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Mechanism of the Reaction $^{28}Si(d, p)^{29}Si$ from 2.0 to 4.2 MeV

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Excitation curves and angular distributions for the reaction $^{28}Si(d,p)^{29}Si$ have been measured over the range of bombarding energy from 2.0 to 4.2 MeV. The proton groups leading to the five lowest states of ^{29}Si were measured. The results are analyzed by the methods of fluctuation theory and of average angular distribution using the Hauser-Feshbach formula and the distorted-wave Born-approximation theory. Ratios of the direct reaction to the compound nucleus cross section obtained from the two methods are compared. The average total level width $\langle\Gamma\rangle = 29$ keV, the average level space of spinless $D_0 = 1.38$ keV, spin cutoff parameter $\sigma^2 = 2.5$, nuclear temperature $t = 1.8$ MeV, and moment of inertia $\mathscr{I} = 0.6\times 10^{-42}$ MeV sec^2 of ^{30}P around 14.78-MeV excitation energy are obtained.

I. INTRODUCTION

In a previous work[1] investigating the reaction $^{24}Mg(d,p)^{25}Mg$, the separation of the contribution of direct interaction (DI) from that of a compound nuclear (CN) process was quite consistent with the methods of fluctuation analysis and distorted-wave Born approximation (DWBA) calculation. Here, we attempt to make a similar separation for the reaction $^{28}Si(d,p)^{29}Si$. Therefore, we analyze the present experimental results in the same way and compare the spectroscopic factors S with those extracted by Fujimoto, Kikuchi, and Yoshida.[2] Comparison is also made for the values of $(2I_f+1)S$ and $|C_j|^2$ calculated by Bromley, Gove, and Litherland,[3] where I_f is a spin of a residual nucleus and C_j is proportional to the fractional amplitude of the eigenfunction of spin j. It is found that the agreement is good for the $k=\frac{3}{2}$ assignment of the states at 1.28 and 3.07 MeV, and $k=\frac{1}{2}$ of the ground state and the 2.03- and 2.43-MeV states of ^{29}Si.

In the present work, analyses are made in the following four steps:
(1) The average level width $\langle\Gamma\rangle$ is estimated by the method of autocorrelation for all of the measured proton groups.
(2) The contribution from a direct reaction $Y_D = \sigma_D/\langle\sigma\rangle$ for p_0 is obtained with the help of the probability distribution suggested by Ericson[4] and Brink and Stephen,[5] and from the relation of Dallimore and Hall.[6] Here σ_D is the cross section of the direct reaction part and $\langle\sigma\rangle$ is an average experimental cross section.
(3) After the direct reaction part is subtracted from the average experimental angular distribution for p_0, the ratio Γ/D_0 and spin cutoff parameter σ^2 are obtained from the statistical compound nuclear theory,[7] where D_0 is a level spacing of zero-spin levels.
(4) Using the values obtained for Γ/D_0 and σ^2, angular distributions for the compound nuclear process for the other proton groups than p_0 are calculated. Then the calculated angular distributions are subtracted from the average experimental angular distribution, and finally, the remaining parts of angular distributions are compared with DWBA calculations.

II. EXPERIMENTAL METHOD AND RESULTS

A beam of deuterons was accelerated by the 5-MV Van de Graaff generator of Tohoku University, with an energy resolution estimated to be 0.1%. A target was prepared by vacuum evaporation of SiO_2 onto a carbon backing of about 10 $\mu g/cm^2$, and the thickness of SiO_2 was about 100 $\mu g/cm^2$, which corresponds to an energy loss of about 14 keV for 3-MeV deuterons. The over-all energy resolution for the present measurement was about 15 keV, which meets well the requirement for fluctuation analysis. The protons were detected simultaneously at two or three angles with surface-barrier detectors, and the experiments were performed in the same way as in the previous one.[1]

Excitation functions for the p_0, p_1, p_2, p_3, and p_4 groups were measured over the angular range from 15 to 165° in steps of 15°, and the energy of incident deuterons was varied from 2.0 to 4.2 MeV in steps of 20 keV. Absolute cross sections at $E_d = 2.08$ MeV were measured at forward angles by comparison with the Rutherford scattering, and the errors were estimated to be about 15%. Figure 1 shows a typical energy spectrum obtained at $E_d = 3.8$ MeV and $\theta_{lab} = 135°$, and Fig. 2 shows the example of excitation function for the indicated particle group at $\theta = 150°$. Solid curves in the figure show the average cross sections $\langle \sigma(E) \rangle$ estimated by the method of Lee *et al.*[1]

