Nuclear effects on neutrino emissivities from nucleon-nucleon bremsstrahlung

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The rates of neutrino pair emission by nucleon-nucleon (*NN*) bremsstrahlung are calculated with the inclusion of the full contribution from a nuclear one pion exchange potential (OPEP). We compute the contributions from the neutron-neutron (*nn*), proton-proton (*pp*), and neutron-proton (*np*) processes for physical conditions encountered in supernovae and neutron stars, both in the degenerate (D) and nondegenerate (ND) limits. We find a significant reduction of these rates, especially for the *nn* and *pp* processes, in comparison with the case when the whole nuclear contribution was replaced by constants, representing the high-momentum limits of the expressions of the nuclear potential. Furthermore, we also perform the calculations by including contributions due to the ρ meson exchange between nucleons, in the OPEP. This may be relevant for processes produced in the inner core of neutron stars, where the density may exceed several times the standard nuclear density, and the short-range part of the *NN* interaction should be taken into account. These corrections lead to an additional suppression of the neutrino emission rates between (8 and 36)%, depending on the process [*nn* (*pp*) or *np*] and physical conditions (temperature and degeneracy of the nucleons).

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For a quantitative understanding of the competition between different neutrino emission processes determining the evolution and properties of the neutron stars (NS) accurate calculations of the corresponding neutrino production rates are very required. A detailed description of these processes and the recent achievements in this domain are presented in many good reviews like, for instance, [1–3] and the references therein. At present, there is a qualitative understanding of the importance of these processes according to specific density-temperature ranges. For example, within a smooth composition model of ground state matter, the main contribution to neutrino emission from deep layers of the crust of a NS comes from plasmon decay at very high temperatures and from neutrino-pair bremsstrahlung and Cooper pairing of neutrons at $T \leqslant 10^9$ K. For other density-temperature ranges ($T \leqslant 10^{10}$ K, $\rho \leqslant 10^{10}$ g cm^{-3}), the electron-positron pair annihilation and the photoneutrino emission may produce comparable emission rates. In nonsuperfluid cores of the NS there are many neutrino reactions that can be classified as follows: (I) baryon direct URCA processes, (II) baryon modified URCA processes, (III) baryon bremsstrahlung processes, (IV) lepton modified URCA processes, and (V) Coulomb bremsstrahlung processes. The neutrino emissivities of many reactions are reliably calculated being only slightly dependent on a particular microscopic model of strong interactions. However, other reactions are dependent on the adopted model of *NN* interaction and for these there is still room for improvement in the calculation of the emission rates. In particular, we refer to the neutrino pair emission from *NN* bremsstrahlung (NNB) processes: $N+N \rightarrow N+N+\nu+\overline{\nu}$ with $N=n$ or p.

In the early 1990's Suzuki's calculations [4] showed that these bremsstrahlung processes have, besides the modified baryon URCA processes, a significant role in the cooling mechanisms of the NS [5,6]. Contrary to the modified URCA processes, the NNB processes have no threshold and, for certain temperature-density ranges, they may be more important for the neutrino pair production than the former.

The main difficulty in the calculation of their emission rates is the appropriate treatment of the strong *NN* interaction which is responsible for the ν production together with the weak interactions. In the early calculations this interaction has been treated either by computing the overlap integrals associated with the initial and final nucleon wave function [5] or through the use of a Fermi liquid parametrization [7]. Subsequently, several authors have deduced a *NN* potential based on the one pion exchange (OPE) approximation [8,9] and used it in calculations of the neutrino emissivities of the modified URCA and NNB processes. Freeman and Maxwell used a *NN* interaction consisting of a long-range OPE tensor and a short-range part parametrized with nuclear Fermi liquid parameters. The resulting emissivities were larger than those obtained by Bahcal and Wolf [5] and Flowers *et al.* [7] with an order of magnitude.

