Alpha-Structure Amplitudes for the $1p$ Shell

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Structure amplitudes for transfer of a $(1p)^4$ group with spatial symmetry [4] are given for $1p$-shell targets with initial and final states represented by intermediate-coupling wave functions. The strength is fragmented, a feature likely to be even stronger in heavy nuclei as is shown for the $(2p, 1f)$ region.

I. DEFINITIONS

α-structure amplitudes are needed in calculating α decay widths or in interpreting transfer reactions such as $(d, {}^6\mathrm{Li})$ and its inverse. In these cases two protons and two neutrons are transferred in a state of zero spin and isospin, completely symmetric in spatial coordinates (symmetry [4]). In the $1p$ shell there are only three such states, one each for orbital angular momentum $L_\alpha = 0$, 2, and 4. It is useful to express the α creation operators as coupling between neutrons and protons so that the α amplitudes can be expressed in terms of the separate neutron and proton two-nucleon amplitudes.

Since each neutron and proton pair should be in a spatially symmetric state (and hence should have spin zero), only orbital angular momenta $L = 0$ and 2 are allowed in the $1p$ shell. These states are then coupled to form a 4-fermion creation operator with orbital angular momentum L_α, defined as

$$\chi^{L_\alpha(L_a \times L_b)^\dagger} \equiv [A_\pi^{L_a\dagger} \times A_\nu^{L_b\dagger}]^{L_\alpha} . \tag{1}$$

The creation operators on the right represent the singlet ($S = 0$) states for protons π and neutrons ν. The α-structure operators which have isospin zero are linear combinations of operators of the form of Eq. (1), namely

$$\begin{aligned}
\chi^{L_\alpha=0^\dagger} &= (\tfrac{5}{9})^{1/2}\chi^{0(S\times S)^\dagger} + (\tfrac{4}{9})^{1/2}\chi^{0(D\times D)^\dagger} , \\
\chi^{L_\alpha=2^\dagger} &= (\tfrac{7}{18})^{1/2}(\chi^{2(S\times D)^\dagger} + \chi^{2(D\times S)^\dagger}) + (\tfrac{2}{9})^{1/2}\chi^{2(D\times D)^\dagger} , \\
\chi^{L_\alpha=4^\dagger} &= \chi^{4(D\times D)^\dagger} ,
\end{aligned} \tag{2}$$

where S and D on the right denote $L = 0$ and $L = 2$, respectively.

An alternative way to obtain the $\chi^{L_\alpha\dagger}$ operators exhibits their connection with the SU(3) representation of Elliott[1] and leads to a sum rule for the α amplitudes, which is explored later. If one considers the lowest prolate Nilsson orbital in the asymptotic limit of large deformation, the spatial wave function is a $1p$ function p_0 with a projected orbital angular momentum $\Lambda = 0$. The full Nilsson level is represented by the 4-fermion intrinsic state $\chi_0^\dagger$ (as in Elliott's treatment)

$$\chi_0^\dagger = (p_0\uparrow\nu)^\dagger(p_0\downarrow\nu)^\dagger(p_0\uparrow\pi)^\dagger(p_0\downarrow\pi)^\dagger , \tag{3}$$

where spin and isospin labels are given. This is clearly a function of spatial symmetry [4] with $S = 0 = T$. By noting that

$$(p_0\uparrow)^\dagger(p_0\downarrow)^\dagger = -(\tfrac{1}{3})^{1/2}A_0^{S\dagger} + (\tfrac{2}{3})^{1/2}A_0^{D\dagger} \tag{4}$$

one can show that $\chi_0^\dagger$ can be expressed in terms of the $\chi^{L_\alpha\dagger}$ of Eq. (2) as

$$\chi_0^\dagger = \sum_{L_\alpha} C_{L_\alpha}\chi^{L_\alpha\dagger} , \tag{5}$$

with $C_0 = 1/\sqrt{5}$, $C_2 = -2/\sqrt{7}$, and $C_4 = 4/\sqrt{70}$.