III. ANALYSIS AND DISCUSSION

A. Fluctuation Analysis

Channel cross correlation functions[4, 8, 9] defined by

$$C_{cc'}(0) = \frac{\langle \sigma_c(E)\sigma_{c'}(E)\rangle}{\langle \sigma_c(E)\rangle \langle \sigma_{c'}(E)\rangle} - 1\,, \quad (1)$$

for all the combinations of proton groups p_0, p_1, p_2, p_3, and p_4, and autocorrelation functions[4, 8, 9] defined by

$$C_{cc}(\epsilon) = \frac{\langle \sigma_c(E)\sigma_c(E+\epsilon)\rangle}{\langle \sigma_c(E)\rangle \langle \sigma_c(E+\epsilon)\rangle} - 1\,, \quad (2)$$

for all the measured proton groups were calculated in the same way as in the previous work.[1] In these equations, $C_{cc'}(0)$ is a channel cross correlation between channels c and c', and $\sigma_c(E)$ and $\sigma_{c'}(E)$ are experimental cross sections for channels c and c', respectively, at an energy E, and $\langle \sigma_c(E)\rangle$ and $\langle \sigma_{c'}(E)\rangle$ are average cross sections estimated by the method of Lee *et al.*[1] Table I shows the channel cross correlations for all the combinations of the groups. In this table, the symbol $(0+1)$, for example, stands for the cross correlation between the excitation functions of the p_0 and p_1 groups. The table also contains the channel correlations[10] in parentheses, expressed by

$$C_{cc'}{}^2 = \frac{C_{cc'}(0)}{[C_{cc}(0)C_{c'c'}(0)]^{1/2}}\,. \quad (3)$$

There are strong correlations for p_3 and p_2, and for p_3 and p_4, which will be discussed in another paper. Figure 3 shows typical features of the autocorrelation functions $C_{cc}(\epsilon)$ at 90°. The values of Γ were obtained from the values of ϵ, where $C_{cc}(\epsilon)$ equals one half of $C_{cc}(0)$, and are shown in Table II. The notation $\overline{\Gamma}_{exp}$ means the average of

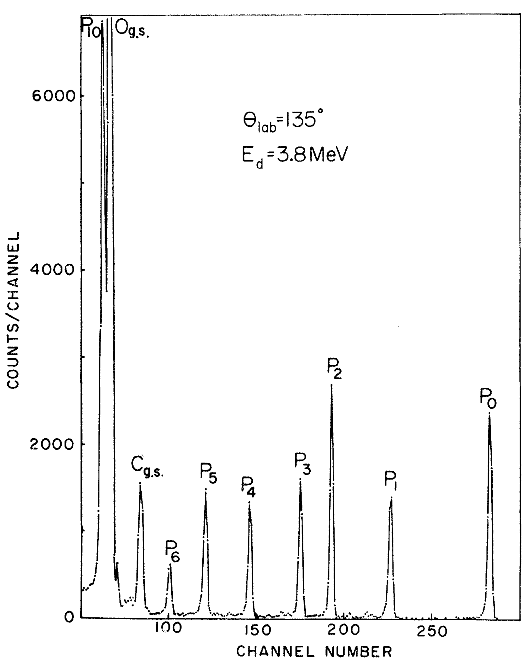

FIG. 1. Typical spectrum for the $^{28}Si(d,p)^{29}Si$ reaction at $E_d = 3.8$ MeV and $\theta_{lab} = 135°$.

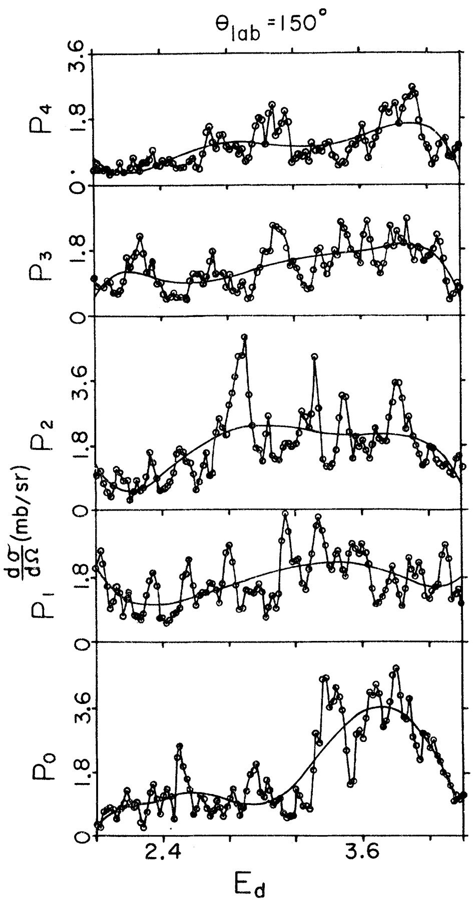

FIG. 2. Typical excitation curves for protons at $\theta_{lab} = 150°$.

TABLE I. Channel cross correlations and channel correlations for several combinations of the measured proton groups. The symbol $(c+c')$ stands for the cross correlation between the excitation functions for the reactions $^{28}Si(d,p_c)$ and $^{28}Si(d,p_{c'})$. The values channel correlation is given in parentheses. A represents the average value over all angles.

