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Spin and Parity Determinations in Tungsten Nuclei Using Polarized Deuterons*

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States in the odd-mass isotopes of tungsten, ^{183,185,187}W, have been studied by the W($\vec{d}$,p) reaction initiated by polarized deuterons of 12.08- and 15.0-MeV bombarding energy. Vector-analyzing-power angular distributions were obtained for the strongly excited states. The use of measured analyzing powers for states of known J^π as calibration standards for the assignment of new J^π values is discussed. This technique is found to be preferable, at the present stage, to a reliance on the distorted-wave Born approximation (DWBA). The phase, but not the magnitude, of the oscillations in the DWBA calculations for the vector-analyzing powers is in qualitative agreement with the data at 12 MeV for spins of $\frac{5}{2}$ or less. At 15 MeV, this is not the case. The dependence of the DWBA calculations on optical-model parameters and Q value is discussed. A number of spin-parity assignments or preferences are made. In particular, candidates for $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states are located in each nucleus. The pairs of odd-parity levels are discussed as possible fragments of the first two members of the rotational band built on the $\frac{1}{2}-[501]$ Nilsson orbital, but the difficulties associated with this assignment are also pointed out.

I. INTRODUCTION

In a recent investigation[1] of the single-particle levels of the odd tungsten nuclei, a large number of states up to about 1500 keV were assigned spins and parities and arranged into rotational levels built on several different Nilsson orbitals. The work in Ref. 1 used primarily the (d,p), (d,t), and $(^3\text{He},\alpha)$ single-neutron-transfer reactions. Similar studies had previously been completed in lighter rare-earth nuclei,[2-3] and the energy systematics of a number of hole excitations in the tungsten isotopes established.[4] On the other hand, little study has previously been made of the detailed energy systematics for the particle excitations. Furthermore, at the deuteron energy 12 MeV used in Ref. 1, the (d,p) angular distributions are not as characteristic of transferred orbital angular momentum as those observed in (d,t). Hence, while nearly all the states observed in the (d,t) and $(^3\text{He},\alpha)$ spectra were rather easily grouped into rotational bands, it was only possible to assign Nilsson orbitals for the strong low-lying (d,p) states [states generally seen also in (d,t)]. In fact, in ^{183,185,187}W the strongest states in the entire (d,p) spectra were left unassigned as to spin, parity, and Nilsson configuration.

There are a number of Nilsson orbitals expected to occur in this region that should produce large (d,p) cross sections to some of their (generally low spin) rotational band members. It was in an attempt to investigate the previously unassigned levels and to search for the expected Nilsson orbitals that the present study using the (d,p) reaction with polarized deuterons was initiated.

Polarized deuterons have been used for (d,p) spectroscopy before, particularly in the calcium region.[5,6] No attempt has heretofore been made to use this technique for J^π assignments in heavy, deformed nuclei. The present work was therefore also initiated to determine if the use of the $(\vec{d},p)$ reaction is fruitful in such nuclei, and to investigate if it is amenable to a straightforward distorted-wave analysis.

The acquisition and interpretation of the present kind of data on heavy deformed nuclei is at an early stage. At present, the technique of using the measured vector-analyzing powers for known states to calibrate the (l,j) dependence of this quantity as a function of angle appears to be preferable for making new spin and parity assignments to reliance on the distorted-wave Born-approximation (DWBA) calculations. The present study has shown that, if the possible spin-parity assignments for a given level can be limited from other sources of information such as decay data, (n,γ) studies, and (d,p) angular distributions, then the use of polarized beams in stripping reactions on such targets as studied here can successfully yield determinations of the level assignments.

II. EXPERIMENTAL PROCEDURES

The deuteron beams were obtained from the Lamb-shift polarized-ion source[7] at the Los Alamos Scientific Laboratory (LASL) tandem Van de Graaff facility, which produced intensities of 50–100 nA on target. Approximately 75–82%[7] of the target current was selected in the nuclear magnetic substates $m=+1$, 0, and -1 as desired by means of a nuclear spin filter.[7] No refocusing was required when the magnetic substate was changed. Electrically isolated collimating slits were placed about 25 cm in front of the target. Upon change of substate the beam intensity remained constant, and the currents read on each slit jaw were essentially unchanged. Thus, negligible systematic errors are expected due to variations in beam spot profile or position.

The detector arrangement consisted either of a single detector located about 7.5 cm from the target (the runs on ^{183}W) or of a ΔE-E counter telescope system with the same solid angle (^{185,187}W data). To maximize counting rates a large solid angle (≈3 msr) was employed, along with thick targets. The targets were produced by vacuum evaporation of enriched material (^{182}W, ^{184}W, ^{186}W), were typically 400–600 μg/cm^2, and were rotated 45° with respect to the beam. Typical energy resolution for most of the runs was 25–35 full width at half maximum (FWHM). For some of the forward angle data, large dead times due to elastic scattering adversely affected the over-all resolution by a few keV, even though beam intensities were reduced for these runs to a few nA.

The zero of the angular scale was set by measuring, with unpolarized deuterons, the elastic scattering on both sides of the beam direction at several angles and by defining equal angles as those which produced equal counting rates.

The signals from the ΔE-E telescope were routed into two analog-to-digital converter (ADC) units, and particle identification was achieved using an on-line SDS-930 computer.[8] Mass gates were set on the outgoing protons and deuterons, and negligible leakage was observed. The length of each run was determined both by a Faraday cup located several feet behind the target and by the scaled elastic peak from a monitor detector placed at 80° to the beam direction and in the reaction plane. At this angle the elastic scattering is calculated to have essentially zero analyzing power.

After each run the beam polarization was measured using the quenching ratio method.[9] The accuracy of the measurement was ±1.5%. In its simplest form this method makes use of the fact that pure nuclear magnetic substate selection takes place in the nuclear spin filter for those deuterium atoms in the metastable $D(2s)$ atomic state. The total output beam current i consists of the nuclei selected in the appropriate state and the nominally unpolarized background beam current i_b. The background beam current can be measured by quenching the $D(2s)$ atoms to the ground state in the ion source. In normal operation, the fraction of deuterons in a pure nuclear magnetic substate is then given by $p_Q=(i-i_b)/i=(1-i_b/i)$, where i/i_b is usually referred to as the quenching ratio.[9]

Spin-1 beams from polarized-ion sources have cylindrical symmetry and can have rank-one polarization $\langle S_Z\rangle = p_Z$ and rank-two polarization $3\langle S_Z{}^2\rangle - 2 = p_{ZZ}$, where S_Z is the component of deuteron spin S along the axis of quantization. The observables p_Z and p_{ZZ} are given by

$$p_Z=\frac{N_1-N_{-1}}{N_1+N_0+N_{-1}+n_B},$$

$$p_{ZZ}=\frac{N_1+N_{-1}-2N_0}{N_1+N_0+N_{-1}+n_B},$$

in terms of the number N_M of nuclei selected in the substate M, and the number of n_B of unpolarized nuclei. The quantities p_Q, p_Z, and p_{ZZ} can be simply related for pure state cases:

$$p_Q=p_{ZZ}=p_Z \qquad (N_1=1,\ N_0=N_{-1}=0);$$

$$p_Q=p_{ZZ}=-p_Z \qquad (N_{-1}=1,\ N_1=N_0=0);$$

$$p_Q=-\tfrac{1}{2}p_{ZZ},\ p_Z=0 \quad (N_0=1,\ N_1=N_{-1}=0).$$

The spin direction $\vec{S}$ of the beam emerging from the ion source was precessed 90° so that it coincided with the direction of $\pm\vec{k}_{\rm in}\times\vec{k}_{\rm out}$ in the present experiment.

Using the single detector system described above, spectra were obtained with the deuteron beam polarized successively in each nuclear magnetic substate. From these data, peak areas exclusive of background were determined and the analyzing powers calculated.

