Absolute Normalization in Distorted-Wave Born-Approximation Analysis of Two-Nucleon-Transfer-Reaction Cross Sections

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The normalization constants for two-nucleon-transfer reactions are determined by the direct and parametrization methods. The results are compared between themselves and with experiment. Both methods give rise to calculated values which fall short by at least a factor of 3.

1. INTRODUCTION

For the direct reaction $A+a \rightarrow b+B$ $(b=a+x)$, there appears in the zero-range distorted-wave Born-approximation (DWBA) expression for the amplitude,[1] a universal normalization constant which depends only on the structure of the incident and outgoing projectiles and on the nucleon-nucleon interaction. Although this constant is available theoretically from a microscopic formulation of the transfer process, general reluctance to include the strength and exact form of the interaction potential as well as the detailed structure of the projectiles in the direct-reaction analysis of data is the reason why absolute magnitudes of the predicted cross sections are undefined.[2-4] A direct quantitative comparison between theoretical predictions and experimental data is, in principle, possible, but in practice, this has seldom been done. The effective normalization factor, wherever required, as in the development of two-particle sum rules,[4] is extracted empirically.

Recently, normalization constants for some single-nucleon-transfer reactions (SNTR) have been computed accurately, via a number of methods[5-9] and were found to agree quite closely with the empirically determined values. The same has not been done for the two-nucleon-transfer reactions (TNTR). Because of the wealth of spectroscopic information available from a quantitative analysis of TNTR cross sections and because the single-step direct-reaction theory has to be tested as comprehensively as possible (since its ultimate value depends on how well it accounts for experimental observation of reaction phenomena), absolute normalization of cross sections should be attempted and expected. This point is underscored by the sometimes large differences in magnitude obtained with various optical-model parameters and nuclear wave functions in the analysis of experimental data. The inherent purpose of this work therefore, is to spotlight attention on a detailed consideration of absolute cross sections of TNTR analyzed through DWBA.

In SNTR, two methods are frequently employed to evaluate the normalization constant theoretically, viz. (i) the empirical parametrization method of Thompson and Hering[7] and (ii) the direct method of Rook[5] and Lim.[6] By (i), estimates of the normalization factor are obtained from a Hulthen parametrization of the abx vertex function. The long- and short-range parameters, commonly labeled α and β, respectively, are derived by fitting the threshold and asymptotic behavior of the light clusters. In Ref. 7, Thompson and Hering postulated that for SNTR involving $1s$ shell projectiles, there exists a universal shape parameter $\beta = 1.36$ fm^{-1}, linked to some fundamental nucleon-nucleon parameter. Because this assumption gave strikingly good agreement with experiment, Hering *et al.*[10] ventured to extend this parametrization method to TNTR. Their calculations with $\beta = 1.36$ fm^{-1} lead to normalization factors more than an order of magnitude too small when directly confronted with experiment. In method (ii) the wave functions of

TABLE I. Values of the spectroscopic factors of the $1s$-shell projectiles.

Reaction	$b_{ST}{}^2$
(p,d), (d,p) (n,d), (d,n)	1
(d,t), (t,d) $(d,{}^3\mathrm{He})$, $({}^3\mathrm{He},d)$	$\frac{3}{2}$
(t,α), (α,t) $({}^3\mathrm{He},\alpha)$, $(\alpha,{}^3\mathrm{He})$	2
(p,t), (t,p) $(n,{}^3\mathrm{He})$, $({}^3\mathrm{He},n)$	1
$(p,{}^3\mathrm{He})$, $({}^3\mathrm{He},p)$ (n,t), (t,n)	$\frac{1}{2}$
(d,α), (α,d)	1

the projectiles and the internucleon force must be known so a direct evaluation of the constant can be made. It is therefore obvious that the direct method is tied in with a microscopic formulation of the transfer reaction, wherein effects of the finite range of the nucleon-nucleon interaction and of the light-particle wave functions are explicitly included in the analysis. By appealing to the direct method, we find agreement with experimental TNTR constants to be substantially improved but still a factor of 4 too low, e.g. in the (p, t) reactions.

2. DWBA, THE DIRECT METHOD AND HULTHEN PARAMETRIZATION

The theory of TNTR is well covered in other papers.[11-14] In this section, we develop enough of the microscopic analysis of TNTR to show the relationship between the direct method and the parametrization method.

