

Bull. Am. Phys. Soc. **12**, 500 (1967).

¹⁹L. Van der Zwan and K. W. Geiger, Nucl. Phys. **A152**, 481 (1970).

²⁰G. Marr, Ph.D. dissertation, The Ohio State University, 1968 (unpublished). (Available from University Microfilms.)

²¹T. A. Welton, in *Fast Neutron Physics Part II*, edited by J. B. Marion and J. L. Fowler (Interscience, New York, 1960), Chap. V F.

²²R. L. Walter, in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, 1971), p. 317.

²³L. C. Northcliffe and R. F. Schilling, Nucl. Data **A7**, 233 (1970).

²⁴J. T. Klopcic and S. E. Darden, Phys. Rev. C **3**, 2171 (1971); see also their errata Phys. Rev. C **4**, 1494(E) (1971). Our $^9\text{Be}(\alpha, n_0)$ data at $E_\alpha = 2.65$ MeV are published in their errata.

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Elastic Scattering of α Particles by ^9Be and Highly Excited States of $^{13}\text{C}^\dagger$

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The elastic scattering of α particles by ^9Be has been studied in the bombarding energy range of 1.7–6.2 MeV. 10 excitation functions in 50-keV steps and five angular distributions near 5 MeV were measured. The elastic scattering data in the 3.5–5.0-MeV region were analyzed with the compound-nucleus theory using a computer code which evaluates the cross section in the “single-level” approximation. Analysis in terms of seven resonances reproduces many of the features of the data. Tentative spin and parity assignments are given.

I. INTRODUCTION

The states of ^{13}C below the $^9\text{Be} + \alpha$ separation energy (at 10.651 MeV) are relatively well known, having been investigated extensively both experimentally and theoretically.¹ However, in the region of excitation above this energy virtually no spin and parity assignments have been made and information concerning level widths and level positions is incomplete.² The reactions $^9\text{Be}(\alpha, \alpha)^9\text{Be}$ and $^9\text{Be}(\alpha, n)^{12}\text{C}$ can be used to investigate this region of ^{13}C , which appears as the compound nucleus in both reactions. Polarization measurements have been made on the latter reaction by De Martini, Soltesz, and Donoghue and are reported, together with an analysis of both polarization and (α, n) cross-section data, in the preceding paper.³

In the present work, the region has been investigated through a study of elastic scattering of α particles by ^9Be . This reaction has previously been studied by Taylor, Fletcher, and Davis⁴ who analyzed data taken above 9 MeV with an optical model but reported only fragmentary data below 9 MeV. In the present work, 10 excitation functions in the laboratory energy region of 1.7 to 6.2 MeV and five angular distributions between 5.0 and 5.5 MeV were measured. The fluctuations in the measured excitation functions are suggestive of compound-nucleus formation and the data were therefore analyzed with a compound-nucleus

theory. Since most compound-nucleus analyses of elastic scattering data have been concerned with cases of spin-0 or spin- $\frac{1}{2}$ projectiles scattering from even- A nuclei, the present study entailed extensive calculations with a computer program especially written for this analysis. The results of the measurements and of these calculations are presented below.

II. EXPERIMENTAL PROCEDURE

The α -particle beam from the Ohio State University 6-MV Van de Graaff accelerator was magnetically analyzed to provide an absolute energy known to within ± 5 keV. The beam was collimated by two slit boxes on the beam line and entered the chamber through a 1-mm-diam circular aperture. The beam was collected in a Faraday cup and measured with a Brookhaven Instrument Company precision current integrator. Typically, the beam currents used in the measurement of the data were 0.1 to 0.5 μA . The integrator was checked and found to be accurate to $\pm 0.2\%$ for this current range. It was also determined that for the charge collection geometry employed, electrostatic or magnetic suppression was not necessary.

The elastically scattered particles were measured simultaneously at four angles, using silicon surface-barrier detectors (with 300- μm depletion depths) mounted in a 23-in.-diam scattering chamber. The platform on which the detectors and

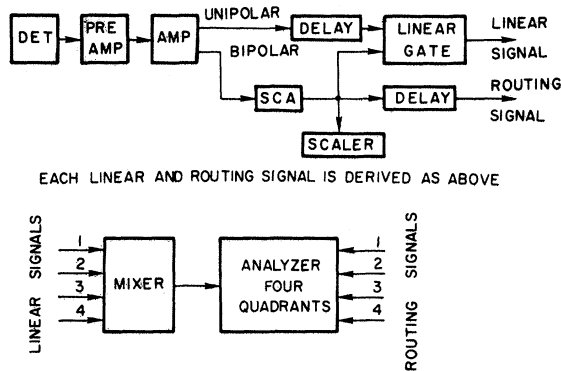


FIG. 1. Electronics block diagram. The following abbreviations are used: amplifiers, AMP; detector, DET; and single-channel analyzer, SCA.

their collimators were mounted could be rotated remotely with an angular uncertainty of $\pm 0.2^\circ$ for each of the detectors with respect to the beam axis. A fifth detector was used for determining 0° by measuring the yield on either side of the beam axis. The solid angles subtended by the detectors at forward angles were smaller than those subtended by the detectors at back angles; this allowed all detectors to operate at similar counting rates. The angular acceptance of the detec-

tors ranged from $0^\circ 19'$ for forward angles to $0^\circ 46'$ for the detectors past 90° . Except for purposes of checking symmetry about the beam axis, the target was positioned with its normal 40° to the beam direction so that all detectors could view the beam spot.

