Multichannel Three-Body Model of ^{6}Li

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The binding energies of the lowest isospin singlet ($J^\pi=1^+$) and triplet ($J^\pi=0^+$) states of ^{6}Li are calculated on a multichannel three-body model in which the internal structure of the α particle is approximately represented by a two-state system. The n-α interaction is given by a two-channel l-dependent potential that is obtained by fitting low-energy scattering data. The np interaction is taken to be the Pease-Feshbach potential.

The Faddeev equations are generalized to allow spin and internal structure of the particles and are solved in the separable t-matrix approximation for bound-state energies. The results obtained are in good agreement with experiments and the effect of the α internal structure is discussed.

I. INTRODUCTION

The three-body model of ^{6}Li was first treated in great detail by Wackman and Austern.[1] They solved the three-body Schrödinger equations by the Rayleigh-Schrödinger variational technique. The calculated electromagnetic moments and the charge radius of the ground state are in general, in good agreement with experiments. The energy levels are, however, shifted upward from the experimental values by about 3.5 MeV. Moreover, the spacing between the 0^+ level and the ground-state level is found to be too small that the 0^+ level lies below the 3^+ level in contrast to experiment.

The problem has also been considered by Shanley[2] in his study of d-α scattering. He applied the formalism of Amado[3] with separable two-body potentials to solve the three-body problem. While he obtained rather good fits to the d-α scattering data, the binding energies he found were a little too small even before the repulsive Coulomb energy was included. Shanley attributed these discrepancies to the repulsive s-wave N-α interaction he had chosen.

More recently, Shah and Mitra[4] calculated the binding energies for the lowest isospin singlet and triplet states of ^{6}Li by solving the Faddeev equations with separable two-body potentials. They found that the ground-state ($T=0$) energy is too low and the excited-state ($T=1$) energy is too high.

In all the existing works, an assumption is made that the α particle is a structureless elementary particle. While this assumption simplifies the calculation greatly, its justification is not really convincing.

As has been pointed out by Fulco and Wong[5] in their paper on the ground state of the three-α system, the inelastic channels of the α-α interaction may have very significant effects on the three-body binding energy. In fact, the present authors have made a calculation on the multichannel three-α model of ^{12}C and found that the binding energies for both the lowest 0^+ and 2^+ states are in good agreement with experiments.[6] It was also pointed out in Ref. 6 that even though the effect of internal structure of α particles on the three-body binding energy is not very large within the framework of the model itself, a similar single-channel potential that fits the same two-body scattering data may give different binding energies of the three-particle system.

In this paper we shall consider the ($np\alpha$) system as a multichannel three-particle problem by assuming that the internal structure of the α particle may be approximately represented by a two-level system. This means that the α particle in the nucleus maintains its identity although it may be either in its ground state or in its excited state. With a phenomenological N-α potential obtained by a multichannel analysis of the low-energy two-body scattering data, this picture has effectively taken into account all the inelastic-channel contributions except for breakup of the α particle.

In Sec. II, we make a complete angular momentum reduction of the Faddeev equations that is generalized to allow the spin and internal-structure quantum numbers. In Sec. III, we describe the np potential to be used and how the multichannel n-α potential is obtained. The results of the calculation are discussed in Sec. IV.

II. GENERALIZED FADDEEV EQUATIONS

We shall consider a general three-body system of particles that may have different spin or internal-structure states. The latter will be specified by the set of quantum numbers (r, m_r). Thus for an elementary particle $r = m_r = 0$ and for a two-state particle $r = \frac{1}{2}$, $m_r = \pm\frac{1}{2}$. These quantum numbers can be treated in exactly the same manner as the spin quantum numbers except that they do not couple with the angular momentum operators. Thus, if we work in a representation in which particles j and k form a subsystem with particle i left free, then the internal state of the subsystem is characterized by $\vec{R}_i = \vec{r}_j + \vec{r}_k$ and $\vec{\mathcal{R}} = \vec{R}_i + \vec{r}_i$ is the corresponding quantum number for the three-body system.

