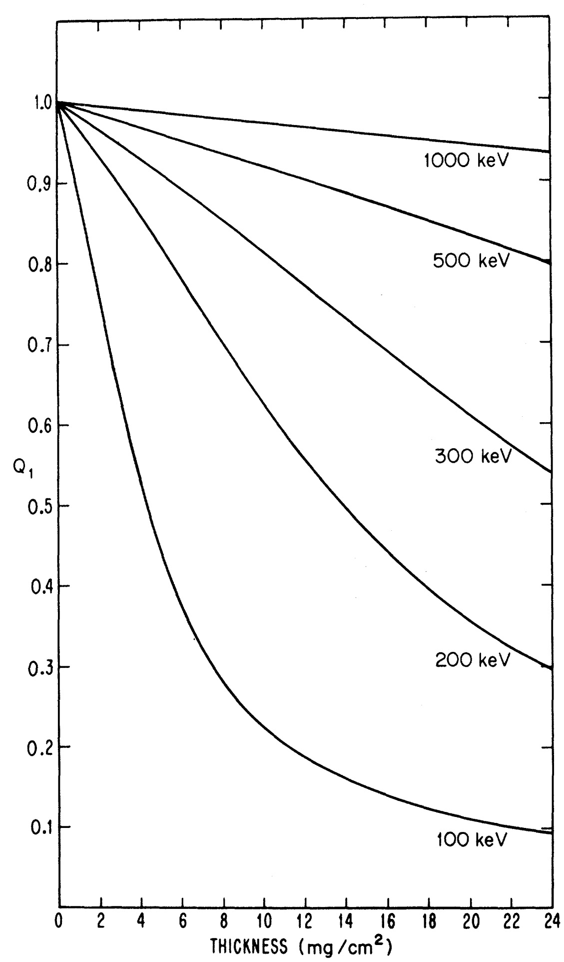

FIG. 5. The multiple scattering attenuation factor for the $P_1(\cos\theta)$ term in the β-ray angular distribution as a function of source thickness and β-ray energy.

5. β- AND γ-RAY COUNTING SYSTEMS

Some of the most persistent problems in accurate nuclear orientation β-ray measurements have been the operation of the detectors in a helium temperature environment, the γ-ray response of the β detectors, and the distortion of the β spectrum by backscattering from the β detectors. Our β-ray counting system has been described briefly in the literature[8]; detailed discussions can be found in thesis form.[9] A similar counter-telescope system has recently been described by Kantele and Passoja.[10]

As described in Sec. 3, the first problem was solved by heating the detectors. In principle it is possible to correct for the γ-ray contamination by taking additional spectra with a β-ray stopper posi-

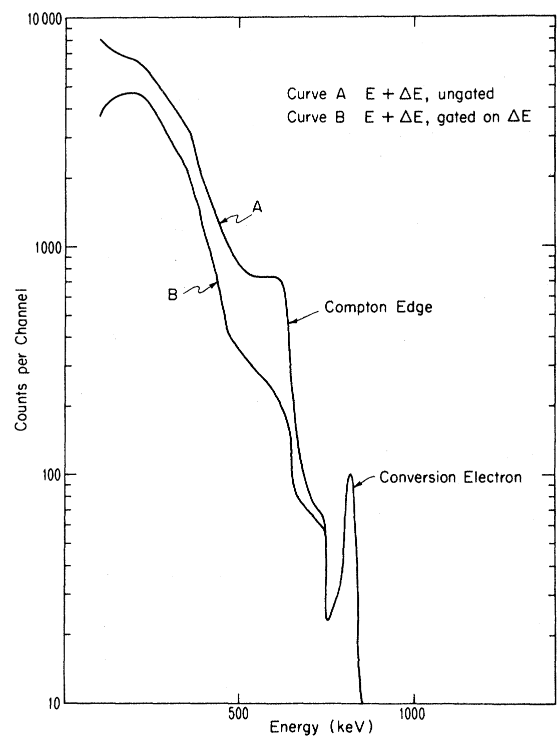

FIG. 6. A plot showing the response of the detector system to the β rays and γ rays from ^{58}Co. Curve A gives the sum of the E and ΔE pulses with no requirement that there be a count in ΔE; curve B gives the sum of the pulses in E and ΔE with the requirement that there be a count in ΔE.

tioned between the source and detectors; however, this is difficult when the detectors are inside a helium cryostat and the γ-ray contamination is large. This problem can be substantially reduced by the use of a detector-telescope arrangement consisting of a thin transmission detector immediately in front of a thick stopping detector. By summing the outputs of the 200-μm ΔE detector (Model 1002, Simtec Ltd., Montreal) and the 1-mm E detector (Model K-11S-Simtec) and gating the pulse-height analyzer (PHA) on events in the ΔE detector, it is possible to reduce the γ-ray response by roughly a factor of 5. This arrangement gives the β-ray stopping power of the combined thicknesses of the E and ΔE counters and the γ-ray response of the ΔE detector. Figure 6 illustrates the use of this arrangement with a ^{58}Co source. Curve A is the ungated spectrum with both positron and γ-ray response. Curve B is taken with gating on ΔE so that the γ-ray response is largely eliminated. The residual γ-ray response shows up in the greatly reduced Compton edge of the 811-keV γ ray.