$c\,c'$ \ Scattering angle	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	A
0+0	0.176±0.042 (1.000)	0.120±0.028 (1.000)	0.139±0.032 (1.000)	0.097±0.022 (1.000)	0.091±0.021 (1.000)	0.056±0.013 (1.000)	0.120±0.028 (1.000)	0.090±0.021 (1.000)	0.128±0.030 (1.000)	0.140±0.033 (1.000)	0.264±0.065 (1.000)	0.129±0.035 (1.000)
1+0	−0.036±0.044 (−0.247)	−0.001±0.027 (−0.010)	0.002±0.018 (0.027)	−0.003±0.018 (−0.042)	0.007±0.018 (0.094)	0.007±0.018 (0.137)	−0.007±0.018 (−0.066)	−0.019±0.018 (−0.268)	0.012±0.018 (0.113)	−0.003±0.027 (−0.024)	−0.022±0.044 (−0.110)	0.011±0.028 (0.104)
1+1	0.121±0.026 (1.000)	0.077±0.016 (1.000)	0.040±0.008 (1.000)	0.050±0.011 (1.000)	0.061±0.013 (1.000)	0.047±0.010 (1.000)	0.095±0.020 (1.000)	0.056±0.012 (1.000)	0.088±0.019 (1.000)	0.114±0.025 (1.000)	0.153±0.033 (1.000)	0.082±0.020 (1.000)
2+0	0.023±0.036 (0.101)	0.024±0.022 (0.171)	0.007±0.015 (0.063)	0.019±0.015 (0.219)	0.008±0.015 (0.102)	0.010±0.015 (0.204)	0.011±0.015 (0.114)	0.023±0.015 (0.298)	0.044±0.015 (0.387)	0.019±0.022 (0.130)	0.025±0.036 (0.106)	0.019±0.019 (0.172)
2+1	−0.029±0.025 (−0.153)	−0.015±0.016 (−0.134)	0.002±0.011 (0.033)	0.005±0.011 (0.082)	0.006±0.011 (0.094)	0.015±0.011 (0.334)	0.032±0.011 (0.374)	0.009±0.011 (0.148)	0.009±0.011 (0.096)	0.001±0.016 (0.008)	−0.005±0.025 (−0.028)	0.012±0.016 (0.135)
2+2	0.296±0.069 (1.000)	0.164±0.036 (1.000)	0.090±0.019 (1.000)	0.075±0.016 (1.000)	0.067±0.014 (1.000)	0.043±0.009 (1.000)	0.077±0.016 (1.000)	0.066±0.014 (1.000)	0.101±0.021 (1.000)	0.152±0.033 (1.000)	0.211±0.047 (1.000)	0.122±0.033 (1.000)
3+0	−0.007±0.044 (−0.026)	0.012±0.027 (0.082)	−0.001±0.018 (−0.008)	0.010±0.018 (0.085)	0.016±0.018 (0.173)	0.018±0.018 (0.269)	0.011±0.018 (0.110)	0.015±0.018 (0.175)	−0.006±0.018 (−0.054)	−0.034±0.027 (−0.240)	−0.012±0.044 (−0.056)	0.013±0.028 (0.116)
3+1	0.008±0.031 (0.036)	0.021±0.019 (0.179)	0.009±0.013 (0.131)	0.005±0.013 (0.060)	−0.008±0.013 (−0.106)	0.003±0.013 (0.049)	0.010±0.013 (0.113)	−0.017±0.013 (−0.251)	−0.006±0.013 (−0.066)	0.009±0.019 (0.071)	0.040±0.031 (0.247)	0.012±0.019 (0.119)
3+2	0.034±0.025 (0.098)	0.019±0.016 (0.111)	0.035±0.011 (0.338)	0.030±0.011 (0.296)	0.009±0.011 (0.113)	0.014±0.011 (0.239)	0.007±0.011 (0.088)	−0.019±0.011 (−0.258)	−0.023±0.011 (−0.235)	−0.019±0.016 (−0.129)	0.001±0.025 (0.005)	0.019±0.016 (0.174)
3+3	0.409±0.099 (1.000)	0.178±0.039 (1.000)	0.119±0.026 (1.000)	0.137±0.030 (1.000)	0.094±0.020 (1.000)	0.080±0.017 (1.000)	0.083±0.017 (1.000)	0.082±0.017 (1.000)	0.095±0.020 (1.000)	0.143±0.031 (1.000)	0.172±0.038 (1.000)	0.145±0.041 (1.000)
4+0	0.017±0.036 (0.087)	0.033±0.022 (0.289)	−0.007±0.015 (−0.063)	0.002±0.015 (0.020)	−0.004±0.015 (−0.048)	−0.020±0.015 (−0.315)	−0.024±0.015 (−0.290)	0.019±0.015 (0.325)	0.026±0.015 (0.271)	−0.022±0.022 (−0.142)	−0.016±0.036 (−0.074)	0.017±0.019 (0.175)
4+1	0.034±0.025 (0.209)	0.009±0.016 (0.098)	0.003±0.011 (0.050)	0.011±0.011 (0.152)	0.016±0.011 (0.235)	0.002±0.011 (0.034)	0.006±0.011 (0.082)	−0.000±0.011 (0.000)	0.003±0.011 (0.038)	−0.005±0.016 (−0.036)	0.035±0.025 (0.212)	0.011±0.016 (0.104)
4+2	−0.009±0.021 (0.035)	0.004±0.013 (0.030)	−0.002±0.009 (−0.022)	0.004±0.009 (0.045)	−0.012±0.009 (−0.168)	−0.009±0.009 (−0.162)	−0.012±0.009 (−0.181)	−0.006±0.009 (−0.120)	0.014±0.009 (0.164)	0.020±0.013 (0.124)	−0.010±0.021 (−0.052)	0.009±0.013 (0.100)
4+3	−0.015±0.025 (−0.050)	0.013±0.016 (0.093)	0.019±0.011 (0.184)	0.007±0.011 (0.058)	−0.012±0.011 (−0.142)	−0.001±0.011 (−0.013)	0.015±0.011 (0.218)	0.029±0.011 (0.520)	0.029±0.011 (0.351)	0.054±0.016 (0.345)	0.045±0.025 (0.257)	0.022±0.016 (0.203)
4+4	0.219±0.049 (1.000)	0.109±0.023 (1.000)	0.090±0.019 (1.000)	0.105±0.022 (1.000)	0.076±0.016 (1.000)	0.072±0.015 (1.000)	0.057±0.012 (1.000)	0.038±0.008 (1.000)	0.072±0.015 (1.000)	0.171±0.038 (1.000)	0.178±0.039 (1.000)	0.108±0.028 (1.000)

