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Pion-Oxygen Elastic Scattering in the 3-3 Resonance Region*

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The total and differential cross sections for π^--^{16}O elastic scattering in the 3-3 resonance region are calculated using the first-order optical potential derived from the multiple-scattering theory of Kerman, McManus, and Thaler. In order to include the 3-3 resonance and generate a reasonable off-shell behavior of the π-nucleon transition matrix, a separable model is used to construct the π-nucleon transition matrix. The optical potential also includes a proper transformation of the π-nucleon transition matrix from the π-nucleon to the π-nucleus c.m. frames. The results are compared with recent π^--^{16}O elastic-scattering data and with earlier π^--^{12}C calculations.

I. INTRODUCTION

With the advent of meson factories, there will be an increase in the study of nuclear structure using the π meson as a probe. The basic π-nucleon interaction has relatively well-determined phase shifts that show the interaction to be resonant and dominated by the $J=\frac{3}{2}$, $T=\frac{3}{2}$ channel over a fairly wide energy range. In order to extract detailed information about nuclear structure, a reliable method of calculation should be established, in which the resonant features and the general dependence of the π-nucleon interaction are correctly incorporated.

In an earlier paper,[1] we studied first-order π-nucleus optical potentials defined in the multiple-scattering theory of Kerman, McManus, and Thaler[2]; several models for the off-shell π-nucleon transition matrices were used. The π-nucleon collision matrices employed in the Kisslinger and Laplacian (or local) optical models were seen to diverge with increasing off-shell momenta, where-

as a separable potential model for the πN interaction did display a reasonable fall off. Hence it was concluded that the off-shell effects, and the 3-3 resonance, were reasonably well incorporated into the π-nucleus optical potential by adopting the separable π-nucleon interaction constructed by Landau and Tabakin.[3] Clearly, improved models of the π-nucleon interaction could be invoked, but our numerical studies indicated that the π-nucleus cross sections are not sensitive to the detailed structure of the off-shell π-nucleon transition matrices as long as some appropriate off-shell fall-off is included. Another important feature of our earlier work is the inclusion of a proper transformation of the π-nucleon transition matrix from the π-nucleon c.m. frame to the π-nucleus c.m. frame.

The first-order optical potential, which was constructed from the separable π-nucleon interaction, was applied earlier to π^--^{12}C elastic scattering. That study provided insight into the basic features of π^--^{12}C data. For example, the downward shift in the peak of the total cross section, the variation of the diffractive structure in the differential cross section with energy, and the shift in the zero of the real part of the forward-scattering amplitude were examined and discussed in detail. Furthermore, we concluded that the downward shift in the peak of the total cross section is not an exotic effect, but is mainly due to a broadening of the imaginary part of the forward-scattering amplitude, caused by multiple scattering and nuclear size.

In this work, we apply the first-order optical potential of Ref. 1 to π^--^{16}O elastic scattering. (There have been several other calculations for π^--^{16}O elastic scattering using an optical-potential approach[4,5] or Glauber theory.[6]) The ^{16}O calculation provides additional insight into the detailed mechanism of π-nucleus collisions. Also, we hope to determine the dependence of π-nucleus elastic scattering on the number of nucleons. Although higher-order effects such as π absorption by a pair of nucleons, interaction of the N^* isobar with nucleons in the nucleus, and two-nucleon correlation effects are not yet included, the main features of the data seem to be already accounted for by the first-order optical potential. However, to extract detailed information about nuclear structure, these effects should ultimately be included.

In Sec. II, the definition of the first-order optical potential is briefly reviewed (for details, see Ref. 1), and in Sec. III, the results of our π^--^{16}O calculation are presented and discussed. A comparison of the π^--^{16}O calculation with recent data[7] suggests that the general features of the total and differential cross sections are obtained using the first-order optical potential. Also, a comparison of this π^--^{16}O calculation with our earlier π^--^{12}C calculation indicates that the results of these two calculations are systematically related.