The other major obstacle to precision β-ray spectroscopy is the backscattering of the β rays from the detector. The backscattering coefficient for electrons in Si is 20 to 30% and thus the spectrum becomes progressively more distorted as the energy decreases. The problem can in principle be solved by measuring the detector response to monoenergetic electrons and then reconstructing the incident spectrum from the measured one. In an orientation measurement, such as is reported here, the relative efficiency of the detector system as a function of energy need not be known, since it cancels out when the β-ray anisotropy is calculated. This fact allowed us to use the β-ray telescope to discriminate directly against backscattered electrons.

First consider an ideal geometry where the ΔE and E detectors are in contact so that any electron passing through ΔE will strike E and any electron backscattering from E will strike ΔE for a second time. We wish to keep only those events where the β ray comes to rest in E. This can be accomplished by summing the outputs of the E and ΔE detectors and gating the PHA on only those events corresponding to a single passage through ΔE. To this end we made measurements of single-passage energy-loss probabilities for electrons. Some of these are shown in Fig. 7.

Such a family of curves enables us to calculate by convolution the total energy-loss probability for a double passage through the ΔE detector with various incident and backscattered energies. Figure 8 shows some curves which are applicable to the 474-keV positron decay of ^{58}Co. Since in this range the rate of energy loss, dE/dx, increases with decreasing energy, it is only necessary to adjust the system for sufficient backscattering suppression with the highest incident energies to insure that any lower energies will be automatically covered. Most of the data of this experiment were taken with the ΔE energy window set to reject events with an energy loss above 130 keV. This gives a suppression of backscattered events by about a factor of 8 in the worst case, which is denoted as curve A in Fig. 8. The suppression factor is about 16 in the more realistic case B.

Our system departed from ideal geometry in that the ΔE detector was separated from the E detector by about 0.64 mm. As a result some of the backscattered electrons missed the sensitive region of the ΔE detector. The detectors were both circular with an area of 100 mm^2. The collimator was 6.4 mm in diameter. The geometrical efficiency was calculated in two steps. Consider a parallel beam of electrons, defined by the collimator, passing through the thin detector and striking the thick detector. The beam is spread somewhat by scattering in the ΔE detector. From

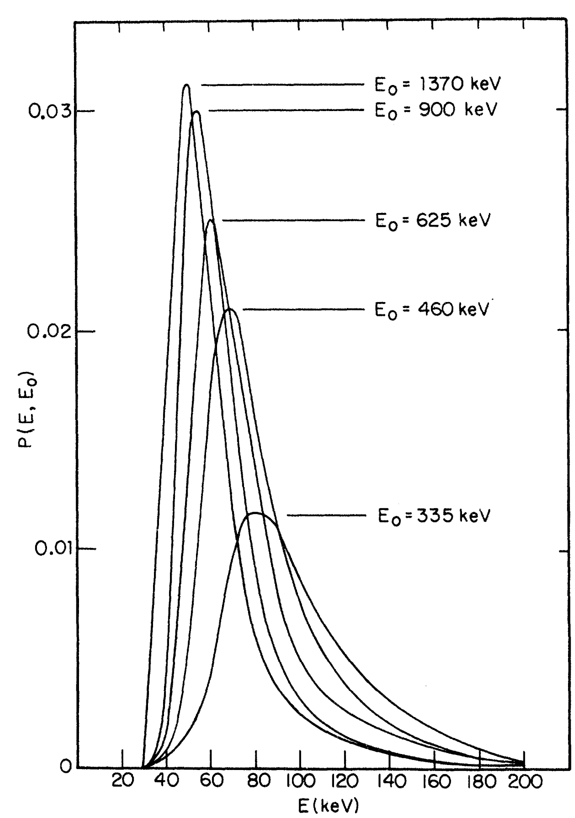

FIG. 7. Energy-loss-probability function for a single passage through the thin detector for several incident energies.

the calculated intensity distribution on the face of the E detector, the probability of the backscattered electrons striking the sensitive region of the thin detector can be calculated. We used the experimental work of Frank[11] for electron transmission and backscattering and the mathematical results of Konijn *et al.*[12-14] to evaluate our geometrical efficiency. For all energies the geometrical efficiency was at least 93% and for high incident energy electrons this efficiency exceeded 97%. Combining the geometrical efficiency with that calculated from the energy loss and bearing in mind that the one is best for high-energy β rays while the other is best for low, we conclude that over the whole energy range the antibackscatter discrimination was better than 90%.

The minimum thickness of the ΔE detector was determined by mechanical considerations, since it had to withstand cycling between room temperature and 4°K. A 100-μm detector disintegrated on the first cycle; our 200-μm detector withstood countless thermal cycles with no deterioration in performance. It was also necessary to have the detector thick enough so that the energy-loss signal from the least ionizing particles would be above the electronic noise. In our system a 200-μm thickness was required for minimum ionizing electrons. The energy calibration of our β detectors was determined *in situ* from the Compton edge of the 811-keV γ ray.

The γ-ray anisotropy was measured with a 30-cm^3 trapezoidal Ge(Li) detector placed outside the Dewar at 0° to the axis of orientation. The detector was used side on for practical reasons. At 0° the linear polarization vanishes so that the polarization sensitivity of the detector in this geometry was not a problem. A 7.6-cm NaI(Tl) scintillation counter was placed at 90° as an added check on the γ-ray anisotropy.