The structure amplitude[2] for α transfer $\mathfrak{A}$ is defined to be the reduced matrix element of $\chi^{L_\alpha\dagger}$ between states of angular momentum I and I_0, namely

$$\mathfrak{A} \equiv \langle ITk(N) \| \chi^{L_\alpha\dagger} \| I_0Tk_0(N-4)\rangle , \tag{6}$$

where N is the number of $1p$ nucleons and k and k_0 are other state labels such as energy. The reduced matrix element times the Clebsch-Gordan coefficient $(I_0L_\alpha M_0M_\alpha|IM)$ is equal to the matrix element for the magnetic substates indicated by the M values. The general structure amplitude is also reduced in isospin space, which is irrelevant here since the operator is isoscalar. The structure amplitude $\mathfrak{A}$ can be expressed in terms of structure amplitudes for two nucleons in isovector states since $\chi^{L_\alpha\dagger}$ consists of terms of the form of Eq. (1). The reduced matrix element of the operator in Eq. (1), evaluated for $T_3 = T$, is obtained by inserting a complete set of states of $(N-2)$ $1p$ nucleons between the neutron and proton operators and leads to a sum weighted by a Racah coefficient

$$\langle ITk(N)\| \chi^{L_\alpha(L_a\times L_b)^\dagger}\| I_0 T k_0(N-4)\rangle = [(2L_\alpha+1)(2T+1)(2T+3)^{-1}]^{1/2} \times \sum_{I_1 k_1} (2I_1+1)^{1/2} W(I_0 L_b I L_a; I_1 L_\alpha)\langle ITk\| A^{L_a\dagger}\| I_1 T+1 k_1\rangle \times \langle I_1 T+1 k_1\| A^{L_b\dagger}\| I_0 T k_0\rangle . \quad (7)$$

Only intermediate states with isospin $(T+1)$ contribute, and the two-nucleon isovector structure amplitudes on the right-hand side of Eq. (7) are reduced matrix elements both in ordinary and isospin spaces. Some of the numerical values given in Table I were evaluated in this way, with existing[3] two-nucleon amplitudes used in order to provide a helpful numerical check.

II. CALCULATION AND RESULTS

The calculation was carried out with a shell-model program developed for the Argonne IBM-360/75 computer by Gloeckner. In this program, neutrons are coupled to states $(J_n k_n)$, protons to states $(J_p k_p)$; these are then coupled to produce states $I(J_p k_p \times J_n k_n)$, where I is the total angular momentum. In this representation, the matrix-element form of Eq. (1) is

$$\langle I(J_p k_p\times J_n k_n)\| \chi^{L_\alpha(L_a\times L_b)^\dagger}\| I_0(J_{0p} k_{0p}\times J_{0n} k_{0n})\rangle = [(2J_p+1)(2J_n+1)(2I_0+1)(2L_\alpha+1)]^{1/2} \begin{pmatrix} J_p & J_n & I \\ J_{0p} & J_{0n} & I_0 \\ L_a & L_b & L_\alpha \end{pmatrix} \times \langle J_p k_p(N_\pi)\| A_\pi^{L_a\dagger}\| J_{0p} k_{0p}(N_\pi-2)\rangle \langle J_n k_n(N_\nu)\| A_\nu^{L_b\dagger}\| J_{0n} k_{0n}(N_\nu-2)\rangle , \quad (8)$$

where N_ν and N_π refer to the numbers of neutrons and protons, respectively, and the 3×3 array is a $9j$ coefficient. The matrices of the $\chi^{L_\alpha^\dagger}$ of Eq. (2) were constructed in the basic representation. Then the energy matrices of effective 1p-shell interactions[4] were diagonalized and the $\chi^{L_\alpha^\dagger}$ matrices were transformed to the diagonalized representation. The (6–16)2B interaction of Ref. 4 was used for nuclei below mass number $A=10$, and the (8–16)POT interaction was used for $A\geq 10$.

The numerical results for the possible 1p-shell targets are listed in Table I. The angular momenta I_R of the states of the residual nucleus are given in the first column. The isospin is naturally the same as that of the target. The calculated excitation energies (second column) are intended solely to give a rough orientation since they are not reliable for highly excited states. The percentage for each I_R indicates the fraction of the total intensity to all 1p states which is contained in the listed entries. When only one such state is possible, no percentage entry is given. One point of interest is that the transition strength is spread over a wide range of excitation energy. Two pertinent examples are the pickup from ^{16}O to the second $I_R=4$ state of ^{12}C and from ^{15}N to the third $I_R=\frac{7}{2}$ state of ^{11}B. Since such states are usually above the particle-emission threshold, much of the transition strength is not experimentally observable.