The latter are expressed in terms of the spherical tensors $iT_{11}(\theta)=(\sqrt{3}/2)\langle\vec{S}\cdot n\rangle$, where n is a unit vector along $\vec{k}_{\rm in}\times\vec{k}_{\rm out}$. The measured values of iT_{11} were calculated according to the expression

$$iT_{11}(\theta)=\frac{R_1(\theta)-R_{-1}(\theta)}{R_1(\theta)+R_0(\theta)+R_{-1}(\theta)}\ \frac{0.866}{p_Q},$$

where $R_M(\theta)$ is the counting rate (peak area divided by accumulated beam charge) at the angle θ with the beam polarized in substate M. This expression assumed that the method of M-state selection produced pure states. For the counting rates R_0 and R_{-1} this is not precisely the case. Furthermore, there were small variations in beam polarization p_Q from run to run. Although the errors introduced by ignoring these effects would have been small compared with statistical uncertainties, the appropriate corrections were included in the calculation of $iT_{11}(\theta)$. Rank-two analyzing powers are not reported here, because they were mostly zero within statistics, and because a preliminary investigation indicated no J-dependent sensitivity in the DWBA to these observables.

The measurements were carried out at 12.08 MeV (the energy used in Ref. 1) and at 15 MeV. Data were obtained typically for about seven angles in the range 30–125°. Relative cross sections were obtained for all well resolved strong states up to ≈2 MeV of excitation in each nucleus. The conflicting requirements of counting rate and resolution precluded the analysis of a few groups, particularly in ^{187}W, that had been well resolved in the spectrograph data of Ref. 1.

Typical spectra taken on each of the targets with the deuterons polarized in the $M=\pm1$ magnetic substates are shown in Figs. 1–3. The spectra at other angles and at 15 MeV are similar except for somewhat higher backgrounds at angles forward of 45°. The states seen rather strongly in the present data are labeled. Numerous others (mostly hole excitations) were observed in the earlier spectrograph data.

In the case of ^{183}W, the intensities for each nuclear magnetic substate were also added and appropriately normalized to produce over-all cross sections (relative scale) at each angle, giving cross-section angular distributions for the states at 1154, 1476, and 1558 keV.

III. DISTORTED-WAVE CALCULATIONS

Distorted-wave (DW) predictions for the vector-analyzing-power angular distributions were obtained with the code DWUCK, written by P. D. Kunz of the University of Colorado. The principal uncertainty in using the code concerns the choice of optical-model parameters. It has been shown[10] for the (d,p) reaction that incorrect spectroscopic factors are determined from a DW analysis on a deformed heavy nucleus if optical parameters are chosen that fit the elastic scattering data. Such parameter sets probably include strong coupling effects involving primarily the low-lying 2^+ state in such a nucleus. This coupling is of less impor-

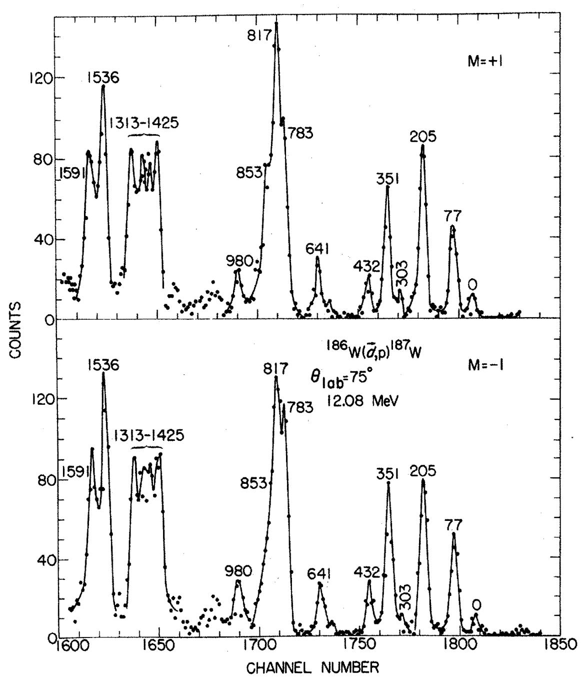

FIG. 1. Spectra of protons scattered at 75° in the reaction ^{186}W$(\vec{d},p)^{187}$W. The peaks are labeled by excitation energy in keV.

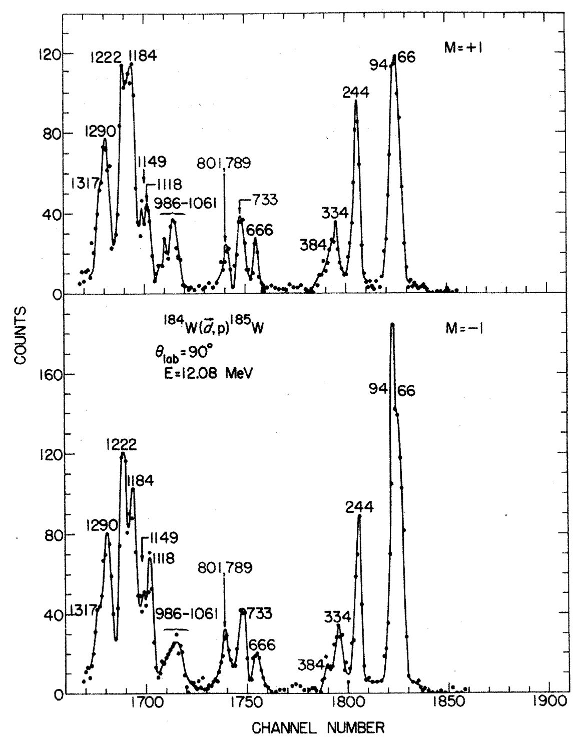

FIG. 2. Spectra of protons scattered at 90° in the reaction ^{184}W$(\vec{d},p)^{185}$W. The peaks are labeled by excitation energy in keV.

tance in direct stripping reactions, and consequently, parameters should be chosen that are independent of such effects in the elastic channel. The alternatives have either been to use parameters obtained from the standard prescriptions in the literature for optical parameters for a spherical nucleus of the correct mass and charge, or to use a coupled-channel calculation.

It has not yet been established, however, if either approach is reasonable for deformed nuclei for the calculation of effects occurring with the use of polarized beams. We have chosen the former alternative and have investigated the effects of parameter variation. The calculations were performed in the local energy approximation with no finite range or radial cutoffs. Partial waves up to $l=25$ were included, and it was determined that no change in the calculations occurred with an increased number. The spin-orbit force was included in a term of Thomas form. In general, different parameter sets gave very similar predictions, typically altering the size of maxima without altering their relative signs or positions, or slightly altering the average slope of the analyzing-power distribution.

Figure 4 gives examples of calculations for typical final states with several different optical parameter sets. (See Table I for a tabulation of the parameters used.) Included in the comparison is one with no spin-orbit interaction in the entrance or exit channels, but only in the bound state. In fact, similar analyzing-power distributions are also obtained with no spin-orbit term anywhere. The comparisons illustrated in the figure are typical of those resulting from other parameter variations not shown. For many, in fact, the deviations of the analyzing-power patterns from those calculated with potential $P1, D1$ are essentially indistinguishable from the solid curves in Fig. 4. In particular, rather large variations of the parameters of the imaginary part of the potential have little effect.

The dependence on Q value of the calculated analyzing powers is also weak (compare $P1, D1$ and $P1, D1^*$). Therefore, the same set of DWBA calculations was used throughout this work. It should be noted that the weak Q dependence, however, may only be characteristic of the low l transfers involved here. Recent calculations in the lead region for $l=4$ transfer have indicated considerably stronger dependence on Q value.[11]

As will become evident, the measured iT_{11} angular distributions for states of known spin and parity are often in poor agreement with the DW calculations. For low spins $J \leq \frac{5}{2}$ and at 12 MeV, the DW predictions are usually adequate for a choice between the two spin alternatives for a given transferred l value. For higher spins and at 15 MeV, however, this is not reliably the case. This is possibly due to greater sensitivity to the non-Coulomb part of the optical potential and therefore to the detailed choice of the parameters for the latter.