The linear signals from each of the detectors were routed into quadrants of a 512-channel pulse-height analyzer. Several electronic configurations were used during the course of the experiment, one of which is shown schematically in Fig. 1. With this configuration, each detector output is isolated from the noise due to the others.

The preparation of pure thin beryllium targets presented considerable difficulty. Sturdy and rugged self-supporting foils can be made with a thickness of the order of 100 keV for 5-MeV α particles; however, thinner films become brittle and crack. The targets finally used in this experiment were made by vacuum evaporation of beryllium metal onto a thin film of Formvar, and the thickness of these targets ranged from 5 to 24 keV for 5-MeV α particles. Unfortunately, the oxygen and carbon in the Formvar added unwanted contaminants and, additionally, made target thickness measurements more difficult. The elastically scattered α particles from ^9Be , ^{12}C , and ^{16}O are unresolved for angles smaller than 25° (lab),

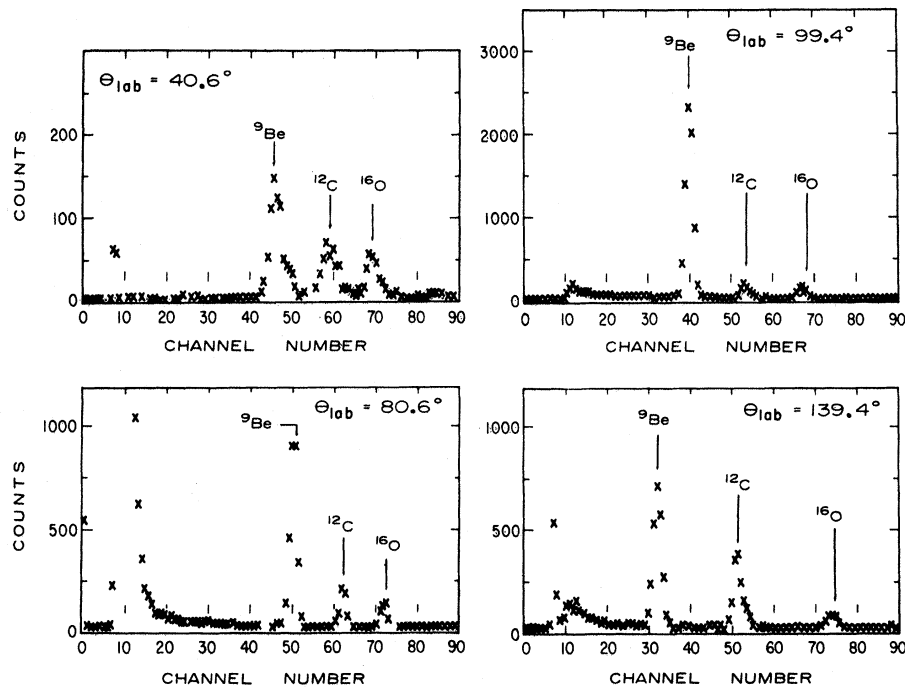


FIG. 2. Particle spectra of α particles on a ^9Be target with Formvar backing. The bombarding energy is 3.85 MeV and the target is approximately 24 keV thick.

preventing reliable reduction of data from the more forward angles. Typical spectra of α particles on the Formvar-backed ${}^9\text{Be}$ target are shown in Fig. 2. In extracting the yields from such spectra, backgrounds were estimated by averaging counts on either side of the ${}^9\text{Be}$ peak.

III. EXPERIMENTAL RESULTS

The 10 excitation functions, shown in Figs. 3 and 4, were obtained at various center-of-mass angles including those for which the Legendre polynomials of orders 1 through 5 are zero. The solid lines are the compound-nucleus calculations discussed in Sec. IV. The data were taken in 50-keV steps with several targets, all approximately 24 keV thick for 4.29-MeV α particles. The relative uncertainty in the cross section is about $\pm 5\%$ for all the data shown, as determined from the reproducibility of the data. This uncertainty includes statistical errors of 1 to 3%, background subtraction uncertainties of 1 to 2%, angular uncertainties of $\pm 0.2^\circ$, and errors due to target non-uniformity.

An absolute target thickness was measured experimentally for one of the several targets used, with the others being determined by a comparison of excitation-function yields at various angles. Two independent experiments were performed in determining the absolute target thickness. In the

first method, the resonance at 4.28 MeV in the ${}^{12}\text{C}(\alpha, \alpha_0){}^{12}\text{C}$ reaction was used. The target energy loss was unfolded from the observed shift in excitation functions obtained for the reaction initiated at the front of the target (carbon buildup during beam bombardment) and from the carbon in the Formvar backing. In the second method, deuteron elastic scattering measurements were used and the observed yields were normalized to the data of Renken.⁵ Both measurements agree well within the experimental uncertainties and give a target thickness of 1.79×10^{18} atoms/cm² with an uncertainty of $\pm 10\%$.

Possible errors due to charge collection, solid-angle uncertainties, analyzer dead-time corrections, and routing errors are less than or equal to $\pm 1\%$. When combined with the target-stopping uncertainty, the uncertainty for the absolute cross section is about $\pm 13\%$ for the excitation-function data.