In DWBA and following the formalism of Towner and Hardy,[12] the differential cross section for the pickup reaction $A(a, b)B$ can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{pu} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{2s_b+1}{2s_a+1}\sum_{JM\sigma_a\sigma_b}\left|\sum_{n_1 n_2 LST} b_{ST} S_{AB}{}^{1/2}(T_B N_B TN|T_A N_A)\begin{bmatrix} l_1 & l_2 & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J\end{bmatrix} B^{LSJT}_{M\sigma_a\sigma_b}\right|^2 . \tag{1}$$

In this formula, μ, k, s, and σ are, respectively, the reduced mass, the momentum, the spin, and the z projection of spin of the projectile. b_{ST} arises from the spin and isospin overlaps between projectiles and the transferred pair of nucleons (Table I contains the values of $b_{ST}{}^2$ for a number of TNTR and their counterparts in SNTR) and $S_{AB}{}^{1/2}$ is the spectroscopic amplitude which is a measure of the overlap of wave functions of the target nucleus and final nucleus with two nucleons. The Clebsch-Gordan coefficient couples the isospin of the residual nucleus T_B to that of the transferred pair T to form the target isospin T_A. The quantity $B^{LSJT}_{M\sigma_a\sigma_b}$ describes the kinematical aspects of the reaction and is defined by

$$B^{LSJT}_{M\sigma_a\sigma_b}(\theta) = \sum G(l_a j_a, l_b j_b, \sigma_a, \sigma_b, LJM) P_{l_b\lambda_b}(\cos\theta) I^{l_1 l_2 L l_a j_a l_b j_b ST}, \tag{2}$$

where the sum runs over all incoming and outgoing partial waves $l_a j_a$, $l_b j_b$ and

$$\lambda_b = \sigma_b - \sigma_a - M.$$

$G(l_a j_a, l_b j_b, \sigma_a, \sigma_b, LJM)$ involves various coupling coefficients and $I^{l_1 l_2 L l_a j_a l_b j_b ST}$, the radial integral over distorted waves and form factor, is

$$I^{l_1 l_2 L l_a j_a l_b j_b ST} = D(S, T) I^{l_1 l_2 L}_{\Lambda}. \tag{3}$$

The factor $D(S, T)$ allows for different strengths in singlet and triplet states in the interaction potential and is given by

$$D(0, 1) = 1 - 0.5(b + h),$$

$$D(1, 0) = -1 + 1.5(b + h),$$

where b and h are the Bartlett and Heisenberg exchange components in the nuclear force. The matrix element

$$I^{l_1 l_2 L}_{\Lambda} = \int\int\int \chi^{(-)*}_b(\vec{r}_b)\phi^*_b(\vec{r}_1, \vec{r}_2, \vec{r}_a)[V_{a1} + V_{a2}]\,\phi^{l_1 l_2 L}_{\Lambda}(\vec{r}_1, \vec{r}_2)\chi^{(+)}_{aA}(\vec{r}_{aA})\, d\vec{r}_{12} d\vec{r}_{aA} d\vec{r}_b \tag{4}$$

with

$$\phi^{l_1 l_2 L}_{\Lambda}(\vec{r}_1, \vec{r}_2) = \tfrac{1}{2} g \sum_{\lambda_1\lambda_2}(l_1\lambda_1 l_2\lambda_2|L\Lambda)\sum_{m=0}^{1}(-)^m P^m_{12}\phi^{n_1 l_1 j_1}_{\lambda_1}(\vec{r}_1)\phi^{n_2 l_2 j_2}_{\lambda_2}(\vec{r}_2). \tag{5}$$

P^m_{12} is the operator which makes m interchanges between transferred nucleons 1 and 2. (n_1, l_1, j_1) and (n_2, l_2, j_2) characterize the orbits of the two transferred nucleons when they were bound in the target nucleus. If $(n_1, l_1, j_1) \equiv (n_2, l_2, j_2)$, then the constant $g = 1$, otherwise it is $\sqrt{2}$. Removing the P^m_{12} operator by commuting it with the interaction terms, then letting it operate on ϕ^*_b and finally summing over m, we have

$$I^{l_1 l_2 L}_{\Lambda} = \int\int\int \chi^{(-)*}_b(\vec{r}_b)\phi^*_b(\vec{r}_1, \vec{r}_2, \vec{r}_a)[V_{a1} + V_{a2}]\, g \sum_{\lambda_1\lambda_2}(l_1\lambda_1 l_2\lambda_2|L\Lambda)\phi^{n_1 l_1 j_1}_{\lambda_1}(\vec{r}_1)\phi^{n_2 l_2 j_2}_{\lambda_2}(\vec{r}_2)\chi^{(+)}_{aA}(\vec{r}_{aA}). \tag{6}$$

We focus attention on $I^{l_1 l_2 L}_{\Lambda}$ since it contains the theoretical normalization factor we desire.