II. FIRST-ORDER OPTICAL POTENTIAL

The π-nucleus transition matrix, $T=[A/(A-1)]A'$, is obtained from the optical-potential operator U by solving the integral equation,

$$T' = U + U\frac{P_0}{E - K_\pi - H_A + i\epsilon}T', \tag{1}$$

with A denoting the nucleon number, K_π the kinetic-energy operator for the π meson, and H_A the nuclear Hamiltonian. By virtue of P_0 (the projection operator onto the ground state of the nucleus), no intermediate nuclear excited states are present in Eq. (1). Effects of the excited states are included instead in the exact optical potential U, which is defined by another integral equation,

$$U = U^0 + U^0\frac{(1-P_0)}{E - K_\pi - H_A + i\epsilon}U, \tag{2}$$

with $U^0=(A-1)\tau(E)$. Here $\tau(E)$ is the transition matrix for the scattering of a π meson from a nucleon bound in the nucleus. In terms of the basic pion-nucleon interaction (v), $\tau(E)$ can be written as,

$$\tau(E) = v + v\frac{Q}{E - K_\pi - H_A + i\epsilon}\tau(E). \tag{3}$$

(It is assumed here that the π-N interaction can be adequately represented by a potential v.)

Using the multiple-scattering and impulse approximations,[2] the first-order optical potential is obtained;

$$\begin{aligned} U &\simeq U^0 = (A-1)\tau(E) \\ &\quad \text{(multiple scattering approximation)} \\ &\simeq (A-1)t(\omega) \\ &\quad \text{(impulse approximation)}, \end{aligned} \tag{4}$$

where $t(\omega)$ is the free pion-nucleon transition matrix evaluated at some appropriate energy variable ω.

In order to solve Eq. (1), we need the many-body matrix elements of U^0: $\langle k', 0|U^0|k, 0\rangle$. Here $|k\rangle$ represents the state of a meson with momentum $\vec{k}$ and isospin m_I, and $|0\rangle$ denotes the nuclear ground state. For a spin-zero nucleus the first-

order optical potential can be written as,

$$\langle k'0|U^0|k0\rangle = \sum_{\alpha=0,1}(A-1)(k'0|t_\alpha(\omega)0_\alpha|k0)$$
$$= \sum_{\alpha=0,1}(A-1)\int(k',\vec{p}-\vec{q}|t_\alpha(\omega)|k,\vec{p}) \times F_\alpha(\vec{p}-\vec{q},\vec{p})\,d\vec{p}, \quad (5)$$

where $F_\alpha(\vec{p}',\vec{p}) = \langle 0|\sum_{j=1}^{A}\delta(\vec{p}_j'-\vec{p}')0_\alpha\delta(\vec{p}_j-\vec{p})|0\rangle$ and $0_0=1$, $0_1=\vec{\imath}\cdot\vec{\tau}$. Here $\vec{q}=\vec{k}'-\vec{k}$ and $\vec{k},\vec{p}$ denote the initial-pion and struck-nucleon momenta, respectively.

Since the nucleus is large compared to the π-nucleon interaction range, F_α in Eq. (5) varies more rapidly, as a function of nucleon momentum $\vec{p}$, than does the T matrix element. We can thus approximate the integral be evaluating t_α at some average value of momentum $\vec{p}=\vec{p}_0$ and factoring it outside of the integral:

$$\langle k'0|U^0|k0\rangle \simeq \sum_\alpha (A-1)(k',\vec{p}_0-\vec{q}|t_\alpha(\omega)|k,\vec{p}_0)\rho_\alpha(\vec{q}), \quad (6)$$

where the form factor $\rho_\alpha(\vec{q}) \equiv \int F_\alpha(\vec{p}-\vec{q},\vec{p})\,d\vec{p}$ is related to the Fourier transform of the densities of neutrons and protons within the nucleus. To fully define t_α in (6), we now choose ω to be the on-energy-shell π-nucleon collision energy $\omega = (k_0^2+m_\pi^2)^{1/2}+(p_0^2+m_N^2)^{1/2} = E_\pi(k_0)+E_N(p_0)$. Here $\vec{k}_0$ is the on-shell pion momentum in the π-nucleus c.m. and $\vec{p}_0=-\vec{k}_0/A$ is the momentum of a nucleon "frozen" in a nucleus of momentum $-\vec{k}_0$.