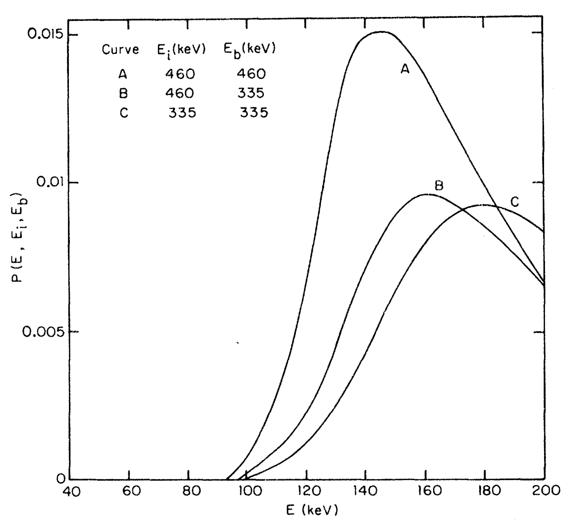

FIG. 8. Energy-loss-probability function for two passages through the thin detector for several incident and backscattered energies.

The Ge(Li) solid-angle corrections were estimated to be $Q_2 = 0.98$, $Q_4 = 0.95$ on the basis of published tables.[15] These values are probably good to 1% although our geometry was not well represented in the tables of Ref. 15. The slower variation of the orientation parameter B_1 with respect to B_2 means that the uncertainty in B_1 due to these corrections was only about $\frac{1}{3}$%. The finite solid-angle correction, Q_1, for the β telescope was calculated from geometry to be 0.986. Through focusing effects the polarizing field increased the effective solid angle by 7%. This had a small effect on Q_1, reducing it to 0.985.

6. EXPERIMENTAL PROCEDURE AND DATA ANALYSIS

The β- and γ-ray anisotropies were determined by comparing the count rates when the nuclei were oriented and unoriented. The "cold" data were taken in 100-min runs for about 10 h after demagnetization. The sample was then warmed to 1°K by the admission of He exchange gas to the salt chamber. After a few minutes this was pumped away and "warm" normalization data were accumulated for a comparable time.

The sample was inclined at about 45° to the vertical so that nominal β detector angles were 45 and 135° depending on the sign of the applied vertical polarizing field. Because of parity violation in the β decay, the β-ray anisotropy changes sign when the direction of orientation is reversed. By combining the data taken with field up and field down it was possible to separate the measured β-ray distribution into an even and odd part under reversal of the polarizing field. The odd term can be written as the product $\rho_\beta(v/c)Q_1A_1B_1\cos\theta$, where ρ_β is the fraction of true β rays in the measured warm spectrum.

The orientation parameters can, in principle, be calculated from the sample temperature and the hyperfine interaction strength. Sample temperatures can be accurately determined from γ-ray anisotropy measurements of well understood systems such as ^{60}Co in iron. Hyperfine fields have been accurately determined in many cases by nuclear magnetic resonance (NMR). Unfortunately, an NMR measurement samples only a fraction of the nuclei in a sample while β- and γ-ray angular distribution measurements average over all the radioactive nuclei. With implanted sources the variation of the average internal field can be quite large. This can be due to poor surface conditions either before or after implantation, radiation damage caused by the implantation process, unusual sites occupied by the implanted atoms, or any combination of these.[16] Annealing after implantation might improve the situation; however, for β-ray spectroscopy purposes it is important to avoid diffusion of the activity and subsequently increased multiple scattering. For this reason the ^{58}Co source was not annealed.

In this experiment the observed ^{58}Co γ-ray anisotropies were about 15% smaller than expected from the known thermal behavior of the system and the known hyperfine interaction of Co in Fe. This was verified after the β-ray measurements were completed by orienting the ^{58}Co sample with a thermally diffused sample of ^{60}Co in Fe.

For the purposes of data analysis two extreme models can be adopted. The first assumes that the ^{58}Co nuclei experience a reasonably narrow range of internal fields such that the average value yields the observed γ-ray anisotropy. This would require an average field of 200 kG rather than the NMR value of 288 kG. This corresponds to the simple approach of using the observed γ-ray results to deduce a value of B_1.

The other extreme model assumes that most of the nuclei see the NMR field and a fraction see essentially zero field. This fraction is just the ratio of the observed and expected ^{58}Co γ-ray anisotropies. The observed β-ray anisotropy is too low by the same factor. The analysis proceeds by multiplying the observed β- and γ-ray anisotropies by the correction factor determined from the measurements with ^{60}Co. B_1 is then calculated from the corrected γ-ray data and used with the corrected β-ray data. The difference in the final A_1 values determined from these two modes of analysis is only 6%; this can be compared with the original 15% discrepancy in the γ-ray data. The similar temperature dependence of B_1 and B_2 partially compensates for the uncertainty caused by the internal-field distribution.

