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Effect of Woods-Saxon Wave Functions on Cross Sections for Charged-Pion Photoproduction from $^{16}O^{\dagger}$

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The cross sections for charged-pion photoproduction from $^{16}O(0^+, g.s.)$ leading to the four low-lying bound states $J^P = 2^-$, 0^-, 3^-, 1^-; $T = 1$ in ^{16}N are calculated in the Woods-Saxon basis. Two-particle–two-hole correlations in the ground-state wave function of ^{16}O are taken into account. Calculations in the Woods-Saxon basis yield more realistic results for the cross sections near the first pion-nucleon resonance region, where high momentum transfers occur, than those obtained in the conventional harmonic-oscillator basis.

I. INTRODUCTION

Recently, many attempts have been made to study the effect of Woods-Saxon (WS) wave functions[1] on shell-model calculations[2-4] of the spectra of certain nuclei, on inelastic electron scattering cross sections,[5] and on the diffraction minima in elastic electron scattering.[6,7] The results of shell-model calculations indicate that the effect of WS wave functions on level spacings can be as large as those obtained by adding a core-polarization correction to the interaction, and the results from electron scattering show that realistic WS single-particle states can predict the second diffraction minima in ^{16}O and ^{12}C, in agreement with experimental data, while the harmonic oscillator (HO) cannot. In the light of these studies, the author deems it necessary to do a somewhat similar study of the effect of the more physical WS basis on the cross sections for charged-pion photoproduction from ^{16}O, since high momentum transfers, as in the case of electron scattering, occur near the first pion-nucleon resonance region.

In earlier studies[8,9] of the reaction

$$\gamma + {}^{16}O(0^+, g.s.) \to \pi^+ + {}^{16}N(J^P = 2^-, 0^-, 3^-, 1^-; T = 1), \tag{1}$$

in which the final nuclear states are the four low-lying bound states of ^{16}N, we have shown that the energy dependence of the cross section is an important feature which strongly reflects the details of the structure of the initial and final nuclear states. The introduction of two-particle–two-hole (2p-2h) correlations in the ground-state wave function of ^{16}O and the use of Kuo wave functions with "screening" for ^{16}N states reproduced[9] the right order of magnitudes but the dependence on energy did not reflect the experimental shape.[10] An important factor to be taken into account, regarding the magnitude of the cross section, is the effect of final state interactions (FSI) of the outgoing pion with the residual nucleus. Saunders[11] finds that around 250-MeV incident photon energy the FSI is more significant for neutral than for charged-pion photoproduction and, in one particular case of charged-pion photoproduction, viz. $^{88}Sr(\gamma, \pi^-)$-

TABLE I. Expansion of WS wave functions in terms of single-particle oscillator components is given below for the first five terms.

nlj	E_{nl} (MeV)	b (fm)	Oscillator amplitudes (n') 1	2	3	4	5
$1p_{3/2}$	−21.83	1.6	0.999	0.007	0.025	−0.023	−0.002
		1.76	0.988	0.148	0.050	−0.008	−0.006
$1p_{1/2}$	−15.67	1.6	0.998	0.004	0.051	−0.022	0.003
		1.76	0.988	0.139	0.073	−0.001	0.004
$1d_{5/2}$	−4.14	1.6	0.992	−0.062	0.090	−0.050	0.018
		1.76	0.990	0.087	0.108	−0.020	0.010
$2s_{1/2}$	−3.27	1.6	−0.006	0.971	−0.151	0.147	−0.089
		1.76	−0.120	0.979	0.012	0.154	−0.047
$1d_{3/2}$	+0.94	1.6	0.976	−0.128	0.143	−0.080	0.047
		1.76	0.986	0.006	0.150	−0.044	0.036

^{88}Y, he finds the effect to be of some importance only in the forward angles (<30°) and there too the reduction in the differential cross section is at most of the order of 20–25%. The present investigation is therefore performed leaving the FSI and examining instead the effect of a realistic WS basis for the nuclear states on the energy dependence of the cross section.

We present here, the results of our investigation on the energy dependence of the cross section for reaction (1) using Perez[12] particle-hole wave functions for ^{16}N and taking the 2p-2h correlations in the ground-state wave function of ^{16}O into account. We find that the WS basis reproduces the right order of magnitude and shape of the experimental cross sections. In Sec. II we outline the approach and in Sec. III we summarize and discuss the results of the present study.