Γ over all the angles. The average value of the level width $\bar{\Gamma}$ of the compound nucleus was deduced from $\bar{\Gamma}_{exp}$ using the relation[11]

$$\bar{\Gamma} = [\bar{\Gamma}_{exp}^{\ 2} - (\Delta E)^2]^{1/2}, \qquad (4)$$

where ΔE is the experimental energy resolution. The total average coherence width $\langle \Gamma \rangle$ for all the proton groups was obtained as 29 ± 3.5 keV.

B. Probability Distribution for p_0

In order to evaluate the contribution from a direct reaction, the probability distribution of reduced variable $Y_S = \sigma/\langle\sigma\rangle$ is used. This distribution is independent from a reaction channel, and depends on the number N of uncorrelated effective channels and also on the relative strength of the direct process $Y_D = \sigma_D/\langle\sigma\rangle$ according to the expression[4,5]

$$P_{N,Y_D}(Y_s) = \left(\frac{N}{1-Y_D}\right) Y_s^{N-1} \exp\left[-\frac{N(Y_s+Y_D)}{1-Y_D}\right] \times \frac{J_{N-1}(2iN\sqrt{Y_s Y_D}/1-Y_D)}{[iN\sqrt{Y_s Y_D}/(1-Y_D)]^{N-1}}, \qquad (5)$$

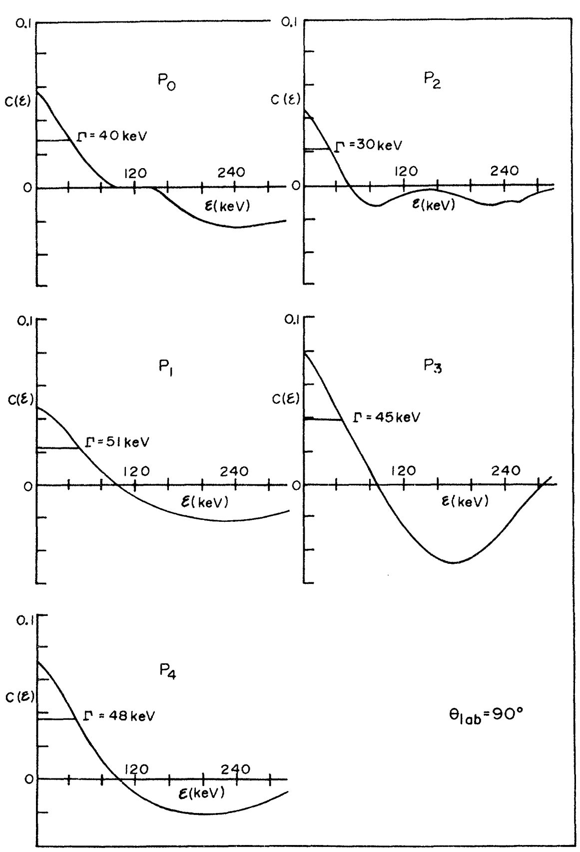

FIG. 3. Typical autocorrelation functions at $\theta_{lab} = 90°$.

TABLE II. Average level widths obtained from autocorrelations. (The different quantities are explained in the text.)

