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Anomalous Parity States in $g_{9/2}$-Shell Nuclei and the Thankappan-True Core-Coupling Hamiltonian

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It is shown that the observed features of the anomalous parity states in $g_{9/2}$-shell odd-mass nuclei can be qualitatively explained if core-quasiparticle coupling interactions of the type $-\lambda\vec{J}_c\cdot\vec{j}_{qp}$ and $-\chi\vec{Q}_c\cdot\vec{Q}_{qp}$ are taken into account.

The highly successful model involving quasiparticle-phonon coupling[1] has not worked well in the mass region of $A=60-110$, with a particular lack of success in explaining those $g_{9/2}$ nuclei with $41 \leqslant (N \text{ or } Z) \leqslant 49$. For example, it has not been possible to explain the behavior of the low-lying $\frac{5}{2}^+$ and $\frac{7}{2}^+$ "anomalous parity" states on this basis. The purpose of the present paper is to show that the trends observed in recent experiments[2-5] for the $g_{9/2}$ anomalous parity states can be understood in terms of an extension of the quasiparticle-phonon core-coupling model following the suggestion of Thankappan and True.[6]

According to the shell model, the $g_{9/2}$ neutron and proton odd-mass nuclei should have a $\frac{9}{2}^+$ quasiparticle state as either the ground or as a low-lying excited level and this is borne out by the data. The problem of the anomalous parity states started with the observation of a low-energy $\frac{7}{2}^+$ level in these nuclei which cannot be a $g_{7/2}$ quasiparticle state, as this state should appear a few MeV higher in energy. In several nuclei, a second $\frac{7}{2}^+$ state has been found with $E_{\text{ex}} \approx 500$ keV. An even more complicated behavior is observed for the $\frac{5}{2}^+$ level which occurs at low excitation energies at the beginning of the $g_{9/2}$ shell and moves towards higher excitation energies as the shell fills. Finally, it should be noted that higher angular momentum states have not been found so far at excitation energies below 600 keV, which is a crucial point for the discussion below.

The first quasiparticle-phonon coupling calculations[1] (QPC) were completely unable to explain these phenomena. These calculations were then improved by the inclusion of backward (quasihole-phonon) coupling[7] (EQPC) and by the extension of this latter approach to include the effect of the static moment of the quadrupole phonon.[8] The improved calculations predict four low-energy (200-600-keV) phonon levels of spin and parity $\frac{5}{2}^+$, $\frac{7}{2}^+$, $\frac{11}{2}^+$, and $\frac{13}{2}^+$, with the $\frac{9}{2}^+$ level of the multiplet pushed up from the interaction with the $\frac{9}{2}^+$ quasiparticle. The low-energy high-spin states have been searched for experimentally, so far without success. The levels of ^{111}Ag were populated[3] by β decay of ^{111m}Pd ($J^\pi=\frac{11}{2}^-$), and those of ^{73}Ge and ^{99}Tc by Coulomb excitation,[4,5] all with negative results insofar as low-lying $\frac{11}{2}^+$ or $\frac{13}{2}^+$ states are concerned. These three spectra are summarized in Fig. 1 along with the predictions of EQPC for the positive-parity states and the extension of EQPC to include the effect of the quadrupole moment. The QPC prediction would have given all five levels of the multiplet at about 900 keV.[9] The inclusion of the $E2$ static moment can help explain the movement of the $\frac{5}{2}^+$ level with shell filling.

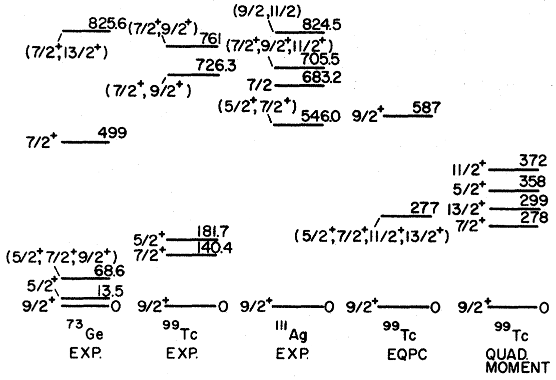

FIG. 1. Comparison of the low-energy positive-parity states of ^{73}Ge, ^{99}Tc, and ^{111}Ag with two different theoretical predictions. Further discussion and references for the theoretical spectra are given in the text.