IV. NILSSON-MODEL PREDICTIONS

The low-lying levels in the odd W isotopes have been arranged[1] into rotational bands built on various Nilsson[12] orbitals. For intrinsic excitations above the Fermi surface the orbitals lying above

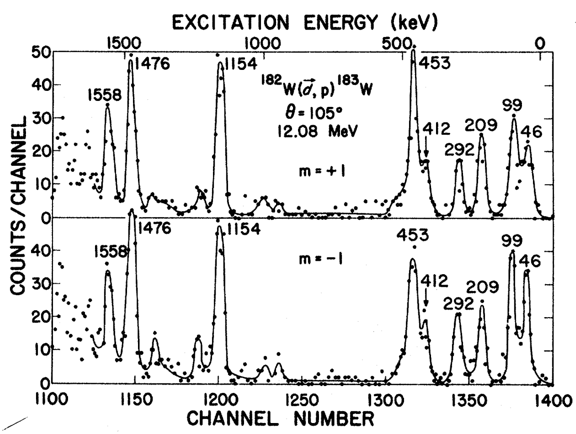

FIG. 3. Spectra of protons scattered at 105° in the reaction $^{182}W(\vec{d}, p)^{183}W$. The peaks are labeled by excitation energy in keV.

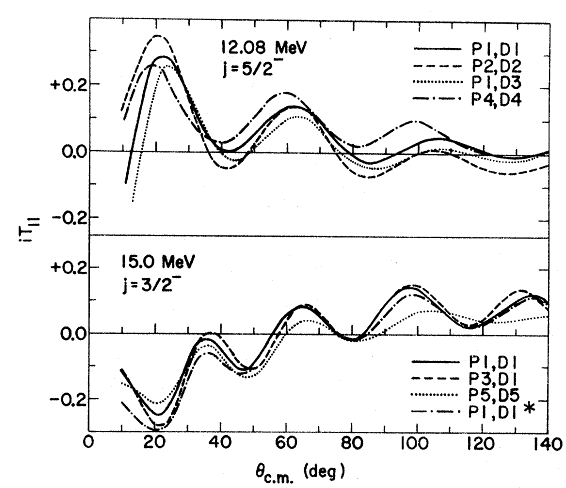

FIG. 4. Comparison of analyzing powers calculated with the DWBA for various choices of optical-model parameters. The parameter sets are listed in Table I. The set labeled $P1,D1^*$ is the same as the set $P1,D1$ but calculated with a Q value of 1.76 MeV instead of 2.76 MeV as in the other calculations.

those already assigned are expected to be the $\frac{3}{2}-[501]$, $\frac{1}{2}-[501]$, and $\frac{5}{2}-[503]$ with odd parity and the $\frac{1}{2}+[651]$, $\frac{3}{2}+[642]$, and $\frac{1}{2}+[640]$ with even parity.

Most of the (d,p) strength is expected to be concentrated[12] in one or two states of each of these rotational bands. This has the double consequence of leading to isolated strong states that are easy to observe, while rendering the patterns of cross sections to the successive rotational band members for each orbital very similar and difficult to use for orbital identification.

It has been shown[1-4, 13, 14] that Coriolis mixing can have extremely important effects on single-particle-transfer cross sections in this mass region. It acts essentially to shift cross sections among the levels of a given spin and generally to concentrate it in fewer levels. Following the general procedures of Ref. 1 the mixing among the Nilsson orbitals listed above was calculated. The qualitative results are insensitive to the details of the calculations. They indicate, for the even parity orbitals, that one should expect either one or two strong states of each spin $\frac{5}{2}^+$ and $\frac{9}{2}^+$. The total cross section at 90° is 400 μb/sr for the $\frac{5}{2}^+$ levels and ~250 μb/sr for the $\frac{9}{2}^+$ level. Both these numbers are increased somewhat if Nilsson wave functions calculated assuming a hexadecapole deformation are used. For the odd-parity levels the calculations predict much larger cross sections in each nucleus, namely, one extremely strong $\frac{1}{2}^-$ state ($\sigma \sim 800$ μb/sr), either one or two (depending on the degree of mixing) strong $\frac{3}{2}^-$ states with total cross section ~1000 μb/sr, and one moderately strong $\frac{5}{2}^-$ state (~400 μb/sr).

TABLE I. Optical-model parameters.

Potential	V (MeV)	r_r (fm)	a_r (fm)	W_d (MeV)	r_i (fm)	a_i (fm)	VSOR[a]
P1[b]	55.0	1.25	0.65	17.5	1.25	0.47	26.3
P2	55.0	1.25	0.65	17.5	1.25	0.47	0
P3	53.0	1.25	0.65	17.5	1.25	0.47	26.3
P4	55.0	1.35	0.65	17.5	1.25	0.47	26.3
P5	60.0	1.15	0.65	17.5	1.25	0.47	26.3
D1[b]	104.5	1.15	0.81	17.3	1.34	0.68	13.9
D2	104.5	1.15	0.81	17.3	1.34	0.68	0
D3	104.5	1.15	0.71	17.3	1.34	0.68	13.9
D4	104.5	1.25	0.81	17.3	1.34	0.68	13.9
D5	110.0	1.05	0.81	17.3	1.34	0.68	13.9
Bound state[b]		1.25	0.65				32.0

[a] VSOR is a factor used in the spin-orbit term. The latter is defined as

$$V_{so}(r) = V\left(\frac{\text{VSOR}}{45.2}\right)\frac{1}{r}\frac{df}{dx}\ \vec{L}\cdot\vec{S}\ .$$

[b] These potentials are taken from Ref. 1 and are used for all DWBA calculations except for the examples shown in Fig. 4.

V. RESULTS

A. States of Known Spin and Parity

The magnitudes of the vector-analyzing powers measured in this work are disappointingly small when compared with those obtained for single-particle states in the same mass region.[11, 15] It therefore appears difficult to use polarized deuteron beams exclusively to unambiguously assign J^π values for deformed states with small cross sections in this mass region. As stated previously, however, when other data exist that limit the possibilities to a few J^π values, the phase and general slope of the analyzing-power angular distributions (in comparison with those for states of known J^π) are sometimes sufficient to reduce the remaining candidates and in some cases to provide reasonably firm J^π assignments. Since DW calculations have a typical period of oscillation of 40°, data at every 15° were considered sufficient for such determinations. The data represent a compromise between low count rates due to the small cross sections, and a sufficiently thin target to resolve certain states.

In this section we shall consider the vector-ana-

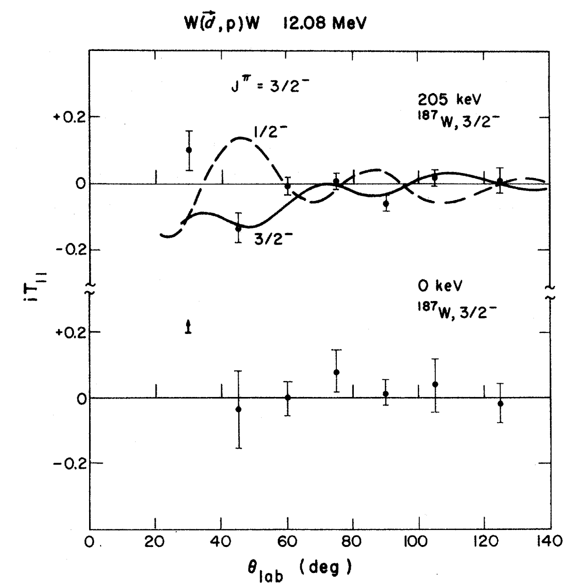

FIG. 5. Results for known $\frac{3}{2}^-$ states. In Figs. 5–13 the bombarding energy and the excitation energy, residual nucleus, and spin-parity if known of the final state are given with each set of data. The solid and dashed curves are DWBA calculations for the indicated spin and parity; these curves have been adjusted relative to the data as described in Sec. V A. The difference between laboratory and center-of-mass angles is indistinguishable in these figures.

lyzing-power angular distributions for the states of previously assigned spin and parity, some of which have been selected as calibration states. Although the use of DW predictions has been of secondary importance in determining new J^π preferences or assignments, we compare them in this section with the analyzing-power angular distributions for the calibration states.