Scattering angle θ_{Lab}	Γ (keV) P_0	P_1	P_2	P_3	P_4
15°	30	30	36	20	20
30°	22	24	34	28	34
45°	40	16	26	32	24
60°	26	20	24	34	40
75°	32	32	28	36	44
90°	40	51	30	45	48
105°	28	32	24	32	28
120°	44	36	48	44	36
135°	36	36	34	32	34
150°	28	26	28	28	28
165°	36	32	32	28	42
$\bar{\Gamma}_{exp}$ (keV)	33	33	31	33	34
$\bar{\Gamma}$ (keV)	29	29	27	29	31
$\langle\Gamma\rangle$ (keV)			29		
$\Delta\Gamma$ (keV) (frd error)			±3.5		

where J_{N-1} is the cylindrical Bessel function of imaginary argument of the $(N-1)$th order. The distribution of $P_{N,Y_D}(Y_s)$ for large N is rather similar to that for large Y_D and shows sharp rise around $\sigma/\langle\sigma\rangle = 1$. Furthermore, the distribution is sensitive to Y_D only if N is small. The value of N can be considered to be N_{max} for a wide

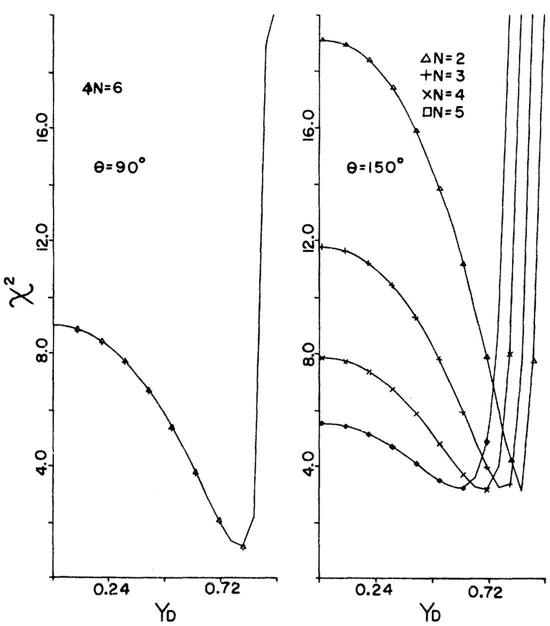

FIG. 4. χ^2 distributions as a function of Y_D for p_0 at $\theta_{lab} = 90$ and 150°.

range of angles (90± 40°), where $N_{max}=\frac{1}{2}(2i+1)\times(2I+1)(2i_f+1)(2I_f+1)$[4,5] and i, I, i_f, and I_f are the spins of projectile, target, outgoing particle, and residual nucleus, respectively. We see now that among all proton groups the smallest value of N_{max} is found for the group P_0. Therefore we evaluate the direct reaction contribution only for P_0. The χ^2 test was used to estimate the direct reaction contribution to the reaction. For a given N,

$$\chi^2(Y_D)=\sum\frac{[f(Y_s)-P_{Y_D}(Y_s)]^2}{P_{Y_D}(Y_s)}, \qquad (6)$$

where $f(Y_s)$ is the experimental frequency of Y_s at an interval of 0.2, and $P_{Y_D}(Y_s)$ is the theoretical prediction from Eq. (5). Calculations were performed by changing Y_D in steps of 0.01 between 0.01 and 0.90, and the results are shown in Fig. 4, where we took $N=6$ for $\theta=45$, 60, 70, 90, 105, 120, and 135°, and $N=2$, 3, 4, and 5 for the other angles. The values of Y_D were obtained for minimum values of χ^2, and are shown in Table III. In this table, only the cases of $N=5$, 4, and 3 for $\theta=30$ and 150°, and of $N=3$ and 2 for $\theta=15$ and 165° are shown, as the present reaction $^{28}Si(d,p)^{29}Si$ is one-channel reaction at scattering angles of 0 and 180°. Figure 5 shows the theoretical and the experimental distributions as a function of Y_s. For comparison, the relation[6,12]

$$C_{cc}(0)=\frac{1}{N}\,(1-Y_{D*}^2) \qquad (7)$$

was used to calculate Y_{D*}, where the definition of Y_{D*} is the same as Y_D and * means the direct reaction part obtained from another way than the probability distribution. The Y_{D*} values obtained are also listed in the table. The consistency between Y_D and Y_{D*} is very excellent within the errors which were estimated from the relation

$$\frac{\Delta C_{cc}(0)}{C_{cc}(0)}=\frac{2Y_D^*\,\Delta Y_D^*}{1+Y_D^*}=\left(\pi\,\frac{1+C_{cc}(0)}{n}\right)^{1/2}. \qquad (8)$$

The quantity n is a sample size, which is the present experiment $n=2.2$ MeV/29 MeV = 76.

C. Average Angular Distribution for p_0

After the direct reaction contribution Y_D for p_0 was estimated from the probability distribution, the cross section $\sigma_D(\theta)$ for direct reaction was subtracted from the average experimental differential cross section $\langle d\sigma/d\Omega(\theta)\rangle_{exp}$ for p_0. By fit-

TABLE III. Direct contributions Y_D and Y_{D*} obtained by the methods of probability distribution and autocorrelation (Refs. 6 and 12), respectively. Column 1 gives the experimental differential cross sections. The values of columns 2 and 3 are obtained by taking the difference between the values of column 1 and the direct differential cross sections $d\sigma_D(\theta)/d\Omega$ and $d\sigma_{D*}(\theta)/d\Omega$ calculated from Y_D and Y_{D*}, respectively.