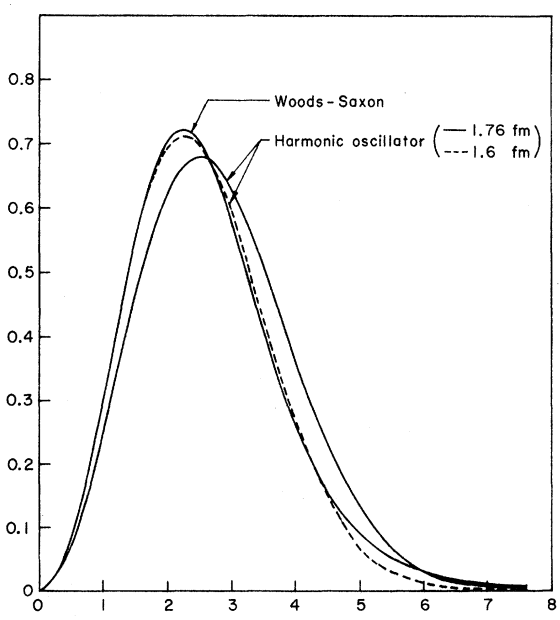

FIG. 1. The $1p$ WS radial wave function is shown in comparison with the $1p$ HO radial wave function.

II. WOODS-SAXON BASIS

A. Single-Particle Potential

The single-particle radial wave functions are calculated numerically, using subroutine BDSTS,[13] assuming a standard, realistic WS potential, including a Coulomb term, of the form:

$$v(r) = -Uf(r) + \left(\frac{\hbar}{m_\pi c}\right)^2 U_s g(r)(\vec{l}\cdot\vec{\sigma}) + V_c(r, R_c), \tag{2}$$

where

$$f(r) = \left[1 + \exp\left(\frac{r-R_0}{a}\right)\right]^{-1},$$

$$g(r) = \frac{1}{r}\frac{df}{dr},$$

$$V_c(r, R_c) = Z_A Z_\mu e^2 h(r),$$

$$Z_A, Z_\mu = \text{core, particle charge},$$

$$R_0 = R_c = r_0 A^{1/3},$$

$$h(r) = \begin{cases} \dfrac{1}{r} & \text{for } r \geq R_c, \\ \dfrac{1}{2R_c}\left(3 - \dfrac{r^2}{R_c^2}\right) & \text{for } r < R_c, \end{cases}$$

with the following values of the geometrical parameters: $r_0 = 1.25$ fm and $a = 0.65$ fm. The central and spin-orbit potential strengths, U and U_s, respectively, are fitted to the single-particle energies of the spectra of neighboring $(A+1)$ and $(A-1)$ nuclei and these are taken to be the same as those given in Table I of Ref. 12.

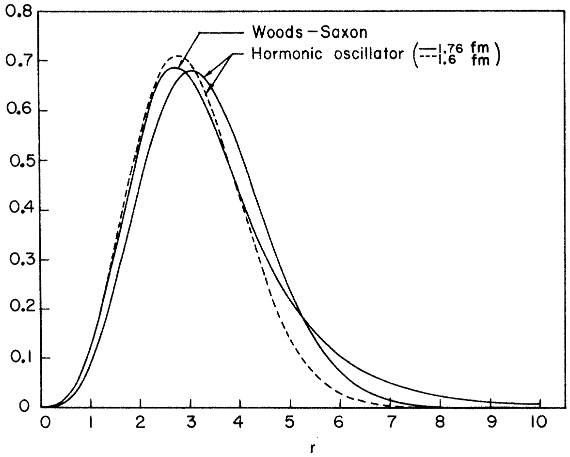

FIG. 2. The 1d WS radial wave function is shown in comparison with the 1d HO radial wave function.

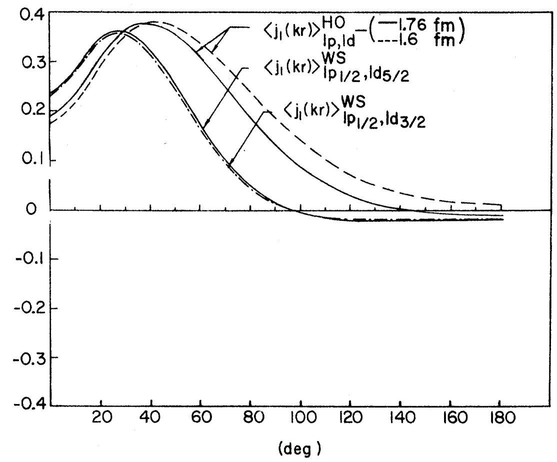

FIG. 4. The radial integrals $\langle j_1(kr)\rangle_{1p,1d}$ as a function of the angle of pion emission in the WS and HO basis.