While the intercomparison of measured cross-section angular distributions is a common practice, a similar approach in regard to analyzing powers obtained using polarized beams has been tested to a much lesser extent[5] and not at all in deformed nuclei. Theoretically it is not expected that the analyzing powers will depend on configuration, and the evidence presented below, especially for the 12-MeV data, for states of known spin and parity tends to confirm this expectation and allow some confidence in its application to unknown states. Also, as mentioned earlier, the Q-value dependence of the analyzing powers is calculated to be weak. No difficulty should then be expected in comparing standard measured analyzing powers to those corresponding to different Q values and occurring in neighboring nuclei.

Figures 5–7 show the data at 12 MeV for those states that have firm J^π assignments, that are reasonably strongly populated in (d,p), and that are well resolved from interfering states. The 454-keV level in ^{183}W is not fully resolved from the levels at 487 and 412 keV. However, the former is a $\frac{13}{2}^+$ state which is very weakly populated except near 125°, while the latter is also a $\frac{7}{2}^-$ level. Also shown in the figures, once for each spin, is the DW calculation of the analyzing powers.

The absolute magnitudes of the calculated DW vector-analyzing powers are considerably more positive than the measured values. As Figs. 5–7 show, however, the phase of the oscillations in the calculated distributions, and their general slope is in qualitative agreement with the data. This is clearly more so for $J=\frac{3}{2}^-, \frac{5}{2}^-$ than for $J=\frac{7}{2}^-$. We have only used the DW results to test whether or not the predictions oscillate in phase with the data. For this purpose in the figures we have lowered the DW curves to approximate the observed magnitudes of the vector-analyzing powers. This additive lowering varies in magnitude from -0.045 to -0.17 and, to main consistency, has been kept constant for a given value of J^π. No significance should therefore be attached to the absolute magnitude of the calculated analyzing powers, only to their phase and relative magnitudes of oscillation.

Although the error bars are large for the weak transition to the ^{187}W $\frac{3}{2}^-$ ground state (Fig. 5) the similarity of its analyzing-power angular distribu-

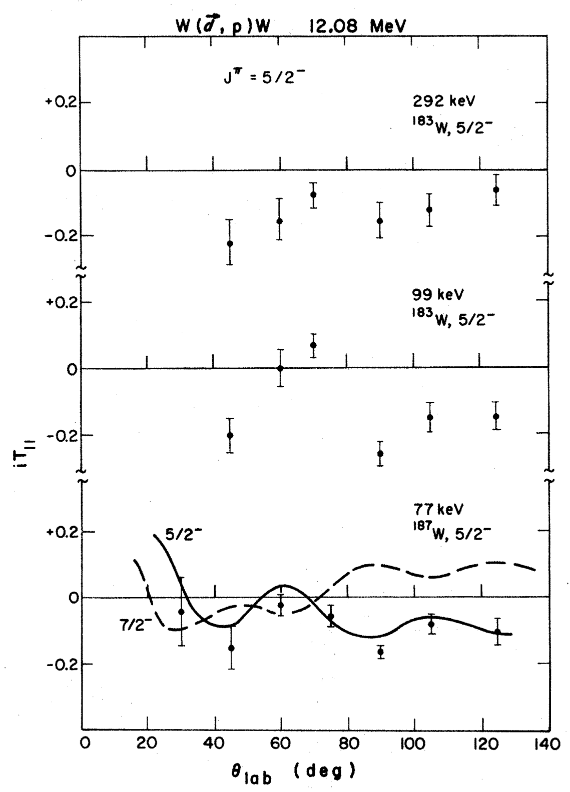

FIG. 6. See caption for Fig. 5.

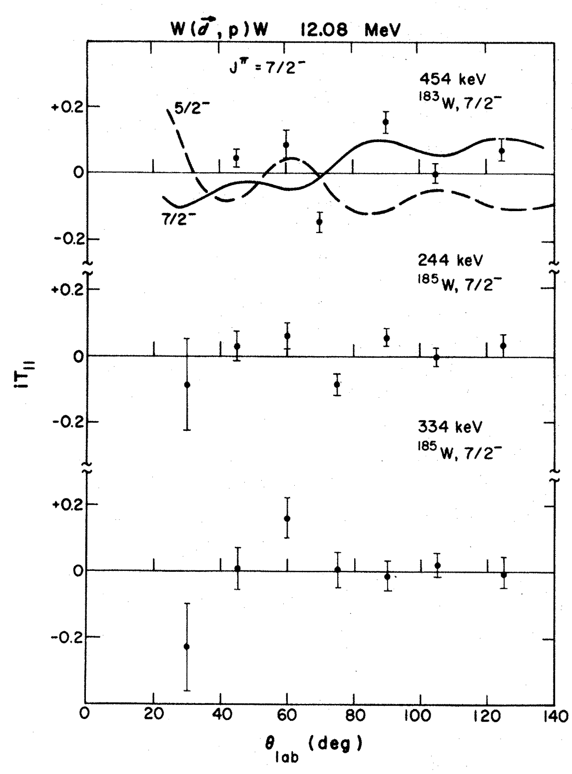

FIG. 7. See caption for Fig. 5.

tion to that for the 205-keV $\frac{3}{2}^-$ level is clear. It is also clear for the latter state that the phase of the DWBA curve for $J^\pi = \frac{3}{2}^-$ yields better agreement than that for $\frac{1}{2}^-$. The latter is out of phase with the data. The calculated $\frac{3}{2}^-$ curve reproduces the tendency but not the relative extent of the positive increase in analyzing power near 30° at both excitation energies.

For the $\frac{5}{2}^-$ states (Fig. 6), there is mutual consistency in the experimental distributions. The calculated $\frac{5}{2}^-$ curve at 12 MeV is in good over-all agreement with the data and is clearly favored over $\frac{7}{2}^-$.

Finally, for the $\frac{7}{2}^-$ states shown in Fig. 7, the analyzing-power angular distributions are again mutually consistent. Furthermore, the measured distributions for these states are rather accurately out of phase with those for the $\frac{5}{2}^-$ levels. Perhaps because of the large spin-orbit term, the DW fit is poorer for this spin, but the general slope of the data is reproduced better than with the calculation for $J^\pi = \frac{5}{2}^-$.

As comparison standards in what follows we have chosen the strong and well resolved transitions to the $\frac{3}{2}^-$ 205-keV (^{187}W) and $\frac{5}{2}^-$ 77-keV (^{187}W) levels. Their vector-analyzing-power angular distributions were shown in Figs. 5 and 6, respectively. The 783-keV (^{187}W) and 1154-keV (^{183}W) levels will be assigned relatively unambiguous $\frac{1}{2}^-$ and $\frac{5}{2}^+$ spin-parity values, respectively, below. The analyzing powers for these levels will then also be used as standards. For ^{187,185}W, the comparisons of measured analyzing powers for some of the previously unassigned levels with the DWBA calculations are collected in Fig. 8.

B. States of Unknown Spin and Parity

In discussing the assignment of spins and parities to levels below, use will be made of a number of other studies.[16-22] These have been used to limit as much as possible the allowed J^π assignments after which the present analyzing-power studies were used to obtain definite spin-parity determinations or preferences for a number of levels. In the subsections below, it is clearly cumbersome to illustrate all possible comparisons for spin choices for each level discussed. Certain examples will be shown in detail and the results for others summarized. The data for all states for which definite new spin-parity results were obtained, however, are shown.