Various comments should be made about Eq. (6). First of all, the factorization approximation (assuming t_α to be a slowly varying function of $\vec{p}$) is perhaps not a good enough approximation in the resonance region. Therefore we have corrected approximately for the effect of nucleon motion by averaging the on-shell π-nucleon transition matrix over the nucleon motion. Secondly, to solve Eq. (1), the off-shell matrix elements ($k' \neq k$) of t_α should be known. A separable π-nucleon interaction which exactly fits the on-shell scattering data has been used to calculate these off-shell matrix elements. We feel that this separable model accounts for the 3-3 resonance and is a physically reasonable way to generate off-shell matrix elements.

Also, another point that should be stressed is the transformation of the transition matrix from the π-nucleon c.m. frame to the π-nucleus c.m. frame. It is best to solve Eq. (1) in the *π-nucleus* c.m. frame, whereas the phase-shift for the π-nucleon scattering, and hence the on-shell π-nucleon transition matrices, are tabulated in the *π-nucleon* c.m. frame. The various models for the off-shell behavior give the transition matrix in the π-nucleon c.m. frame; this matrix must be properly transformed to the π-nucleus c.m. frame. The result of this transformation is a mixing of different π-nucleon partial waves, which in our previous work[1] was shown to be of great importance. The transition matrices in the π-nucleon and the π-nucleus c.m. frames are related by $(k',\vec{p}_0-\vec{q}|t(\omega)|k,\vec{p}_0) = \gamma(\kappa'|\tilde{t}(\tilde{\omega})|\kappa)$, where we have chosen $p_0=-k_0/A$ (frozen nucleon in the nucleus). Here $\tilde{\omega}$, κ', and κ are determined using the Lorentz transformation from the π-nucleon c.m. system to the π-nucleus c.m. system.[1] The factor γ is

$$\gamma = \left[\frac{E_\pi(\kappa)E_\pi(\kappa')E_N(\kappa)E_N(\kappa')}{E_\pi(k)E_\pi(k')E_N(k/A)E_N(k'/A)}\right]^{1/2},$$

where E_π and E_N are energies of the π meson and the nucleon, respectively. (Note that we have assumed that the nucleon is frozen in the nucleus, both before and after collision.)

In addition to the factor γ, the angle between $\vec{k}$ and $\vec{k}'$ is not the same as the angle between $\vec{\kappa}$ and $\vec{\kappa}'$. Using the invariance of the four-momentum transfer, we have

$$\vec{\kappa}\cdot\vec{\kappa}' = E_\pi(\kappa)E_\pi(\kappa') - E_\pi(k)E_\pi(k') + \vec{k}\cdot\vec{k}'.$$

An important effect of this relation is that any partial wave of the π-nucleon transition matrix contributes to the lower partial waves of the π-nucleon transition matrix as seen in the π-nucleus c.m. frame.

Having discussed these transformations, let us return to Eq. (6). For isospin-zero nuclei like ^{16}O and ^{12}C, the form factors for neutrons and protons can be considered to be the same (which is a good approximation for light nuclei, since effects of Coulomb force would be small.) In this case, the first-order optical potential becomes

$$\langle\vec{k}',\pi^\pm,0|U^0|\vec{k},\pi^\pm,0\rangle = (A-1)\rho(\vec{k}'-\vec{k}) \times (\vec{k}',\vec{p}_0-\vec{q}|\tfrac{1}{3}t_{1/2}(\omega)+\tfrac{2}{3}t_{3/2}(\omega)|\vec{k},\vec{p}_0), \quad (7)$$

where $t_{1/2}$ and $t_{3/2}$ are the isospin $\frac{1}{2}$ and $\frac{3}{2}$ π-nucleon transition matrices, respectively. (In this paper we do not include the π-nucleon D and F waves, which we had included in our earlier π^--^{12}C calculation.[1] The contribution of the π-nucleon D and F waves to the π^--^{12}C cross sections was found to be very small in the 3-3 resonance region.) The nucleon form factor $\rho(\vec{k}'-\vec{k})$ used in our calculation is of the form,

$$\rho(\vec{q}) = [1-\tfrac{1}{2}\alpha q^2 a_{\text{c.m.}}{}^2(2+3\alpha)^{-1}] \times \exp(-q^2 a_{\text{ch}}{}^2/4), \quad \text{where } \alpha=(A-4)/6.$$

The parameters a_{ch} and $a_{c.m.}$ are selected from the analysis of electron-^{16}O elastic scattering[8] to be $a_{ch} = 1.70$ fm, and $a_{c.m.} = 1.76$ fm; this removes the finite proton size. Thus, after a partial wave decomposition of U^0 and T', the Lippmann-Schwinger equation (1) becomes a one-dimensional integral equation,

$$T_l'(k', k; E(k_0)) = U_l^0(k', k; \omega(k_0)) + \int dk'' k''^2 \frac{U_l^0(k', k''; \omega(k_0))\, T_l'(k'', k; E(k_0))}{E(k_0) - E(k'') + i\epsilon}. \quad (8)$$

Equation (8) is solved numerically using a matrix-inversion technique developed by Haftel and Tabakin,[9] now modified to deal with complex potentials.