The coordinates we have used are $\vec{r}_a$, $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_b$, and $\vec{R}_A$ which are, respectively, the coordinates of the

incident particle, the two picked-up nucleons, the center of mass of the emitted projectile, and the center of mass of the target nucleus A, all referred to the center of mass of the residual nucleus B as origin. ϕ_b^* is the internal wave function of b and $\chi_b^{(-)}$ and $\chi_{aA}^{(+)}$ are the distorted waves in the exit and entrance channels.

If we specialize to the (p, t) reaction for clarity, assume Gaussian forms for the light particle wave function and nuclear force, and define the new variables

$$\vec{R} = \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2), \qquad \vec{p} = \vec{r}_p - \vec{R}, \tag{7}$$

then

$$\phi_b = \phi_t(\vec{r}_1, \vec{r}_2, \vec{r}_p) = N_t e^{-\eta_t^2(r_{12}^2 + r_{1p}^2 + r_{2p}^2)} = \phi^{10}(3\eta_t^2 r_{12}^2)\phi^{10}(4\eta_t^2 p^2), \tag{8}$$

where

$$\phi^{10}(\theta x^2) = \left(\frac{\theta}{\pi}\right)^{3/4} e^{-\theta x^2/2} \tag{9}$$

and

$$V_{p1} = V_0 e^{-k^2 r_{p1}^2} \tag{10}$$

so

$$V_{p1} + V_{p2} = V_0 e^{-k^2(p^2 + r_{12}^2/4)}(e^{-k^2 \vec{p}\cdot\vec{r}_{12}} + e^{k^2 \vec{p}\cdot\vec{r}_{12}}). \tag{11}$$

Two approximations are usually made in regard to the form of the interaction terms in Eq. (11). In the Glendenning or "zero-interaction range" approximation the $\vec{r}_{12}$ dependence is dropped and Eq. (11) is written as

$$V_{p1} + V_{p2} = 2V_0 e^{-k^2 p^2}, \tag{12}$$

while in the Chant-Mangelson approximation

$$V_{p1} + V_{p2} = 2V_0 e^{-k^2(p^2 + r_{12}^2/4)}. \tag{13}$$

The bound-state wave functions $\phi_{\lambda_1}^{n_1 l_1 j_1}(\vec{r}_1)$ and $\phi_{\lambda_2}^{n_2 l_2 j_2}(\vec{r}_2)$ are usually expanded as a series of eigenfunctions of an harmonic-oscillator potential and then transformed into relative and center-of-mass coordinates. Thus, in the Chant-Mangelson approximation

$$I_\Lambda^{l_1 l_2 L} = \sum_{\substack{n_1 n_2 \\ n l \lambda \\ N L' \Lambda'}} C_{n_1} C_{n_2} (l\lambda L' \Lambda' | L\Lambda) g\langle nl, NL' : L | n_1 l_1, n_2 l_2 : L\rangle \times \int\int\int \chi_t^{(-)*}(\vec{r}_t)\phi^{10}(3\eta_t^2 r_{12}^2)\phi^{10}(4\eta_t^2 p^2) 2V_0 e^{-k^2(p^2 + r_{12}^2/4)} \times \phi_\lambda^{nl}(\tfrac{1}{2}\nu r_{12}^2)\phi_{\Lambda'}^{NL'}[2\nu(\vec{r}_t + \rho\vec{p})^2]\chi_p^{(+)}(\gamma\vec{r}_t + \mu\vec{p}) d\vec{r}_t d\vec{p} d\vec{r}_{12}, \tag{14}$$

where ν is the oscillator constant and

$$\vec{R} = \vec{r}_t + \rho\vec{p}, \quad \vec{r}_{pA} = \gamma\vec{r}_t + \mu\vec{p}, \tag{15}$$

$$\rho = -\tfrac{1}{3}, \quad \gamma = \frac{B}{A}, \quad \mu = \frac{2}{3}\frac{A+1}{A}. \tag{16}$$