1. ^{187}W

(a) 640-keV level. This level is populated in (d,p) with $l=3$ or 4.[1] The polarization data at 12 and 15 MeV are shown in Fig. 9 in comparison with that for the known $\frac{5}{2}^-$ level at 77 keV, with which it is seen to be in phase and in good agreement. From Fig. 8, it is seen that at 12 MeV the phase of the $\frac{5}{2}^-$ calculation is in agreement with the data for the 640-keV level and the $\frac{7}{2}^-$ is not. Comparisons at 12 and 15 MeV with calculations for $l=4$ indicate poorer agreement with the data than for $l=3$, $J^\pi = \frac{5}{2}^-$. The $\frac{7}{2}^+$ assignment is inconsistent with other evidence as well. The state is not seen in (n,γ)[17,18] and decays[19] most strongly to the $\frac{7}{2}$ $\frac{7}{2}-[503]$ level, and more weakly to the $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ levels of the $\frac{1}{2}-[510]$ band. A $\frac{5}{2}^-$ assignment is, therefore, indicated.

(b) 783-keV level. This level is populated in (d,p) with $l \leq 2$ and rather strongly by primary radiation in the (n,γ) reaction at both thermal and

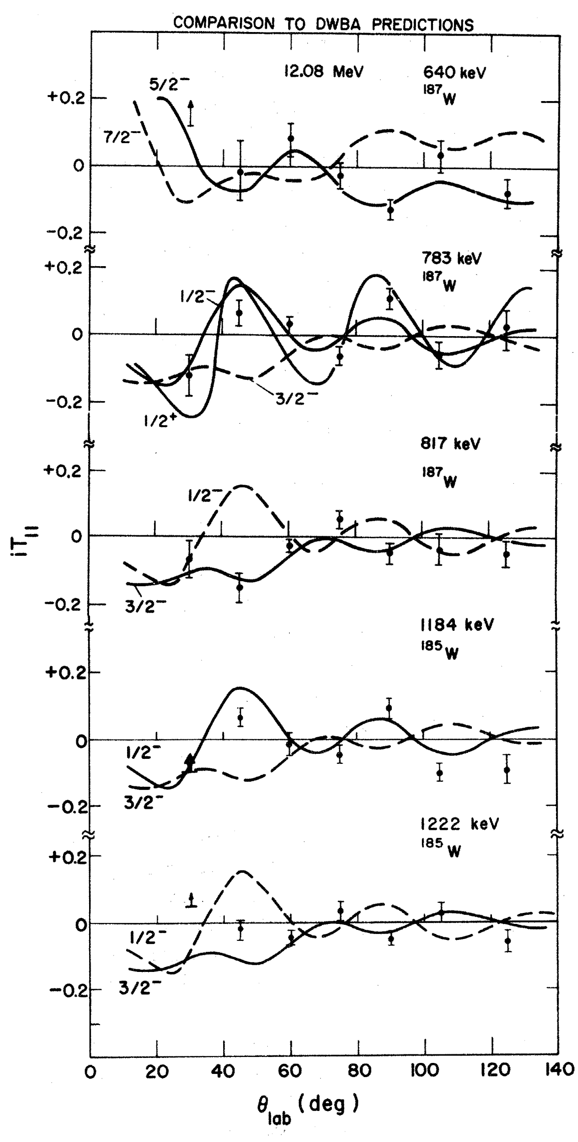

FIG. 8. See caption for Fig. 5.

resonant neutron energies. Thus the spin is either $\frac{1}{2}^-$ or $\frac{3}{2}^-$. At both 12 and 15 MeV the measured analyzing-power angular distributions for this level and the known $\frac{3}{2}^-$ levels are opposite in phase to one another, and therefore a $\frac{1}{2}^-$ assignment is suggested. (This comparison can be easily seen at 12 MeV in Fig. 10, discussed below in dealing with the 817-keV level.) The DW calculations shown in Fig. 8 are also consistent with the $\frac{1}{2}^-$ assignment just made; the curve for $J^\pi=\frac{1}{2}^-$ is in phase with the data while the $\frac{3}{2}^-$ curve is not.

(c) 817-keV level. This state is populated in (d,p) with $l \leq 2$. The analyzing-power data are illustrated in Fig. 10, where they are compared to those for the 205- and 783-keV levels. The comparison shows a definite preference for a $\frac{3}{2}^-$ assignment over a $\frac{1}{2}^-$. Since a $\frac{1}{2}^+$ state is calculated to exhibit an analyzing-power angular distribution similar to a $\frac{1}{2}^-$ state (see Fig. 8, 783-keV level, for example), the $\frac{1}{2}^+$ possibility may also be eliminated. The state is populated (weakly) in (n,γ)[17,20] but only at the 19-eV resonance and so can be assumed to have spin and parity of $(\frac{1}{2}, \frac{3}{2})^\pm$ or $\frac{5}{2}^+$. The above comparisons rule out $\frac{1}{2}^\pm$. The $\frac{3}{2}^+$, $\frac{5}{2}^+$ possibilities are eliminated by the decay data for this state.[19] The level is observed to deexcite primarily to the $\frac{3}{2}^-$ ground state and the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ levels of the $\frac{1}{2}-[510]$ rotational band. Weaker decays are reported to several $\frac{5}{2}^-$ and $\frac{7}{2}^-$ levels. The $\frac{3}{2}^+$ choice would require an $M2$ transition to compete with several $E1$ branches, and the $\frac{5}{2}^+$ choice would require an $E3$ or $M2$ transition to the $\frac{1}{2}\frac{1}{2}-[510]$ level to compete favorably with an $E1$ decay to the $\frac{3}{2}^-$ member of the same band and with decay to a $\frac{3}{2}^-$ level in the ground-state band.

(d) 1536- and 1591-keV levels. These have analyzing powers similar at both 12 and 15 MeV to the 817-keV levels and are also populated with $l \leq 2$ in (d,p). The former is observed in (n,γ).[17,20] On this basis we tentatively assign a J^π of $\frac{3}{2}^-$ to the 1536-keV level but can only suggest that $J^\pi \neq \frac{1}{2}^\pm$ for the 1591-keV state.

2. ^{185}W

All the states to be considered in ^{185}W are populated in (d,p) with $l \leq 2$. In general, the measured analyzing powers are less distinctive than in ^{187}W or ^{183}W, especially at 12 MeV. Furthermore, there is no information available on the decay of the states of interest in ^{185}W and little firm information from (n,γ) studies.

(a) 666-keV level. This state lies low enough to be seen also in (d,t) and the angular distribution data there favor a transferred angular momentum of $l=1$ or 2. At both energies the data for the 1154-keV tentative $\frac{5}{2}^+$ state in ^{183}W (see below) agree fairly well with those for this level. To a lesser extent, particularly at 15 MeV, those for the 205-keV $\frac{3}{2}^-$ level agree also. The distribution for the 783-keV ^{187}W level assigned above as $\frac{1}{2}^-$ is in disagreement with that for the 666-keV level. At 12 MeV the DWBA calculations favor a $\frac{5}{2}^+$ assignment over a $\frac{3}{2}^+$ choice. There is no decay information available but the state is seen very weakly in (n,γ).[17,20] The state is tentatively assigned as $[\frac{3}{2}^-, \frac{5}{2}^+]$ on the basis of these facts.

