Phys. Rev. C 3, 1271 (1971).
[14]J. Borggreen, G. Løvhøjden, and J. C. Waddington, Nucl. Phys. A131, 241 (1969).
[15]R. D. Rathmell and W. Haeberli, in *Proceedings of the Third Polarization Symposium, Madison, 1970* (see Ref. 5), p. 791.
[16]J. R. Erskine, private communication.
[17]C. Samour, J. Julien, R. N. Alves, S. De Barros, and J. Morgenstern, Nucl. Phys. A123, 581 (1969).
[18]R. R. Spencer and K. T. Faler, Phys. Rev. 155, 1368 (1967).
[19]H. H. Bolotin and D. A. McClure, Argonne National Laboratory Report No. ANL-7620, 1969 (unpublished).
[20]M. Beer, R. Chrien, and O. A. Wasson, private communication.
[21]R. F. Casten and W. R. Kane, Bull. Am. Phys. Soc. 17, 17 (1972); and to be published.
[22]C. W. Reich and R. C. Greenwood, U. S. AEC Report No. ANCR-1016, 1971 (unpublished), p. 54; and C. W. Reich, private communication.
[23]F. A. Rickey, Jr., and R. K. Sheline, Phys. Rev. Letters 17, 1057 (1966); D. L. Allan, H. C. Britt, and F. A. Rickey, Jr., Phys. Letters 27B, 11 (1968).
[24]T. Udagawa, T. Tamura, G. Igo, J. Chai, and R. F. Casten, in Proceedings of the Argonne Conference on Two-nucleon Transfer Reactions, March, 1972 (unpublished); and Nucl. Phys. (to be published).

PHYSICAL REVIEW C VOLUME 7, NUMBER 3 MARCH 1973

Electron Scattering from ^{51}V

G. A. Peterson*
Department of Physics, University of Massachusetts, Amherst, Massachusetts 01002

and

K. Hosoyama, M. Nagao, A. Nakada, and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 4 October 1972)

Form factors for the excitation of the ^{51}V 0.32-, 0.93-, 1.61-, 1.81-, and 2.70-MeV members of the $(1f_{7/2})^3$ multiplet, and of the 2.40- and 3.91-MeV states have been measured for momentum transfers up to 1.8 fm^{-1} by using 183- and 250-MeV electrons. Comparisons have been made with form factors calculated by using harmonic-oscillator shell-model wave functions. For the $(1f_{7/2})^3$ multiplet, effective charges from 1.79e to 2.01e were obtained for the $E2$ components of the excitations, and 1.69e for the $E4$ component of the 2.70-MeV excitation. Higher multipole components of each excitation required smaller effective charges in the strict $(1f_{7/2})^3$ configuration. An analysis of the elastic scattering data using a Fermi charge distribution yielded the parameters $c = 3.94 \pm 0.03$ fm, $t = 2.22 \pm 0.06$ fm, and 3.58 ± 0.04 fm for the rms radius.

I. INTRODUCTION

Deeply penetrating high-energy electrons are excellent probes of nuclear structure. The electrons interact with the nucleons only via the electromagnetic force, and the cross sections for the scattering of the electrons can be related directly to the reduced matrix elements of the charge and current-density operators between the initial and final nuclear states. Detailed radial information is provided which is very difficult to obtain by other means.

We have used high-energy electron scattering to investigate some of the low-lying levels of $^{51}_{23}V_{28}$. This nucleus is thought to be well described in terms of the shell-model $(1f_{7/2})^3$ proton configuration and a closed $1f_{7/2}$-shell neutron configuration. The energy levels can be calculated closely from the two-body $(1f_{7/2})^2$ matrix elements of the effective nuclear interaction taken from neighboring nuclei, such as ^{50}Ti, and the spins and parities are correctly predicted.[1,2] However, if $E2$ transition rates are calculated by assuming that only the valence protons take part, it is found that the experimental values are larger, as is the case for most, if not all nuclei. It is customary to evoke the concept of "effective charge" to explain the observed enhanced transition probabilities. Each valence proton is assumed to have a charge larger than that of a free proton in order to account for polarization of the core by the valence protons, or from a microscopic viewpoint, the effective charge accounts for virtual excitations of particle-hole core states which admix with the independent-particle states. The use of effective charge permits tractable calculations to be made by using simple shell-model descriptions with valence particles.

Although several multipolarities could be involved for each ^{51}V transition within the $(1f_{7/2})^3$ configuration, other studies[2,3] have obtained information only about the $E2$ components. The high-energy electrons used in this experiment gave sufficient momentum transfer to excite $E4$ and $E6$ multipoles, as well as $E2$.

The results were compared with harmonic-oscillator shell-model calculations in order to deduce the $E2$ effective charges, e_2^*, and also in the case of the excitation of the 2.70-MeV ($\frac{15}{2}^-$) state, to deduce a value of the $E4$ effective charge, e_4^*. However, if this e_4^* value is used for other excitations, and if e_6^* is assumed to be approximately the same as e_4^*, a large discrepancy exists between the experimental results and the calculations.

From an analysis of the elastic scattering data, ground-state charge-distribution parameters were obtained. Also the form factors for the excitation of the states at 2.40 and 3.91 MeV have been compared with harmonic-oscillator shell-model calculations.

II. EXPERIMENTAL METHOD

The experiment was carried out at the Tohoku University 300-MeV Electron Linear Accelerator Laboratory. The magnetic deflection system, the spectrometer, and other apparatus have been described before,[4] as well as the 33-channel solid-state detection system,[5] and we forego further description here.

Beams of 183- and 250-MeV electrons were used to bombard a chemically pure ^{51}V target of thickness 99.3 mg/cm^2. ^{51}V is naturally 99.75% isotopically abundant. Experimental runs taken with, and without, the target vibrating in the beam, showed that the target was uniform in thickness to better than 1%. Differential cross sections were measured relative to the ^{12}C elastic cross section, or to the ^{12}C 4.43-MeV inelastic cross section, whichever was larger for the particular kinematic conditions. A 106.7-mg/cm^2 graphite target was used. The ^{12}C form factors of Sick and McCarthy[6] were checked and used.