The integration over $\vec{r}_{12}$ can then be carried out and noting that l must be zero for the integral to be non-zero, we have

$$I_\Lambda^{l_1 l_2 L} = \int\int \chi_t^{(-)*}(\vec{r}_t) F_x(\vec{r}_t + \rho\vec{p})\phi^{10}(4\eta_t^2 p^2) 2V_0 e^{-k^2 p^2} \chi_p^{(+)}(\gamma\vec{r}_t + \mu\vec{p}) d\vec{r}_t d\vec{p}, \tag{17}$$

where

$$F_x(\vec{r}) = \sum A_N \phi_\Lambda^{NL}(2\nu r^2) \tag{18}$$

with

$$A_N = \sum C_{n_1} C_{n_2} \hat{\Omega}_n g \langle n0, NL: L | n_1 l_1, n_2 l_2: L \rangle . \tag{19}$$

$\langle n0, NL: L | n_1 l_1, n_2 l_2: L \rangle$ is the generalized Talmi coefficient. The overlap factor $\hat{\Omega}_n$ is given by

$$\hat{\Omega}_n = \int \phi^{10}(3\eta_t^2 r_{12}^2) e^{-k^2 r_{12}^2/4} \phi^{n0}(\tfrac{1}{2}\nu r_{12}^2) d\vec{r}_{12}$$

$$= \frac{[2(n-1)!]^{1/2}}{2^{n-1}(n-1)!} (cd)^{3/2} (1-c)^{n-1}, \tag{20}$$

where

$$c = \frac{2\nu}{6\eta_t^2 + k^2 + \nu}$$

and

$$d = \left(\frac{6\eta_t^2}{\nu} \right)^{1/2} .$$

It should be realized that the Glendenning formulation results in a factor Ω_n which differs from $\hat{\Omega}_n$ in not having the k^2 term. Finally, using the zero-range approximation, we find

$$I_\Lambda^{l_1 l_2 L}(\text{z.r.}) = D_0(p, t) \int \chi_t^{(-)*}(\vec{r}_t) F_x(\vec{r}_t) \chi_p^{(+)}(\gamma \vec{r}_t) d\vec{r}_t . \tag{21}$$

The terms in $\vec{p}$ in F_x and $\chi_p^{(+)}$ are dropped before the integration over $\vec{p}$. The constant $D_0(p, t)$ is given by

$$D_0(p, t) = 2V_0 \left[\frac{4\pi\eta_t^2}{(2\eta_t^2 + k^2)^2} \right]^{3/4} \tag{22}$$

and is the normalization factor by the direct method.

In the corresponding parametrization method,[10]

$$I_{\Lambda,\text{TH}}^{l_1 l_2 L} = \int\int \chi_t^{(-)*}(\vec{r}_t) F_x(\vec{r}_t + \rho\vec{p}) \left\{ -\frac{\hbar^2}{2m_{pt}} (\beta^2 - \alpha^2) e^{-\beta\rho} \right\} \chi_p^{(+)}(\gamma \vec{r}_t + \mu\vec{p}) d\vec{r}_t d\vec{p} \tag{23}$$

and

$$I_{\Lambda,\text{TH}}^{l_1 l_2 L}(\text{z.r.}) = D_{0,\text{TH}}(p, t) \int \chi_t^{(-)*}(\vec{r}_t) F_x(\vec{r}_t) \chi_p^{(+)}(\gamma \vec{r}_t) d\vec{r}_t , \tag{24}$$

where

$$D_{0,\text{TH}}(p, t) = -\frac{\hbar^2}{m_{pt}} \left\{ 2\pi\alpha \left(1 + \frac{\alpha}{\beta} \right)^3 \right\}^{1/2} . \tag{25}$$

The parameter α is obtained from the separation energy of the clusters a and x while β is determined from the requirement that the low-momemtum components of the a-x interaction are properly fitted.