(b) 1184-keV level. The data at 12 MeV are compared to those for the transitions to the 205-keV $\frac{3}{2}^-$ and 783-keV $\frac{1}{2}^-$ levels of ^{187}W in Fig. 11. The vec-

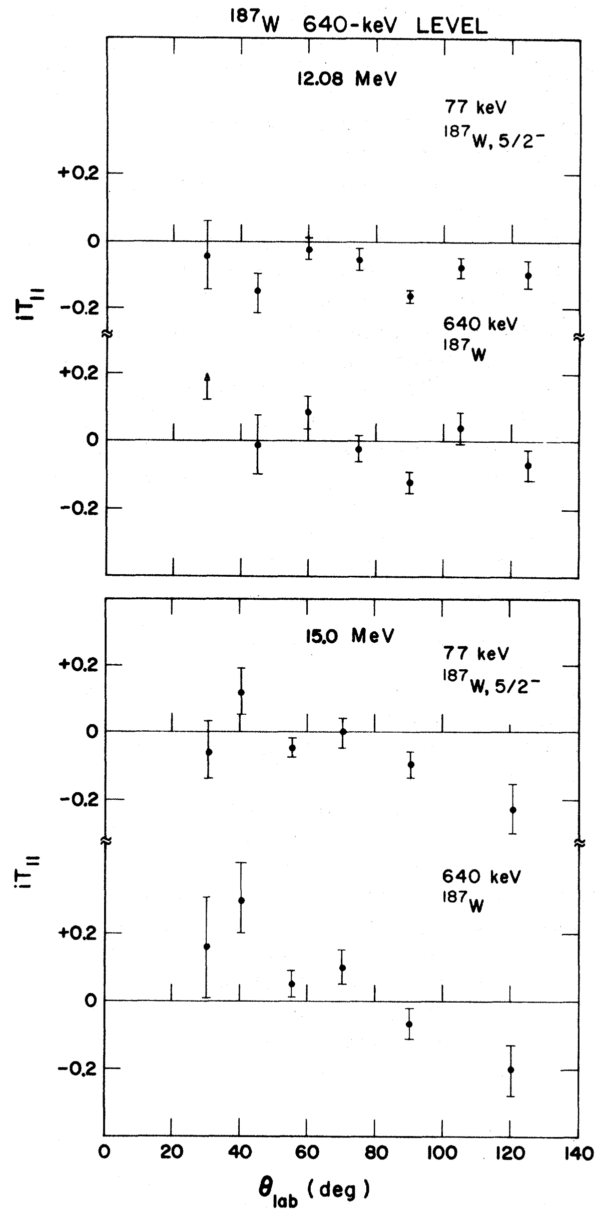

FIG. 9. See caption for Fig. 5.

tor-analyzing-power angular distribution is in good agreement with that for the 783-keV level and in disagreement with that for the $\frac{3}{2}^-$ 205-keV level. At 15 MeV the over-all agreement with the distribution for the 783-keV level is only qualitative. The analyzing powers for a $\frac{5}{2}^+$ state (1154-keV level in ^{183}W discussed below) at 12 MeV show negative values around 50 and 90° (see Fig. 12) in contrast to the data for this state. The $\frac{3}{2}^+$ possibility is not entirely ruled out but is argued against by the poor fit to the data provided by the calculated analyzing powers for $\frac{3}{2}^+$ and on model-dependent grounds by the unlikelihood of any $\frac{3}{2}^+$ level being strongly populated in (d,p) in this region. However, as we shall compare our results to the Nilsson model predictions later, we cannot allow this argument to carry weight here. Hence the state is assigned as $\frac{1}{2}^-$ $(\frac{1}{2}^+, \frac{3}{2}^+)$.

(c) 1222-keV level. For this level there is the auxiliary information that the state is seen strongly in (n,γ) studies,[17,20] therefore indicating a $\frac{1}{2}^-$ or $\frac{3}{2}^-$ assignment. Both of these are, of course, consistent with the (d,p) angular distribution data cited earlier. The present data for this level are shown in Fig. 8 where they can be compared to those for the 817-keV level in ^{187}W assigned above as $\frac{3}{2}^-$; comparison can also be made to those for the 205-keV $\frac{3}{2}^-$ level in ^{187}W in Fig. 5. The agreement is fair but the distribution is definitely out of phase with that for the 783-keV $\frac{1}{2}^-$ level (Fig. 8). Finally, it is noted that the distributions for the 1184- and 1222-keV levels are rather consistently out of phase, again favoring opposite spin assignments provided they are populated with the same transferred orbital angular momentum. Combining the (n,γ) and polarization data, the level can be assigned a J^π value of $\frac{3}{2}^-$.

(d) 1290-keV level. No decay or (n,γ) information exists for this state which is populated, as mentioned, in (d,p) with $l=0,1,2$. DWBA predictions for a $\frac{1}{2}^+$ assignment do not fit the data at either energy. The data for this level resemble that for the 1222- and 666-keV levels. These results imply a probable spin-parity assignment of $\frac{3}{2}^-$ or $\frac{5}{2}^+$ as opposed to $\frac{1}{2}^-$ or $\frac{3}{2}^+$ but no further limitation is possible.

3. ^{183}W

In this nucleus there are strong and well resolved levels at 1154, 1476, and 1558 keV. There is no decay information but some (n,γ) results and some (d,p) angular distributions recorded by Erskine.[16] These provide rather firm limitations on the possible angular momentum transfers and have greatly facilitated the spin-parity assignments

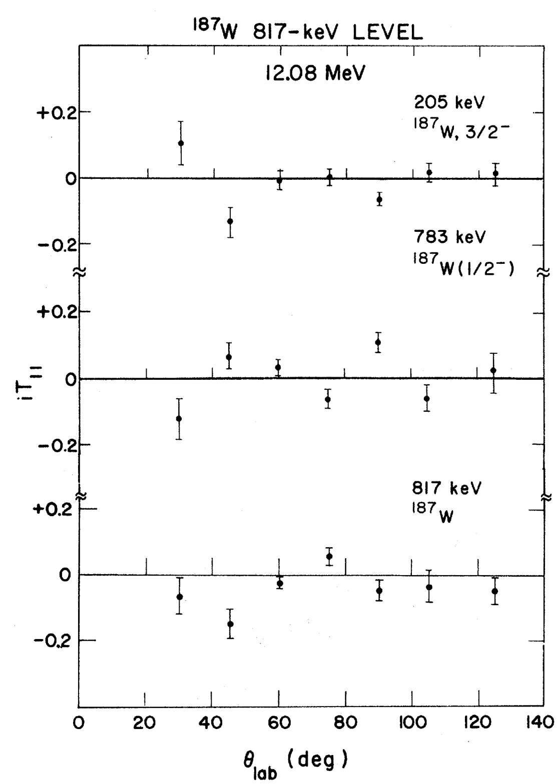

FIG. 10. See caption for Fig. 5.

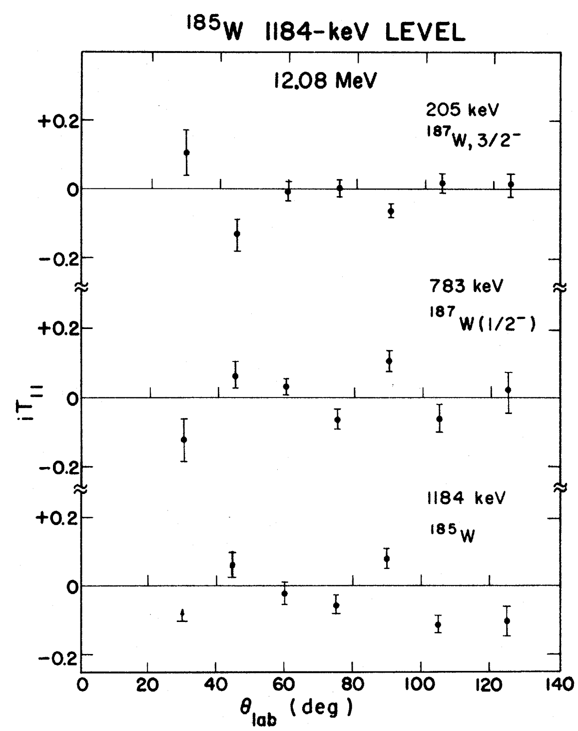

FIG. 11. See caption for Fig. 5.

below.