A total resolution after scattering of $\Delta p/p \approx 1/10^3$ was sufficiently high enough so that the first excited state at 0.32 MeV could be resolved from the much larger elastic scattering peak, except at the most forward angles where the elastic scattering was overwhelmingly larger. Also it was possible to separate the 1.61-MeV peak from the 1.81-MeV peak by using a peak-unfolding program.[7] An early experiment[8] on ^{51}V similar to this one

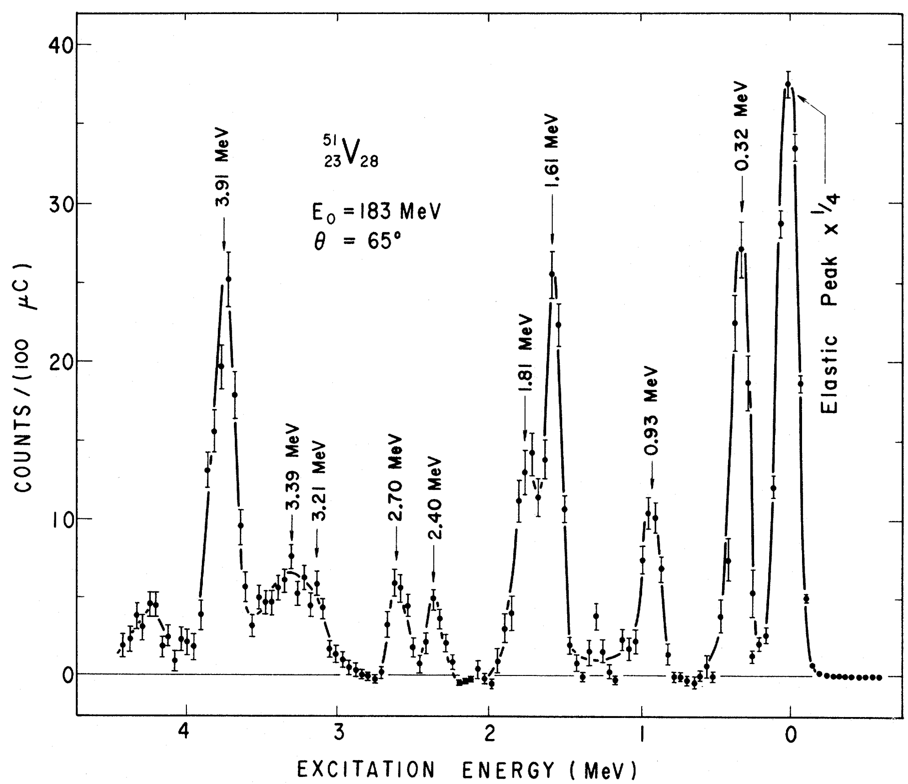

FIG. 1. Radiation corrected spectrum of electrons of initial energy 183 MeV after scattering through an angle of 65° from ^{51}V.

was greatly hampered by lack of adequate resolution. A spectrum of scattered electrons after radiative corrections[6,9] is shown in Fig. 1. The peaks at 2.70 and 3.91 MeV appeared to rise as single peaks with no evidence of broadening due to excitation of nearby levels.

By observing scattered electrons between $\theta = 35$ and 85° for an incident energy $E_0 = 183$ MeV, and between $\theta = 65$ and 90° for $E_0 = 250$ MeV, momentum transfers q from 0.56 to 1.8 fm^{-1} were obtained, where q is defined by

$$q = (2E_0/\hbar c)(\sin\tfrac{1}{2}\theta)(1-\epsilon/E_0)^{1/2} \tag{1}$$

for a nuclear level at an excitation energy ϵ.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A. Elastic Scattering

Figure 2 shows the square of the form factor $|F|^2$, as a function of q for elastic scattering. $|F|^2$ is defined by the relations

$$\frac{d\sigma}{d\Omega} = \sigma_{\mathrm{Mott}}\,|F|^2, \tag{2}$$

and

$$\sigma_{\mathrm{Mott}} = (Ze^2/2E_0)^2(\cos^2\tfrac{1}{2}\theta/\sin^4\tfrac{1}{2}\theta) \times\left(1-\frac{2E_0}{Mc^2}\sin^2\tfrac{1}{2}\theta\right)^{-1/2}, \tag{3}$$

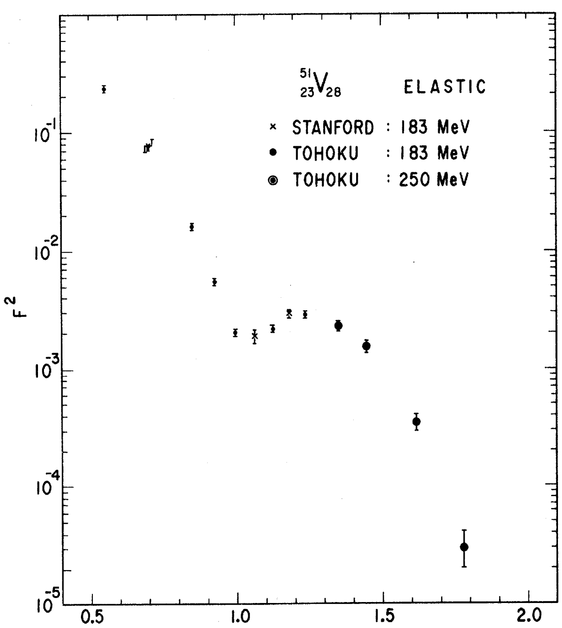

FIG. 2. The square of the form factor for elastic scattering from ^{51}V versus momentum transfer.

where $d\sigma/d\Omega$ is the cross section for elastic scattering, and σ_{Mott} is the Mott cross section for the scattering through an angle θ of an electron of energy E_0 from a point spinless nucleus of mass M. The four highest q points were obtained at 250 MeV and the rest at 183 MeV. It can be seen that there is good agreement between this experiment and the Stanford results.[10] The error bars shown include systematic errors of 1% for target thickness uncertainty and 2% for counter efficiency corrections, both of which were added linearly to the statistical errors.

The points shown in Fig. 2 include contributions from both charge and magnetic scattering. The magnetic contributions estimated[11] in the plane-wave Born approximation by using a single-particle shell model are shown in Fig. 3 for the $M1$, $M3$, and $M5$ multipoles only. The one experimental point shown was obtained by separating the longitudinal and transverse components[12] at a constant effective momentum transfer[13] of 1.52 fm^{-1} by running at 183 MeV and 70°, and at 151.2 MeV and 135°. Typically the magnetic contributions were less than a few percent except for the highest q point where they were larger than the Coulomb contributions.