Scattering angle	$C_{cc}(0)$	N	Y_D	$\bar{Y}_D$	Y_{D*}	$\bar{Y}_{D*}$	(1) $\left\langle\frac{d\sigma}{d\Omega}(\theta)\right\rangle_{exp}$ (mb)	(2) $\left\langle\frac{d\sigma}{d\Omega}(\theta)\right\rangle(1-Y_D)$ (mb)	(3) $\left\langle\frac{d\sigma}{d\Omega}(\theta)\right\rangle(1-Y_{D*})$ (mb)
15°	0.176±0.042	2	0.76	0.69	0.81±0.05	0.75±0.11	2.99	0.94	0.75±0.33
		3	0.61		0.69±0.09				
30°	0.120±0.28	3	0.76		0.80±0.05				
		4	0.69	0.68	0.72±0.08	0.72±0.15	1.35	0.42	0.38±0.20
		5	0.58		0.63±0.10				
45°	0.139±0.032	6	0.31		0.41±0.25		1.04	0.72	0.61±0.25
60°	0.097±0.022	6	0.59		0.65±0.10		1.37	0.58	0.49±0.14
75°	0.091±0.021	6	0.61		0.67±0.09		1.50	0.58	0.49±0.14
90°	0.056±0.013	6	0.81		0.82±0.05		1.17	0.22	0.21±0.06
105°	0.120±0.028	6	0.41		0.53±0.16		1.07	0.63	0.50±0.17
120°	0.090±0.021	6	0.61		0.68±0.09		0.94	0.37	0.30±0.09
135°	0.128±0.030	6	0.40		0.48±0.19		1.20	0.72	0.62±0.23
150°	0.140±0.033	3	0.76		0.76±0.06				
		4	0.71	0.70	0.66±0.10	0.66±0.21	1.77	0.54	0.61±0.37
		5	0.62		0.55±0.15				
165°	0.264±0.065	2	0.66	0.54	0.69±0.09	0.58±0.28	2.35	1.09	1.00±0.66
		3	0.41		0.49±0.21				

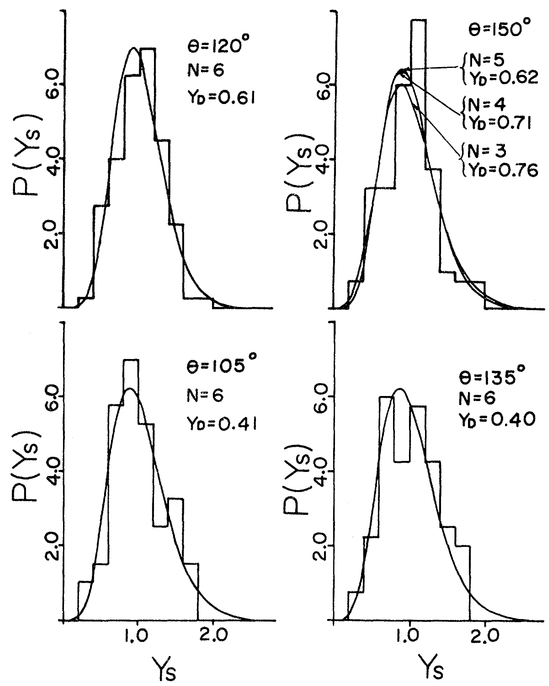

FIG. 5. Theoretical and the experimental distributions as a function of Y_s for p_0 at $\theta = 105$, 120, 135, and 150°.

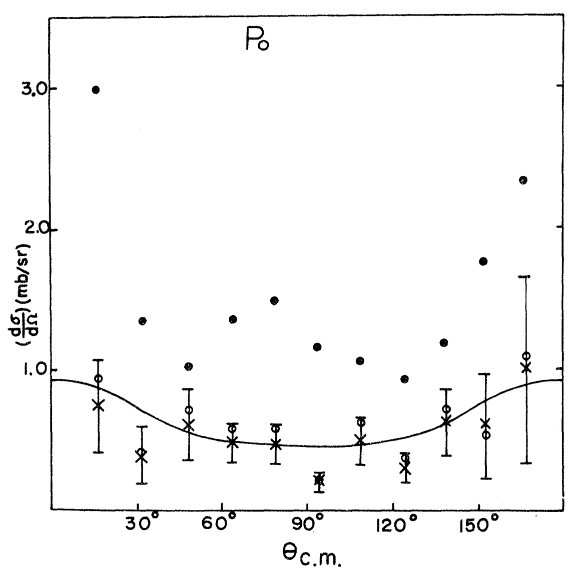

FIG. 6. Differential cross section for the particle group p_0. Full circles indicate the experimental cross section averaged over the energy interval $E_d = 2.0$ to 4.2 MeV. Open circles and crosses indicate the cross section after subtraction of the contribution of direct reaction, referring to the calculated values of column 2 and column 3 of Table III, respectively. The line shows the theoretical CN cross section.