Another point of interest is the relative contribution of different values of L_α. For example, in stripping on ^{10}B, the $I_R=1$ ground state of ^{14}N is predicted to be reached chiefly via $L_\alpha=4$ transfer, while the next $I_R=1$ state should be reached by $L_\alpha=2$ transfer. Another example concerns the two $I_R=1$ states of ^{10}B, which are found experimentally at 0.72 and 2.15 MeV. In stripping on ^{6}Li, the lower state is calculated to be reached mainly by $L_\alpha=0$ transfer while for the upper state $L_\alpha=2$ should dominate. In pickup on ^{14}N, the calculation gives $L_\alpha=2$ dominant in transfer to the lower state and nearly equal amplitudes for $L_\alpha=0$ and $L_\alpha=2$ transfer to the upper state. Of course the observation of transfer with different L_α values is also strongly affected by the penetrability.

The α-structure amplitude refers to four nucleons whose spatial coordinates still have their origin at the center of the shell-model potential well. If one assumes a harmonic-oscillator potential, one can project out that part of the four-nucleon function wherein the intrinsic wave function is in a 1s state and the motion of this α particle with respect to the residual nucleus contains all the quanta of oscillator excitation and the orbital angular momentum of the original four-nucleon function. These factors are known[5] and for our $(1p)^4$ case the spectroscopic factors are

$$S=\tfrac{3}{32}[A/(A-4)]^4\mathfrak{A}^2, \quad (9)$$

TABLE I. α-structure amplitudes for the indicated targets. The angular momentum I_R of the residual state is in column 1, and a calculated excitation energy is given in column 2. The last three columns list the α-structure amplitudes for $L_\alpha = 0$, 2, and 4; the percentage at the bottom of each group of amplitudes is equal to the sum of the squares of these amplitudes divided by the corresponding total for that particular value of I_R.

I_R	E_{calc} (MeV)	$L_\alpha=0$	$L_\alpha=2$	$L_\alpha=4$
	Pickup for target $^{16}O(0, 0)$			
0	0	−0.8902		
	13.47	0.4497		
		99%		
2	4.65		−2.0952	
	15.73		0.4498	
	18.13		−0.4604	
			96%	
4	13.50			−2.8367
	23.70			−0.9516
				99%
	Pickup for target $^{15}N(\frac{1}{2}, \frac{1}{2})$			
$\frac{1}{2}$	1.71	−0.7949		
		95%		
$\frac{3}{2}$	0		1.1186	
	5.39		−0.5742	
	11.44		0.3556	
	13.36		0.3352	
			99%	
$\frac{5}{2}$	4.35		−0.9464	
	8.11		−0.4395	
	10.69		−0.5874	
			96%	
$\frac{7}{2}$	5.85			1.8300
	12.65			0.1717
	15.08			0.8700
				95%
$\frac{9}{2}$	12.73			1.2511
				94%
	Pickup for target $^{14}N(1, 0)$			
1	0.90	−0.1064	0.5901	
	2.38	0.4770	−0.5105	
	6.19	−0.4042	−0.2039	
		97%	96%	
2	3.34		−0.9812	
	5.53		0.0335	
	10.28		−0.3721	
			99%	
3	0		−0.1842	1.3786
	4.72		−0.3499	−0.1002
	7.68		−0.4498	0.5906
			81%	86%
4	5.72			−1.0590
	12.23			−0.7412
				98%
5	12.36			−0.5512

I_R	E_{calc} (MeV)	$L_\alpha=0$	$L_\alpha=2$	$L_\alpha=4$
	Pickup for target $^{14}C(0, 1)$			
0	0	−0.8177		
	11.05	0.2443		
		98%		
2	4.16		1.4863	
	5.81		0.8230	
	9.16		−0.0598	
	10.26		−0.3768	
			98%	
4	11.63			−1.5466
	15.70			0.1337
	16.70			0.7664
				99%
	Pickup for target $^{13}C(\frac{1}{2}, \frac{1}{2})$			
$\frac{1}{2}$	3.01	−0.7369		
	11.49	0.0046		
	12.18	0.2123		
		98%		
$\frac{3}{2}$	0		−0.9982	
	5.09		−0.7406	
	9.93		−0.1747	
			98%	
$\frac{5}{2}$	2.64		−0.7296	
	7.48		0.4179	
			82%	
$\frac{7}{2}$	6.19			0.7460
	10.24			0.5266
	12.43			0.7525
				98%
$\frac{9}{2}$	11.10			0.9057
				99%
	Pickup for target $^{12}C(0, 0)$			
0	0	−1.0830		
		98%		
2	3.41		1.2257	
	14.43		−0.3453	
			95%	
4	11.29			−1.2758
				99%
	Pickup for target $^{11}B(\frac{3}{2}, \frac{1}{2})$			
$\frac{1}{2}$	1.07		−0.0538	
	9.97		0.0415	
			86%	
$\frac{3}{2}$	0	−0.6731	0.8314	
	10.87	0.0500	−0.0772	
	11.82	−0.1143	0.1166	
		99%	98%	