We were able to determine which of these models was more correct by comparing the ^{58}Co γ-ray

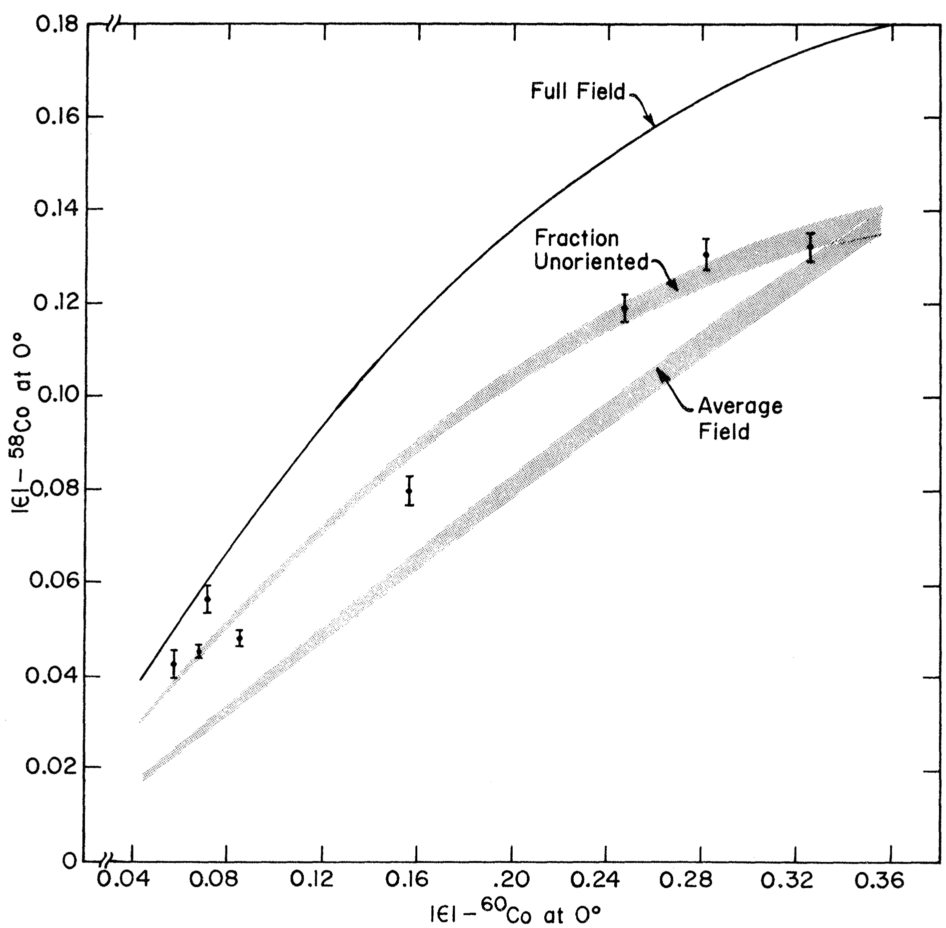

FIG. 9. Plot of the ^{58}Co γ-ray anisotropy versus the ^{60}Co γ-ray anisotropy for several source temperatures.

anisotropy in the implanted foil with the ^{60}Co anisotropy for a diffused source over a wide temperature range. The results of these measurements are summarized in Fig. 9. This figure was plotted by using the measured results at the lowest temperature to predict the behavior at higher temperatures according to the two models. The widths of the plotted curves reflect the statistical uncertainty in the lowest-temperature data. The data are in very good agreement with the hypothesis of the unoriented fraction. As a result, this model was used in the remainder of the data analysis. For this sample 84% of the nuclei see the full field.

The sample holder was designed to hold the sample at 45° to the vertical; however, since it was impossible to clamp the salt pills firmly because of resultant heat leaks, there was some doubt about the actual value of the detector angle θ. The β-ray asymmetry is proportional to $\cos\theta$, which varies quite rapidly around 45°. Since the relative variation of $P_2(\cos\theta)$ is 5 to 6 times larger than that of $\cos\theta$ in this angular region, a better value for $\cos\theta$ was obtained by comparing the γ-ray anisotropy at 0°, measured with the Ge(Li) detector, with the anisotropy of the Compton distribution in the thick detector. The thick detector was operated in anticoincidence with the thin so as to obtain a pure γ-ray response. Since $P_2(\cos\theta)$ is stationary at 0°, a small angular error in the position of the Ge(Li) detector is unimportant. After correcting for the finite solid angle of the β detector and the contribution of the weak 864-keV γ ray to the Compton distribution, we found the following angles for the three independent sets of data: 45.4 ± 1.0, 44.2 ± 1.8, and $46.0\pm1°$.

The data of this experiment were taken over a period of several months and are divided into four sets which differ from one another through minor changes in the experimental conditions. In terms of the detector angles mentioned previously, there are only three independent sets of data. For the purpose of analysis the β spectra were summed in energy bins. The theoretical positron spectrum is shown in Fig. 10 along with the location of the energy bins.

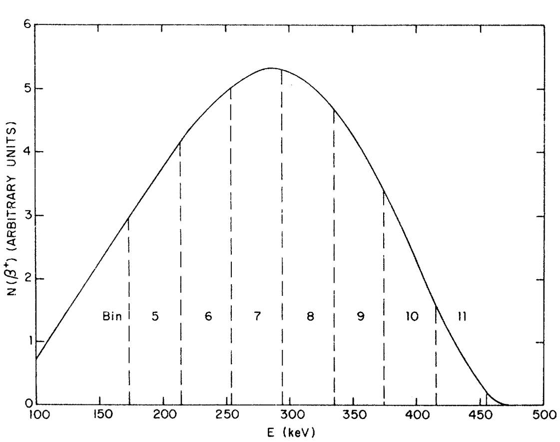

FIG. 10. Theoretical spectrum of the 474-keV positron group of ^{58}Co showing the location of the energy bins used in the analysis.

Figure 11 shows the average β-ray anisotropies divided by (v/c) from data sets 2 and 3. The data shown are averages of the 17 demagnetizations. The errors shown are statistical; the actual variations within the data are consistent with statistics. χ^2 was calculated for each bin of each data set and the average normalized value was 1.4; this corresponds to a probability of 25%.