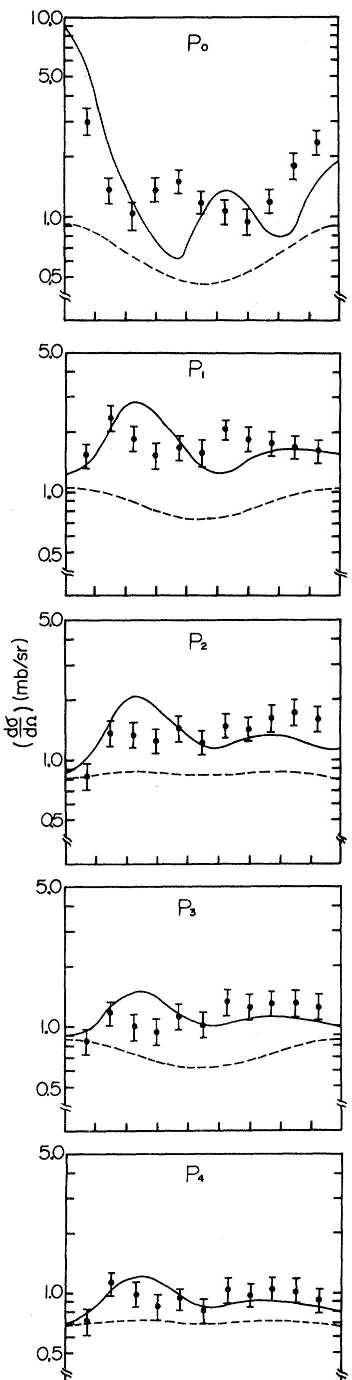

FIG. 7. Contribution of CN and DI cross section for the (d, p) reaction. Experimental points represent the average cross section for the energy interval $E_d = 2.0$ to 4.2 MeV. The solid curve represents the sum of the calculations of DWBA and Hauser-Feshbach theory. The dash curve represents the result of Hauser-Feshbach theory.

TABLE IV. Optical-model parameters for calculation of the transmission coefficients and DWBA.

	V (MeV)	W (MeV)	r_0 (fm)	r_{0I} (fm)	a (fm)	a_2 (fm)	r_c (fm)
d [a]	103.4	11.28	1.29	0.75	0.56	1.16	1.5
p [b]	54	13.5	1.25	1.25	0.65	0.47	1.25

[a] D. P. Gurd, G. Roy, and H. G. Leighton, Nucl. Phys. 120, 94 (1968).

[b] F. G. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

ting the theoretical average differential cross section $\langle d\sigma/d\Omega(\theta)\rangle_{\text{theor}}$:

$$\left\langle \frac{d\sigma}{d\Omega}(\theta)\right\rangle_{\text{theor}} = \frac{D_0}{\Gamma}\frac{\lambda\!\!\!^{-2}}{4(2I+1)(2i+1)}\frac{1}{2\pi\langle N_\mu^2\rangle_\mu} \times \sum_{sls'l'J} A_{cc'}^{LJ}(\theta)\frac{T_c T_{c'}}{(2J+1)\exp[-J(J+1)/2\sigma^2]} \quad (9)$$

to the remaining experimental cross section for p_0, the quantities Γ/D_0 and σ^2 were evaluated. In Eq. (9), $A_{cc'}^{LJ}(\theta)$ is expressed by

$$A_{cc'}^{LJ}(\theta) = (-)^{s'-s}\bar{Z}(lJlJ;SL) \times \bar{Z}(l'Jl'J;SL)P_L(\cos\theta),$$

and the other notation is the same as described in Ref. 7. Transmission coefficients for the protons and deuterons were calculated by assuming appropriate optical potentials shown in Table IV. The choice of spin cutoff parameter σ^2 determines the shape of angular distribution, and Γ/D_0 and $\langle N_\mu{}^2\rangle_\mu$ give the absolute value of the cross section. The result is shown in Fig. 6. Putting $\langle N_\mu{}^2\rangle_\mu = 1$, we obtain $\Gamma/D_0 = 21$ and $\sigma^2 = 2.5$ for the best fit. From the analysis of autocorrelation functions, $\langle\Gamma\rangle$ was extracted and then D_0 was found. Using this value for D_0 in Erba's level density formula,[13] the level density parameter a was calculated. From the relations,

$$\sigma^2 = \frac{\mathcal{J}t}{\hbar^2}, \quad U = at^2 - t, \quad (10)$$

where U is the excitation energy of the compound nucleus minus pairing energy,[14] the nuclear temperature t, and the nuclear moment of inertia $\mathcal{J}$ were obtained. The results are shown in Table V. The temperature is very close to that extracted from the level density.[15] For comparison, we also show the values $\hbar^2/2\mathcal{J}$ obtained by Bromley, Gove, and Litherland[16] for ^{29}Si, which were calculated from the modified rotational spectrum. Our value of $\hbar^2/2\mathcal{J} = 0.36$ is just the average of Bromley's values of 0.40 for $k=\frac{3}{2}$ and 0.31 for $k=\frac{1}{2}$, which indicates that the shape of ^{30}P nuclei is very close to that of ^{29}Si nuclei.