TABLE I *(Continued)*

I_R	E_{calc} (MeV)	$L_\alpha=0$	$L_\alpha=2$	$L_\alpha=4$
	Pickup for target $^{11}B(\frac{3}{2},\frac{1}{2})$ *(Continued)*			
$\frac{5}{2}$	7.40		0.2424	−0.2696
	9.15		−0.2788	0.2122
			84%	98%
$\frac{7}{2}$	4.79		−0.3355	0.8612
			99%	99%
	Stripping for target $^{11}B(\frac{3}{2},\frac{1}{2})$			
$\frac{1}{2}$	0		1.1186	
$\frac{3}{2}$	6.36	0.6397	−0.0552	
	Pickup for target $^{10}B(3, 0)$			
1	0		−0.0020	0.0615
	5.06		−0.0258	0.0888
			68%	86%
2	5.23		0.2318	−0.1827
3	2.14	−0.5518	0.9086	−0.5826
	Stripping for target $^{10}B(3, 0)$			
1	0		−0.1842	1.3786
	3.62		0.9286	−0.0198
			99%	99%
2	6.99		−0.3887	0.4132
3	10.14	−0.4096	−0.0043	−0.0826
	Pickup for target $^{10}Be(0, 1)$			
0	0	−0.8719		
		98%		
2	3.53		0.8479	
			98%	
	Stripping for target $^{10}Be(0, 1)$			
0	0	−0.8177		
		86%		
2	6.83		−0.3803	
			99%	
	Pickup for target $^{9}Be(\frac{3}{2},\frac{1}{2})$			
$\frac{1}{2}$	0.63		0.2493	
$\frac{3}{2}$	0	−0.7556	0.7502	
	Stripping for target $^{9}Be(\frac{3}{2},\frac{1}{2})$			
$\frac{1}{2}$	0		−0.9982	
	8.78		−0.1133	
	13.81		0.4598	
			99%	
$\frac{3}{2}$	3.59	−0.7585	0.2290	
	10.43	−0.0921	0.0693	
	14.00	0.1046	−0.1152	
		97%	93%	
$\frac{5}{2}$	7.40		−0.0586	0.7204
	13.18		0.0395	0.0183
			60%	93%

I_R	E_{calc} (MeV)	$L_\alpha=0$	$L_\alpha=2$	$L_\alpha=4$
	Stripping for target $^{9}Be(\frac{3}{2},\frac{1}{2})$ *(Continued)*			
$\frac{7}{2}$	11.08		0.3396	−0.1836
			98%	97%
	Stripping for target $^{7}Li(\frac{3}{2},\frac{1}{2})$			
$\frac{1}{2}$	1.71		0.7741	
			97%	
$\frac{3}{2}$	0	−0.6731	0.8314	
	5.39	−0.3856	−0.4264	
	11.44	−0.1880	0.2428	
	13.36	−0.1576	0.1765	
		94%	96%	
$\frac{5}{2}$	4.35		−0.9594	−0.0238
	8.11		−0.0770	−0.1147
	10.69		0.1768	−0.2166
			93%	90%
$\frac{7}{2}$	5.85		0.1295	−0.3726
	12.65		−0.0318	−0.2910
	15.08		−0.0374	−0.4354
			87%	89%
$\frac{9}{2}$	12.73			−0.7224
				96%
	Stripping for target $^{6}Li(1, 0)$			
1	0.90	0.8956	−0.3176	
	2.38	0.1919	0.6566	
	6.19	0.0643	−0.0027	
		93%	90%	
2	3.34		0.7804	
	5.53		0.4572	
	10.28		−0.0617	
			97%	
3	0		−0.0020	0.0615
	4.72		0.7543	0.0944
	7.68		−0.1016	0.2651
			90%	35%
4	5.72			−0.1630
	12.23			−0.6963
				89%
5	12.36			−0.4812
	Stripping for target $^{4}He(0, 0)$			
0	0	0.9929		
		99%		
2	3.41		0.9886	
	14.43		0.1010	
			99%	
4	11.29			0.9638
				93%

where A is the mass number and $\mathcal{A}$ is the structure amplitude of Eq. (6) and Table I.