However, it still cannot predict the $\frac{5}{2}^+$ state at low energies as is often observed. Also, at least one high-spin state is always predicted to be at low energy, in disagreement with the experimental data.

We suggest that many of the theoretical difficulties can be overcome by the addition of a dipole-dipole interaction term $-\lambda\vec{J}_c\cdot\vec{j}_{qp}$ to supplement the static-moment interaction term (which is equivalent to adding a term of the form $-\chi\vec{Q}_c\cdot\vec{Q}_{qp}$, where the subscripts c and qp refer to core phonon and quasiparticle, respectively). In effect we are postulating the quasiparticle extension of the core-coupling interaction of Thankappan and True,[6]

$$H_{\text{int}} = -\lambda\vec{J}_c\cdot\vec{j}_{qp} - \chi\vec{Q}_c\cdot\vec{Q}_{qp}\,. \tag{1}$$

For the single j-shell case, the quasiparticle operators are,

$$\begin{aligned}(Q_{qp})_\mu &= \frac{1}{\sqrt{5}}\langle j\|Q\|j\rangle \sum_{mm'}(-1)^{j-m'}(jmj-m'|2\mu)(1-2V_j{}^2)a^\dagger_{jm}a_{jm'}\\ &\qquad +(\text{terms containing } a^\dagger_{jm}a^\dagger_{jm'} \text{ and } a_{jm}a_{jm'})\,;\\ (j_{qp})_\mu &= \frac{1}{\sqrt{3}}\langle j\|J\|j\rangle\sum_{mm'}(-1)^{j-m'}(jmj-m'|1\mu)a^\dagger_{jm}a_{jm'}\,.\end{aligned} \tag{2}$$

The justification for using only one quasiparticle level j (namely $g_{9/2}$) is as follows. First, the $g_{9/2}$ level is well separated from the other positive-parity quasiparticle levels (e.g., $g_{7/2}$ or $d_{5/2}$) by the well-known spin-orbit interaction. Also, the quadrupole-phonon states based on the $g_{9/2}$ level are of positive parity and thus do not admix with the phonon states based on the negative-parity quasiparticle levels in this region. Thus, as long as we confine ourselves to the discussion of these "phonon" levels based on the $g_{9/2}$ state *relative to their unperturbed position,* the single j model should be adequate. The reduced matrix elements follow the notation of de Shalit and Talmi.[10] The expectation value of H_{int} for the quasiparticle-phonon states under consideration is then easily evaluated as

$$E_{\text{int}} = -\tfrac{1}{2}\lambda[I(I+1)-30.75] - \chi\{[I(I+1)-30.75]^2 + [I(I+1)-30.75] - 198\}\left(-\frac{5.4b^2}{2578.3}\langle 2\|Q^2\|2\rangle(1-2V_{g_{9/2}}{}^2)\right). \tag{3}$$

In this expression,

$$\langle 2\|Q^2\|2\rangle = \frac{5}{(16\pi)^{1/2}}\frac{Q_{\text{exp}}}{(2220|22)} = 1.32Q_{\text{exp}}\,,$$

b^2 is the usual harmonic-oscillator wave-function parameter and is given as $0.01A^{1/3}$ b, and I is the angular momentum of the phonon coupled state.