The WS wave functions may be expanded in terms of the conventional HO wave functions $\phi_{nl}(r)$ as

$$\psi_{nlj}(r)=\sum_{n'} a_{nn'(lj)}\phi_{n'l}(r)\,. \tag{3}$$

The value of the oscillator size parameter

$$b=[\hbar/m\omega]^{1/2} \tag{4}$$

may be chosen at our convenience. In earlier studies[8,9] of reaction (1) in the HO basis, the value of $b=1.76$ fm fitted to the Stanford electron-scattering data[14] was used. Using averaged values for the proton and neutron shell-model potentials of Ref. 12, the value obtained[15] for the rms radius of ^{16}O is

$$\langle r^2\rangle^{1/2}=2.42\text{ fm}\,, \tag{5}$$

as compared with an experimental value[14] of 2.65 fm (corresponding to $b=1.76$ fm). The "equivalent" size parameter value corresponding to (5) is $b=1.6$ fm. Since the geometrical parameters used are resonable, the discrepancy between the two values of b can be attributed to the artificial deepening of the WS well, an artifice required to bind the $d_{3/2}$ particle states.

In Table I we present a description of the WS single-particle wave functions by giving the explicit expansion of each level into single-particle HO components for the first five terms, corresponding to the values of $b=1.6$ fm and $b=1.76$ fm.

In Figs. 1, 2, and 3 we plot the 1p, 1d, and 2s WS wave functions, respectively. The difference between $1p_{3/2}$ and $1p_{1/2}$, as well as that between $1d_{5/2}$ and $1d_{3/2}$ WS wave functions is small and hence they are not shown in Figs. 1 and 2. The most prominent feature of the WS wave functions

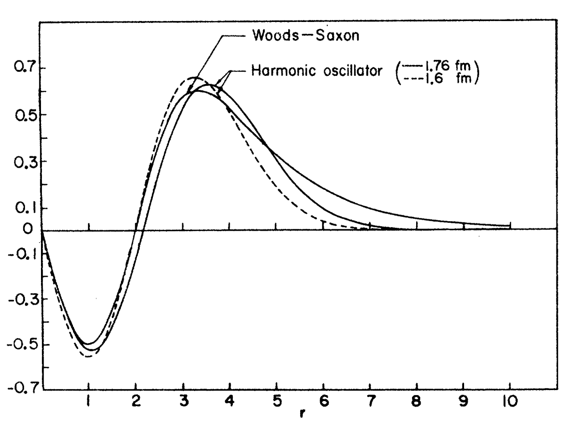

FIG. 3. The 2s WS radial wave function is shown in comparison with the 2s HO radial wave function.

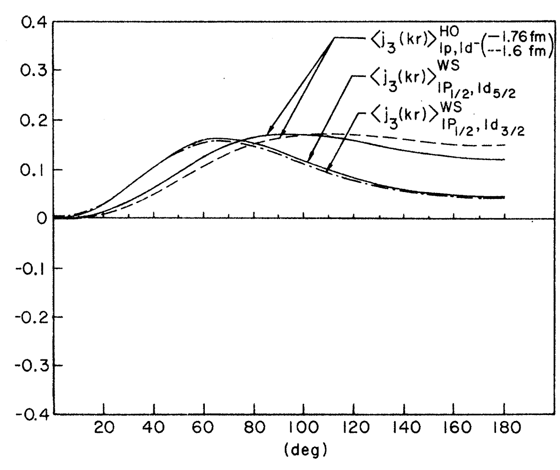

FIG. 5. The radial integrals $\langle j_3(kr)\rangle_{1p,1d}$ as a function of the angle of pion emission in the WS and HO basis.

is the extent to which the $1d$ and especially $2s$ functions protrude beyond their HO counterparts. Further, as is to be expected from Table I, the HO wave functions with "equivalent" size parameter value of 1.6 fm have a better overlap with their WS counterparts than those with $b = 1.76$ fm.