(a) 1154-keV level. This has not been observed in (n,γ) or decay studies.[17-20] It is populated in (d,p) with $l=2$.[16] In Fig. 12, a comparison is shown of the data for this state to the DW predictions for the two allowed spins. The phase of the 12-MeV calculations distinctly favors a $\frac{5}{2}^+$ assignment. As noted before, the DWBA predictions at 15 MeV are less reliable. Figure 12 shows, in any case, that up to 70° the curve for $\frac{5}{2}^+$ is in good agreement with the data while the $\frac{3}{2}^+$ curve is not. Given the $l=2$ transferred orbital angular momentum, a $\frac{5}{2}^+$ preference is indicated, with $\frac{3}{2}^+$ definitely less likely. We have recently learned[22] that this conclusion is also consistent with some (n,γ) results using a variant of the average-resonance-capture technique in which primary capture radiation leading to this state was not observed, contrary to the expectation if the J^π were $\frac{3}{2}^+$.

(b) 1476-keV level. According to Ref. 16, this level is populated with $l=1$ or 2 and the present cross-section angular-distribution data slightly favor $l=1$. As it is in any case rather strongly populated by primary γ radiation in neutron capture studies,[17,20] the $l=1$ (d,p) population is assured and J^π values of $\frac{1}{2}^-$ or $\frac{3}{2}^-$ determined. Figure 13 shows a comparison with the data for the population of the 205-keV $\frac{3}{2}^-$ and 783-keV $\frac{1}{2}^-$ levels in ^{187}W and also includes the DW curves for the allowed J^π values. A preference is evident, particularly at the forward angles, for the $\frac{1}{2}^-$ assignment.

(c) 1558-keV level. This state is populated in (d,p) with orbital angular momentum transfer of $l=1$ or 2, with the former preferred.[16] Once again, the present data favor the $l=1$ assignment but in any case the state is observed rather strongly in resonant neutron capture[17,20,21] indicating the $l=1$ choice and a J^π of $\frac{1}{2}^-$ or $\frac{3}{2}^-$. Figure 13 compares the data for transitions to this level with those to the standard $\frac{1}{2}^-$ and $\frac{3}{2}^-$ levels and to the corresponding DW predictions. The results are not conclusive but favor the $\frac{3}{2}^-$ assignment. The analyzing powers for $\theta<80°$ for the 1476- and 1558-keV states are opposite in phase. Given that both states are $\frac{1}{2}^-$ or $\frac{3}{2}^-$ the only consistent J^π choice is $\frac{3}{2}^-$ for this level and $\frac{1}{2}^-$ for the 1476-keV state. Our conclusion is again consistent with the recent (n,γ) study[22] cited above in which the 1558-keV state is observed to decay most strongly to the $\frac{5}{2}^-$ level at 903 keV, rather than to several $\frac{1}{2}^-$, $\frac{3}{2}^-$ levels at lower energies.

The assignments discussed above are summarized in Table II. Where firm choices were not possible, the alternatives allowed by the present data are listed. Also given in the table are Nilsson orbital assignments suggested as an important com-

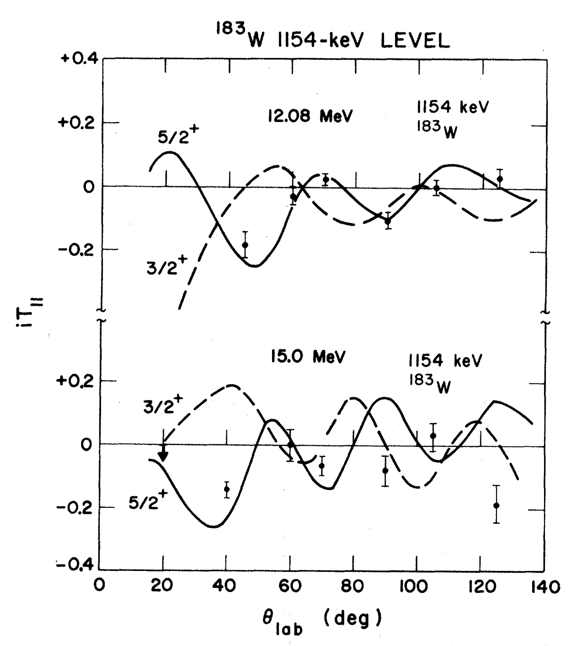

FIG. 12. See caption for Fig. 5.

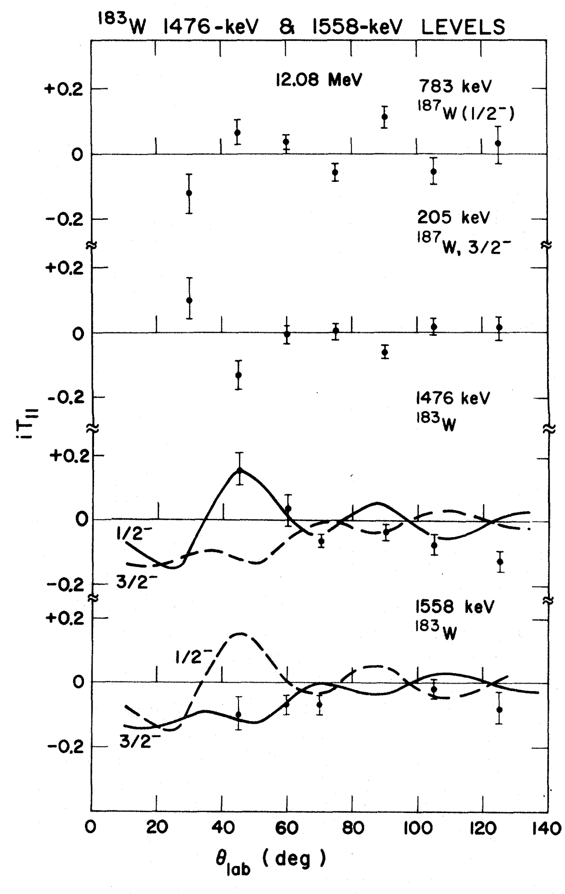

FIG. 13. See caption for Fig. 5.

ponent in the wave function of each level. These are given assuming the preferred J^π assignments in cases in which two or more such assignments are listed. The Nilsson assignments are discussed in the following section.

VI. DISCUSSION

A. Level Assignments

All of the assignments (favored or otherwise) given in Table II are consistent with the expectations of the Nilsson model discussed earlier. A consistent pattern is the occurrence of a pair of close-lying states in each nucleus with spin-parity of $\frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively. One can speculate that the pair corresponds to the first two members of the rotational band based on the $\frac{1}{2}-[501]$ orbital. The consistent decrease in energy of this pair with increasing mass is in agreement with the variation of the energy of the $\frac{1}{2}-[501]$ orbital with deformation in the Nilsson model. A difficulty with these assignments is that the $\frac{3}{2}-[501]$ orbital is expected to be lower in energy than the $\frac{1}{2}-[501]$. Clearly no candidates exist (at least in ^{185,187}W) for the $\frac{3}{2}-[501]$ at a lower energy. On the other hand, the measured 90° cross sections to the candidates for the $\frac{1}{2}\,\frac{1}{2}-[501]$ state are ≤ 300 μb/sr; that is, they are only $\sim\frac{1}{3}$ the expected strength, and represent no more than a fragment of the wave function whose energy might be considerably depressed.

TABLE II. Summary of assignments. See text for a more detailed discussion.

State (keV)	$J^{\pi\,a}$	Plausible Nilsson assignments [b]
	^{187}W	
640	$\frac{5}{2}^-$	$\frac{5}{2}\,\frac{5}{2}-[503]$
783	$\frac{1}{2}^-$	$\frac{1}{2}\,\frac{1}{2}-[501]$
817	$\frac{3}{2}^-$	$\frac{3}{2}\,\frac{1}{2}-[501]$
1536	$[\frac{3}{2}^-]$	
	^{185}W	
666	$[\frac{3}{2}^-, \frac{5}{2}^+]$	
1184	$\frac{1}{2}^-, (\frac{1}{2}^+, \frac{3}{2}^+)$	$\frac{1}{2}\,\frac{1}{2}-[501]$
1222	$\frac{3}{2}^-$	$\frac{3}{2}\,\frac{1}{2}-[501]$
1290	$[\frac{3}{2}^-, \frac{5}{2}^+]$	
	^{183}W	
1154	$\frac{5}{2}^+ (\frac{3}{2}^+)$	$\frac{5}{2}\,\frac{1}{2}+[651]$
1476	$\frac{1}{2}^-$	$\frac{1}{2}\,\frac{1}{2}-[501]$
1558	$[\frac{3}{2}^-]$	$\frac{3}{2}\,\frac{1}{2}-[501]$

[a] Where a single entry is displayed it is considered a firm assignment. Where the second of two entries is enclosed in parentheses it is considered as definitely less likely. Where entries are enclosed in square brackets, they are the only spin-parity assignments considered consistent with all the available information, but the quality of the data or the lack of sufficient information [e.g., (n, γ) data] precludes a firm assignment.