After the magnetic contributions were subtracted,

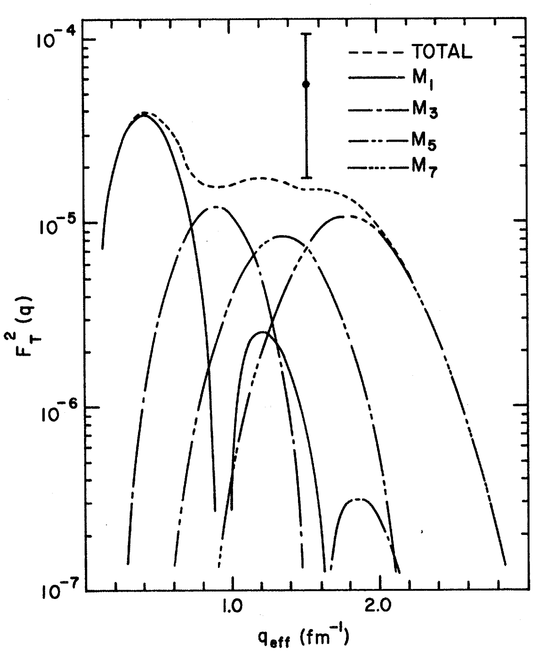

FIG. 3. The square of the form factor for elastic magnetic scattering from ^{51}V calculated by using a single-particle shell model in the plane-wave Born approximation versus the effective momentum transfer. Only $M1$, $M3$, and $M5$ components are shown.

TABLE I. Fermi distribution parameters in fm.

	Electron scattering				Muonic x rays
	This work	Refs. 14,15	Ref. 16	Ref. 17	Ref. 18
c	3.94 ± 0.03	3.98	3.90 ± 0.05		
t	2.22 ± 0.06	2.2	2.38 ± 0.16		
$(\langle r^2\rangle)^{1/2}$	3.58 ± 0.04	3.60	3.632 ± 0.036	3.62 ± 0.09	3.63 ± 0.06

a distorted-wave calculation showed that a two-parameter Fermi charge distribution with half-density radius $c = 3.94 \pm 0.03$ fm and skin thickness $t = 2.22 \pm 0.06$ fm gave a good fit to the remaining Coulomb form factors. The value for t is slightly smaller than the value[14] t' for ^{48}Ca of 2.31 fm, where t' is a three-parameter Fermi-distribution parameter. The third parameter $w = -0.03$ is small for ^{48}Ca, so $t \approx t'$. The ^{51}V parameters yield an rms radius of 3.58 ± 0.04 fm where 55% of the error was estimated to come from uncertainties in the subtraction of magnetic contributions and in the ^{12}C normalization form factors. Our results are in agreement with early results from Stanford,[10,15] more recent results from Amsterdam[16] and Darmstadt,[17] and with muonic x-ray results,[18] as indicated in Table I. Our value of the rms radius yields a harmonic-oscillator length parameter $b = 2.00 \pm 0.03$ fm. Before a more accurate charge distribution for ^{51}V can be determined, careful work to determine the magnetic contributions over a large range of q is needed.

B. Excitation of $(1f_{7/2})^3$ States

Figure 4 shows an energy level diagram for ^{51}V taken from the recent results of Goodman and Donahue.[19] It has been known for many years[1,2]

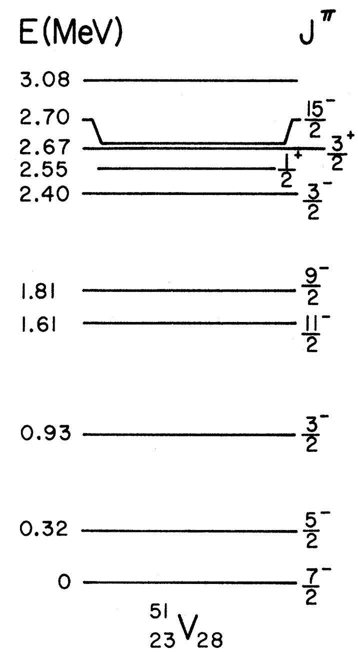

FIG. 4. Nuclear level scheme of ^{51}V taken mainly from the results of Ref. 19.

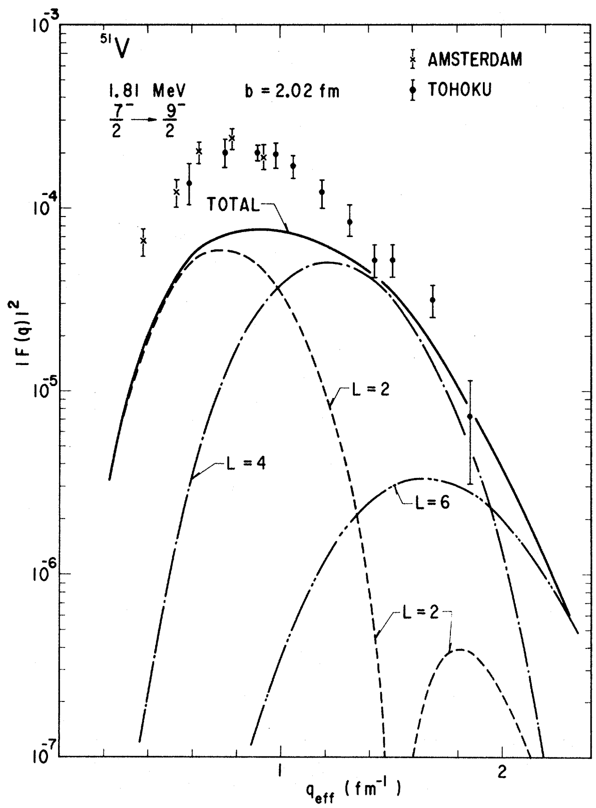

FIG. 5. The square of the form factor for the excitation of the 1.81-MeV level of ^{51}V by inelastic electron scattering versus effective momentum transfer. The calculated form factors are for a $(1f_{7/2})^3$ configuration of protons of unit charge in a harmonic-oscillator potential.

that the ground state, the first four excited states, and possibly the state at 2.70 MeV, can be described by the shell model in which it is assumed that the three $1f_{7/2}$ valence protons of ^{51}V move in the potential well of the doubly magic ^{48}Ca core. These six states are the only ones allowed in this $(1f_{7/2})^3$ configuration because of the necessity of antisymmetric wave functions enforced through fractional parentage coupling of a $1f_{7/2}$ proton to pairs of $1f_{7/2}$ protons in states with $J=0$, 2, 4, and 6. The other levels shown in Fig. 4 presumably involve other configurations and core excitations.