Hannestad and Rafelt [9] derived a scattering kernel that governs the bremsstrahlung and the inelastic scattering and give an analytic approximation formula. They use an OPE *NN* potential in a perturbative approach, which applies only to densities below 10^{14} g cm^{-3}. Their calculations revealed again that the bremsstrahlung processes cannot be ignored as a source of ν_μ, ν_τ in supernovas, besides elastic and inelastic scattering of neutrinos on nucleons and electrons, pair annihilation e^+e^-, etc.

Moreover, in Ref. [10] the authors replaced the full contribution of the *NN* interaction by a constant factor. This simplifies the calculation of the emission rates by NNB and it would be useful to estimate how large is the error in doing that.

However, for an accurate computation of the neutrino pair emissivities from these processes an accurate treatment of the nuclear potential is certainly required. In a recent work [11] a method was developed for treating explicitly the momentum dependence of the OPEP. The authors showed that in particular physical conditions, characterized by temperatures $T \leqslant m_\pi^2/m \sim 20$ MeV, the integral over the nuclear matrix elements (NME) collapses, in a good approximation, to an integral which is independent of angles. The remaining part of the nuclear contribution, which depends only on the nucleon and neutrino energies, can be easily integrated numerically. As a first application, this method was used to the computation of the neutrino emission rates for the like-nucleon bremsstrahlung processes in the (D) and (ND) regimes. The aim was to compare (for $T=6$) their results with the results obtained by the authors of Ref. [10] where the whole nuclear contribution was reduced to a constant factor ζ. Both in the (D) and (ND) limits it was found that the inclusion of the full contribution of a *NN* OPE potential produces neutrino emissivities which are about two times larger than the case when the *NN* interaction is reduced to a constant $\zeta=1$.

In this brief review we extend the method developed in Ref. [11] to calculate the neutrino emission rates from the (*nn*), (*pp*), and (*np*) bremsstrahlung processes taking into account the full contribution from a nuclear OPEP. The calculations are performed for several temperatures and are relevant for physical conditions encountered in supernovas and NS, both in the degenerate (D) and nondegenerate (ND) limits. Furthermore, we also include in the calculation the short-range part of the *NN* interaction by adding the contribution of ρ mesons in the expression of the OPEP. The general effect of these nuclear contributions is a suppression of the corresponding neutrino emission rates which depends on the specific process and the physical conditions (temperature and degeneracy).

First, we describe briefly our method of calculation. If one considers only two-nucleon collisions the total volumetric emissivity for the neutrino pair production by nucleon bremsstrahlung is given by Fermi's golden rule formula:

$$\epsilon_{\nu\bar{\nu}} = \frac{2\pi}{\hbar}\int\left[\Pi_1^4 \frac{d^3\mathbf{p}_i}{(2\pi)^3}\right]\frac{d^3\mathbf{q}_\nu}{(2\pi)^3 2\omega_1}\frac{d^3\mathbf{q}_{\bar{\nu}}}{(2\pi)^3 2\omega_2}\omega(s\Sigma|M|^2) \times(2\pi)^3\delta^4(P)F(f), \tag{1}$$

where $F(f)=f_1 f_2(1-f_3)(1-f_4)$ is the product of Fermi-Dirac distribution functions for the initial (1,2) and final (3,4) nucleons with $f_i=(\exp^{E_i-\mu_i/T}+1)^{-1}$.

In Eq. (1) $\mathbf{p}_i$, $i=1,4$ and $\mathbf{q}_\nu, \mathbf{q}_{\bar{\nu}}$ are the nucleon and neutrino momenta, respectively, $\omega=\omega_1+\omega_2$ are the neutrino energies; s is a symmetry factor taking into account the symmetry of identical particles ($s=1/4$ for *nn* and *pp*, and $s=1$ for the *np* channels) and E_i and μ_i are the energies and chemical potentials of the nucleons.