The spectroscopic factors also indicate how well $1p$ nuclei would serve as projectiles for transfer of a group with spatial symmetry [4]. Table II contains those cases that have large spectroscopic factors and includes cases in which the residual projectile is left in an excited state. For several of these possible reactions, the spectroscopic factors are equal to or greater than that for the well-known (^{16}O, ^{12}C) reaction. For most of these, the projectile is one of the lighter nuclei, for which the factor $[A/(A-4)]^4$ increases the spectroscopic factor significantly. However, Rotter[6] has emphasized that for projectiles like those of Table II other spatial symmetries can contribute to the transfer reaction. For example, in (^{16}O, ^{12}C) the structure amplitude for [31] symmetry is $\mathcal{A}=-0.40$, about half the value $\mathcal{A}=-0.89$ for symmetry [4].

There are pickup and stripping sum rules for the squares of the α-structure amplitudes based on Eq. (5). The pickup sum rule is

$$\sum_{I_0 k_0 L} C_L^2(2L+1)^{-1}\langle ITk(N)\|\chi^{L^\dagger}\|I_0Tk_0(N-4)\rangle^2 = (2I+1)^{-1}\sum_M \langle IMTk|\chi_0^\dagger\chi_0|IMTk\rangle\,. \tag{10}$$

The right-hand side is the probability that the single-particle level p_0 of Eq. (3) is fully occupied by both neutrons and protons in the target. This quantity can be evaluated with the target wave functions, and the probability varies from 1 for ^{16}O to 0.58 for ^{14}C and 0.48 for ^{12}C. The companion stripping sum rule, wherein the left-hand side is weighted by $(2I+1)/(2I_0+1)$ in the sum over I, measures the probability that the level p_0 is completely empty; and this probability varies from 1 for ^{4}He to 0.16 for ^{12}C. These sum rules are probably of limited use experimentally since much of the transfer strength is not observable. However, they do provide a limit for the transfer strength and were very useful in checking the calculated values of Table I.

TABLE II. Spectroscopic factors $S_L[4]$ for $1p$-shell projectiles with large α amplitudes. The ingoing and outgoing particles (not necessarily in that order) are listed in column 1. If the final state is not the ground state, its I value is given in parentheses. Pickup and stripping are indicated by P and S in column 2.

Particles	Reaction	$S_0[4]$	Particles	Reaction	$S_2[4]$
^{16}O, ^{12}C	P, S	0.23	^{16}O, ^{12}C(2)	P	1.31
^{14}C, ^{10}Be	P, S	0.24	^{15}N, ^{11}B	P, S	0.41
^{12}C, ^{8}Be	P	0.55	^{14}C, ^{10}Be(2)	P	0.80
^{11}B, ^{7}Li	P, S	0.26	^{12}C, ^{8}Be(2)	P	0.72
^{10}B(1), ^{6}Li	S	0.59	^{8}Be(2), ^{4}He	S	1.47
^{10}Be, ^{6}He	P	0.55			$S_4[4]$
^{9}Be, ^{5}He	P	0.57	^{16}O, ^{12}C(4)	P	2.39
^{8}Be, ^{4}He	S	1.50	^{15}N, ^{11}B($\frac{7}{2}$)	P	1.09
			^{14}N, ^{10}B	P, S	0.69

III. DISCUSSION

The α-structure factor is an aid in determining how well direct transfer with symmetry [4] describes such reactions as (d, ^{6}Li), (t, ^{7}Li), (^{3}He, ^{7}Be), and their inverses. Some experimental measurements for $1p$-shell targets have been treated with distorted-wave Born-approximation (DWBA) analysis. Both the (^{3}He, ^{7}Be) reaction[7] and the (d, ^{6}Li) reaction[8] show larger cross sections for exciting the first 2^+ state than for exciting the ground state of ^{12}C in pickup on ^{16}O. Similarly in pickup on ^{10}B, the excited 3^+ state is reached more strongly than the 1^+ ground state of ^{6}Li. This is qualitatively what one would expect from the relevant entries of Table I. However, as emphasized in Ref. 8, the cross section to the ^{6}Li ground state is at least 1 order of magnitude greater than indicated by the very small α-structure amplitude of Table I. Aside from this discrepancy, these $1p$ data[7,8] seem consistent with structure factors obtained by Rotter,[6] which are quite similar to the values in Table I for these few cases. While processes other than direct α transfer seem to be present in these reactions,[9] direct transfer may dominate at higher energies and one may hope that with more experience in handling the DWBA parametrization it will be possible to test the spectroscopic information quantitatively.