The second term in Eq. (1) for the interaction Hamiltonian can be shown to originate from the $\frac{1}{2}\chi Q\cdot Q$ interaction in a microscopic theory.[7] This identifies the parameter χ as the strength of the quadrupole-quadrupole force. A reasonable estimate is[11]

$$\chi b^4 = \frac{240}{A^{5/3}}\,. \tag{4}$$

The experimental data on the anomalous parity states in the $g^{9/2}$ shell suggest that λ should be negative. In order to predict no low-lying high-spin states for ^{73}Ge, ^{99}Tc, and ^{111}Ag, a value of λ near -0.025 MeV is required. In their calculations for ^{63}Cu, Thankappan and True[6] chose $\lambda > 0$. This change of sign of λ may be correlated with the quasirotational character of the nuclei under consideration.

The occupation probabilities, $V_{g_{9/2}}{}^2$, can be estimated in this region from data obtained by studies of single-nucleon-transfer reactions. In particular, the (d,p) reaction on isotopes of Ge and Se gives some information about the neutron single-particle state-occupation probabilities.[12] The occupation probabilities for the proton $g_{9/2}$ shell were obtained by assuming a behavior similar to the neutrons from beginning to end of the shell.

The results of the numerical calculations for the proton $g_{9/2}$ region are presented in Fig. 2(a) as a function of the occupation probability, $V_{g_{9/2}}{}^2$. An experimental quadrupole moment of -0.6 b was assumed and an A_{av} of 105 was used. The experimental data for ^{99}Tc, ^{105}Rh, and ^{111}Ag are presented as corresponding to $V_{g_{9/2}}{}^2 = 0.3$, 0.5, and 0.7, respectively. In Fig. 2(b) we present the results of a similar calculation for the neutron $g_{9/2}$ region. An A_{av} of 78 and $Q_{\text{exp}} = -0.4$ b were assumed. The experimental data for ^{73}Ge and ^{81}Se are presented as corresponding to $V_{g_{9/2}}{}^2 = 0.3$ and 0.8.

Note that both the movement of the $\frac{5}{2}^+$ level with shell filling and its appearance in some nuclei at low excitation energy is explained. The energy of the $\frac{7}{2}^+$ state is nearly flat as a function of the $g_{9/2}$ occupation probability. The higher-spin levels are now predicted to occur at higher energies, in accordance with the data available. The prediction for the $\frac{9}{2}^+$ state is expected to be inadequate, since it should be pushed up in energy due to the repulsive effect of the interaction with the $\frac{9}{2}^+$ quasiparticle state.

The experimental data for the $g_{9/2}$-shell nuclei often reveal at least two $\frac{7}{2}^+$ states. We suggest that the lower of these is to be identified with the phonon states described above. The other $\frac{7}{2}^+$ state usually lies considerably higher in energy with $E_{\text{ex}} \approx 500$ keV. Kisslinger[13] and more recently Juriyama, Marumori, and Matsuyanagi[14] have suggested that $\frac{7}{2}^+$, low-energy states in this mass region may be due to the $(j-1)$-seniority three state of the $(g_{9/2})^3$ configuration. This could be the explanation of the higher $\frac{7}{2}^+$ state.

Some further comments about the modified quasiparticle-phonon scheme described herein are in order. The recent Coulomb-excitation experiment[5] on ^{73}Ge supports the validity of the proposed model. As shown in Fig. 2(b), the calculation for ^{73}Ge with $Q_{\text{exp}} = -0.4$ b and $\lambda = -0.025$ MeV correctly predicts the over-all features of the positive-parity states below 900 keV. Just as important is the confirmation of the coupling scheme provided by the measured upwards reduced transition probabilities, $B(E2)\uparrow$. The measured values to the four excited states of the band are all about one half of the measured value for the 0^+ to 2^+ transition in the neighboring even isotopes of Ge.