To check the possibility that narrower structure was being averaged over, several excitation functions were measured with a target approximately 5 keV for 5-MeV α particles. The data were

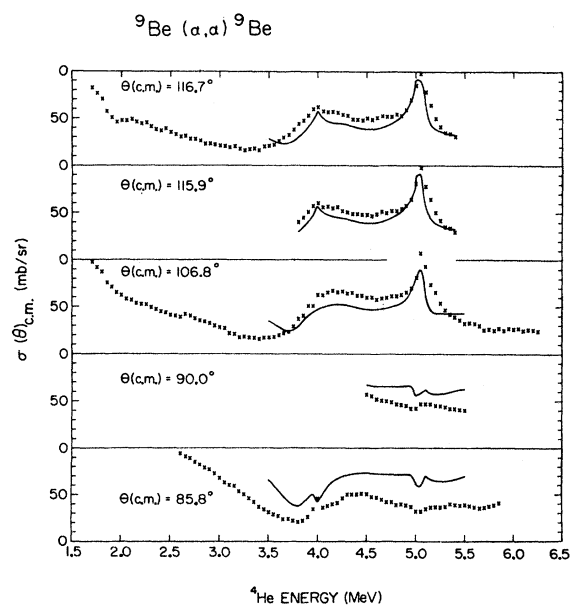


FIG. 3. Excitation curves of ${}^9\text{Be}(\alpha, \alpha_0){}^9\text{Be}$ at 85.8, 90.0, 106.8, 115.9, and 116.7° c.m. The solid lines are the results of compound-nucleus calculations for the set of level parameters given in Table I.

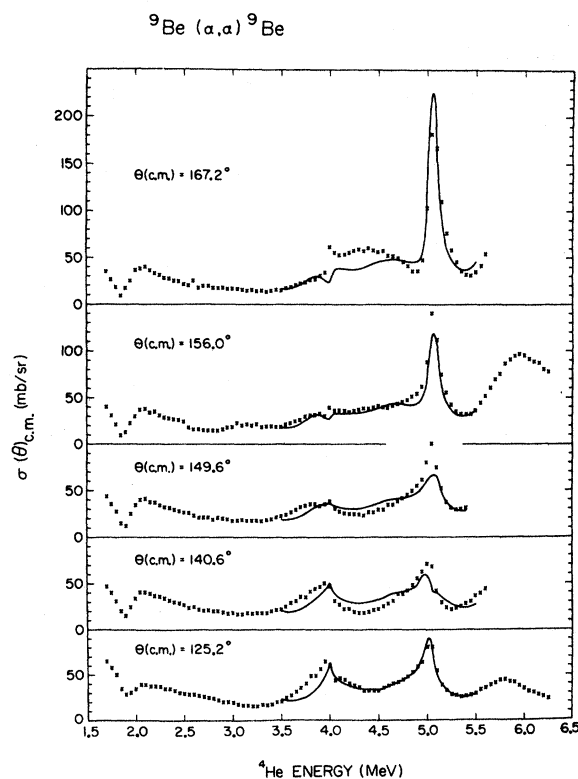


FIG. 4. Excitation curves of ${}^9\text{Be}(\alpha, \alpha_0){}^9\text{Be}$ at 125.2, 140.6, 149.6, 156.0, and 167.2° c.m. The solid lines are the results of compound-nucleus calculations for the set of level parameters given in Table I.

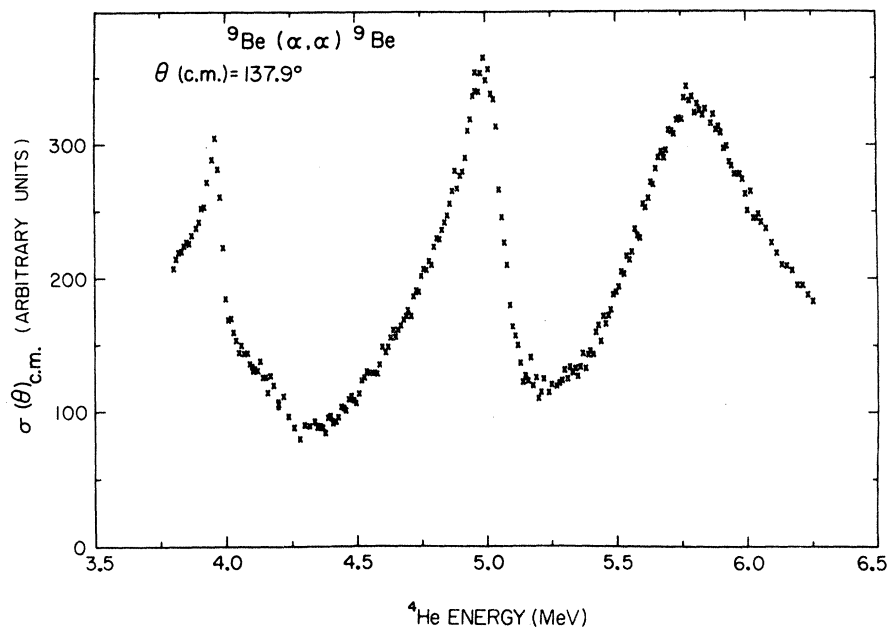


FIG. 5. Thin-target excitation curve of ${}^9\text{Be}(\alpha, \alpha){}^9\text{Be}$ at 137.9° c.m. The target was approximately 5 keV thick for 5-MeV α particles.

taken in 10-keV steps and one such excitation function, at 137.9° (c.m.), is shown in Fig. 5. Since no absolute normalization was made, the cross section is plotted in arbitrary units. When compared to the data taken with the 24-keV targets (Figs. 3 and 4), additional fluctuations do appear; however, the general structure is the same and none of the resonance features are further resolved.