The relevant radial integrals

$$\langle j_l(kr)\rangle_{\mathrm{p,h}} = \int_0^\infty R_{n_\mathrm{p} l_\mathrm{p} j_\mathrm{p}}(r)\, j_l(kr) R_{n_\mathrm{h} l_\mathrm{h} j_\mathrm{h}}(r)\, dr \tag{6}$$

are plotted in Figs. 4, 5, and 6 as a function of the pion angle for WS (ψ_{nlj}) and HO (ϕ_{nl}) single-particle radial wave functions, for an incident photon energy of 260 MeV. The differences in values of the radial integrals

$$\langle j_l(kr)\rangle_{1p_{1/2},\,1d_{5/2}},\quad \langle j_l(kr)\rangle_{1p_{1/2},\,1d_{3/2}},$$

$$\langle j_l(kr)\rangle_{1p_{3/2},\,1d_{5/2}},\quad \text{and } \langle j_l(kr)\rangle_{1p_{3/2},\,1d_{3/2}}$$

are small and therefore only two of these four integrals are shown in Figs. 4 and 5, for $l = 1$ and 3, respectively. On comparison with the radial integrals obtained in the HO basis, we notice that the radial integrals in the WS basis are not only shifted to the left but are also smaller in value, with the differences between the two becoming large at large angles.

B. Particle-Hole States in ^{16}O

Most of the extensive calculations of the negative-parity levels of ^{16}O,[16-19] in the particle-hole model of Elliott and Flowers,[20] were performed using the infinite HO as the single-particle basis. The studies of Holder and Flowers[5] and of Perez,[12] on the other hand, were carried out using a WS potential with spin-orbit coupling as an approximation to the nuclear self-consistent field. In the former, attention was given to the lowest octupole excitation and the giant-resonance levels of ^{16}O, while in the latter, the excited states of ^{16}O chosen for study are essentially the same as those chosen by Gillet and Vinh Mau[16] in their calculation in the HO basis.

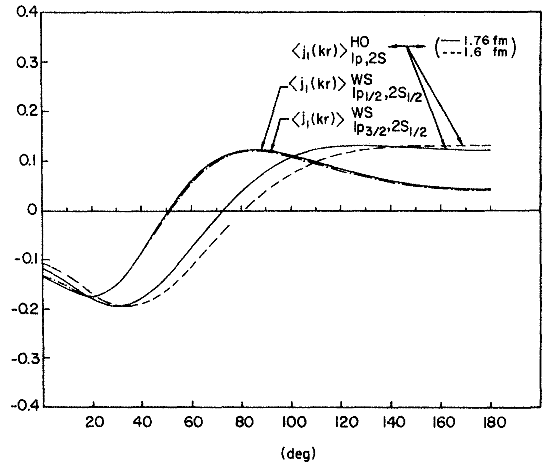

FIG. 6. The radial integrals $\langle j_1(kr)\rangle_{1p,2s}$ as a function of the angle of pion emission in the WS and HO basis.

The particle-hole wave functions of Perez have an isospin mixing of $T = 0$ and $T = 1$ states due to the Coulomb potential V_c. After simple isospin projection,[15] the $T = 1$ particle-hole wave functions of ^{16}O are obtained and these are tabulated and compared with those of Kuo[19] obtained in the HO basis in Table II. The major difference between the WS and HO particle-hole calculations is that less configuration mixing occurs in the WS case.

The wave functions for the four low-lying, $T = 1$, bound states of ^{16}N are taken from the wave func-

TABLE II. Wave functions for low-lying $T = 1$ bound states of ^{16}N.

J^P	Model	$2s_{1/2}\,1p_{1/2}^{-1}$	$1d_{5/2}\,1p_{1/2}^{-1}$	$1d_{3/2}\,1p_{1/2}^{-1}$	$2s_{1/2}\,1p_{3/2}^{-1}$	$1d_{5/2}\,1p_{3/2}^{-1}$	$1d_{3/2}\,1p_{3/2}^{-1}$
0^-	IPM	1.000	···	···	···	···	···
	KUO	0.997	···	···	···	···	0.076
	PEREZ	0.999	···	···	···	···	0.047
1^-	IPM	1.000	···	···	···	···	···
	KUO	0.975	···	0.073	0.064	−0.198	0.009
	PEREZ	−0.998	···	−0.007	0.128	0.078	0.021
2^-	IPM	···	1.000	···	···	···	···
	KUO	···	0.962	−0.097	0.048	0.226	0.106
	PEREZ	···	−0.981	0.071	−0.057	−0.149	−0.071
3^-	IPM	···	1.000	···	···	···	···
	KUO	···	0.942	···	···	−0.329	−0.073
	PEREZ	···	−0.985	···	···	0.171	−0.043

tions for the analogous levels in ^{16}O under the assumption of good isospin. In the absence of residual interaction, these four levels of ^{16}N are assigned to the configurations with a proton-hole in the $1p_{1/2}$ shell and a neutron-particle in the $1d_{5/2}$ or $2s_{1/2}$ shell. This scheme is called the independent-particle model (IPM) for ^{16}N states.