The π-nucleus scattering amplitude is obtained from the on-shell π-nucleus collision matrix as

$$f_{k_0}(\cos\theta) = -\left(\frac{A}{A-1}\right) 2\mu \sum_l (2l+1) \times T_l'(k_0, k_0'; E(k_0)) P_l(\cos\theta),$$

where

$$\mu = \frac{E_\pi(k) E_A(k)}{E_\pi(k) + E_A(k)},$$

E_π and E_A being energies of the π-meson and the nucleus, respectively.

In terms of this π-nucleus scattering amplitude, the differential and total cross sections are,

$$\frac{d\sigma}{d\Omega}(\cos\theta) = |f_{k_0}(\cos\theta)|^2;$$

and

$$\sigma_{tot} = \frac{4\pi}{k_0} \mathrm{Im} f_{k_0}(\cos\theta = 1).$$

III. DISCUSSION AND RESULTS

A. Total Cross Sections and Forward-Scattering Amplitude

Consider the total cross sections first. In Fig. 1 the total cross-sections (σ_{tot}) for π^--^{16}O scattering at different π-meson kinetic energies (T_π^{lab}) are plotted. For comparison, σ_{tot} for π^--^{12}C is also plotted. We see that the two curves are very similar. Both calculations show a downward shift in the total cross section peak from the corresponding 3-3 resonance peak (~180 MeV). The amount of shift (~65 MeV) is approximately the same for both calculations (σ_{tot} peaks at 112 MeV for ^{16}O and at 120 MeV for ^{12}C).

The relation between the ^{16}O and ^{12}C results can be understood by noting that the optical potential is quite absorptive in the resonance region. Thus the nucleus can be considered to be almost black, and consequently the total cross section will be proportional to the square of the nuclear radius. For a constant nuclear density $R \propto A^{1/3}$ and therefore σ_{tot} will be proportional to $A^{2/3}$. In addition, for a perfectly black nucleus, σ_{tot} will be independent of energy. Although the calculated total cross sections do vary with energy, σ_{tot} for π^--^{16}O and π^--^{12}C do, to a good approximation, scale according to a simple black-disk rule (Fig. 1):

$$(\sigma_{tot})_{\pi^-\text{-}^{16}O} = (R_{^{16}O}/R_{^{12}C})^2 \times (\sigma_{tot})_{\pi^-\text{-}^{12}C} = (16/12)^{2/3} \times (\sigma_{tot})_{\pi^-\text{-}^{12}C}.$$

In addition to the above mentioned relation between σ_{tot}, both π^--^{16}O and π^--^{12}C calculations show that σ_{tot} peaks about 65 MeV below the corresponding 3-3 resonance peak. In order to understand the origin of the shift, we have plotted the imaginary part of the forward-scattering amplitude [Im$f(0)$] vs T_π^{lab} (Fig. 2). The Im$f(0)$ has a very broad peak at about 210 MeV, quite close to the 3-3 resonance energy. The broadening is due to the nuclear size and the multiple scattering of pions from nucleons in the nucleus. Since $\sigma_{tot} = (4\pi)/k \mathrm{Im} f(0)$, the peak of σ_{tot} will always be at