The three-body states in general have three parts: the internal structure part

$$|\gamma_i\rangle_i \equiv |r_i m_{r_i}; (r_j r_k) R_i m_{R_i}\rangle_i , \tag{1}$$

the spin part

$$|\beta_i\rangle_i \equiv |[s_i(s_j s_k) S_i]\, \mathcal{S} m_{\mathcal{S}}\rangle_i , \tag{2}$$

and the spatial part $|\vec{p}_i \vec{q}_i\rangle_i$, where $\vec{p}_i$ and $\vec{q}_i$ have their standard meaning. As is well known, one can permute ijk cyclically to obtain the states in different representations. The transformation between i and j representation is given by

$$|\vec{p}_j \vec{q}_j; \beta_j; \gamma_j\rangle = \sum_{S_i} \sum_{r_i m_{ri}} \sum_{\mathcal{R} m_{\mathcal{R}}} {}_i\langle \beta_i | \beta_j \rangle_j \times \begin{pmatrix} r_i & R_i & \mathcal{R} \\ m_{r_i} & m_{R_i} & m_{\mathcal{R}} \end{pmatrix} \begin{pmatrix} r_j & R_j & \mathcal{R} \\ m_{r_j} & m_{R_j} & m_{\mathcal{R}} \end{pmatrix} {}_i\langle (r_i R_i) \mathcal{R} m_{\mathcal{R}} | (r_j R_j) \mathcal{R} m_{\mathcal{R}} \rangle_j |\vec{p}_i \vec{q}_i; \beta_i; \gamma_i\rangle_i . \tag{3}$$

Since we are going to calculate the energies of bound states with definite spin-parity J, we shall for convenience use the three-body state in the following form with obvious notation

$$|p_i q_i; \alpha_i; \gamma_i\rangle_i = |p_i[(s_j s_k) S_i l_i] \mathcal{J}_i; q_i (s_i L_i) I_i; JM; \gamma_i\rangle_i , \tag{4}$$

where, as in (1) and (2), we have denoted the set of angular momentum quantum numbers collectively by the symbol α_i.

With the three-body states defined as in (4), the generalized multichannel Faddeev equations can be written in the form

$$\Psi_\alpha^{(i)}(pq\gamma, z) = \Phi_\alpha^{(i)}(pq\gamma z) - \frac{1}{4} \sum_{j \neq i} \sum_{\alpha_j \gamma_j} \int_0^\infty dp_j^{\,2} \int_0^\infty dq_j^{\,2} \frac{p_j q_j}{p_j^{\,2} + q_j^{\,2} + \epsilon_{r_j m_{rj}} + \epsilon_{R_j m_{Rj}} - z} \times K^{(ij)}(pq\alpha\gamma | p_j q_j \alpha_j \gamma_j) \Psi_{\alpha_j}^{(i)}(p_j q_j \alpha_j \gamma_j z) , \tag{5}$$

where

$$\Psi_\alpha^{(i)}(pq\gamma z) \equiv {}_i\langle pq\alpha\gamma | T^{(i)}(z) | n\rangle , \tag{6a}$$

$$\Phi_\alpha^{(i)}(pq\gamma z) \equiv {}_i\langle pq\alpha\gamma | T_i(z) | n\rangle , \tag{6b}$$

$$K^{(i,j)}(pq\alpha\gamma | p_j q_j \alpha_j \gamma_j) \equiv {}_i\langle pq\alpha\gamma | T_i(z) | p_j q_j \alpha_j \gamma_j\rangle_j = \sum_{\alpha_i} {}_i\langle pq\alpha\gamma | T_i(z) | p_i q_i \alpha_i \gamma_i\rangle_i \, {}_i\langle p_i q_i \alpha_i \gamma_i | p_j q_j \alpha_j \gamma_j\rangle_j , \tag{6c}$$