C. Cross-Section Calculation

The ground-state function of ^{16}O is taken to be deformed as:

$$|0^+, \text{g.s.}\rangle = \alpha\,|0\text{p-0h}\rangle + \beta\,|(1p_{1/2}{}^{-2})_{J=0,T=1}(1d_{5/2}{}^2)_{0,1}\rangle + \gamma\,|(2s_{1/2}{}^2)_{0,1}(1p_{1/2}{}^{-2})_{0,1}\rangle\,, \tag{7}$$

where the values of the parameters α, β, and γ are given in Table III.

The nuclear photopion-production transition operator for the reaction (1) is given in the impulse approximation by:

$$\mathfrak{J} = \sum_{n=1}^{A} t_n = \sum_{n=1}^{A} (\vec{\sigma}_n \cdot \vec{K} + L)\tau_n^{(-)} \exp(i\vec{k}\cdot\vec{r}_n)\,, \tag{8}$$

where $\vec{K}$ and L are the single-nucleon photopion-production amplitudes of Chew *et al.*[21] and $\vec{k}(=\vec{\nu}-\vec{\mu})$ is the momentum transfer to the nucleon.

The differential cross section for reaction (1), calculated assuming (7) for the ground-state wave function of ^{16}O and particle-hole states $|f\rangle$ for ^{16}N, is given by

$$\frac{d\sigma}{d\Omega}(0^+ \to f) = (2\pi)^{-2}\mu\mu_0 \overline{\sum_{M_f}} |\langle f|\mathfrak{J}|0^+, \text{g.s.}\rangle|^2\,, \tag{9}$$

where $\mu = |\vec{\mu}|$ and μ_0 are the momentum and energy of the outgoing pion, respectively, and the bar over the sum denotes the average over photon polarization. The details of the calculations can be found in Refs. 8, 9, 22, and 23.

III. RESULTS AND DISCUSSION

The theoretical total cross sections obtained in the WS and HO basis for the reaction (1) are plotted in Fig. 7 as a function of the incident photon energy, corresponding to the three ground-state wave functions of ^{16}O given in Table III. The experimental data are those of Meyer, Walters, and Hummel.[10] A large reduction in the cross section is produced by the change of basis (from HO to WS). The differences between WS and HO basis calculations can be understood from Table IV. The reductions in cross sections due to transitions to the four levels of ^{16}N add up to produce an almost factor of 2 reduction in the total (sum of the four) cross section. It should be noted that large reductions in the cross sections occur especially when transitions to 2^- and 3^- levels are considered and this can be attributed to the $\langle j_l(kr)\rangle_{1p,1d}$ radial integrals in the WS basis for $l=1$ and 3; of these, the $l=1$ radial integrals almost vanish for $\theta_{\text{c.m.}} > 90°$ (Fig. 5).

In Fig. 8, the theoretical cross sections obtained in the WS basis are plotted. The curves using the Perez particle-hole wave functions of ^{16}N are labeled as PEREZ. The reduction in cross section between IPM(PS) and PEREZ (PS) is small compared to that between curves labeled I in Fig. 7

TABLE III. Ground-state wave functions of ^{16}O. PS denotes pure-shell-model wave function. II and III are wave functions obtained by K. H. Purser, W. P. Alford, D. Cline, H. W. Fulbright, H. E. Gove, and M. S. Krick, Nucl. Phys. A132, 75 (1969).