Figure 5 shows experimental form factors for excitation of the 1.81-MeV state where $|F|^2$ for inelastic scattering is defined by Eqs. (2) and (3) if M is set equal to infinity. There is good agreement between these results and the Amsterdam results.[3] The calculated form factors shown were obtained from the expression[12]

$$|F|^2=\frac{4\pi}{Z^2}(2L+1)\left|\int \rho_{\rm tr}\, j_L(qr)\, r^2 dr\right|^2, \qquad (4)$$

where L is the multipolarity, r is the nuclear radial coordinate, $j_L(qr)$ is a spherical Bessel function of order L, and $\rho_{\rm tr}$ is a transition charge density given by

$$\rho_{\rm tr}=eK(L,j,l,J)|\langle f_{7/2}|r^2 Y_{L,M}(\theta,\phi)|f_{7/2}\rangle|^2. \qquad (5)$$

$K(L,j,l,J)$ includes[8] Racah and fractional parentage coefficients for $j=\frac{7}{2}$, $l=3$, and $J=\frac{9}{2}$. The second factor involves only $1f_{7/2}$ wave functions and is the same for all transitions within the $1f_{7/2}$ shell of the same multipolarity. These plane-wave Born-approximation form factors were calculated for various values of the harmonic-oscillator length parameter b. The value $b=2.02\pm0.06$ fm gave good low-q fits to all of the inelastic data, and is nearly the same as the ground-state value. The calculated form factors have $L=2$, 4, and 6 electric multipole components, the only

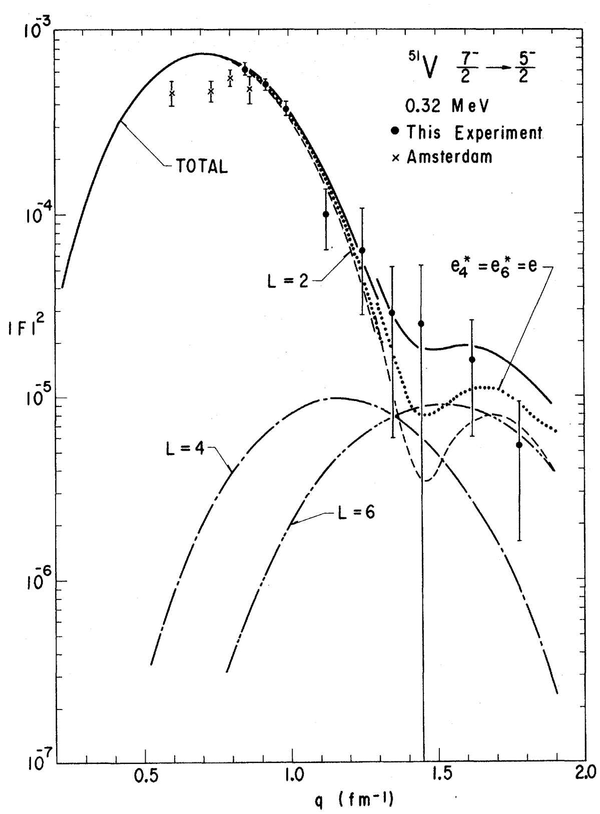

FIG. 6. The square of the form factor for the excitation of the 0.32-MeV level of ^{51}V by inelastic electron scattering versus momentum transfer. The calculated form factors are for a $(1f_{7/2})^3$ configuration of protons in a harmonic-oscillator potential. The curves for the $L=2$, $L=4$, and $L=6$ multipoles and their total were calculated for an effective charge of 1.84e. The curve labeled $e_4^*=e_6^*=e$ is the sum of the form factors assuming the effective charge is 1.84e, e, and e for the $L=2$, 4, and 6 multipoles, respectively.

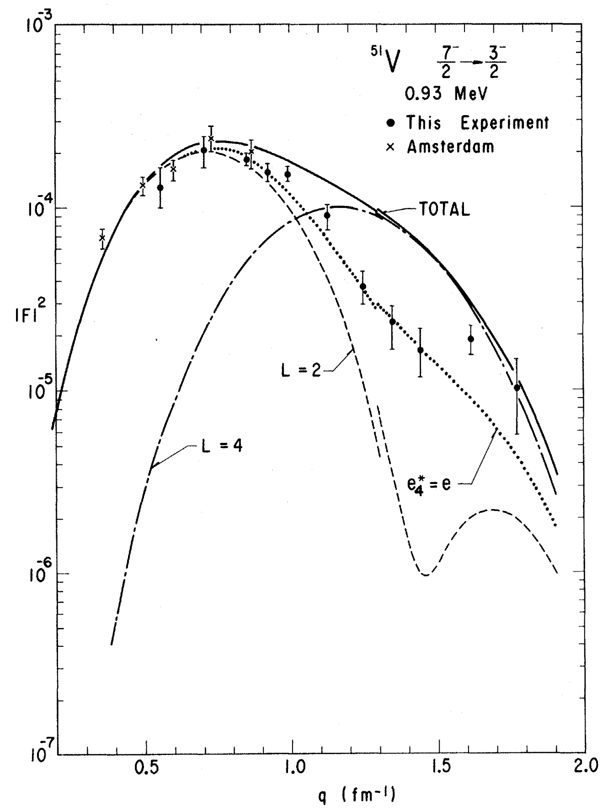

FIG. 7. Form factors for the 0.93-MeV excitation. See the caption to Fig. 6. The total curve was calculated assuming an effective charge of 2.01e.

ones allowed for a single-particle electron scattering operator acting on a $1f_{7/2}$ proton. The transitions are expected to be of a Coulomb character, as magnetic transitions involving spin-flips are not allowed if the proton remains in the $(1f_{7/2})^3$ configuration. A run at two different angles but at the same q, as mentioned in Sec. III A, is consistent with this, although other methods have detected small $M1$ components in ground-state transitions from the 0.32- and the 1.81-MeV states.[19]

The curves of Fig. 5 were calculated using the plane-wave Born approximation. For the incident electron energies of this experiment the Born approximation is fairly accurate and Coulomb-distortion corrections to the incoming and outgoing electron waves are not large for ^{51}V. Nevertheless, a distorted-wave calculation for the $E2$ and $E4$ components was carried out for all $(1f_{7/2})^3$ states by using the distorted-wave Born-approximation (DWBA) code written by Tuan, Wright, and Onley.[20] A transition charge density[21] of the form

$$\rho_{tr} = r^L (p_1 + p_2 r^2 + p_3 r^4) e^{-r^2/(b^2 + g^2)} \qquad (6)$$

was used, where $g = 0.427$ fm accounts for finite proton size and recoil, and the parameter p_1, p_2, and p_3 are obtained to agree with Eq. (5). This form of ρ_{tr} simplifies the DWBA computations. The DWBA corrections were not large except near Born zeroes.