In the nonrelativistic limit $E_i \sim m+\mathbf{p}_i^2/2m$. If one defines the chemical potential as $\hat{\mu}=\mu-m$ and one introduces the nondimensional quantities [12] $y=\hat{\mu}/T$; $u_i=\mathbf{p}_i^2/2mT$, the expressions of the Fermi-Dirac functions read $f_i=(\exp^{u_i-y_i}+1)^{-1}$. The (D) limit is achieved for $y \gg 1$, while in the (ND) limit for $y \ll 1$. For $|M|$ we use the spin-summed NME derived in the OPE approximation (see for instance Ref. [9]). In the nonrelativistic approximation one can neglect the neutrino momenta in comparison with the nucleon momenta and one can write:

$$s\Sigma_{\text{spins}}|M_{NN}|^2 = s64G^2 g_A^2\left(\frac{f}{m_\pi}\right)^4\frac{\omega_1\omega_2}{\omega^2}F_{NN} \tag{2}$$

For the *nn* and *pp* processes F_{NN} reads:

$$F_{nn} = \left[\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2}\right)^2 + \left(\frac{\mathbf{l}^2}{\mathbf{l}^2+m_\pi^2}\right)^2 + \frac{\mathbf{k}^2\mathbf{l}^2 - 3(\mathbf{k}\cdot\mathbf{l})^2}{(\mathbf{k}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)}\right] \tag{3}$$

while for the *np* process:

$$F_{np} = \left[\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2}\right)^2 + 2\left(\frac{\mathbf{l}^2}{\mathbf{l}^2+m_\pi^2}\right)^2 - 2\frac{\mathbf{k}^2\underline{l}^2 - (\mathbf{k}\cdot\mathbf{l})^2}{(\mathbf{k}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)}\right], \tag{4}$$

where m_π is the pion mass, $g_A=1.26$, $f\simeq 1$; $\mathbf{k}=\mathbf{p}_1-\mathbf{p}_3$ and $\mathbf{l}=\mathbf{p}_1-\mathbf{p}_4$ are the nucleon direct and exchange transfer momenta, respectively. The third terms in the above expressions are the exchange ones and come from the interference of two different reaction amplitudes. They contain the term $(\mathbf{k}\cdot\mathbf{l})^2$ which has kinematic constraints. With good approximation [9] we replace in the calculations $(\mathbf{k}\cdot\mathbf{l})^2/(\mathbf{k}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)$ by its average over the phase space: $\langle(\mathbf{k}\cdot\mathbf{l})^2\rangle \sim 0$ in the (D) regime and $\langle k\cdot l\rangle \sim 1.085/3$ in the (ND) limit. We would like to mention that this last correction was not used in the calculations of Ref. [11].

One needs to handle NME having the above dependence on nucleon transfer momenta. The OPE ansatz is a good approximation for a *NN* potential at distances between nucleons $\geqslant 2$ fm, while at shorter distances the two-pion exchange effects become important. The effect of the short-range part of the *NN* interaction can be estimated by mimicking the two-pions exchange by exchange of one heavier meson $\rho(m_\rho \sim 770$ MeV) between nucleons. This affects only the direct exchange part in the $|M|^2$ expressions which has to be modified as follows [13]:

$$\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2}\right)^2 \to \left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2} - C_P\cdot\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\rho^2}\right)^2. \tag{5}$$

Furthermore, the calculation procedure has the following main steps:

(1) Perform the integral over the neutrino phase-space, leading to a ω^5 dependence in the emissivity formula.