where $\epsilon_{r_j m_{rj}}$ and $\epsilon_{R_j m_{Rj}}$ are the internal-state energies of the corresponding subsystems. In order to solve the Eqs. (5) for bound states, we need to decompose the angular momentum states involved in $K^{(ij)}(pq\alpha\gamma | p_j q_j \alpha_j \gamma_j)$. This can be done by (i) expressing the two-body t matrix in three-body Hilbert space in terms of the ordinary two-body t matrix, (ii) inserting a complete set of states

$$\sum_{\vec{p}_j \vec{q}_j} |\vec{p}_j \vec{q}_j \beta_j \gamma_j\rangle\langle \vec{p}_j \vec{q}_j \beta_j \gamma_j |$$

into (6c), and (iii) carrying out the angular integrations by rotating the coordinate axes from space-fixed system to body-fixed system. The procedure is essentially similar to those introduced by Omnes[7] and by Ahmadzadeh and Tjon.[8] The details of the calculation can be found in a thesis by Chuu[9] and here we simply

give the reduced form of the kernel:

$$K^{(ij)}(pq\alpha\gamma\,|\,p_jq_j\alpha_j\gamma_j)=\delta_{JJ_j}\delta_{MM_j}\sum_{R_im_{R_i}}\sum_{\mathcal{R}m_{\mathcal{R}}}\delta_{rr_i}(-1)^{3r_i+2r_j+r_k-R_j+2m_R}\sqrt{(2R_i+1)(2R_j+1)}$$

$$\times(2\mathcal{R}+1)\begin{Bmatrix} r_i & r_k & R_j \\ r_j & \mathcal{R} & R_i\end{Bmatrix}\begin{pmatrix} r_i & R_i & \mathcal{R} \\ m_{r_i} & m_{R_i} & -m_{\mathcal{R}}\end{pmatrix}\begin{pmatrix} r_j & R_j & \mathcal{R} \\ m_{r_j} & m_{R_j} & -m_{\mathcal{R}}\end{pmatrix}$$

$$\times\frac{4\pi^{3/2}}{q}\,\frac{U(L_{ij},U_{ij},p_j^{\,2})}{\alpha_{ij}\beta_{ij}p_jq_j}\sum_{S_il_i}\sum_{\mathcal{L}}(2\mathcal{L}+1)(-1)^{l_i+l_j-L_i-L_j+s_j+s_k-S_i}$$

$$\times[(2l_j+1)(2S_j+1)(2I_j+1)(2\mathcal{J}_j+1)(2S_i+1)(2I_i+1)(2\mathcal{J}_i+1)]^{1/2}$$

$$\times\sum_{\mathcal{S}}(-1)^{2\mathcal{S}}(2\mathcal{S}+1)\begin{Bmatrix} s_i & s_k & S_j \\ s_j & \mathcal{S} & S_i\end{Bmatrix}\begin{Bmatrix} s_i & L_i & I_i \\ S_i & l_i & \mathcal{J}_i \\ \mathcal{S} & \mathcal{L} & J\end{Bmatrix}\begin{Bmatrix} s_j & L_j & I_j \\ S_j & l_j & \mathcal{J}_j \\ \mathcal{S} & \mathcal{L} & J\end{Bmatrix}$$

$$\times\sum_{n_{l_i}n_{L_i}}\sum_{n_{L_j}}\begin{pmatrix} l_j & L_j & \mathcal{L} \\ 0 & n_{L_j} & -n_{L_j}\end{pmatrix}\begin{pmatrix} l_i & L_i & \mathcal{L} \\ n_{l_i} & n_{L_i} & -n_{L_j}\end{pmatrix}Y^*_{l_in_{l_i}}\left(\theta_{p_ip_j},\frac{\pi}{2}+(-)^P\frac{\pi}{2}\right)$$

$$\times Y^*_{L_in_{Li}}(\theta_{q_ip_j},\pi)Y_{L_jn_{Lj}}(\theta_{p_jq_j},0)t_i^{\,l'l_i,\,SS_i,\,RR_i}(pp_i,z-q^2-\epsilon_{r_im_{ri}}),\tag{7}$$