Clearly the experimental points in Fig. 5 lie higher than the curves which were calculated assuming that each proton had a unit charge e. In order to obtain agreement between experiment and calculation, it is necessary to assume that the proton had an effective charge larger than e in order to account for virtual excitations of the core by the valence nucleons. It is expected that the effective charge for a particular multipole should be the same for all low-lying levels within a particular configuration, but that different multipoles may have different effective charges.[22]

Figure 6 shows the results of a DWBA calculation for the excitation of the 0.32-MeV state assuming an effective charge $e^* = (1.84 \pm 0.07)e$ for all multipoles, where the error was estimated from the quality of the fit to the low-q points. The breaks

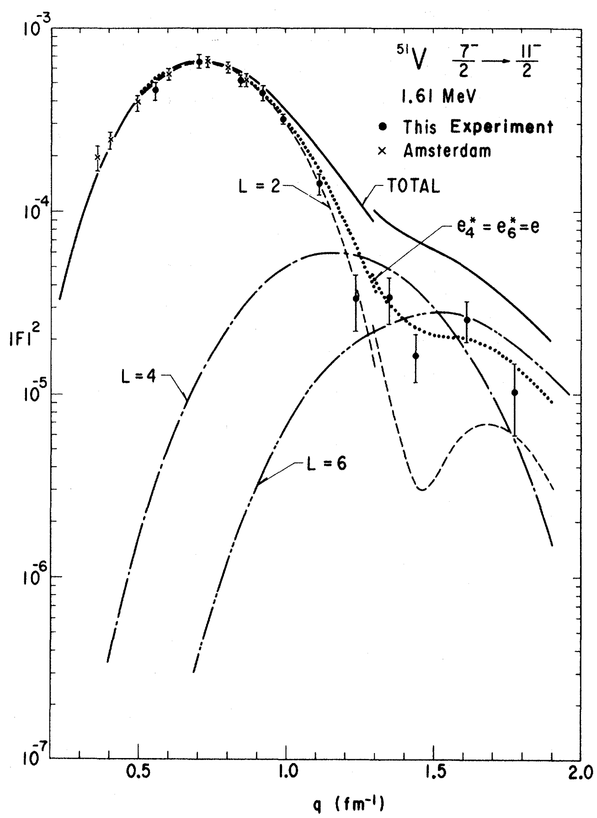

FIG. 8. Form factors for the 1.61-MeV excitation. See the caption to Fig. 6. The total curve was calculated for an effective charge of 1.79e.

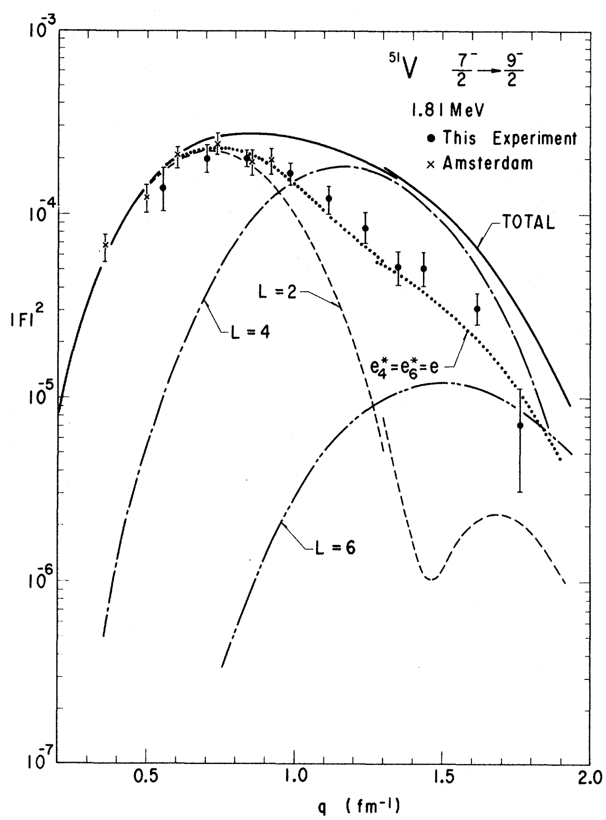

FIG. 9. Form factors for the 1.81-MeV excitation. See the caption to Fig. 6. The total curve was calculated for an effective charge of 1.88e.

in the curves at 1.3 fm^{-1} result because of the different Coulomb distortion for 183- and 250-MeV electrons. Also shown in Fig. 6 is a total of the form factors for each multipole calculated for $e_2^* = 1.84e$ and $e_4^* = e_6^* = e$. This is the only excitation where there is disagreement with the Amsterdam data.[3] However no distortion corrections for different incident energies E_0 were applied to the published Amsterdam data.

In Fig. 7 is shown the results for the second excited state at 0.93 MeV where only $L=2$ and 4 electric multipoles are allowed by the differences in J. Here a value of $e_2^* = (2.01 \pm 0.10)e$ was obtained. There is a more sizable $E4$ component for this excitation than for the 0.32-MeV excitation. If $e_4^* = e_2^*$, then it can be seen that the curve labeled "total" is higher than the experimental points where the $E4$ component is expected to be large. If e_4^* is set equal to e, as indicated by the dotted curve, better agreement with experiment is obtained.

Figure 8 shows the good agreement between recent Amsterdam data[23] and the present results for the 1.61-MeV excitation, and the good agreement between experiment and calculations for $L=2$. A value of $e_2^* = (1.79 \pm 0.05)e$ was obtained. Here again the experimental points are lower than expected where the higher multipole components are calculated to be large if e_4^* and e_6^* take on values about as large as e_2^*. Better agreement is obtained by setting $e_4^* = e_6^* = e$. The same effect appears for the 1.81-MeV excitation, as shown in Fig. 9, for which a value $e_2^* = (1.88 \pm 0.10)e$ was obtained.

Our values of e_2^* for the first four excited states are nearly the same, varying from 1.79e to 2.01e, consistent with the interpretation of these levels as being members of a nearly pure $(1f_{7/2})^3$ configuration multiplet. Our values are higher than the value of 1.61e deduced by Talmi[2] from an examination of results of Coulomb excitation of ^{51}V

TABLE II. Effective charges for the lowest multipole of excitation of the states of the $(1f_{7/2})^3$ multiplet of ^{51}V.