Several four-particle transfer reactions with $1p$-shell projectiles have been investigated experimentally. Not only the well-known (^{16}O, ^{12}C) reaction and its inverse but also the (^{12}C, ^{8}Be) pickup[10] seem to be well described by symmetry [4] transfer, as would be expected from Table II. The reaction ^{12}C(^{10}B, ^{6}Li)^{16}O provides a good example[11] of four-nucleon transfer with spatial symmetry other than [4], since the α-structure amplitude of Table I is very weak for this projectile. The general features of the cross section differ from those of α transfer, and the authors of Ref. 11 point out that the strong excitation of a 2^- state in ^{16}O is consistent with transfer of four nucleons with spatial symmetry [31] since this is a strong amplitude for both (^{10}B, ^{6}Li) and [^{16}O(2^-), ^{12}C].

The representation of α-structure operators in the form of Eqs. (1) and (5) can be readily extended to other spaces such as the $(2s, 1d)$ and $(2p, 1f)$ configurations. However, in these regions the spin-orbit splitting of the single-particle levels is very important in determining the configurations used to describe the nuclear wave functions.

TABLE III. α strengths associated with the indicated configurations. These strengths are expressed as a percentage of the total $2p\,1f$ α strength for the given L_α.

L_α	$(f)^{4-n}(p)^n$ $(n \geq 2)$	$(f_{7/2}p_{3/2}p_{1/2})^4$	$(f_{7/2}p_{3/2})^4$	$(f_{7/2})^{4-n}(p_{3/2})^n$ $(n \geq 2)$	$(f_{7/2})^4$
0	92.2	63.4	24.0	22.5	0.3
2	88.4	58.6	20.6	18.4	0.2
4	84.5	48.3	15.2	13.0	0.2
6	80.5	34.7	10.0	7.9	0.3
8	58.0	20.1	6.1	3.1	0.3
10	0	7.5	3.5	0	0.7
12	0	2.0	2.0	0	2.0

Often the shell-model space is truncated in order to obtain a feasible calculation of the nuclear eigenfunctions (e.g. by omitting the $1f_{5/2}$ level in the $2p$, $1f$ region). It is therefore illuminating to expand the α-structure operators into a jj representation even though one loses their simplicity in the SU(3) representation. In this way one can see what fraction of the α strength is contained in a subspace of the jj representation.

This has been done for the $2p1f$ region. For each L_α, Table III shows what percentage of the total strength $|\chi^{L_\alpha}|^2$ is contained in each indicated configuration. The percentages in the second column are the totals for all configurations with two or more $2p$ nucleons. Aside from $L_\alpha = 10$ and 12, which cannot be formed with this many $2p$ nucleons, there is a strong concentration of strength into these configurations. This feature is a natural extension of our experience with two-neutron transfer,[12] for which the $2p$ contribution is much more important than the $1f$ contribution. Since the α operator can be written in the form of coupled singlet pairs of neutrons and protons as in Eq. (1), the preponderance of the $2p$ contribution from two-nucleon amplitudes enters quadratically here. In addition to this effect a further enhancement of the $2p$ dominance may result from consideration of the radial wave function in a DWBA calculation. The third column of Table III gives the effect of omitting the $1f_{5/2}$ orbital from the $2p1f$ space, and the fourth column gives the effect of limiting the space to $2p_{3/2}1f_{7/2}$. In the latter case, less than one fourth of the total strength remains. Column five shows that in this smaller space the $2p_{3/2}$ contribution is again dominant. Finally, column six shows how little α strength remains if the nuclear states are restricted to $(1f_{7/2})^n$ configurations. The decompositions of Table III indicate that a microscopic description of α transfer within the $2p1f$ shell probably requires the inclusion of all single-particle levels and certainly the $2p$ levels. One would also expect a large fragmentation of this α strength, with a large amount at experimentally unobservable excitation energies. The fragmentation amongst the spherical states should help in an experimental identification of any states of strong deformation, since these should contain four-nucleon groups similar to those of the complete α operator.

In the $1p$ shell, complications would come from admixtures of $(2s, 1d)^n$ configurations. However, one should first see how well the reactions are described by pure $(1p)^4$ transfer.

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