where

$$\alpha_{ij}\equiv\left[\frac{m_im_j}{(m_i+m_k)(m_j+m_k)}\right]^{1/2},\tag{8}$$

$$\beta_{ij}\equiv(1-\alpha_{ij}^{\,2})^{1/2},\tag{9}$$

$$U_{ij}=(\alpha_{ij}q_j+q)^2/\beta_{ij}^{\,2},\tag{10a}$$

$$L_{ij}=(\alpha_{ij}q_j-q^2)/\beta_{ij}^{\,2},\tag{10b}$$

and P is the cyclic permutation of the particle indices i and j.

It is noted that the azimuthal angles appearing in the spherical harmonics differ from those given in Ref. 8. This is because we have chosen the direction of $\vec{p}_j$ to be the z axis, and $\vec{q}_j$ lies in the xz plane. It therefore follows from the relations

$$\vec{p}_i=-\alpha_{ij}\vec{p}_j-(-1)^P\beta_{ij}\vec{q}_j,\tag{11a}$$

$$\vec{q}_i=(-1)^P\beta_{ij}\vec{p}_j-\alpha_{ij}\vec{q}_j\tag{11b}$$

that the azimuthal angles $\phi_{p_jq_i}=\pi$ and $\phi_{p_jp_i}=\pi/2+(-1)^P\pi/2$. If one fixes the z axis to be the direction of $\vec{p}_2$ instead of $\vec{p}_j$, then the results reduce to those in Ref. 8 except for $\phi_{q_ip_j}$ which involves an error as has been pointed out by Balian and Brezin.[10] The step function U in (7) is defined as

$$U(L_{ij},U_{ij},p_j^{\,2})=\begin{cases}1, & \text{if } L_{ij}\le p_j^{\,2}\le V_{ij}\\ 0, & \text{otherwise}\end{cases}\tag{12}$$

and n_L and n_l are the components of the angular momentum operators along the body-fixed axis.

The Faddeev equations with kernel given by (7) represent a set of coupled integral equations with two continuous variables $p_j^{\,2}$ and $q_j^{\,2}$. Before we attempt to solve it, we shall make the so-called separable t-matrix approximation introduced by Ball and Wong,[11] so that one of the variables can be explicitly integrated out. For bound-state problems, the two-body t matrix can be expressed in terms of the complete set of eigenfunctions ϕ_n of the homogeneous Lippman-Schwinger equation. Thus

$$t_i^{\,l'l_i,\,SS_i\,RR_i}(pp_i,z-q^2-\epsilon_{r_im_{ri}})=\sum_n C^{(i)}_{nl_i}(p_iS_iR_im_{R_i}z)\phi^{(i)}_{nl}(pSRm_Rz),\tag{13}$$

where $\phi^{(i)}_{nl}$ satisfies the equation

$$\lambda^{(i)}_{nl}(z)\phi^{(i)}_{nl}(pSRm_Rz)=\frac{1}{2}\sum_{R_i}\sum_{S_il_i}\int dp_i^{\,2}\,\frac{p_iv^i_{ll_i}(pSRm_R,p_iS_iR_im_{R_i})}{p_i^{\,2}-z}\,\phi^{(i)}_{nl_i}(p_iS_iR_im_{R_i}z)\tag{14}$$

with the orthogonality condition

$$\frac{1}{2}\sum_{R}\sum_{lS}\int_0^\infty dp^2\,\frac{p\phi_{nl}^{(i)}(p,SRm_Rz)\phi_{ml}^{(i)}(pSRm_Rz)}{p^2-z}=\delta_{nm}\,. \tag{15}$$

The expansion coefficients $C_{nl}^{(i)}$ in (13) are given by

$$C_{nl}^{(i)}(p_iS_iR_im_{R_i}z)=\frac{\lambda_{nl}^{(i)}(z)}{1-\lambda_{nl}^{(i)}(z)}\,\phi_{nl}^{(i)}(p_iS_iR_im_{R_i}z)\,. \tag{16}$$