Energy (MeV)	J^π	e_2^*/e	e_4^*/e
0.32	$\frac{5}{2}^-$	1.84 ± 0.07	...
0.93	$\frac{3}{2}^-$	2.01 ± 0.10	...
1.61	$\frac{11}{2}^-$	1.79 ± 0.05	...
1.81	$\frac{9}{2}^-$	1.88 ± 0.10	...
2.70	$\frac{15}{2}^-$	...	1.69 ± 0.06

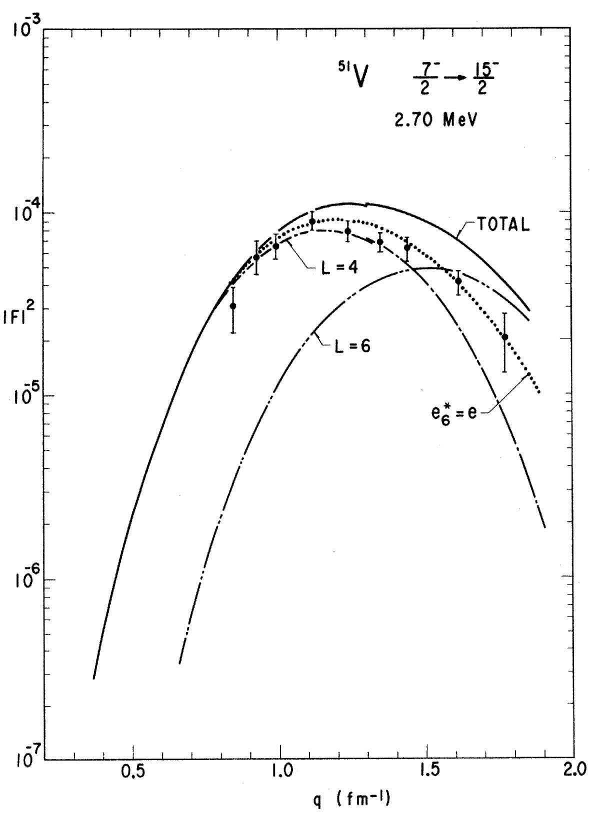

FIG. 10. Form factors for the 2.70-MeV excitation. See the caption to Fig. 6. The total curve was calculated for an effective charge of 1.69e.

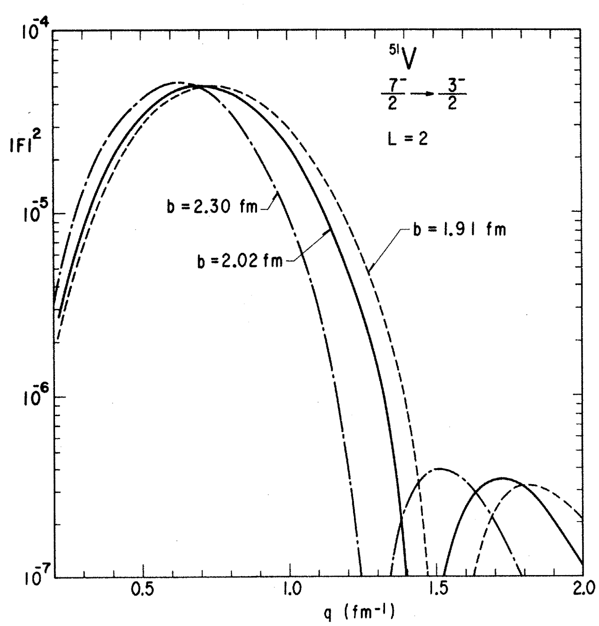

FIG. 11. $E2$ form factors for a $\frac{7}{2}^-$ to $\frac{3}{2}^-$ transition in an $(1f_{7/2})^3$ proton configuration in a harmonic-oscillator well calculated for various harmonic-oscillator length parameters.

by ^{12}C ions, and higher than the Amsterdam results,[3] where $B(E2)$ values were obtained at low q by using the Tassie model.

In Fig. 10 is shown the results for the excitation of the 2.70-MeV state which can proceed by $L=4$ and 6 only. We obtained for the first time a value of the $E4$ effective charge, $e_4^* = (1.69 \pm 0.06)e$. This is slightly smaller than the e_2^* values. Again better agreement between calculation and experiment is obtained if the higher multipole effective charge e_6^* is set equal to e. Also if this value of e_4^* obtained for the 2.70-MeV state is used in the calculation of the $L=4$ contributions of other excitations, poor agreement with experiment results. The effective charge values are summarized in Table II.

Thus it appears that in the excitation of the $(1f_{7/2})^3$ states, the lowest possible multipole components have effective charges that vary between about $1.7e$ and $2.0e$, and the higher multipole components may have smaller effective charges. The calculations can not be made to agree with experiment if the parameter b is increased by reasonable values. This is illustrated in Fig. 11 in which is shown the results of a Born-approximation calculation for three different values of b. If b is made larger so that $|F|^2$ is reduced at higher values of q, then $|F|^2$ is increased for lower values of q and a poor fit to the low-q data would result.

The use of the pure $(1f_{7/2})^3$ configuration could be questioned. For example Lips and McEllistrem[24] have added $1f_{5/2}$ and $2p_{3/2}$ mixtures to their wave functions in order to account for the small $M1$ components experimentally observed. As will be evident from the next section, it is unlikely that admixtures, such as $2p_{3/2}$, could account for the decrease of the form factors at large q. Instead the form factors would be larger at high q for a $2p_{3/2}$ admixture than for a pure $(1f_{7/2})^3$ configuration. Also different potential wells could be tried, e.g., the Woods-Saxon. On the other hand, if our application of the shell model is essentially correct, then the effect conceivably could arise from the way the state is excited by the electron, the lowest multipole receiving most of the effective charge and hence restricting the number of particle-hole states available for other multipoles.