^{16}O model	α	β	γ
I(PS)	1.00	0.00	0.00
II Expt. ^{16}O(d, t)^{15}O	0.87	0.26	0.27
III Expt. ^{16}O(d, ^{3}He)^{15}N	0.82	0.54	0.20

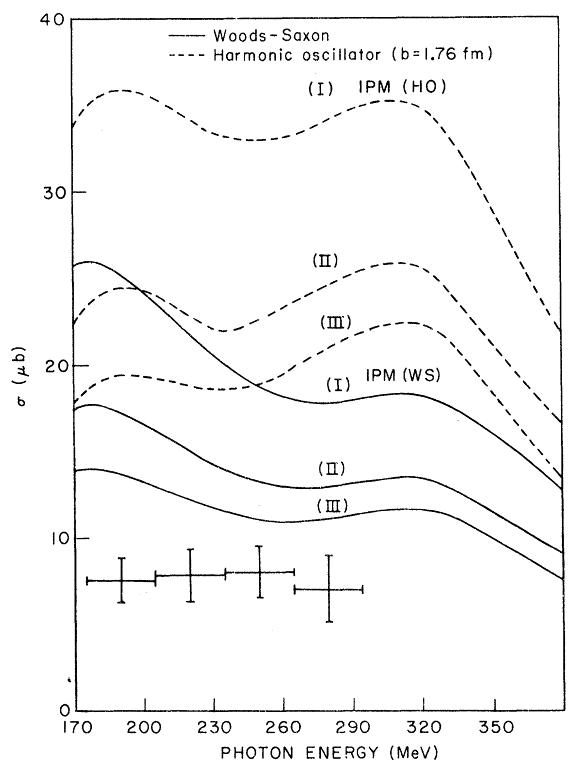

FIG. 7. Total cross section for the reaction ^{16}O(γ,π^+)-^{16}N in the IPM. The solid line curves and the dashed line curves correspond, respectively, to the results obtained in the WS and HO ($b=1.76$ fm) basis. I, II, and III correspond to the three ground-state wave functions of ^{16}O given in Table III.

TABLE IV. Cross sections for the reaction $^{16}O(\gamma, \pi^+)^{16}N$. The incident photon energy is 260 MeV and the nuclear models used are IPM, KUO, and PEREZ. The value of b is 1.76 fm. The values in brackets are for b = 1.6 fm. Kuo wave functions here contain "screening" corrections.

^{16}O model	^{16}N model	Cross section (μb/sr) for ^{16}N states (J^P) 0^-	1^-	2^-	3^-	Total
I(PS)	IPM(HO)	0.442 (0.405)	4.068 (3.693)	13.349 (17.927)	15.215 (12.062)	33.074 (37.897)
	IPM(WS)	0.412	2.718	5.926	9.041	18.097
	KUO(HO)	0.420 (0.347)	3.990 (3.606)	7.095 (9.430)	9.008 (9.428)	20.513 (22.812)
	PEREZ(WS)	0.417	2.719	4.480	7.602	15.218
II	IPM(HO)	0.279 (0.254)	2.775 (2.508)	9.086 (12.251)	11.123 (11.564)	23.263 (26.577)
	IPM(WS)	0.260	1.894	4.033	6.694	12.881
	KUO(HO)	0.263 (0.211)	2.732 (2.456)	4.428 (5.918)	6.469 (6.738)	13.892 (15.323)
	PEREZ(WS)	0.264	1.899	2.634	5.609	10.406
III	IPM(HO)	0.257 (0.235)	2.512 (2.279)	7.114 (9.554)	9.553 (9.862)	19.436 (21.931)
	IPM(WS)	0.239	1.707	3.158	5.836	10.940
	KUO(HO)	0.242 (0.197)	2.471 (2.231)	3.047 (4.027)	5.456 (5.622)	11.216 (12.076)
	PEREZ(WS)	0.243	1.711	1.931	4.875	8.760

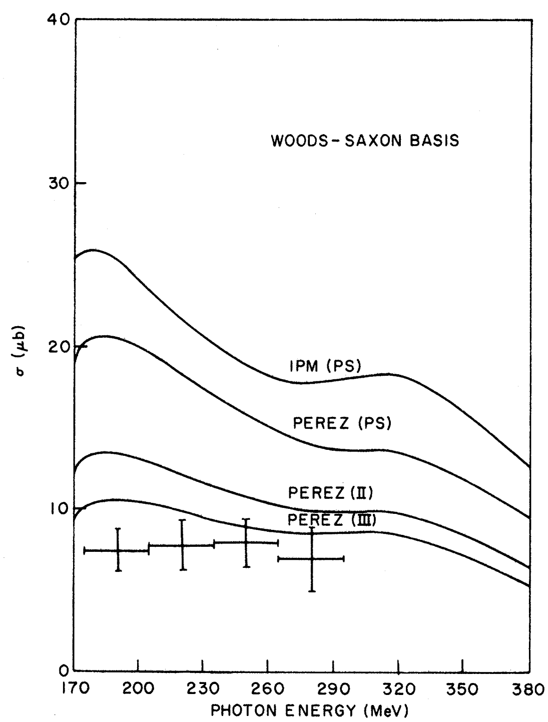

FIG. 8. Total cross section for the reaction $^{16}O(\gamma, \pi^+)$-^{16}N in the WS basis. The nuclear models used are IPM and PEREZ.