By solving (14) for ϕ_{nl}, we can separate the variables p and q in $t_i^{l_il_iSS_iRR_i}$ and hence the generalized Faddeev equations are reduced to a set of coupled integral equations in one dimension:

$$\chi_{n\alpha}^{(i)}(qr_im_{r_i}z)=\eta_{n\alpha}^{(i)}(qr_im_{r_i}z)+\sum_{j\neq i}\sum_{\alpha_j}\sum_{r_jm_{r_j}}\sum_{n_j}\int_0^\infty dq_j^{\,2}K_{n\alpha n_j\alpha_j}^{(ij)}(qr_im_{r_i},q_jr_jm_{r_j};z)\chi_{n_j\alpha_j}^{(i)}(q_jr_jm_{r_j}z)\,. \tag{17}$$

In Eq. (17), the function $\chi_{n\alpha}^{(i)}$ satisfies

$$\Psi_\alpha^{(i)}(pq\gamma z)=\Phi_\alpha^{(i)}(pq\gamma z)+\sum_{n_j}\frac{\lambda_{n_jl}^{(i)}(z-q^2-\epsilon_{r_im_{ri}})\phi_{n_jl}^{(i)}(pRm_R,z-q^2-\epsilon_{r_im_{ri}})}{1-\lambda_{n_jl}^{(i)}(z-q^2-\epsilon_{r_im_{ri}})}\chi_{n_j\alpha}^{(i)}(qr_im_{r_i}z)\,. \tag{18}$$

The kernel is given by

$$K_{n\alpha,\,n_j\alpha_j}^{(ij)}=\sum_{R_jm_{R_j}}\int_{L_{ij}}^{U_{ij}}dp_j^{\,2}\,\frac{\lambda_{n_jl_j}^{(j)}(z-q_j^{\,2}-\epsilon_{r_jm_{rj}})}{1-\lambda_{n_jl_j}^{(j)}(z-q_j^{\,2}-\epsilon_{r_jm_{rj}})}W(\alpha\alpha_jr_im_{r_i}\gamma_jp_jq_jz)\phi_{n_jl_j}^{(j)}(p_jR_jm_{R_j},z-q_j^{\,2}-\epsilon_{r_jm_{rj}}) \tag{19}$$

and the inhomogeneous part

$$\eta_{n\alpha}^{(i)}=\sum_{j\neq i}\sum_{\alpha_j\gamma_j}\int_0^\infty dq_j^{\,2}\int_{L_{ij}}^{U_{ij}}dp_j^{\,2}\,W(\alpha\alpha_jr_im_{r_i}\gamma_jp_jq_jz)\Phi_{\alpha_j}^{(j)}(p_jq_j\gamma_jz)\,. \tag{20}$$