The data must also be corrected for backscattering in the source and for γ-ray contamination. Three assumptions were made in order to evaluate the source backscattering correction. The probability of backscatter is independent of energy; the energy-loss probability in the scattering material is flat from zero to the incident energy; the directional information carried by the backscattered positron is lost. These assumptions are supported by existing data.[17] Measurements made in our geometry with a ^{137}Cs source showed that the total probability for backscattering in the source foil was 20%.

The evaluation of ρ_β, the β-ray weight, is the major source of systematic error in the experiment. The straightforward approach is to place a β-ray stopper between the source and detector and to measure the residual response. Figure 12 shows the results of such a measurement. Here we have plotted a typical β spectrum taken with

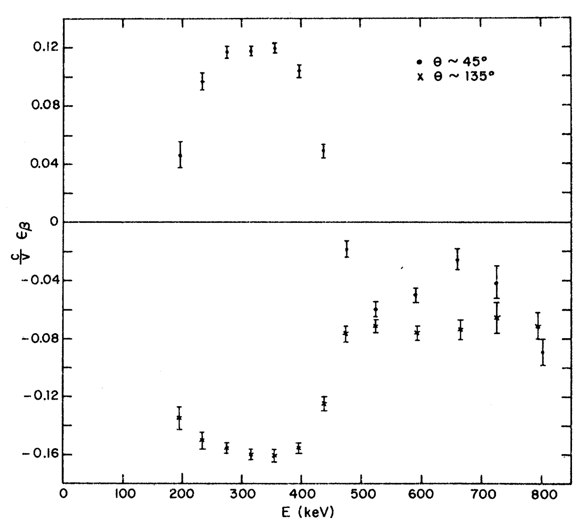

FIG. 11. The average β-ray anisotropies derived from data sets 2 and 3.

the full antibackscatter requirement and a γ-ray contamination spectrum taken with the same electronic configuration and a 1.6-mm-thick aluminum disk above the thin window in the bottom of the 1°K tail.

Figure 13 shows the apparent β-ray asymmetry

$$C_1 = \rho_\beta Q_1 A_1$$

and the corrected anisotropy Q_1A_1 calculated using the measured β-ray weights for one of the data sets (analysis I). Source backscattering corrections of 1.6, 3.0, 5.4, 9.4, and 16.0% were applied to bins 9, 8, 7, 6, and 5, respectively. For the proper values of ρ_β and backscattering correction, Q_1A_1 should have been a constant over the positron spectrum. At the upper end of the spectrum in bins 10 and 11, where the positron intensity was decreasing, the decrease in Q_1A_1 indicated that the γ-ray intensity had been underestimated. The missing γ-ray response was due to Compton electrons generated in the source and its environs which were stopped with the β rays in the aluminum disk mentioned in the previous paragraph.

A second analysis for the correction for the γ-ray contamination was made by using the fact that Q_1A_1 should be constant over the whole spectrum. The measured γ-ray contamination for each bin was multiplied by a constant factor, f, so that the average Q_1A_1 for bins 11 and 12 was equal to the average Q_1A_1 for bins 8 and 9. The values of f required for the four sets of data ranged between 1.45 and 1.75. Figure 13 also shows the results of this analysis (analysis II). The relative γ-ray contamination for bins 10 and 11 was large; that for bins 8 and 9 was small. Thus an increase of 50% in the γ-ray contamination to correct the Q_1A_1 of bins 10 and 11 only changed the Q_1A_1 of bins 8 and 9 by 10%. Multiplying all the γ intensities by f gives an A_1 for bins 5 and 6 that was too large but that was constant for the higher bins. The second analysis overestimated the γ-ray contaminations; the first analysis underestimated the γ-ray response.

The low-energy bins, such as 5 and 6, were most sensitive to the accuracy of the source backscattering correction. Bins 10 and 11 were not very sensitive to the source backscattering corrections but were very sensitive to the correction for γ-ray contamination. The most intense parts of the β spectrum, bins 8 and 9, were least sensitive to these corrections. Thus, bins 8 and 9 were the most reliable and were used to determine the final value of A_1.

Table I summarizes the statistical errors in the four sets of data. The major sources of statistical error were: counting statistics in the raw β-ray asymmetries, error in the angle determination, and the error in B_1 arising from γ-ray counting statistics. Table II summarizes the averages

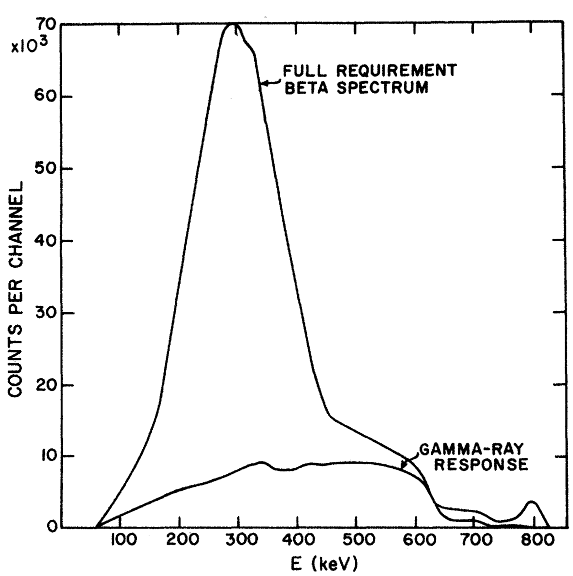

FIG. 12. The β spectrum observed with the antibackscattering requirement imposed on the telescope and γ-ray contamination determined by placing a 1.6-mm aluminum β-ray stopper between the source and the detector.