Special attention has been given to the region just above 5 MeV because of the large resonance feature in the excitation functions seen there. Angular distributions measured at five energies in this region are shown in Fig. 6. The solid lines are again compound-nucleus calculations discussed in Sec. IV. Somewhat thicker targets were used in measuring these angular distributions than were used in measuring the excitation curves in Fig. 3 and 4. The relative uncertainty is about $\pm 5\%$ for these data and the absolute uncertainty (primarily in the target thickness) is about $\pm 15\%$.

IV. COMPOUND-NUCLEUS ANALYSIS

A. Theory

The differential cross section for elastic scattering of charged particles near an isolated single level of the compound nucleus has been given explicitly by Blatt and Biedenharn.⁶ It is evident from the present data that to explain the cross

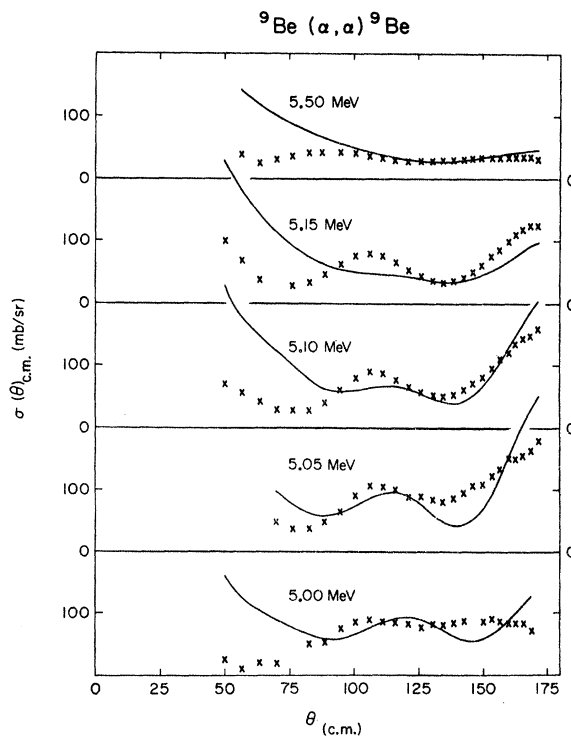


FIG. 6. Angular distributions of ${}^9\text{Be}(\alpha, \alpha){}^9\text{Be}$ at 5.00, 5.05, 5.10, 5.15, and 5.5 MeV. The solid lines are the results of compound-nucleus calculations for the set of level parameters given in Table I.

section in the energy region of interest, several levels must be included. As a first approximation, one might make the assumptions that all the levels to be included are of different spins and parities and that contributions from "distant levels" are negligible. The formula of Blatt and Biedenharn, corrected for the Huby time-reversal phase factor,⁷ may then easily be extended to a multiple-level theory in this "single-level" approximation.

The terms which occur in the multiple-level formula may be grouped and interpreted as follows. First is a contribution from pure resonance scattering to the cross section. The pure resonance scattering has the same form as a reaction cross section and expresses the cross section as a sum over L of Legendre polynomials $P_L(\cos\theta)$. The restrictions on the sum over L are determined from the selection rules for non-vanishing Z coefficients.⁸ For two levels of opposite parity, L may have odd values as well as even; whereas for the case of two levels with the same parity or the case of a single level, L may have only even values. In addition to pure resonance scattering, there is also a contribution from potential scattering. As assumed by Blatt and Biedenharn, the potential scattering in the present work is taken to be the scattering from a charged hard sphere with $R = 1.2(A_1^{1/3} + A_2^{1/3}) = 4.4$ fm. Rutherford scattering has been separated explicitly from the potential scattering so that the latter may be written as rapidly convergent sums over l . In our analysis, these sums have been truncated at $l=6$. The final two contributions to the cross section come from the interference between resonance scattering and pure Rutherford scattering, and from a finite nuclear-size correction to this interference term.

Besides the assumption of hard-sphere potential scattering, it has also been assumed that the level shift is negligible and may be omitted. This approximation is good only for narrow states and may introduce some uncertainty in the width and resonance position of broad levels. Finally, to handle the possibility that all the levels included might not be of different spin and parity, additional interference terms have been included in the cross-section formula which are strictly valid only if the levels of the same J^π are separated by at least a full level width.

B. Application to ${}^9\text{Be}(\alpha, \alpha_0){}^9\text{Be}$

A computer code has been written⁹ to evaluate the elastic scattering cross section discussed above. In addition the code allows one to calculate the differential cross section for the ${}^9\text{Be}(\alpha, n)$ re-

actions in the "single-level" approximation. The elastic scattering data were compared to calculations by minimization of the quantity χ^2 given by

$$\chi^2 = \sum_{i=1}^N \left[\frac{(\sigma_{\text{exp}} - \sigma_{\text{cal}})^2}{(\Delta\sigma_{\text{exp}})^2} \right]_i.$$

Here N is the number of data points, σ_{exp} and σ_{cal} are the measured and calculated cross section, respectively, and $\Delta\sigma_{\text{exp}}$ is the uncertainty in the measured cross section. A search routine incorporated in the code minimized χ^2 by varying the reduced partial widths for both l values which could contribute to a given level, as well as the total width of each level. The resonance energy and spin and parity of each level were held constant during any one search. It should be noted that no other adjustable parameters, such as background terms or an arbitrary data normalization factor, were used in the analysis. To obtain a set of consistent level parameters which would give a "best fit" to the 10 excitation functions, several of the excitation functions were fitted simultaneously, although a better fit could usually be obtained if each excitation function was considered separately.