An oft-used approximation to simulate finite-range effects is the LEA method of Buttle and Goldfarb.[15] The analysis is, in first order, exactly as in the zero-range approximation with a modified normalization factor,[6]

$$D_{\text{LEA}}(p, t) = R D_0(p, t) \tag{26}$$

with

$$R_{\text{TH}} = \left(1 - \frac{\alpha^2}{\beta^2} \right)^{-1} \tag{27}$$

and

$$R_D = \exp\left(\frac{m S_{pt}}{3\hbar^2 (2\eta_t^2 + k^2)} \right), \tag{28}$$

where S_{pt} is the separation energy of p from t.

For (d, α) reactions, the expression for the nuclear matrix $I_\Lambda^{l_1 l_2 L}$ differs from that for (p, t) only by the introduction of a second multiplicative factor, $\hat{\Omega}_d$, defined by

$$\hat{\Omega}_d = \int \phi^{10}(4\eta_\alpha^2 r_{34}^2) e^{-k^2 r_{34}^2/4} \phi^d(\vec{r}_{34}) d\vec{r}_{34} . \tag{29}$$

The deuteron wave function is ϕ^d while $\phi^{10}(4\eta_\alpha^2 r_{34}^2)$ is the r_{34} factor in the α-particle wave function, which is assumed to be of Gaussian form. The normalization and R factors for such TNTR are, for the direct method:

$$D_0(d, \alpha) = 4V_0 \left[\frac{8\pi\eta_\alpha^2}{(4\eta_\alpha^2 + k^2)^2} \right]^{3/4} ; \tag{30}$$

$$R_D = \exp\left(\frac{mS_{d\alpha}}{2\hbar^2(4\eta_\alpha{}^2 + k^2)}\right). \quad (31)$$

The form of these factors in the parametrization method are unchanged except for the replacement of the appropriate (d, α) quantities.

3. RESULTS AND DISCUSSION

In Ref. 7, a common short-range parameter $\beta = 1.36$ fm^{-1} was derived from SNTR and this value was utilized in the computation of the normalization constants labeled TH displayed on Table II.

In the direct method, the results quoted are obtained using the following[6, 16, 17]:

$$V_0 = -62.2 \text{ MeV}, \quad k^2 = 0.379 \text{ fm}^{-2}, \quad (32)$$

$$\phi_d = N_d\{e^{-\eta_1{}^2 r^2} + 0.37e^{-\eta_2{}^2 r^2}\} \quad (33)$$

with

$$\eta_1{}^2 = 0.28 \text{ fm}^{-2}, \quad \eta_2{}^2 = 0.039 \text{ fm}^{-2},$$

$$\eta_t{}^2 = 0.0635 \text{ fm}^{-2}, \quad \eta_\alpha{}^2 = 0.140 \text{ fm}^{-2},$$

and

$$\nu = 0.20 \text{ fm}^{-2}. \quad (34)$$

Our results indicate that the $D_0{}^2$ from the two methods differ by a factor of 6 in TNTR although agreeing in SNTR. The peripheral model of Borbely, Baryshnickov, and Blokhintsev[18, 19] allows the extraction of $D_0{}^2$. Their value for $D_0{}^2(d, \alpha)$ agrees with that from the direct method.

In LEA, the agreement between methods (i) and (ii) is improved for (d, α) reactions. However, the discrepancy remains for (p, t) TNTR. Besides, the improvement in (d, α) is achieved with an uncomfortably large $R_{\text{TH}}{}^2 = 15.2$.

Table III lists the experimental values of the normalization constants for representative SNTR and TNTR. These were extracted from

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{expt}} = D_{\text{expt}}{}^2\left(\frac{d\sigma}{d\Omega}\right)_{\text{DWBA}}, \quad (35)$$

where, in $(d\sigma/d\Omega)_{\text{DWBA}}$, $D_0{}^2$ is taken to be 10^4 MeV^2 fm^3. A direct comparison with our theoretical values cannot be made as yet since these analyses were conducted using the Glendenning formulation, i.e., Ω_n and Ω_d were used rather than $\hat{\Omega}_n$ and $\hat{\Omega}_d$. As the overlap factors Ω_n and $\hat{\Omega}_n$ are dependent upon n, we have defined the factors

$$K_n = \frac{\hat{\Omega}_n}{\Omega_n} \quad (36)$$

and

$$K_d = \frac{\hat{\Omega}_d}{\Omega_d} = \begin{cases} 1 & [\text{for } (p, t)] \\ 0.67 & [\text{for } (d, \alpha)] \end{cases}. \quad (37)$$