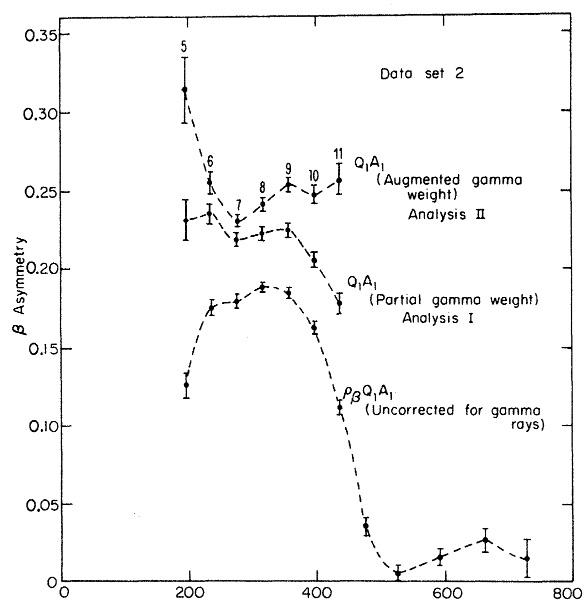

FIG. 13. A plot showing the β-ray asymmetry uncorrected for γ rays ($\rho_\beta Q_1A_1$), the Q_1A_1 determined by using the measured γ-ray response to determine ρ_β (analysis I), and the Q_1A_1 determined by increasing the γ-ray weight so as to make the asymmetry in the high bins a constant (analysis II).

of the values of A_1 from bins 8 and 9, the statistical errors, and χ^2 for both modes of analysis. The reproducibility shown by the χ^2 values is excellent.

The two modes of analysis allow us to set an upper and lower bound on A_1. Using the values from Table II we have

$$(0.2304 \pm 0.0038) \leq A_1 \leq (0.2563 \pm 0.0042),$$

where the errors quoted are statistical standard deviations. The presence of a systematic error such as that represented by the different results given by the two methods used for γ-ray response corrections makes it difficult to assign a unique value with associated error for comparison with other measurements. In the absence of further experimental information bearing on the γ-ray correction, we take the mean of the two values as our final result. The quoted error was obtained by taking $\frac{1}{4}$ of the difference between the two limiting values and combining this in quadrature with the statistical error. The choice of $\frac{1}{4}$ of the difference reflects the fact that a range of $\pm 2\sigma$ about the mean almost certainly encompasses the correct result in the same sense that a range of $\pm 2\sigma$ contains the result with 95% probability in the case of purely statistical errors. Thus our final result becomes

$$A_1 = 0.243 \pm 0.007.$$

A nuclear orientation experiment is sensitive to higher-order corrections to the β decay through $P_2(\cos\theta)$ terms in the β-ray angular distribution. Figure 11 shows that the negative anisotropies were larger than the positive anisotropies. After subtracting off the $P_2(\cos\theta)$ term due to the γ rays there was a nonzero A_2 term due to the β rays which was positive for two of the data runs and negative for the other two data runs. The fact that this A_2 term differed significantly from data run to data run suggests that it is an instrumental and not a physical effect. Several explanations have been considered. The favored explanation is that the source moves slightly as a result of magnetic forces on the salt pill during the demagnetization cycle. The excellent consistency obtained for the $P_1(\cos\theta)$ term in the presence of the varying residual $P_2(\cos\theta)$ term indicates that this effect does not preclude an accurate measurement of the β-ray asymmetry.

TABLE I. A summary of the various contributions to the statistical error in the β-ray asymmetry.

Data set	1	2	3	4
$\rho_\beta A_1 Q_1$	1.25%	1.2%	1.38%	1.96%
$P_1(\cos\theta)$	1.7%	3.1% [a]	3.1% [a]	1.7%
B_1	1.0%	1%	1%	1.7%
Resultant error	2.3%	4.6%	4.6%	3.1%

[a] Correlated errors since they are based on same measurement.

7. RESULTS AND CONCLUSIONS

In this final section we shall first compare our asymmetry with previous work on ^{58}Co and then calculate the isospin impurity and Coulomb matrix element disregarding the possibility of significant second-forbidden contributions. Finally a brief discussion of second-forbidden corrections will be given.

The results of the equivalent circular polarization experiment calculated from our β asymmetry is

$$\tilde{A}_1 = -0.1615 \pm 0.0046.$$

The most recent circular-polarization result is that of Pingot[2] who found

$$\tilde{A}_1 = -0.167 \pm 0.004.$$

Pingot's value is seen to be in excellent agreement with that reported here. In Fig. 14 we present all results to date for the β asymmetry or circular polarization correlation in ^{58}Co plotted versus the time of publication.[18] The nuclear orientation results, denoted by "0," have been converted to equivalent circular-polarization values. The significant improvement represented by the work of Pingot and that reported here is clear. The large scatter in the previous circular-polarization results makes the usual weighted averages subject to doubt so that the only way to tie down the value of $\tilde{A}_1$ to significantly better accuracy is to have new individual measurements of the required quality. Those of Pingot and of the present authors are such; their excellent agreement, coming as they do from such widely differing experimental situations, suggests an absence

TABLE II. Average values of A_1 determined from bins 8 and 9. Analysis I refers to the use of the measured γ-ray contamination. Analysis II refers to the use of the augmented γ-ray weights.