(2) Conversion to the center-of-mass system, since it is more convenient to perform the integrals. For this, one introduces the new variables [12]:

$$\mathbf{p}_+ = \frac{\mathbf{p}_1+\mathbf{p}_2}{2};\ \mathbf{p}_- = \frac{\mathbf{p}_1-\mathbf{p}_2}{2};\ \mathbf{p}_{3c} = \mathbf{p}_3+\mathbf{p}_+;\ \mathbf{p}_{4c} = \mathbf{p}_4+\mathbf{p}_+. \tag{6}$$

Now, one defines the following dimensionless quantities:

$$u_i = \frac{\mathbf{p}_i^2}{2mT},(i=1,4);\quad u_+ = \frac{\mathbf{p}_+^2}{2mT};\quad u_- = \frac{\mathbf{p}_-^2}{2mT};\quad u_{3c} = \frac{\mathbf{p}_{3c}^2}{2mT};$$

$$\cos\gamma_1 = \frac{\mathbf{p}_+\mathbf{p}_-}{|\mathbf{p}_+||\mathbf{p}_-|};\quad \gamma_c = \frac{\mathbf{p}_+\mathbf{p}_{3c}}{|\mathbf{p}_+||\mathbf{p}_{3c}|};\quad \cos\gamma = \frac{\mathbf{p}_-\mathbf{p}_{3c}}{|\mathbf{p}_-||\mathbf{p}_{3c}|};$$

$$u_- = u_{3c} + \frac{\omega}{2T}. \tag{7}$$

(3) Treatment of the momentum dependence part of the spin summed NME and calculation of the corresponding

emissivities. The idea is to handle the expressions of F_{NN} such that to separate a main (constant) part, which is just its high-momentum limit and a correction part which can be integrated over the nucleon momenta with good precision. As an example we address the *nn* (*pp*) processes in the (D) limit, since the treatment of the *np* process and/or the (ND) limit is similar:

$$F_{nn} = \left(3 - m_\pi^2 \frac{A_1 - A_2(c_g + s_g \cos\phi)^2}{B_1 - B_2(c_g + s_g\cos\phi)^2 + B_3(c_g + s_g\cos\phi)^4}\right) \equiv (3 - F_{nn}^{(D)\mathrm{corr}}), \quad (8)$$

where $c_g = \cos\gamma_1 \cos\gamma_c$, $s_g = \sin\gamma_1 \sin\gamma_c$, and ϕ is the difference between the azimuthal angles corresponding to γ_1 and γ_c (in spherical coordinates). As we mentioned above the main part of the nuclear potential contribution (the constant term) is just 3, i.e., the limit to which Eq. (3) converges when the pion mass is neglected compared to the nucleon momentum transfer. However, this limit is a good approximation for $|\mathbf{q}|^2 \gg m_\pi^2$ and occurs for $T > 20$ MeV (taking into account that $\mathbf{q}^2 \sim 3Tm_\pi$). The coefficients A_i and B_j are polynomials of degree eight in the product $(m_\pi^2 mT)$, depending only on variables x and z but otherwise independent of the angles. B_3 is also binomial having a coefficient proportional to $(2mT)^4$, while B_1 and B_2 contain terms proportional to m_π^4, m_π^6, and m_π^8. Therefore, whenever $mT < m_\pi^2$ (or equivalent, $T < 20$ MeV), the term with the coefficient B_3 can be neglected because it is small compared to the other terms. This approximation can also be checked numerically. The results of integration over M_{nn}^{corr} with and without the term B_3 differ each other by a small amount, less than 2% but greatly simplifies the calculation. The remaining integral over ϕ can be performed analytically yielding to:

TABLE I. The total volumetric emission rates from the *nn*, *pp*, and *np* bremsstrahlung processes in the (ND) limit for several values of *T*. The calculations were performed with (π) and $(\pi+\rho)$ meson exchange included in the OPEP.