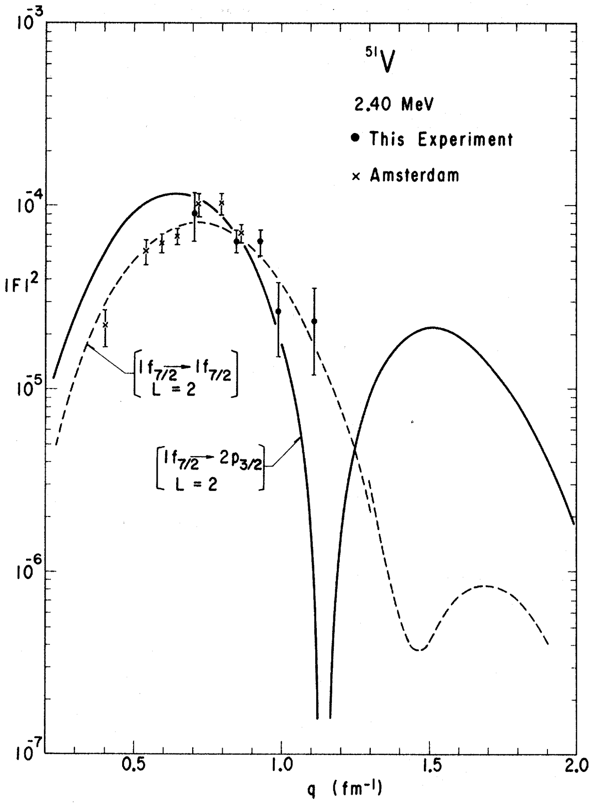

FIG. 12. $E2$ form factors for the 2.40-MeV excitation calculated for a proton transition from a $1f_{7/2}$ to a $2p_{3/2}$ configuration and for a transition within the $1f_{7/2}$ shell.

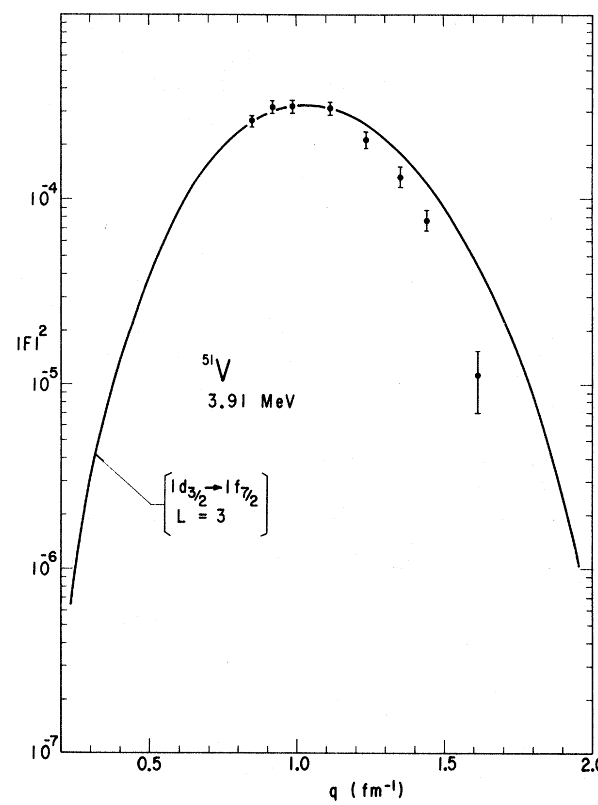

FIG. 13. $E3$ form factor for the 3.91-MeV excitation calculated for a $1d_{3/2}$ to $1f_{7/2}$ transition.

For example if the core is deformed, the $E2$ effective charge may be larger than the $E4$ or $E6$ effective charge for a particular transition. It is also likely that the effective charge may have a different radial distribution than the bare protons. An interference of the multipoles will not account for the effect because only the scattered electron is observed, and not the decay photon.[12]

C. Excitation of the 2.40- and 3.91-MeV States

The 2.40-MeV $(\frac{3}{2}^-)_2$ state showed clear experimental peaks above neighboring unresolvable peaks. In Fig. 12 is shown the form factors of this experiment and of an Amsterdam experiment[23] for the excitation of this state. Also shown are form factors calculated for harmonic-oscillator single-particle transitions from $1f_{7/2}$ to $2p_{3/2}$, and for a transition within the $1f_{7/2}$ shell. In the latter case, the same form-factor shape would result if the transition involved the $1f_{5/2}$ shell as well. Our results show, in agreement with the Amsterdam results, that the $1f_{7/2} \to 2p_{3/2}$ form factor will not fit the data, especially at higher momentum transfers where the second predicted form-factor peak at $q \sim 1.5$ fm^{-1} was not observed, but instead the cross section and hence the form factor fell to statistically unobservable small values as q was increased. Our experiment indicates that the transition may involve configurations largely within the $1f$ shells.

Figure 13 shows the form factor for an $E3$ excitation of a prominent positive-parity state at about 3.91 MeV. The calculated form factor shown for a promotion of a $1d_{3/2}$ core proton to the $1f_{7/2}$ shell results in a $B(E3)$ of 2.1 single-particle units. The assumption of a transition of this type is not unreasonable, since the energy difference[25] between the $1d_{3/2}$ shell and the $1f_{7/2}$ shell is about 5.1 MeV for $A = 51$. In ^{48}Ca there is a 3^- state at 4.51 MeV with a $B(E3)$ of 6.8 single-particle units as determined by an inelastic electron scattering experiment.[26] If both the ^{51}V and the ^{48}Ca transitions proceed predominantly by any core excitation to the $1f_{7/2}$ shell, the blocking effect of the three $1f_{7/2}$ protons of ^{51}V (with five $1f_{7/2}$ holes) should result in a $B(E3)$ for ^{51}V reduced with respect to that of ^{48}Ca (with eight $1f_{7/2}$ holes) by a factor of $\frac{5}{8}$. Furthermore, if only one half of the particles are promoted, either with, or without a spin-flip, then the relative blocking effect should be multiplied by a factor of $\frac{1}{2}$ to give $\frac{5}{16} \simeq 0.31$, as is the observed ratio of the $B(E3)$'s.

Although the shown form factor is calculated for a transition from $1d_{3/2}$ to $1f_{7/2}$, which involves a proton spin-flip, as well as a change of one unit of orbital angular momentum, the same shape form factor would result for a $1d_{5/2}$ to $1f_{7/2}$ transition, which only involves an orbital angular momentum change, a more likely process for the longitudinal excitation observed.

---

*Work supported in part by the Japan Society for the Promotion of Science, and by the Office of Naval Research under Contract No. ONR N00014-67-A-0230-0009.

[1]R. D. Lawson and J. L. Uretsky, Phys. Rev. 106, 1369 (1957); J. E. Schwäger, *ibid.* 121, 569 (1961).