In the laboratory energy range of 3.5 to 5.5 MeV, resonances have previously been observed from ${}^9\text{Be}(\alpha, n)$ reactions¹⁰⁻¹⁴ at the bombarding energies of 4.00, 4.18, 4.50, 5.00, and 5.40 MeV. Many of the levels strongly overlap and, because of the complexity of the excitation functions, the analysis of the elastic scattering data is here primarily concerned with the interval from 3.5 to ≈ 5 MeV.

As a starting point in our calculations, we have used the $J^\pi(l_\alpha)$ assignments of $\frac{5}{2}^+(1)$ and $\frac{3}{2}^+(3)$ deduced by Lietz, Trevino, and Darden^{15, 16} for levels corresponding to $E_\alpha \approx 4.2$ and 4.5 MeV, respectively. Evidence for these two states is supported by De Martini, Soltesz, and Donoghue³ although these authors have recently shown that the ordering of these states is in question. As this was not known at the time of our calculations, we have used only the ordering proposed by Darden¹⁵ for these two states. Information concerning the J^π assignments for other states in this region is sketchy. The (α, α) scattering excitation curves show a pronounced resonance at 5.0 MeV; a similar resonance is observed in the (α, n_1) and (α, n_2) excitation curves,¹³ but contributes weakly or not at all to the (α, n_0) data. De Martini, Soltesz, and Donoghue deduce a significant contribution from a $\frac{5}{2}^-(4)$ state in this energy region, although the width of the state appears to be greater than exhibited by the (α, α) data. If such a state is present, it would show a noticeable dip at 90° (c.m.).

This is in fact evident in our data. The inclusion of a $\frac{5}{2}^-(4)$ level at $E_\alpha = 5$ MeV is consistent with the (α, α) data back to 149° (c.m.), but this or any other negative-parity state up to $\frac{9}{2}^-$ is unable to reproduce the elastic scattering data at 156 and 167.2° . This observation, plus the failure of any positive-parity state in describing the data at 5 MeV, leads to the hypothesis that the resonance feature must be due to the presence of at least two levels. Under this assumption and with a $\frac{5}{2}^-(4)$ state at 5.0 MeV, the best description of the data was obtained with a $\frac{7}{2}^+(3)$ state at $E_\alpha = 5.075$ MeV. Other two-level combinations were also tried, and equally good fits to the data in this region were obtained with a $\frac{7}{2}^-(4)$ level at 5.0 MeV, together with a $\frac{3}{2}^+$, $\frac{5}{2}^+$, or $\frac{7}{2}^+$ level (all $l_\alpha = 3$) at 5.075 MeV. It is difficult to conclude which of these sets are to be favored here at this time.

The narrow ($\Gamma_{\text{c.m.}} \cong 60$ keV) level corresponding to $E_\alpha = 4.00$ MeV in the ${}^9\text{Be}(\alpha, n\gamma_{4.43}){}^{12}\text{C}$ reaction¹¹ is also prominent in our (α, α) excitation curves. This level cannot however explain the broad structure for $E_\alpha < 4$ MeV, nor can the next lowest reported level¹ which appears at $E_\alpha = 2.58$ MeV ($\Gamma_{\text{c.m.}} \cong 200$ keV). To obtain even a qualitative description of the data below 4 MeV, the presence of at least one additional level had to be assumed. Our best fit was obtained with a $\frac{3}{2}^-(0)$ assignment for a level near $E_\alpha = 3.8$ MeV. However, the

quality of the fit suggests that the resonance structure may require contributions from more than one level. Although several combinations of levels were tried in an attempt to explain the data below 4 MeV, without success, there is little merit in attempting to fit the data by just including more and more levels in the calculations. Thus, although our work favors a $\frac{3}{2}^-(0)$ assignment for this region, a clearer understanding of the region below 4 MeV is required before any definiteness may be attached to this assignment.

Attempts to fit the narrow 4.00 -MeV level directly with the computer code were not successful, since the broad levels at 3.8 and 4.2 MeV dominate the absolute cross section in this region. The spin and parity of this level, for which no previous assignments have been made, were obtained, instead, by comparison of theoretical isolated-resonance shapes for many J^π values at all the angles for which data were taken. The results of this comparison indicate that $\frac{3}{2}^-$ is the most likely assignment.

In our attempts to describe the data around 5 MeV, it was necessary to include in the calculations information on the structure near $E_\alpha = 5.4$ MeV. Our best agreement with the data was obtained with an $l_\alpha = 1$ state with J^π being either $\frac{3}{2}^+$ or $\frac{5}{2}^+$, located at $E_\alpha = 5.5$ MeV. In the calculations shown in Figs. 3, 4, and 6, the $\frac{5}{2}^+(1)$ assign-

TABLE I. Resonance parameters used for ${}^9\text{Be}(\alpha, \alpha_0){}^9\text{Be}$ calculations in Figs. 3, 4, and 6.