The function W appearing in (19) and (20) is given by

$$\begin{aligned}W=&\frac{-\pi^{3/2}}{\alpha_{ij}\beta_{ij}q}\,\frac{1}{p_j^{\,2}+q_j^{\,2}+\epsilon_{r_jm_{rj}}+\epsilon_{R_jm_{Rj}}-z}\sum_{R_im_{Ri}}\sum_{\mathfrak{R}m_{\mathfrak{R}}}(-1)^{3r_i+2r_j+r_k-R_j+2m_{\mathfrak{R}}}\\&\times[(2R_i+1)(2R_j+1)]^{1/2}(2\mathfrak{R}+1)\begin{Bmatrix}r_i & r_k & R_j\\ r_j & \mathfrak{R} & R_i\end{Bmatrix}\begin{pmatrix}r_i & R_i & \mathfrak{R}\\ m_{r_i} & m_{R_i} & m_{\mathfrak{R}}\end{pmatrix}\begin{pmatrix}r_j & R_j & \mathfrak{R}\\ m_{r_j} & m_{R_j} & -m_{\mathfrak{R}}\end{pmatrix}\\&\times\sum_{S_il_i\mathfrak{L}}[(2l_j+1)(2S_j+1)(2I_j+1)(2\mathfrak{I}_j+1)(2S_i+1)(2I_i+1)(2\mathfrak{I}_i+1)]^{1/2}(2\mathfrak{L}+1)(-1)^{l_i+l_j-L_i-L_j+s_j+s_k-S_i}\\&\times\sum_{\mathfrak{S}}(-1)^{2\mathfrak{S}}(2\mathfrak{S}+1)\begin{Bmatrix}s_i & s_k & S_j\\ s_j & \mathfrak{S} & S_i\end{Bmatrix}\begin{Bmatrix}s_i & L_i & I_i\\ S_i & l_i & \mathfrak{I}_i\\ \mathfrak{S} & \mathfrak{L} & J\end{Bmatrix}\begin{Bmatrix}s_j & L_j & I_j\\ S_j & l_j & \mathfrak{I}_j\\ \mathfrak{S} & \mathfrak{L} & J\end{Bmatrix}\sum_{n_{l_i}n_{L_i}}\sum_{n_{L_j}}\delta_{ll_i}\delta_{SS_i}\\&\times\begin{pmatrix}l_j & L_j & \mathfrak{L}\\ 0 & n_{L_j} & -n_{L_j}\end{pmatrix}\begin{pmatrix}l_i & L_i & \mathfrak{L}\\ n_{l_i} & n_{L_i} & -n_{L_j}\end{pmatrix}Y_{l_in_{li}}^*\left(\theta_{p_ip_j},\frac{\pi}{2}+(-)^P\frac{\pi}{2}\right)Y_{L_in_{Li}}^*(\theta_{q_ip_j},\pi)\\&\times Y_{L_jn_{Lj}}(\theta_{p_jq_j},0)\phi_{n_li}^{(i)}(p_iR_im_{R_i},z-q^2-\epsilon_{r_im_{ri}})\,.\end{aligned} \tag{21}$$

TABLE I. Parameters of $n\alpha$ potential.

Solution	l	V_1 (MeV)	V_2 (MeV)	V_3 (MeV)	r_0 (fm)
I	0	46.815	31.140	0.205	3.580
	1	21.507	46.865	0.325	3.494
II	0	38.093	34.0	4.358	3.190
	1	21.332	19.082	0.95	3.025

III. TWO-BODY POTENTIALS

For the neutron-proton interaction, we take the Pease-Feshbach[12] potential which has been shown to yield a correct triton binding energy in addition to the low-energy two-nucleon data fitting. It has also been shown in Ref. 1 that this potential and Gammel-Thaler potential give very close results in the ^{6}Li calculation. The analytic form of this potential is

$$V_{np}(\rho) = -V_c \frac{e^{-\sigma_c \rho}}{\sigma_c \rho} - V_T \frac{e^{-\sigma_T \rho}}{\sigma_T \rho} S_{12}, \tag{22}$$

where

$$V_c = 46.96 \text{ MeV}, \quad \sigma_c = 0.847\,06 \text{ fm}^{-1},$$
$$V_T = 23.879 \text{ MeV}, \quad \sigma_T = 0.588\,24 \text{ fm}^{-1}.$$

To obtain a nucleon-α potential for our purpose, we have to make a multichannel analysis of the two-body scattering problem. Since the α particle is assumed to be a two-state particle, the Schrödinger equation for the nucleon-α system can be written as

$$\left[-\frac{\hbar^2}{2\mu}\nabla_\rho{}^2 + \mathcal{H}(r, m_r) + V(\rho, r, m_r)\right]\Psi(\rho, r, m_r) = E\Psi(\rho, r, m_r), \tag{23}$$

where the internal energy operator $\mathcal{H}$ has the form

$$\mathcal{H}(r, m_r) = \begin{pmatrix} \epsilon_\downarrow & 0 \\ 0 & \epsilon_\uparrow \end{pmatrix} \tag{24}$$

and the potential energy operator V is given by

$$V(\rho, r, m_r) = \begin{pmatrix} V_1(\rho) & V_3(\rho) \\ V_3(\rho) & V_2(\rho) \end{pmatrix} = \frac{\hbar^2}{2\mu}\begin{pmatrix} v_1(\rho) & v_3(\rho) \\ v_3(\rho) & v_2(\rho) \end{pmatrix}. \tag{25}$$