D. DWBA Analysis

Using the values of Γ/D_0 and σ^2, calculations of theoretical average differential cross section $\langle d\sigma/d\Omega(\theta)\rangle_{\text{theor}}$ were extended to all measured proton groups. The average experimental differential cross section which remains after subtraction of the theoretical average differential CN cross section was compared with a DWBA theory. The code INS-DWBA2, which takes the Woods-Saxon potential without an l-s force,[17] was used. Values of optical parameters used are shown in Table IV. Calculations were carried out in steps of 500 keV, and the results were averaged over the entire energy range. Fig. 7 shows the comparison between the results of the experiment and the calculation. The agreement is excellent within the errors. Spectroscopic factors were deduced from the relationship,

$$\left\langle\frac{d\sigma}{d\Omega}(\theta)\right\rangle_{\text{exp}} = (2I_f+1)S\left\langle\frac{d\sigma}{d\Omega}(\theta)\right\rangle_{\text{DWBA}} + \left\langle\frac{d\sigma}{d\Omega}(\theta)\right\rangle_{\text{theor}}, \quad (11)$$

where $\langle d\sigma/d\Omega(\theta)\rangle_{\text{DWBA}}$ is an average differential cross section of a DWBA calculation. Table VI shows the values of spectroscopic factor S for the measured protons. For comparison, the S factors estimated from the Nilsson model using $\delta = -0.15$[18] and those obtained by Fujimoto, Kikuchi, and Yoshida,[2] and also the values of $(2I_f+1)S$ and $|C_j|^2$ given by Bromley, Gove, and Litherland[3] are listed in the table. Errors of the

TABLE V. Some parameters characterizing the compound nucleus ^{30}P around 14.78-MeV excitation energy.

$\langle\Gamma\rangle$ (keV)	Γ/D_0	D_0 (keV)	σ^2	a (MeV^{-1})	t (MeV)	$\mathcal{J}\times10^{42}$ (MeV sec^2)	$\hbar^2/2\mathcal{J}$ (MeV)	$\hbar^2/2\mathcal{J}$ [a] (MeV) $k=\frac{3}{2}$	$k=\frac{1}{2}$
29	21	1.38	2.5	4.45	1.8	0.6	0.36	0.41	0.31

[a] Reference 3.

TABLE VI. Spectroscopic factors S for neutron from the $^{28}Si(d,p)^{29}Si$ reaction.

				Present results					Theoretical values $\|C_j\|^{2\,c}$			
									$\delta=-0.15$		$\delta=+0.15$	
Proton group	Level energy (MeV)	I_f	l_n	S	$(2I_f+1)S$	Y_D	S [a]	S [b]	$k=\frac{1}{2}$	$k=\frac{3}{2}$	$k=\frac{1}{2}$	$k=\frac{3}{2}$
p_0	0	$\frac{1}{2}^+$	0	0.99 ± 0.25	1.98	0.60	1.00	1.00	0.34		0.43	
p_1	1.28	$\frac{3}{2}^+$	2	1.20 ± 0.30	4.80	0.50	0.92	2.10	0.22	0.89	0.35	0.97
p_2	2.03	$\frac{5}{2}^+$	2	0.40 ± 0.16	2.40	0.39	0.33	0.45	0.44	0.11	0.22	0.03
p_3	2.43	$\frac{3}{2}^+$	2	0.26 ± 0.16	1.04	0.25	0.08	0.02	0.02	0.89	0.35	0.97
p_4	3.07	$\frac{5}{2}^+$	2	0.12 ± 0.07	0.72	0.25	0.06	0.01	0.44	0.11	0.22	0.03

[a] These values are reduced from the Reference 2.
[b] Reference 18.
[c] Reference 3.

S factors were estimated from the uncertainty of the absolute cross section.[1] Our results are consistent with those of Fujimoto, Kikuchi, and Yoshida,[2] which indicate the $k=\frac{3}{2}$ assignment for the states at 1.28 and 3.07 MeV and $k=\frac{1}{2}$ for the states at 0, 2.03, and 2.43 MeV of ^{29}Si. The direct reaction contribution Y_D, which was extracted from Eq. (11), is also listed in Table VI.

IV. CONCLUSION

The present work was carried out in the excitation region from 13.8 to 16 MeV of the ^{30}P nucleus, for which $\Gamma/D_0\approx21$. A number of states with spin >0 are excited in this region and, hence, Γ/D_J must be larger than 21. In such cases, not only can the theory of fluctuation and probability distribution be used to analyze the experimental data, but also the theory of DWBA, because the average angular distributions have been taken and, therefore, the interference between the DI and CN parts can be neglected.

From the agreement of the spectroscopic factors obtained here with those of Fujimoto, Kikuchi, and Yoshida and because the present value of $\hbar^2/2\mathcal{I}=0.36$, which extracted from the spin cutoff parameter σ^2 and temperature t is reasonably consistent with the result from Bromley, Gove, and Litherland,[3] it can be concluded that the methods of the probability distribution, the fluctuation theory, and the DWBA analysis are quite useful to evaluate the contribution of the direct reaction. However, the probability distribution method is effective only for a small value of N and an experiment with large Y_D. The fluctuation method depends upon the autocorrelation, which has a large uncertainty due to the errors from finite range of data (frd), and the DWBA method can only be used for single-particle transitions. Therefore, all three methods should be considered when one attempts to separate the direct reaction part from the experimental results.

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