E_x (MeV)	$\Gamma_{\text{c.m.}}$ (keV)	E_α (MeV)	E_x (${}^{13}\text{C}$) (MeV)	$\Gamma_{\text{c.m.}}$ (keV)	$J^\pi (l_\alpha)$	$\Gamma_{\alpha l}$ (c.m.) (keV)	
						l_1 ^a	l_2
Other work ^b		Present work					
		3.80	13.28	343	$\frac{3}{2}^-(0, 2)$	194	84
13.41	60	4.00	13.42	58	$[\frac{9}{2}^-(4, 6)]$ ^c	15	0 ^d
13.55	≈ 500	4.20	13.56	685	$\frac{5}{2}^+(1, 3)$ ^e	284	12
13.76	≈ 350	4.50	13.77	247	$\frac{3}{2}^+(1, 3)$ ^e	37 ^f	44
14.13	≈ 200	5.00	14.11	75	$\frac{5}{2}^-(2, 4)$ ^g	0	61
		5.075	14.164	73	$\frac{7}{2}^+(3, 5)$ ^g	46	24
14.39 \pm 0.1	260	(5.50) ^h	(14.46)	400	$[\frac{5}{2}^+(1, 3)]$ ^h	199	0

^a The number in parentheses refers to the l_α values of the incoming α particle; l_1 refers to the lower value.

^b See text, Ref. 1.

^c This assignment is tentative.

^d This partial width fixed at zero in the search.

^e The correlation of these spin assignments with resonance energies depends upon the nature of the states contributing below $E_\alpha = 4$ MeV (see text).

^f The $l_\alpha = 1$ contribution for this state is probably a component of the $\frac{5}{2}^+(1)$ state at $E_\alpha = 4.2$ MeV. Our data are sensitive principally to the l value and hence cannot completely distinguish the J value.

^g A second pair of levels that described this region consisted of a $\frac{7}{2}^-$ state (at $E_\alpha = 5.0$ MeV) plus any one of a $\frac{3}{2}^+$, $\frac{5}{2}^+$, or $\frac{7}{2}^+$ assignments (for $E_\alpha = 5.075$ MeV).

^h This $l_\alpha = 1$ level at $E_\alpha = 5.5$ MeV is included primarily to describe the data near $E_\alpha = 5.0$ MeV (see text). The assignment is highly tentative (see Ref. 3).

ment was used for this state. It should be noted that the inclusion of this level was necessary to provide a reasonable over-all amplitude for the (α, α) cross section at somewhat lower energies; because the agreement with the data is not very good, the evidence for the level and the assignment made here must be considered tentative, particularly as this assignment is not supported by the accompanying analysis of De Martini, Soltesz, and Donoghue.

The solid lines in Figs. 3, 4, and 6 are the compound-nucleus calculations obtained using the level parameters in Table I. These parameters were determined as outlined above by simultaneously fitting the excitation functions at 106.8, 125.2, 140.6, 156.0, and 167.2° (c.m.) over the 3.5- to 5-MeV energy interval. While changes in the parameters could produce significantly better fits to the data at any one angle, the basic features seen at most angles are reproduced by the single set of parameters obtained in this search. It should be noted that no arbitrary normalization has been applied to the calculated cross section; a closer match in normalization might be obtained if a more realistic nuclear potential-scattering background was assumed, rather than the simple hard-sphere approximation employed here.

V. DISCUSSION

Analysis of the elastic scattering data alone is insufficient to uniquely determine the spins and parities of the levels which have been included. We have relied on the earlier assignments for the levels at 4.20 and 4.5 MeV as a starting point. In comparing our present results to other analyses, particularly that of De Martini, Soltesz, and Donoghue³ there are some apparent discrepancies; we shall consider these levels in turn, pointing out any additional supporting information or uncertainties in the assignments.

As mentioned above, the levels used as the starting point in our calculations correspond to $E_\alpha = 4.2$ MeV ($\frac{5}{2}^+$) and 4.5 MeV ($\frac{3}{2}^+$). Using both l_α values contributing to each of these levels, our best fitting favors $l_\alpha = 1$ for the $\frac{5}{2}^+$ level, in agreement with the neutron analyses,^{3, 15} and nearly equal partial widths for $l_\alpha = 1$ and 3 for the $\frac{3}{2}^+$ level, whereas the (α, n_0) analysis is well fitted using $l_\alpha = 3$ alone. As the (α, α) cross section is sensitive mainly to l and not J , the $l_\alpha = 1$ contribution obtained in our search fit is probably only an additional contribution from the $\frac{5}{2}^+(1)$ state and hence, there is probably no disagreement between the two works. De Martini, Soltesz, and Donoghue³ indicate that the positions of these two levels may be interchanged from the energies used in our cal-

ulation, depending upon what levels contribute for $E_\alpha < 4$ MeV. The alternate ordering of these states was not tried in this analysis since their observation was made after these calculations had been concluded. Since our calculations do suggest the presence of a $\frac{5}{2}^-(0)$ state at 3.8 MeV and it is the inclusion of this state in the neutron analysis which gives the ordering of the levels used here, the results of the two works appear to be in agreement. However, as discussed above, the structure for $E_\alpha < 4$ MeV appears to be more complicated than has been assumed here. Until this structure is better explained, the J^π assignment to a level near 3.8 MeV must be treated as tentative, as in the ordering of the two broad positive-parity states. In any case, the 4- to 5-MeV region is dominated by $\frac{5}{2}^+(1)$ and $\frac{3}{2}^+(3)$ states whose widths are in qualitative agreement in the two works.