If we now write the total wave function as

$$\Psi(\rho, r, m_r) = \sum_l \frac{(2l+1)}{k_0 \rho} i^l P_l(\cos\theta)\begin{pmatrix} u_{1l}(\rho) \\ u_{2l}(\rho) \end{pmatrix}, \tag{26}$$

then for incident energies below the α excitation energy the Schrödinger Eq. (23) becomes two

TABLE II. Binding energies of 1^+ and 0^+ states of ^{6}Li. All the energies are in the unit of MeV and corrected for Coulomb repulsion energy 0.88 MeV.

Authors	n-α interaction	n-p interaction	Ground state (1^+)	Excited state (0^+)
Wackman-Austern	Gaussian	Pease-Feshbach	−3.8	−2.71
	Exponential	Pease-Feshbach	0.08	1.27
Shah-Mitra	Separable potential for p-wave interaction only	Yamaguchi-Naqvi	−4.131	0.63
Shanley	Separable potential	S wave, Yamaguchi	−2.47	1.12
Allessandrini	Hebach form separable potential (fits $p_{3/2}$ phase shifts only)	Separable potential Hulthen form in singlet and Yamaguchi form in triplet state	−3.06	−0.317
Present work Solution I	$S+P$-wave square well with internal structure	S wave, Pease-Feshbach	−3.57	−0.45
	$S+P$-wave square well without internal structure	S wave, Pease-Feshbach	−3.38	
Solution II	$S+P$-wave square well with internal structure	S wave, Pease-Feshbach	−2.40	0.74
Experimental			−3.697	−0.13

coupled equations

$$\frac{d^2u_1}{d\rho^2}+\left[k_0^2-\frac{l(l+1)}{\rho^2}-v_1\right]u_{1l}-v_3u_{2l}=0, \quad (27a)$$

$$\frac{d^2u_{2l}}{d\rho^2}-\left[k^2+\frac{l(l+1)}{\rho^2}-v_2\right]u_{2l}-v_3u_{1l}=0, \quad (27b)$$

where we have taken $\epsilon_\dagger=0$ for convenience and defined k by

$$-k^2=k_0^2-\frac{2\mu}{\hbar^2}\epsilon_\dagger. \quad (28)$$

The wave functions are required to satisfy the boundary conditions

$$u_{1l}(0)=u_{2l}(0)=0$$

and behave asymptotically like

$$u_{1l}\to i^l\sin\left(k_0\rho-\frac{l\pi}{2}\right)+i^l\alpha_l e^{i(k_0\rho-l\pi/2)}, \quad (29a)$$

$$u_{2l}\to\beta_l e^{-k\rho}. \quad (29b)$$

The Eqs. (27) for square well can be solved analytically in the resonance approximation.[13] The differential cross section for elastic scattering is given by

$$\frac{d\sigma}{d\Omega}=\left|\sum_l\frac{2l+1}{k_0}\alpha_lP_l(\cos\theta)\right|^2, \quad (30)$$

where

$$\alpha_l=\frac{1}{2i}(e^{2i\eta_l}-1)-e^{2i\eta_l}\frac{\frac{1}{2}\Gamma_l}{E-E_c+\frac{1}{2}i\Gamma_l}. \quad (31)$$

The first term in (31) is due to the direct channel and the second term represents the coupled-channel contributions which is responsible for the resonance at E_c with width Γ_l.

The n-α scattering differential cross sections for six different energies are fitted by an l-dependent potential. With a set of initially chosen potential parameters, the numerical program calculates the square deviations of the cross section at eight different angles. Iteration procedure then starts to minimize square-sum deviation by optimizing the nonlinear potential parameters until the square-sum deviation is less than 10^{-4}. Two sets of parameters obtained in this way are listed in Table I. The α excitation energy $\epsilon_\dagger$ is taken to be 20 MeV.