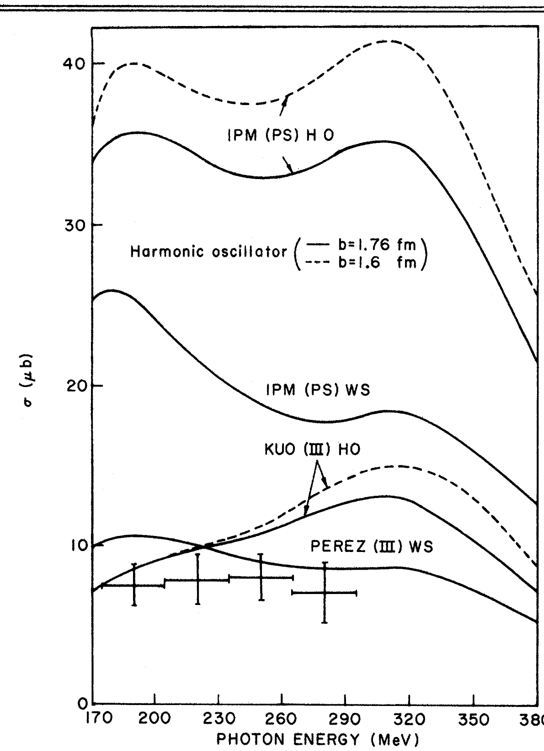

FIG. 9. Total cross section for the reaction $^{16}O(\gamma, \pi^+)$-^{16}N. Comparison is made between the results obtained in WS and HO basis.

for HO and WS basis results for IPM, mainly due to the small configuration mixing in the particle-hole wave functions of Perez.

In Fig. 9, a comparison is made between the theoretical cross sections obtained in the WS basis and the results obtained in the HO basis. The most interesting feature is the flattening of the cross section obtained in the WS basis around the first pion-nucleon resonance region (~320 MeV), resulting in a theoretical reproduction of the right order of magnitude and shape of the experimental data. It is in this energy region that high momentum transfers (400–600 MeV/c) occur. It is to be noted that in comparison with results in the HO basis, the large reduction due to change of basis is compensated for the smaller reduction due to small configuration mixing in the particle-hole wave functions, so that, in the final analysis, the WS basis essentially contributes to only a change in shape of the theoretical cross sections. Further, the dashed line curve, also marked KUO(III)HO, is the one obtained with the "equivalent" size parameter value ($b = 1.6$ fm) and it shows how a reduction in the value of b produces the effect opposite to the change of basis. This sounds paradoxical, since the smaller value of b is the "equivalent" size parameter value, so called since it is deduced from the rms radius of ^{16}O using the WS single-particle states. This paradox can be understood when a study is made of the single-particle wave functions in Figs. 1–3; the curves with "equivalent" b have an almost complete overlap with the WS wave functions only up to ~4 fm and beyond 4 fm we notice that the discrepancy between the two increases, the HO wave function having a reduced tail when compared to the WS wave function. This results in the radial integrals for $b = 1.6$ fm being pushed to the right of the curves for $b = 1.76$ fm, while those with WS wave functions are pulled towards the origin (see Figs. 4, 5, and 6), thus accounting for the opposite effects in the cross sections. Clearly this indicates the extreme sensitivity of the pion-photoproduction cross sections on the nature of the single-particle wave functions.

In conclusion, we wish to observe that with the resolution in the ambiguity of the nuclear states, we are able to almost reproduce the experimental data in magnitude and shape, without resorting to either surface production of pions[8] or the final state interactions. Since FSI are expected to produce a reduction of almost 20–25% only,[11] an explicit calculation of the FSI corrections will enable us to make the correct choice for the ground-state wave function of ^{16}O (Fig. 8).

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