During the course of the data analysis, calculations including the narrow level at $E_\alpha = 4.0$ MeV, with the $\frac{3}{2}^-$ assignment, were made which produced a much better agreement with the experimental cross section than the fit shown in Figs. 3 and 4. However, it was necessary to include a significant $l=6$ contribution in order to obtain this good agreement. The reduced width for this channel was then much larger than the Wigner limit and hence, not physically realistic. The present fit, with $\frac{9}{2}^-$ with $l_\alpha = 4$, matches better than the shapes obtained for other assignments, but the level is still not well accounted for. Because of the small total width of this state, the neutron-polarization data which were taken with thicker targets are not sensitive to this level and no further clarification is obtainable from this source. The excitation curve for neutrons leading to the first excited state of ${}^{12}\text{C}$ shows the presence of this level,¹³ but again there is no information for clarifying its assignment.

De Martini, Soltesz, and Donoghue find a significant contribution for a $\frac{5}{2}^-(4)$ level at $E_\alpha = 5$ MeV, in agreement with one of our two choices for a level here. However, the amplitude of the matrix element for this assignment in the (α, n_0) analysis suggests either a broad level peaking near 5.3 MeV, or, possibly, the presence of two levels of this character in this energy region. The (α, α) excitation curves clearly indicate the presence of narrow structure at 5.0 MeV, as do the ${}^9\text{Be}(\alpha, n_1)$ and ${}^9\text{Be}(\alpha, n_2)$ excitation curves¹³ at 0°. The width of the structure appears to vary with the exit channel. The (α, n_0) yield curve at 0° on the other hand shows a pronounced minimum at this energy, suggesting that this level at $E_\alpha = 5.0$ MeV has a small partial width in the ground-state neutron channel.

A $\frac{5}{2}^-(4)$ level fits the (α, α) data well at forward

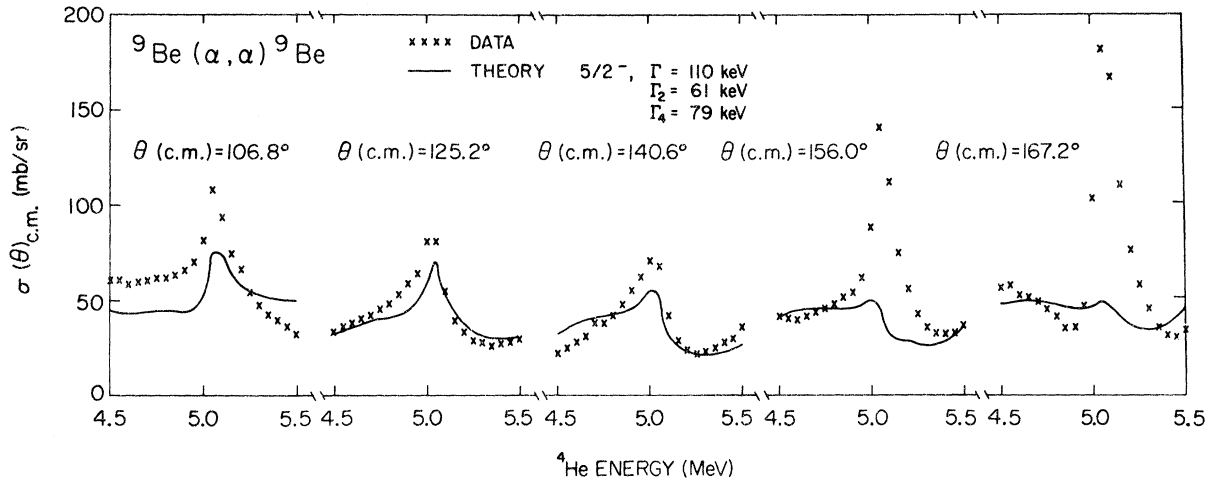


FIG. 7. "Fits" to ${}^9\text{Be}(\alpha, \alpha){}^9\text{Be}$ with a single level at 5.0 MeV. Except for the levels at 5.00 and 5.075, the level parameters are those given in Table I. The necessity of including an additional level in the 5-MeV regions is clear.

angles; however, the backward-scattering-angle data require the presence of a second level of opposite parity approximately one level width away, and this was determined to be a $\frac{7}{2}^+(3)$ state [with the $\frac{5}{2}^-(4)$ companion state assignment]. This is well illustrated in Fig. 7 where the fits to the data in this region were attempted with all the levels listed in Table I, except that the 5.0-MeV structure was described by a single state of $\frac{5}{2}^-(4)$. The J -value assignment for the state at ~ 5.07 MeV depends critically upon the nature of the state at 5.0 MeV. As pointed out above, an alternate description of the data was obtained if the state for $E_\alpha = 5.0$ MeV has a $\frac{7}{2}^-$ assignment, with the required second state at $E_\alpha = 5.07$ MeV having a $J^\pi = \frac{3}{2}^+, \frac{5}{2}^+, \text{ or } \frac{7}{2}^+$ assignment. As the $\frac{5}{2}^- - \frac{7}{2}^+$ levels overlap strongly, their level positions are estimated to be uncertain to at least 30 keV. These levels have large α partial widths (see Table I). The total width of the positive-parity state is accounted for almost entirely in the α channel, whereas the odd-parity member of this pair does have a neutron width (~ 14 keV). It is perhaps the latter state that contributes to the (α, n_1) and (α, n_2) reaction excitation curves.