We have inserted in a separate column on Table III the values of $D'_{\text{expt}}{}^2$ where

$$D'_{\text{expt}}{}^2 = \frac{D_{\text{expt}}{}^2}{(K_n K_d)^2}. \quad (38)$$

We have arbitrarily assumed

$$K'_n = \frac{4K_1\Omega_1 + K_2\Omega_2}{4\Omega_1 + \Omega_2} = \begin{cases} 0.5 \text{ [for } (p, t)] \\ 0.7 \text{ [for } (d, \alpha)] \end{cases} \quad (39)$$

as a suitable average value of K_n. The weights used in its definition are based on a "guesstimate" of the relative contributions of Ω_1 and Ω_2 terms in

TABLE II. SNTR and TNTR normalization factors and the finite-range constant.

Reaction	Method	R^2	$D_0{}^2$	$D_{\text{LEA}}{}^2$
(d, t)	TH	1.26	2.59	3.3
	Direct	1.40	2.88	4.0
	Peripheral		5.0	
$(^3\text{He}, \alpha)$	TH	2.6	7.3	19
	Direct	2.9	12.8	37
	Peripheral		26	
(p, t)	TH	1.39	1.51	2.1
	Direct	1.31	8.5	11.3
(d, α)	TH	15.2	7.6	115
	Direct	2.36	50.5	120
	Peripheral		103	

TABLE III. Experimental values of the normalization constants for SNTR and TNTR.

Reaction	$D_{\text{expt}}{}^2$	$D'_{\text{expt}}{}^2$
(d, t), (t, d), $(^3\text{He}, d)$, $(d, {}^3\text{He})$ [a]	3.3	
$(^3\text{He}, \alpha)$, $(\alpha, {}^3\text{He})$, (t, α), (α, t) [a,b]	28	
Zr(p, t) [c]	22	88
Ca, Sn, Pb(p, t) [d]	39	156
Ca, Sn, Pb(t, p) [e]	22.5	90
Bi$(p, {}^3\text{He})$ [f]	12.9	52
P$(^3\text{He}, p)$ [g]	16	64
Cl$(^3\text{He}, p)$ [g]	20	80
Bi(d, α) [f]	120	480
Pb(d, α) [h]	102	408

[a] Reference 6.
[b] Reference 7.
[c] Reference 3.
[d] Reference 4.
[e] E. R. Flynn *et al.*, Nucl. Phys. A195, 97 (1972).
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the angular distributions.

On comparing our constants with D'^2_{expt}, it is clear that $D_0^2(p,t)$ and $D_{\text{LEA}}^2(p,t)$ are an order of magnitude too small and that $D_{0,\text{TH}}^2(p,t)$ and $D_{\text{LEA,TH}}^2(p,t)$ are even worse off. The comparison with $D^2(d,\alpha)$ values is better but our computed direct-method numbers are still short by a factor of 4.

Within the limitations of the approximations made in the direct method, the first place to look for a possible resolution of the disagreement is in the retention of the dot term in the interaction. If this is carried out specifically for the (p,t) reactions, the direct method gives[11]

$$D_0^{FI} = \frac{D_0}{\{1 - k^4/f^2(k^2 + 2\eta_t^2)\}^{3/2}}$$

with

$$f^2 = k^2 + 6\eta_t^2 + \nu .$$

The (p,t) normalization constant is now off by a factor of 3–4. It is interesting to note[20] that an elaborate exact calculation of the angular distribution for (p,t) reactions is also deficient by a factor of 3. For (d,α) a corresponding improvement would lead to better agreement between theory and experiment.

To explain these developments, it should be pointed out that the normalization factor depends upon many computational parameters used in the direct-reaction analysis. These parameters, such as are in the optical potentials, the form factors, the nuclear wave functions, and the transfer interaction, can quite easily affect our conclusions. On the other hand, if the theoretical factor and the DWBA analysis are correct, the discrepancy suggests that the parameters now being used must be changed.

Finally, it appears clear that the parametrization method is unreliable for TNTR with the present prescription of selecting the values of α and β. Kok and Rinat[8] have criticized the uniform parametrization of the vertex functions, maintaining that their values of β from scattering data and dispersion relations deviate significantly from the constant of Thompson and Hering. We have varied the value of β, without improving matters. It seems proper to ask whether the restriction of relating α to the separation energy should not be dropped. Variation in both α and β may provide the stability and agreement we require.

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