	A_1 Analysis I	A_1 Analysis II
Data set 1	0.2349 (55)	0.2591 (60)
Data set 2	0.2274 (106)	0.2568 (118)
Data set 3	0.2254 (106)	0.2454 (118)
Data set 4	0.2265 (70)	0.2561 (79)
Weighted average over data sets	0.2304 (38)	0.2563 (42)
χ^2 (normalized)	0.43	0.36
$P(\chi^2)$	0.73	0.78

of serious systematic error. Pingot has reported results of comparable precision for several other isotopes.[2,19] The fact that his ^{58}Co result has been corroborated in our orientation experiment to this level of accuracy lends credence to the small errors assigned to his values in these other experiments.

Neglecting second-forbidden effects, the following quantities were deduced from our β-ray asymmetry[1]:

Fermi–Gamow-Teller mixing ratio,

$$x = -0.0063 \pm 0.0056\,,$$

Fermi matrix element (using $\log ft = 6.6$),

$$|M_F| = (0.26 \pm 0.23) \times 10^{-3}\,,$$

Isospin impurity,

$$|\alpha| = (0.10 \pm 0.09) \times 10^{-3}\,.$$

Charge-dependent matrix element,

$$|\langle H_c \rangle| = 0.69 \pm 0.62 \text{ keV}\,.$$

The charge-dependent matrix element was calculated using the expression of Bouchiat[20] for the energy denominator.

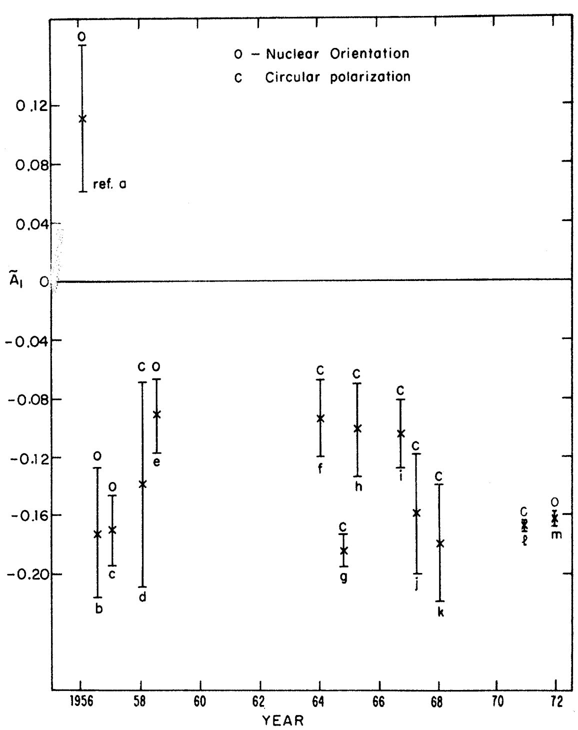

FIG. 14. A summary of all the experiments giving a determination of the ratio of the Fermi to the Gamow-Teller matrix element for the 474-keV positron decay of ^{58}Co. The results are expressed in terms of the asymmetry coefficient, $\tilde{A}_1$, observed in a β-γ circular-polarization-correlation experiment. The sources of the results are listed in Ref. 18.

The very small size of the mixing ratio makes its interpretation in terms of the Fermi–Gamow-Teller mixing of allowed decay open to question because of the possibility of second-forbidden corrections at this level. Bloom[21] has pointed out that a rough idea of the expected second-forbidden effects comes from noting that the average of all the known $\log ft$ values for second-forbidden transitions is about 12 ± 1. Corresponding to a $\log ft$ range of 11 to 13, the value of the contributing matrix element goes from about 2.5×10^{-4} to 0.25×10^{-4}. This is certainly comparable to our experimental value of $|M_F|$ which is $(2.6 \pm 2.3) \times 10^{-4}$.

Coussement and Van Neste[22] have made quantitative estimates of second-forbidden corrections to the decay of ^{58}Co. They write the mixing ratio in the approximate form

$$x = \frac{C_V}{C_A M_{GT}} \left[\int 1 + b \int (r/R)^2 + c \int i(\vec{\alpha} \cdot \vec{r})/R \right]$$

and they calculate that the ratio of the second-forbidden tensor rank-zero terms to $\int 1$ could be as large as 35%. They also conclude that the coefficients b and c have a weak energy dependence of a few percent per MeV so that it would be extremely difficult to separate the allowed and second-forbidden contributions in this case.

Second-forbidden effects in allowed decay can show up as a nonisotropic β-γ angular correlation or as a nonstatistical shape factor. The β-γ correlation appears to be isotropic within experimental error: Wohn and Wilkinson[23] found a correlation coefficient of 0.003 ± 0.004 and Sastry *et al.*[24] found -0.0003 ± 0.0014. Rhode and Johnson[25] found a shape factor of $1 + 0.3/W$. This was a difficult experiment and it should be confirmed before it is taken as evidence of second-forbidden components. The second-forbidden matrix elements of rank zero which compete with the Fermi matrix element appear in the expression for the spectrum shape but not in that for the β-γ angular correlation so that the only experimental evidence bearing on our result is the spectrum shape. Since the expression for the spectrum shape is dominated by the Gamow-Teller matrix element, deviations from it are expected to be extremely small. The best hope for studying the rank-zero correction terms is in the $0^+ \to 0^+$ isospin-forbidden decays where larger deviations from allowed shapes are possible.