It is observed that with a weak coupling V_3, the data can be fitted without spin-orbit potential. This is in contrast to the single-channel potential in which a term of LS coupling is required.[14] It is also noted that p-wave contribution is significant in $n\alpha$ scattering even at very low energies and hence the first resonance at 1.15 MeV is mainly the p-wave resonance. As the higher partial-wave contributions are negligible in the three-body calculation, we have not fitted the scattering data for c.m. energies higher than 3 MeV when d wave must be included.

IV. RESULTS AND DISCUSSIONS

We shall limit ourselves to the calculation of the lowest isospin singlet and triplet bound-state energies. For definiteness, we label the α particle as particle 1, the neutron particle 2, and the proton particle 3. Thus $r_1=r=\frac{1}{2}$, $r_2=r_3=0$ and $s_1=0$, $s_2=s_3=s=\frac{1}{2}$.

The angular momentum states to be included in these calculations can be determined with the help of shell model which requires that the two nucleons are in the $1p$ shell while the four nucleons constituting the α particle have zero orbital angular momentum. Therefore the two nucleons interact only in the triplet-even states for $T=0$ or $J=1^+$ state, and in $T=1$ or $J=0^+$ state they interact only in singlet-even state. Since the d-wave np interaction introduces only a small correction[2] to $J=1^+$ state and is not allowed in $J=0^+$ state, we neglect it in our calculation to save the computation time.

With the two-body potentials given in Sec. III, one can solve the eigenvalue Eq. (14) and use these solutions to calculate from (19) all the matrix elements of the kernel $K^{(ij)}$ as a function of the three-body energy z for a state of definite spin and parity. The bound-state energy is then the z value for which the eigenvalue of $K^{(ij)}$ is unity.

The binding energies are calculated for both sets of the n-α potential parameters. We have found the ground-state energy of ^{6}Li to about 90% accuracy by taking only p-wave contribution of n-α interaction. Furthermore, we have also calculated the ground-state energy by setting all the V's except V_1 to zero in the n-α potential to investigate the importance of the α internal structure. This gives about 6% less binding than the complete potential.

The results after correction for Coulomb energy which is 0.88 MeV are listed in Table II together with those from the previous works for comparison with experiments. It is seen that while our first solution agrees better with experiments than the second solution, they both predict almost the same level spacing which is very close to the experimental value 3.567 MeV. All the other calculations give a level spacing that differs from experiments by a factor of 2 or so except for the results of Shanley[2] which are close to our second solution. Although the results of Alessandrini *et al.*[15] differ from those given by Shah and Mitra,[4]

the calculations are essentially the same in both cases except that the separable potential for n-α p-wave interaction used in Ref. 15 is tuned exclusively to the $p_{3/2}$ phase-shift data while in the work of Shah and Mitra, both the $p_{1/2}$ and $p_{3/2}$ phase shifts are fitted. As has been discussed in Ref. 4, the justification for ignoring the $p_{1/2}$ phase shifts is not very sound, especially in the three-body calculation. The s-wave n-α interaction, on the other hand, has been ignored in both of these calculations on the basis of the Pauli principle. As can be seen in Table II, our calculation shows that this, in fact, introduces only about 10% error in the three-body binding energy.

The effect of hard core in two-body potentials on the three-body binding energy has been investigated by Fuda[16] for the trinucleon case. He found only about 3% difference in the binding energies corresponding to square-well potentials with and without core for the two-nucleon interaction. Thus it should not cause any significant change in the present calculation.

Although the ground-state binding energy is reduced only by about 6% when the α internal structure is neglected, it does not mean that a single-channel square-well potential for $n\alpha$ interaction will produce the same result since V_1 alone does not fit the scattering data anymore.

In view of the above discussion, it appears that ^{6}Li can be fairly well described as a bound $(np\alpha)$ system provided that the internal structure of the α particle is properly taken into account.

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