The level we place at $E_\alpha = 5.5$ MeV, with our assignment of either $\frac{3}{2}^+(1)$ or $\frac{5}{2}^+(1)$, is needed in order to get the magnitude of the cross section to match the data in the 5.0-MeV region. As such, this level primarily represents a background term for the lower-energy data, and both its position and width are not well determined in this experiment. Hence discrepancies between our results and those of De Martini, Soltesz, and Donoghue are probably more apparent than real since the data at 5.5 MeV are not fitted well.

The total width of each level found from the (α, α) elastic scattering data analysis is compared with the previously quoted widths in Table I. The numerical values of the total width and partial width of each level, however, are quite sensitive to the contributions to the cross section from nearby levels and will change if additional levels are required to completely describe the data. Because the level shift and the contributions of distant levels have been neglected in our calculations, the quoted widths must be considered as approximate.

VI. CONCLUSIONS

Measurements have been presented for the elastic scattering of α particles by ${}^9\text{Be}$ in the bombarding-energy range 1.7–6.2 MeV. The data have allowed cross-section information to be extracted to within a relative uncertainty of $\pm 5\%$, and an absolute normalization within $\pm 13\%$. The observed structure indicates the presence of many excited states of ${}^{13}\text{C}$ in the excitation region covered. An attempt has been made to analyze all the data in the 3.5- to 5.0-MeV region with a single set of levels; the fits obtained reproduce the basic features seen experimentally, and the magnitude of the calculated cross section, while not as close to the experimental values at all angles as might be desired, is nevertheless not unreasonable.

It may be possible to improve the parameters by attempting a simultaneous fitting of *all* the elastic scattering data, perhaps including the data for the (α, n_0) channel as well. However, this would require an extensive revision of the search program and even then the computation

time for such an undertaking would be prohibitive. It appears that additional levels and a more realistic direct-reaction component are needed to obtain a more accurate picture of the structure of

${}^{13}\text{C}$ in this region. The levels included in the analysis presented here, however, would appear to be important components in a complete picture of this nucleus in the 13–15-MeV excitation region.

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¹F. Ajzenberg-Selove, Nucl. Phys. A152, 1 (1970).

²J. D. Goss, G. L. Marolt, C. P. Browne, and A. A. Rollefson, Phys. Rev. C 7, 663 (1973).

³D. C. De Martini, C. R. Soltész, and T. R. Donoghue, preceding paper, Phys. Rev. C 7, 1824 (1973).

⁴R. B. Taylor, N. R. Fletcher, and R. H. Davis, Nucl. Phys. 65, 318 (1965).

⁵J. H. Renkin, Phys. Rev. 132, 2627 (1963).

⁶J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. 24, 258 (1952), formula 7.12.

⁷R. Huby, Proc. Phys. Soc. A67, 1103 (1954).

⁸L. C. Biedenharn, J. M. Blatt, and M. E. Rose, Rev. Mod. Phys. 24, 249 (1952).

⁹J. D. Goss, Ph.D. dissertation, Ohio State University, 1970 (unpublished) (available from University Microfilms).

¹⁰J. H. Gibbons and R. L. Macklin, Phys. Rev. 114, 571 (1959).

¹¹J. B. Seaborn, G. E. Michell, N. R. Fletcher, and R. H. Davis, Phys. Rev. 129, 2217 (1963).

¹²R. G. Miller and R. W. Kavanagh, Nucl. Phys. 88, 492 (1966).

¹³A. W. Obst, T. B. Grandy, and J. L. Weil, Phys. Rev. C 5, 738 (1972).

¹⁴J. R. Risser, J. E. Price, and C. M. Class, Phys. Rev. 105, 1288 (1957).

¹⁵S. E. Darden, in *Proceedings of the Second International Symposium on Polarization Phenomena of Nucleons*, edited by P. Huber and H. Schopper (Birkhauser-Verlag, Basel, 1966), p. 433.

¹⁶G. P. Lietz, S. F. Trevino, A. F. Behof, and S. E. Darden, Nucl. Phys. 67, 193 (1965); G. P. Lietz, Ph.D. thesis, University of Notre Dame, 1964 (unpublished).

Effect of the Short-Range Part of the Nucleon-Nucleon Interaction on the Properties of the Three-Nucleon System

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Some of the low-energy properties of the three-nucleon system are calculated with two s -wave spin-dependent interactions, which produce almost identical 3S_1 and 1S_0 two-nucleon phase shifts up to a lab energy of 350 MeV. Both interactions consist of an attractive square well and a short-range repulsion. The repulsive part of one potential is a hard core, for the other it is a nonlocal, separable potential. The hard-core potential produces a triton binding energy of 8.7 MeV, a doublet scattering length of 0.84 fm, and a quartet scattering length of 6.30 fm. The corresponding parameters calculated with the partly nonlocal potential are 8.8 MeV, 0.85 fm, and 6.31 fm. These results suggest that the low-energy properties of the three-nucleon system are insensitive to the details of the short-range part of the two-nucleon interaction.

I. INTRODUCTION

Recently many calculations for the binding energy of the triton have been carried out with so-called realistic potentials. It appears that all of them underbind the triton.¹ The results of most of the recent calculations are summarized in Ta-

ble II of Ref. 1, so we will mention only a few of them here as examples. The Hennel-Delves² variational calculation for the Reid³ soft-core potential gives a value of 7.75 ± 0.5 MeV for the triton binding energy. Their⁴ value for the Hamada-Johnston⁵ potential is 6.5 ± 0.2 MeV. The calculation of Harper, Kim, and Tubis,⁶ which is based