It should finally be noted that this result in ^{58}Co and those of Pingot[2,19] in ^{58}Co, ^{48}V, ^{52}Mn, ^{56}Co, ^{124}Sb, ^{46}Sc, and ^{110m}Ag lead to vanishingly small Fermi matrix elements with very tight error bars. It is important that other nuclei be remeasured with improved accuracy to set tighter limits on the size of the Fermi matrix elements. The fact that at least six nuclei exist with very small measured Fermi matrix elements suggests that the second-forbidden corrections are of this magnitude or smaller, since very exact cancellations are unlikely in so many cases. The only case of a $\Delta J = 0$, $J \neq 0$ isospin-forbidden decay with an experimentally well-confirmed nonzero mixing is ^{134}Cs where Pingot[19] finds $(5.75 \pm 0.10) \times 10^{-4}$ for $|M_F|$ in reasonable agreement with previous measurements. This would correspond to a $\log ft$ of 10.3 if the Fermi matrix element alone were acting. It is interesting to note that the $\log ft$ values of 10.3 or higher corresponding to the Fermi matrix elements in the mixed transitions discussed above greatly exceed those of the isospin-forbidden $0^+ \rightarrow 0^+$ transitions in ^{66}Ge and ^{66}Ga which have $\log ft$'s of 6.8 and 7.9, respectively.[26] They are also somewhat larger than the $\log ft$ values in the $0^+ \rightarrow 0^+$ decays of the deformed nuclei ^{156}Eu, ^{170}Lu, and ^{234}Np.[26] It is clear that further theoretical work is needed on charge-dependent effects and higher-order corrections in order to utilize and interpret these accurate new data.

Now let us summarize the conclusions to be drawn from this experiment. We have measured the β-ray asymmetry from ^{58}Co to an accuracy significantly better than 5% thus providing the first rigorous independent test of the circular-polarization technique. Because of the generally large experimental effects in such an experiment it is easy to make the statistical errors smaller than the systematic uncertainties. The latter have been kept small by various means reflecting the improved techniques now available. The use of a counter telescope removes the error due to backscattering in the β detector and simplifies the corrections for γ-ray response. An ion-implanted source eliminates the correction due to multiple scattering. The corrections for source backscattering and Compton electrons are amenable to experimental study and can be reduced by thinner source foils. The use of γ-ray anisotropy from the same element to determine the orientation parameters reduces the uncertainties due to variation in the hyperfine field over the sample and precision measurements as a function of temperature can largely eliminate any remaining error. It is clear that these methods can be applied to the study of many cases of allowed and forbidden decays to yield accurate new information on nuclear β-decay matrix elements.

ACKNOWLEDGMENTS

We should like to thank Professor K. T. Bainbridge for the use of equipment and materials and for many helpful discussions. We should like to thank Ira Fox for aiding in the data reduction. Much of the analysis of data has been done with computational facilities supported by the Atomic Energy Commission. Finally, one of the authors, (H.R.A.), would like to thank the National Research Council of Canada for financial support during part of this research.

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†Research supported in part by National Science Foundation Grants Nos. GP-15077 and GP-28571.

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Study of V, Mn, and Co Isotopes*

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(Received 13 September 1972)

Nuclei with odd number of protons are studied using the Hartree-Fock-Bogoliubov method. The binding energies, single-particle spectra, quadrupole moments, $B(E2)$ values, and the pickup strengths are calculated. By comparing the results for the neighboring even nuclei one can study the sets of isotones. The nuclei with $N=26$ (^{48}Ti, ^{49}V, ^{50}Cr, ^{51}Mn, and ^{52}Fe), $N=28$ (^{50}Ti, ^{51}V, ^{52}Cr, ^{53}Mn, ^{54}Fe, and ^{55}Co), and $N=30$ (^{53}V, ^{54}Cr, ^{55}Mn, ^{56}Fe, and ^{57}Co) are studied to understand the effects of the addition of a proton on the deformation, Fermi surface, fluctuations of the proton separation energies, pair separation energies, and the configuration mixing.

1. INTRODUCTION

There are essentially two ways in which odd-proton nuclei in the p-f shell have been studied. The first way is to use the macroscopic rotational model.[1] In this model the Coriolis-coupling term is used to couple the various bands emerging from the rotational states based on the single-particle or single-hole excitations. This model successfully explains the ground-state spins and the spectra below 2.5 MeV. However, this model does not give any understanding of the microscopic description of the nuclei.

The second way is the conventional shell-model[2] approach which is microscopic. Though reasonably good agreements are obtained for the spectra of some of the p-f shell nuclei, only the $(f_{7/2})^{\pm n}$ configurations are considered. Besides, in the cases of V and Mn isotopes the $f_{7/2}$ shell for the neutrons is considered closed. However, this model is limited in that to consider more than four particles outside the closed core is a very difficult numerical problem.

To consider large numbers of particles outside the closed core, one has to resort to variational methods. These are the Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) approximations. Since in our early work,[3,4] the HFB method which takes pairing correlations into account is found to be superior over the HF method, we use the same method to study these nuclei. The HFB formalism – which is meant for even-even nuclei – has been extended to the even-odd nuclei in Ref. 5.

The motivation for this work is not only to study these nuclei *per se* but also to see what would be the effects of the addition of a proton on nuclear properties by comparing the present results with the results of our earlier work on the even-even