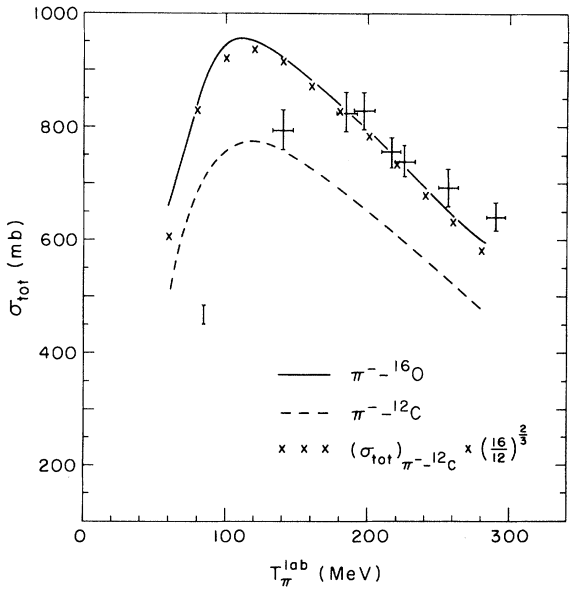

FIG. 1. The total cross section for π^--^{16}O scattering (continuous line) and for π^--^{12}C scattering (dashed line) as a function of π-meson kinetic energy. The crosses are obtained by multiplying $(\sigma_{tot})_{\pi^-\text{-}^{12}C}$ by $(16/12)^{2/3}$ (see text). Data are obtained from Ref. 11.

an energy lower than the energy at which Im$f(0)$ peaks (because of the $1/k$ factor). Furthermore, since the peak of Im$f(0)$ is very broad, the shift in the peak of σ_{tot} from the 3-3 resonance energy is large. Thus, most of the downward shift in the maximum of σ_{tot} is a result of multiple scattering and nuclear size. It is not an exotic phenomenon.

To a lesser extent,[10] however, the shift does depend upon details of the π-nucleon interaction, such as its off-shell behavior. For example, the degree of shift seems proportional to the "nonlocality" of the π-nucleus optical potential (which is related to the πN off-shell behavior); the local Laplacian potential gives the least shift, the "reasonably" nonlocal separable-based potential gives somewhat more of a shift, whereas the very nonlocal Kisslinger model predicts a considerably greater shift.[1]

There is a striking difference between the total cross section data for ^{16}O and our optical-model results. The available data[11] do not show any shift in the peak of σ_{tot} from the resonance energy. This discrepancy between our results and the present data must be resolved. Perhaps after precise information about π^--^{16}O scattering becomes available from the experiments planned at meson factories, and after further development of the theory, this difference will be resolved. Our results indicate a clear downward shift.

In Fig. 2 we have also plotted Im$f(0)$ for the π^--^{12}C calculation. Again, the carbon and oxygen results are similar in nature, with both having a broad maximum near 210 MeV. Also, when Im$f(0)$ for π^--^{12}C is scaled by $(16/12)^{2/3}$, we get Im$f(0)$ for π^--^{16}O, to a good approximation.

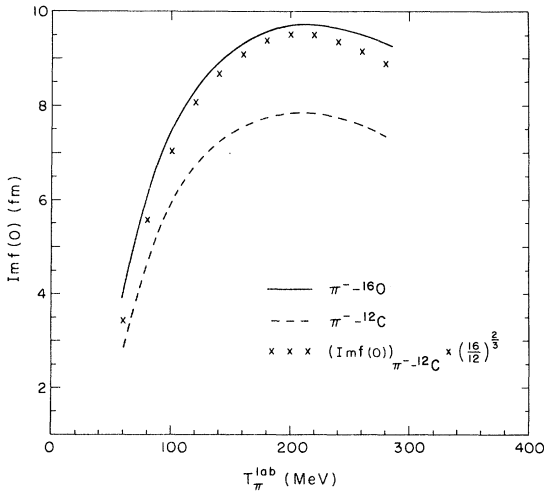

FIG. 2. The imaginary part of the forward-scattering amplitude vs the π-meson kinetic energy. The continuous curve is for π^--^{16}O and the dashed curve is for π^--^{12}C. The crosses are obtained by multiplying $[\text{Im}f(0)]_{\pi^-\text{-}^{12}C}$ by $(16/12)^{2/3}$.

Let us consider the real part of the forward-scattering amplitude. Figure 3 displays Re$f(0)$ for π^--^{16}O and for π^--^{12}C scattering. Both curves show that the zero of Re$f(0)$ is below the 3-3 resonance energy [Re$f_{\pi N}(0)=0$ at 195 MeV]. The amount of shift is about the same for both curves (39 MeV for ^{16}O case and 35 MeV for ^{12}C case). The slopes of the two curves [where Re$f(0)=0$] are -0.049 fm/MeV for the ^{16}O case and -0.041 fm/MeV for the ^{12}C case. Recent π-carbon elastic scattering experiments[12] have indicated a downward shift in the zero of Re$f(0)$ from the 3-3 resonance energy. It would be interesting to see if the π-oxygen elastic scattering experiments now in progress[13] will show such a shift in the zero of Re$f(0)$. In contrast to the optical-model predictions, we note that the Glauber theory calculations do not have a shift in the zero of Re$f(0)$ from the resonance energy – this represents an important difference between the two approaches.