T (MeV)	$\epsilon_{\nu\bar\nu}^{nn}(\pi)/e^{2y}$ (erg s^{-1} cm^{-3})	$\epsilon_{\nu\bar\nu}^{nn}(\pi+\rho)/e^{2y}$ (erg s^{-1} cm^{-3})	$\epsilon_{\nu\bar\nu}^{np}(\pi)/e^{2y}$ (erg s^{-1} cm^{-3})	$\epsilon_{\nu\bar\nu}^{np}(\pi+\rho)/e^{2y}$ (erg s^{-1} cm^{-3})
5	1.87×10^{31}	9.44×10^{30}	7.67×10^{31}	4.99×10^{31}
6	1.06×10^{32}	6.21×10^{31}	4.35×10^{32}	3.01×10^{32}
7	4.43×10^{32}	2.82×10^{32}	1.84×10^{33}	1.32×10^{33}
9	4.39×10^{33}	3.02×10^{33}	1.82×10^{34}	1.37×10^{34}
10	1.14×10^{34}	7.99×10^{33}	4.73×10^{34}	3.69×10^{34}
11	2.67×10^{34}	1.91×10^{34}	1.11×10^{35}	8.99×10^{34}
13	1.18×10^{35}	8.67×10^{34}	4.59×10^{35}	3.86×10^{35}
15	4.17×10^{35}	3.12×10^{35}	1.73×10^{36}	1.47×10^{36}

$$\int F_{nn}^{(D)\mathrm{corr}} d\phi = m_\pi^2 \cdot \frac{3d(2x+z) + 7m_\pi^2}{2\left[d^2(x+z) + 4m^2T^2z^2 + dm_\pi^2(2x+z) + \frac{1}{4}d^2z^2 + m_\pi^4\right]}, \quad (9)$$

where $d = 2mT$, $x = 2u_{3c}$, and $z = \omega/T$. Handling the expression of F_{np} in the same way like the *nn* and *pp* cases one can also perform the integration over angles. The remaining integral from the emissivity formula (1), over the other variables (u_+, u_-, x, z), can be calculated by combining analytical and numerical techniques, for both the (ND) and (D) limits (see, for example, Refs. [10,12]).

We calculated first the neutrino emission rates for the processes $nn\nu\bar\nu$, $pp\nu\bar\nu$, and $np\nu\bar\nu$ using OPEP given by Eqs. (3) and (4), both in (ND) and (D) limits. We were interested in the influence that the inclusion of the full momentum dependence of the potential has on the emission rates, in comparison with the case when the NME are replaced by constants, representing their high-momentum limits. This is a good approximation if $\mathbf{k}^2$, $\mathbf{l}^2 \gg m_\pi^2$. Denoting by ϵ^{hm} the result of the calculations in these high-momentum limits, we found: $\epsilon_{nn(pp)} = 0.67 \times \epsilon_{nn(pp)}^{hm}$ for the (D) limit and $\epsilon_{nn(pp)} = 0.59 \times \epsilon_{nn(pp)}^{hm}$ for the (ND) limit. $\epsilon_{np} = 0.75 \times \epsilon_{nn(pp)}^{hm}$, for the (D) limit and $\epsilon_{np} = 0.85 \times \epsilon_{nn(pp)}^{hm}$, for the (ND) limit.

This shows that the full inclusion of the OPEP has a larger effect on the emission rates of the *nn* and *pp* processes and in the (D) limit. For $T = 6$ MeV we can compare our values for the emission rates with the results obtained by the authors of Ref. [10] for the processes *nn* (*pp*). In terms of their constant factor replacing the nuclear contribution, we found $\zeta \sim 2$ [in the (D) limit] and $\zeta \sim 1.76$ [in the (ND) limit]. This represents a good approximation having in mind that they neglected completely the contribution of the *NN* interaction in their calculations.

The results obtained for the emission rates for several temperatures are given in Tables I and II. The values displayed in the (ND) limit do not include the exponential factors e^y, for the *nn* and *pp* processes, and $e^{y_1+y_2}$ for the *np* process. These factors are related to the chemical potentials μ_n, μ_p which, in turn, depend on the matter density and satisfy the β equilibrium condition. We also have to mention that for getting the contribution of both ν_μ and ν_τ the results from Tables I and II must be multiplied by two.