[b] The Nilsson notation (Ref. 12) is $JK\pi[Nn_z\Lambda]$. The assignments are tentative and the discussion in the text should be consulted. The assignments always assume the favored J^π value. They are meant to suggest only a component in the undoubtedly highly-Coriolis-mixed wave functions. Much of the strength of these orbitals must lie higher in energy (see also Ref. 21).

Another disturbing feature of these $\frac{1}{2}-[501]$ assignments is that the cross sections to the $\frac{3}{2}^-$ rotational states suggested in ^{185,187}W are larger than for the band head rather than in the predicted ratio[12] $\sigma(\frac{3}{2}^-)/\sigma(\frac{1}{2}^-)\sim 0.2$. This latter feature might be due to Coriolis admixtures of the $\frac{3}{2}\,\frac{3}{2}-[501]$ state (at some higher energy) into the $\frac{3}{2}\,\frac{1}{2}-[501]$. In fact such admixtures might also explain the small energy spacing of the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ band members in ^{185,187}W. In ^{183}W, on the other hand, the cross section for the $\frac{3}{2}^-$ candidate at 1558-keV is only ~50% that of the $\frac{1}{2}^-$ level at 1476 keV, in much better agreement with the Nilsson model prediction, and in this nucleus also the $\frac{1}{2}^- - \frac{3}{2}^-$ energy spacing is more reasonable. Taking the 82-keV separation in ^{183}W as approximately the unperturbed spacing, and using a decoupling parameter of $a = 0.75$, the inertial parameter $\hbar^2/2\mathscr{I}$ is 15.6 keV in good accord with unperturbed values for this quantity in other bands in this nucleus (see Ref. 1).

In ^{187,185}W other states are observed (at 1536 keV, possibly at 1591 keV in ^{187}W, and possibly at 1290 keV in ^{185}W) that are candidates for $\frac{3}{2}^-$ levels. If so, their wave functions undoubtedly contain large components of both the $\frac{3}{2}\,\frac{3}{2}-[501]$ and $\frac{3}{2}\,\frac{1}{2}-[501]$ levels.

One other likely odd-parity level was assigned, the $\frac{5}{2}^-$, 640-keV level in ^{187}W. Although the decoupling parameter of the $\frac{1}{2}-[501]$ orbital depresses the energy of the $\frac{5}{2}^-$ band member, it is unlikely that that is the dominant component in the 640-keV wave function. More likely, it is a fragment of the $\frac{5}{2}-[503]$ band head, the main strength of which probably lies somewhat higher in energy.

Possible $\frac{5}{2}^+$ levels are those at 666 and 1290 keV in ^{185}W and at 1154 keV in ^{183}W. Again this is consistent with the Nilsson model predictions in that the $\frac{1}{2}+[651]$ orbital should be characterized by a $\frac{5}{2}^+$ rotational state that is strongly populated in (d, p). However, the energy of this orbital is expected to increase with increasing mass in this region, contrary to the trend established by the above assignments. Once again, the (d, p) cross section indicates that neither level in ^{185}W contains more than a small fragment of the full strength. Evidence adduced in Ref. 21 indicates that the fractionation may spread the strength of these orbitals over as much as 1–2 MeV. Hence the energy-ordering arguments cannot be pressed very far. Apparently, the fractionation among the high-lying particle excitations in these nuclei is more severe than among the hole excitations and precludes a more detailed understanding of the spectra. A similar fractionation of the particle states has been noted in (d, p) and analog resonance studies in the Hf isotopes[23] and in ^{184}W.[21]

B. Polarized-Beam Technique

Several comments may be made about the use of the polarized-beam technique employed in this work. The most consistent approach in nuclei such as the heavy deformed W isotopes appears to be the use of states of known spin and parity to calibrate the expected vector-analyzing-power angular distributions. The agreement among the latter for known states is quite consistent. The phase of the oscillations in the DW calculations also appears to be reliable for $J \leq \frac{5}{2}$ at the lower energy of 12 MeV, but not at all at 15 MeV. The calculated magnitudes of the vector-analyzing powers are considerably more positive than is observed to be the case.

In terms of the DW calculations, an urgent need is for more investigation of the suitability of various optical-model parameter choices for the calculation of observables in reactions initiated with polarized beams. The superiority of "average" versus fitted parameters in the case of unpolarized incoming beams has been established[10] but does not necessarily apply to the present situation. One investigation of the type just suggested is now being carried out[24] and involves the comparison of measured asymmetries in the ^{176}Yb$(\vec{p}, t)^{174}$Yb reaction to those predicted by a coupled-channel calculation. Similar work with single-nucleon transfer is needed if the present technique is to be applied more generally and to deformed nuclei where strong well resolved states of known spin and parity are not available for calibration. Finally, additional experimental information on the stability of measured asymmetries as a function of Q value, nucleus, and configuration is desirable in order to corroborate the use of analyzing powers for known states in making additional assignments for other states.

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Electron Scattering from ^{51}V

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Form factors for the excitation of the ^{51}V 0.32-, 0.93-, 1.61-, 1.81-, and 2.70-MeV members of the $(1f_{7/2})^3$ multiplet, and of the 2.40- and 3.91-MeV states have been measured for momentum transfers up to 1.8 fm^{-1} by using 183- and 250-MeV electrons. Comparisons have been made with form factors calculated by using harmonic-oscillator shell-model wave functions. For the $(1f_{7/2})^3$ multiplet, effective charges from 1.79e to 2.01e were obtained for the $E2$ components of the excitations, and 1.69e for the $E4$ component of the 2.70-MeV excitation. Higher multipole components of each excitation required smaller effective charges in the strict $(1f_{7/2})^3$ configuration. An analysis of the elastic scattering data using a Fermi charge distribution yielded the parameters $c = 3.94 \pm 0.03$ fm, $t = 2.22 \pm 0.06$ fm, and 3.58 ± 0.04 fm for the rms radius.

I. INTRODUCTION

Deeply penetrating high-energy electrons are excellent probes of nuclear structure. The electrons interact with the nucleons only via the electromagnetic force, and the cross sections for the scattering of the electrons can be related directly to the reduced matrix elements of the charge and current-density operators between the initial and final nuclear states. Detailed radial information is provided which is very difficult to obtain by other means.

We have used high-energy electron scattering to investigate some of the low-lying levels of $^{51}_{23}V_{28}$. This nucleus is thought to be well described in terms of the shell-model $(1f_{7/2})^3$ proton configuration and a closed $1f_{7/2}$-shell neutron configuration. The energy levels can be calculated closely from the two-body $(1f_{7/2})^2$ matrix elements of the effective nuclear interaction taken from neighboring nuclei, such as ^{50}Ti, and the spins and parities are correctly predicted.[1,2] However, if $E2$ transition rates are calculated by assuming that only the valence protons take part, it is found that the experimental values are larger, as is the case for most, if not all nuclei. It is customary to evoke the concept of "effective charge" to explain the observed enhanced transition probabilities. Each valence proton is assumed to have a charge larger than that of a free proton in order to account for polarization of the core by the valence protons, or from a microscopic viewpoint, the effective charge accounts for virtual excitations of particle-hole core states which admix with the independent-particle states. The use of effective charge permits tractable calculations to be made by using simple shell-model descriptions with valence particles.