[2]I. Talmi, Phys. Letters 25B, 313 (1967).

[3]P. K. A. de Witt Huberts, L. J. Oostrijk, and G. A. Peterson, Phys. Letters 32B, 599 (1970).

[4]K. Itoh, M. Oyamada, and Y. Torizuka, Phys. Rev. C 2, 2181 (1970).

[5]M. Kimura, Y. Torizuka, K. Shoda, M. Sugawara, T. Saito, M. Oyamada, K. Nakahara, K. Itoh, K. Sugiyama, M. Gotoh, K. Miyashita, and K. Kurahashi, Nucl. Instr. Methods 95, 403 (1971).

[6]I. Sick and J. S. McCarthy, Nucl. Phys. A150, 631 (1970).

[7]K. Nakahara, private communications.

[8]H. W. Kendall and I. Talmi, Phys. Rev. 128, 792 (1962).

[9]H. Nguyen-Ngoc and J. P. Perez-y-Jorba, Phys. Rev. 136, B1036 (1964); H. Crannell, *ibid.* 148, 1107 (1966).

[10]H. Crannell, R. Helm, H. Kendall, J. Oeser, and M. Yearian, Phys. Rev. 121, 283 (1961).

[11]T. A. Griffy and D. U. Yu, Phys. Rev. 139, B880 (1965).

[12]T. de Forest and J. D. Walecka, Advan. Phys. 15, 1 (1966).

[13]D. G. Ravenhall, quoted in R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

[14]R. F. Frosch, R. Hofstadter, J. S. McCarthy, G. K. Nöldeke, K. J. van Oostruum, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. 174, 1380 (1968).

[15]B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev. 101, 1131 (1956).

[16]H. J. Blaauw, W. F. G. Kuiper, C. deVries, and P. K. A. de Witt Huberts, in Contributions to the Conference on Nuclear Structure Using Electron Scattering, Sendai, Japan, 1972 (unpublished).

[17]H. Theissen, H. Fink, and H. A. Bentz, Z. Physik 231, 475 (1970).

[18]J. A. Bjorkland, S. Raboy, C. C. Trail, R. D. Ehrlich, and R. J. Powers, Nucl. Phys. 69, 161 (1965).

[19]A. S. Goodman and D. J. Donahue, Phys. Rev. C 5, 875 (1972).

[20]S. T. Tuan, L. E. Wright, and D. S. Onley, Nucl. Instr. Methods 60, 70 (1968).

[21]M. Nagao, to be published.

[22]P. Federman and L. Zamick, Phys. Rev. 177, 1534 (1969).

[23]P. K. A. de Witt Huberts and J. E. P. de Bie, in Contributions to the Conference on Nuclear Structure Using

Electron Scattering, Sendai, Japan 1972 (unpublished).
[24]K. Lips and M. T. McEllistrem, Phys. Rev. C 1, 1009 (1970).
[25]A. Bohr and B. Mottleson, *Nuclear Structure* (Benjamin, New York, 1969), vol. 1.
[26]R. A. Eisenstein, D. W. Madsen, H. Theissen, L. S. Cardman, and C. K. Bockelman, Phys. Rev. 188, 1815 (1969).

PHYSICAL REVIEW C VOLUME 7, NUMBER 3 MARCH 1973

Neutron Asymmetries and Energy Spectra from Muon Capture in Si, S, and Ca†

R. M. Sundelin* and R. M. Edelstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 5 July 1972)

The energy dependence of the neutron asymmetry parameter for nuclear capture of muons has been measured in Si, S, and Ca for neutron energies of 4–53 MeV. Contrary to most recent experiments, the asymmetry parameter is strongly positive over much of the energy range, with values as much as 9 standard deviations from zero. Because of the unusual nature of these results the experiment was performed twice and with excellent agreement between the two sets of data. The integrated asymmetry parameter for $E_n > 15.6$ MeV is +0.316 ± 0.023 for Si and +0.290 ± 0.034 for Ca, values which are in direct conflict with the "standard" $V-A$ theory. Recent theoretical calculations by Bogan and by Piketty and Procureur obtain positive asymmetries, though not yet in good quantitative agreement with these data. The neutron energy spectra from the three targets are all quite similar. They are consistent with a simple exponential falloff in E_n with a decay constant of 7 MeV. The theoretical calculations cited above are in good agreement with higher-energy regions of the spectra.

I. INTRODUCTION

Since the first days following the discovery of parity violation in the weak interaction,[1] there has been a continuing interest in studying this phenomenon in the process of nuclear capture of muons. For reasons of practicality and difficulty, much of the theoretical work and all of the experiments have concerned themselves, in particular, with the asymmetry in the angular distribution of neutrons following muon capture.

The "standard" $V-A$ theory[2] for muon capture involves six real coupling constants, if second class currents are allowed, so that without *a priori* knowledge of them ≥6 independent combinations must be determined from experiment in order to establish the validity of the theory. At the present time, for the most part, experiments are compatible with the "standard" theory, but they by no means form an overconstraining set from which to derive the coupling constants. Since most measurements of capture rates involve a statistical mixture of Fermi and Gamow-Teller couplings[3] they do not really measure independent combinations of constants (in addition to the difficulties of imprecise nuclear models). Thus, aside from demonstrating parity violation, the neutron asymmetry measurement is important, in principle, for the understanding of muon capture, since it measures an independent combination of constants. To lowest order in nucleon momentum, and neglecting differences in form factors, the capture rate for a statistical mixture of Fermi and Gamow-Teller interactions varies approximately as

$$G_V^2+3G_A^2+G_P^2-2G_PG_A,$$

where G_V, G_A, and G_P are the phenomenological vector, axial vector, and induced pseudoscalar coupling constants; whereas, the neutron asymmetry parameter is characterized by[2]

$$\alpha=\frac{G_V^2-G_A^2+G_P^2-2G_PG_A}{G_V^2+3G_A^2+G_P^2-2G_PG_A}$$

which yields the expected value $\alpha \approx -0.4$.

In reality, the neutron asymmetry parameter is not a sensitive test of weak-interaction theory, because of the uncertainty involved in the nuclear models, for any and all complex nuclei, as well as the final-state interactions of the neutrons produced in the process.[2,4] It is expected that this rescattering reduces the observed asymmetry, but also that higher-energy neutrons will be less affected. It was further realized[5] that there may be local fluctuations in the neutron energy dependence of the asymmetry due to nonstatistical muon capture, as for example in the case of pure Fermi