Furthermore, we perform the calculations including the contribution due to the ρ meson exchange in the expressions of the OPEP [Eqs. (3) and (4)] which accounts for the short-range part of the *NN* interaction. The results obtained are presented in the columns 3 and 4 of Tables I and II, for different temperatures. One can see that the values of the emission rates are suppressed when ρ meson contributions are also included. The suppression varies between (8 and

TABLE II. The same as in Table I but for the (D) limit.

T (MeV)	$\epsilon_{\nu\bar{\nu}}^{nn}(\pi)$ (erg s^{-1} cm^{-3})	$\epsilon_{\nu\bar{\nu}}^{nn}(\pi+\rho)$ (erg s^{-1} cm^{-3})	$\epsilon_{\nu\bar{\nu}}^{np}(\pi)$ (erg s^{-1} cm^{-3})	$\epsilon_{\nu\bar{\nu}}^{np}(\pi+\rho)$ (erg s^{-1} cm^{-3})
5	1.76×10^{33}	1.26×10^{33}	7.30×10^{33}	6.06×10^{33}
6	7.99×10^{33}	5.84×10^{33}	3.03×10^{34}	2.55×10^{34}
7	2.84×10^{34}	2.12×10^{34}	1.18×10^{35}	1.01×10^{35}
9	2.23×10^{35}	1.71×10^{35}	9.25×10^{35}	8.04×10^{35}
10	5.36×10^{35}	4.08×10^{35}	2.21×10^{36}	1.94×10^{36}
11	1.16×10^{36}	8.94×10^{35}	4.81×10^{36}	4.28×10^{36}
13	4.60×10^{36}	3.55×10^{36}	1.91×10^{37}	1.72×10^{37}
15	1.48×10^{37}	1.15×10^{37}	6.14×10^{37}	5.65×10^{37}

36)% depending on the physical conditions (temperature and degeneracy) and the kind of the process considered: *nn* (*pp*) or *np*. The largest effect occurs for the *nn* (*pp*) processes in the (ND) limit. Also, we found that they do not depend much on the density, but, as we already mentioned, they are valid mainly for densities $\rho > \rho_0$.

Concluding, we performed calculations of the rates of neutrino pair emission by NNB processes with the inclusion of the full contribution from a *NN* OPE potential. We considered the *nn*, *pp*, and *np* processes both in the (D) and (ND) limits. These processes, besides the modified baryon URCA processes, are important for the production of $\nu_\mu\overline{\nu_\mu}$ and $\nu_\tau\overline{\nu_\tau}$ pairs and play a significant role in the cooling mechanism of the NS. We used the method developed in Ref. [11], where we showed that for physical conditions encountered in supernovas and NS, the multiple integral appearing in the emissivity formula can be performed exactly, even with the inclusion of the full momentum dependence of the OPEP. To appreciate the nuclear effects of this inclusion we compared the results obtained with our method with those obtained by replacing the expressions of the OPEP by their high-momentum limits. We found rather large deviations especially for the *nn*(*pp*) processes. For densities $\rho > \rho_0$, encountered in the inner core of a NS, the OPE approximation is not sufficient to describe accurately the *NN* interaction. Thus, we also included in the calculations the contribution given by the ρ meson exchange, in the expression of the OPEP. These nuclear short-range effects further suppress the neutrino emission rates by factors between (8 and 36)%, depending on the process [*nn* (*pp*) or *np*] and the physical conditions (temperature and degeneracy of nucleons). The largest effect was found for the *nn*(*pp*) processes in the (ND) limit. It is worth to mention that our method allowing the inclusion of the full contribution of a nuclear OPEP in the calculation of the neutrino emissivities from *NN* bremsstrahlung is rather a general one, and can also be used for the computation of other *NN* model depending neutrino production processes of astrophysical interest. Work on that is underway. Also, there are still some key issues to be clarified for a full evaluation of the bremsstrahlung neutrino emissivities. For example, the contributions of the *NN* correlations in a dense medium and pairing effects are not fully understood at present.

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