A major question arises as to how λ should change with the addition of pairs of inactive particles (e.g., adding pairs of neutrons in odd-proton nuclei). We cannot treat this question in detail here. But we do note that for the Ag isotopes, severe changes of λ may be indicated. The data for the Tc and Rh isotopes, however, do not seem to indicate any serious change in λ. We believe that the value of λ which is required is an indication of the quasirotational nature of the core, with

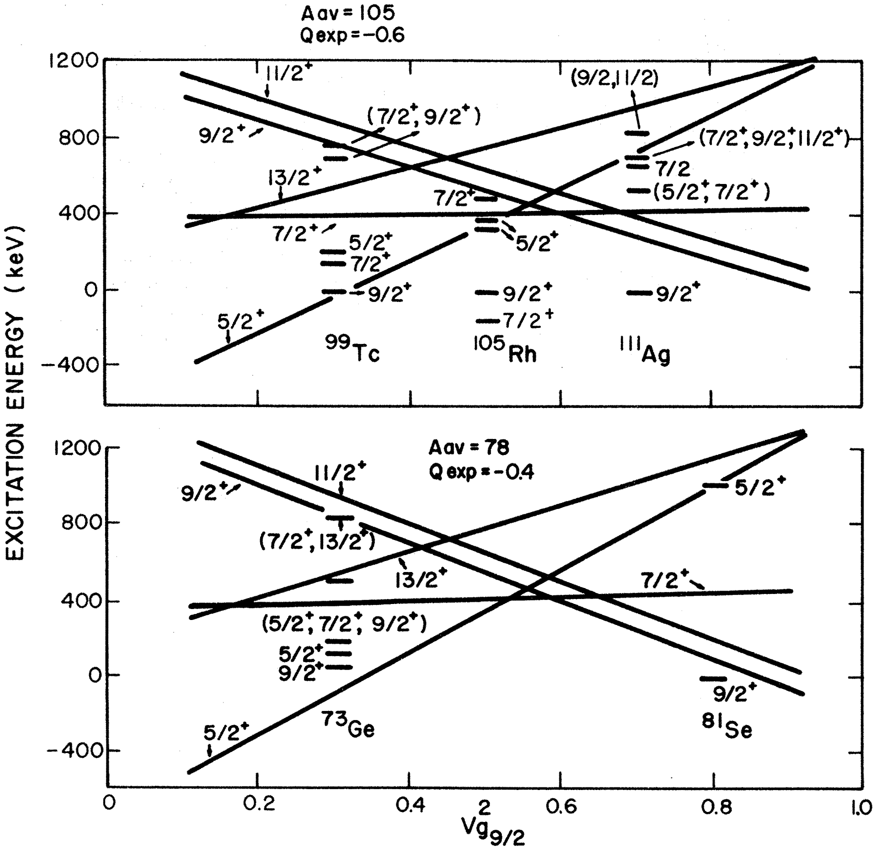

FIG. 2. Results of the calculation of the energy of $g_{9/2}$ nuclei as a function of the occupation probability, $V_{g_{9/2}}{}^2$. A core phonon energy of 600 keV was assumed for both the neutron and proton cases. Placing the unperturbed $g_{9/2}$ quasiparticle at zero energy puts the unperturbed phonon level at 600 keV. A value of $\lambda = -0.025$ was chosen as described in the text. The upper half of the figure corresponds to proton $g_{9/2}$ nuclei, while the lower half is for neutron $g_{9/2}$ nuclei. The limited data available for ^{77}Se and ^{79}Se show that the $\frac{5}{2}^+$ states for these nuclei are in agreement with the predictions of Eq. (3).

a negative value indicating a good quasirotational core. Obviously, this aspect will require a detailed and systematic theoretical and experimental investigation.

To conclude, we have shown that the phonon-quasiparticle coupling scheme can give an adequate explanation of the anomalous parity states in $g_{9/2}$ nuclei provided that a core-coupling interaction of the form given in Eq. (1) is added.

It must be cautioned that the above is essentially a first-order perturbation treatment. For a detailed comparison between theory and experiment, there are several obvious improvements which can be made. These include treating the effect of forward[1] and backward[7] quasiparticle-phonon coupling, adding in backward coupling through the static-phonon-quadrupole moment,[8] coupling also to $g_{7/2}$ and $2d_{5/2}$ quasiparticles, and making the entire theory a self-consistent treatment.

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