B. Differential Cross Sections

In Fig. 4 we have plotted the elastic differential cross section vs the angle of scattering at various energies. The experimental results are from Ref. 7. The theoretical predictions using the optical model agree surprisingly well with the data,

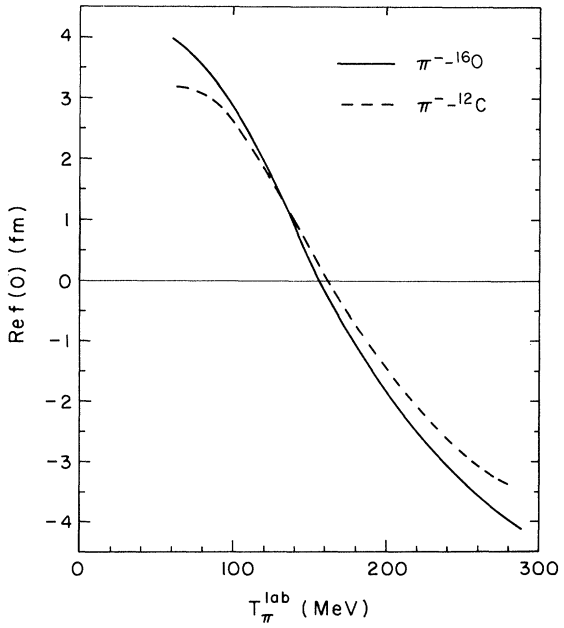

FIG. 3. The real part of the forward-scattering amplitude vs the π-meson kinetic energy. The continuous curve is for the π^--^{16}O case and the dashed curve is for the π^--^{12}C case.

especially considering that this is the lowest-order theory with no adjustable parameters. The position of the first minimum is given to within 3° by our calculation, whereas for ^{12}C (Fig. 5), the calculated first minimum is about 6° lower than the experimental one in the resonance region. We found that the inclusion of Coulomb scattering, or the use of different optical models, does tend to change the height of the minima and maxima in the cross sections, but not their position which is related to the nuclear shape. The discrepancy in the position of the first minimum for carbon thus seems related to the deformation of ^{12}C in its ground state. To get better results for ^{12}C, one should probably use a form factor which accounts for the deformation of ^{12}C.

The theoretical cross sections in Fig. 4 are calculated with our optical potential in which a separable πN interaction is used. The cross sections calculated for ^{16}O depend on the different optical models in much the same way as found in our ^{12}C calculation; the Laplacian and Kisslinger models predict progressively less large-angle scattering, with the separable and Laplacian models rather close for $\theta < 90°$.

Although the oxygen results seem rather good, there are some minor differences with the data worthy of comment. One of them concerns the location of maximum diffractive structure. Experimental differential cross sections appear to have their maximum diffractive structure at $T_\pi^{\rm lab} = 170$ MeV, whereas the calculated $(d\Omega/d\sigma)$ is most diffractive at $T_\pi^{\rm lab} = 160$ MeV. This difference (and other differences such as disagreement around the second maximum) may be accounted for by higher-order corrections to the first-order optical potential. We feel that, rather than adjusting parameters of the first-order optical potential to fit the data, one should try to explain these small deviations by higher-order correc-

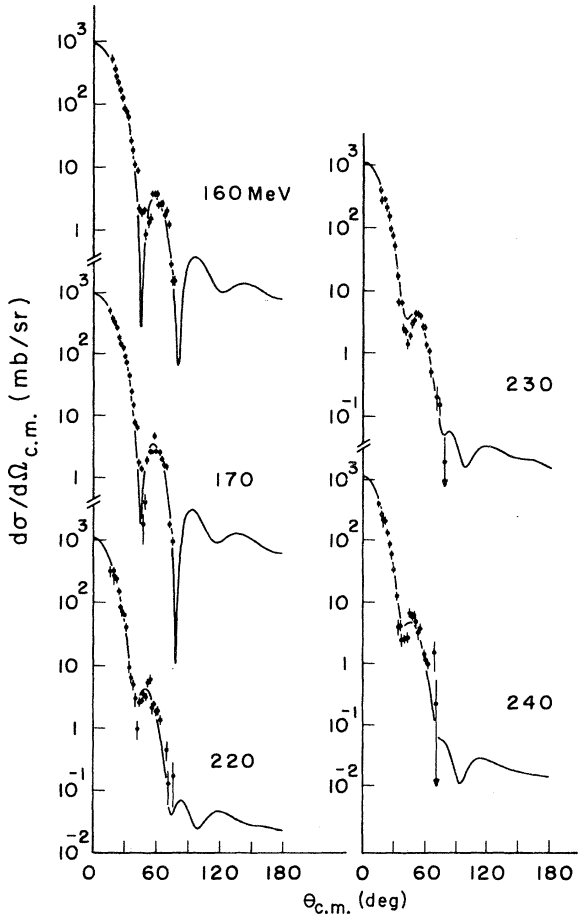

FIG. 4. The differential cross sections for the π^--^{16}O calculation at $T_\pi^{\rm lab} = 160$, 170, 220, 230, and 240 MeV. Data are obtained from Ref. 7.

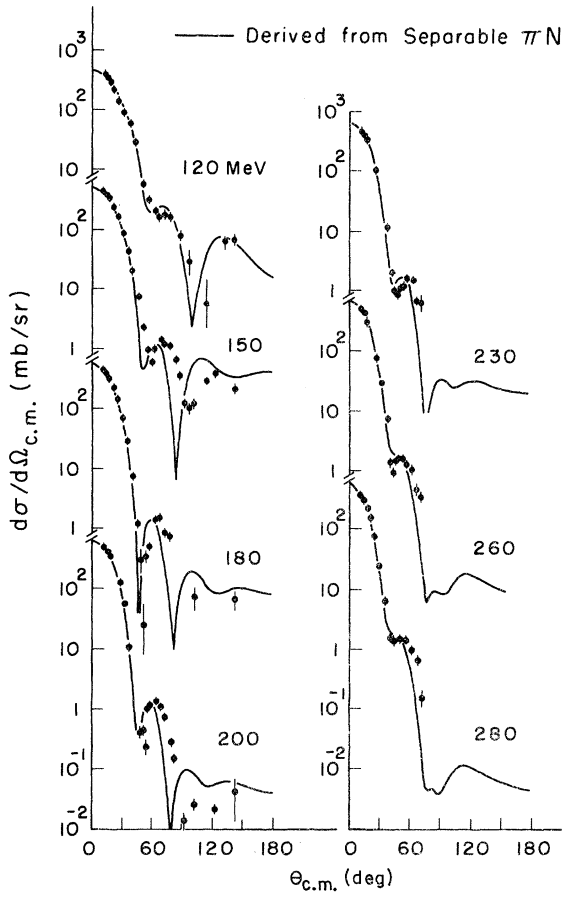

FIG. 5. The differential cross sections for π^--^{12}C calculation at $T^{\rm lab} = 120$, 150, 180, 200, 230, 260, and 280 MeV. Data are obtained from F. Binon *et al.*, Nucl. Phys. B17, 168 (1970).

tions. In this way one can perhaps learn about details of nuclear structure such as nuclear correlations and differences in neutron and proton densities.[14]

To sum up, the first-order optical potential based on multiple-scattering theory and a separable πN interaction has been used to calculate π^--^{16}O total and differential cross sections in the 3-3 resonance region. The peaks in the calculated σ_{tot} for both nuclei exhibit a downward shift from the resonance energy. Most of the shift is explained on the basis of a broadening of Im$f(0)$ due to multiple scattering, and nuclear size. Furthermore, the Re$f(0)$ is shown to have a zero at an energy (~39 MeV) lower than the 3-3 resonance energy. The calculated total cross sections for π^--^{16}O and π^--^{12}C are related by a simple $A^{2/3}$ law, caused by the strong absorptive nature of the optical potential. Finally, the calculated differential cross sections for oxygen are compared with the recent experiment of Bercaw *et al.*; the agreement is quite good.

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