Nucleon QCD sum rules in nuclear matter including four-quark condensates

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We calculate the nucleon parameters in nuclear matter using the QCD sum rules approach in gas approximation. Terms up to $1/q^2$ in the operator product expansion (OPE) are taken into account. The higher moments of the nucleon structure functions are included. The complete set of the nucleon expectation values of the four-quark operators is employed. Earlier the lack of information on these values has been the main obstacle for the further development of the approach. We show that the values of the four-quark condensates are consistent with the assumptions about the convergence of the OPE. Inclusion of these condensates and of the nonlocality of the vector condensate are important for the calculation of the nucleon parameters. The nucleon vector self-energy Σ_v and the nucleon effective mass m^* are expressed in terms of the in-medium values of QCD condensates. The numerical results for these parameters at the saturation value of the density agree with those obtained by the methods of nuclear physics.

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I. INTRODUCTION

The QCD sum rules were invented by Shifman *et al.* [1] to express the hadron parameters through the vacuum expectation values of QCD operators. Being initially used for the mesons, the method was expanded by Ioffe [2] to the description of the baryons. The approach succeeded in describing the static characteristics as well as some of the dynamical characteristics of the hadrons in vacuum—see, e.g., the reviews [3,4].

The basic idea is to consider the correlation function $\Pi_0(q^2)$ describing the propagation of the system with the quantum numbers of the hadron, in the different regions of values of the momentum q^2, where certain information on its behavior is available. The asymptotic freedom of QCD enables to present $\Pi_0(q^2)$ at $q^2 \to -\infty$ as a power series in q^{-2} and the QCD coupling α_s. On the other hand, the imaginary part of $\Pi_0(q^2)$ at $q^2 > 0$ can be described in terms of the observable hadrons. This prompts to consider the dispersion relation for the function $\Pi_0(q^2)$ [1]

$$\Pi_0(q^2) = \frac{1}{\pi}\int \frac{\mathrm{Im}\,\Pi_0(k^2)}{k^2 - q^2} dk^2 \tag{1}$$

at $q^2 \to -\infty$. The coefficients of the expansion of the left-hand side (lhs) of the function $\Pi_0(q^2)$ in powers of q^{-2} are the expectation values of the local operators constructed of the quark and gluon fields, which are called "condensates." Such presentation, known as the operator product expansion (OPE) [5] provides the perturbative expansion of the short-distance effects, while the nonperturbative physics is contained in the condensates. The usual treatment of the right-hand side (rhs) of Eq. (1) consists in "pole+continuum" presentation, in which the lowest lying pole is singled out while the higher states are approximated by the continuum. Thus, Eq. (1) ties the values of QCD condensates with the characteristics of the lowest hadronic state. Such interpretation requires that the contribution of the pole to the rhs of Eq. (1) exceeds the contribution of the continuum.

The OPE of the lhs of Eq. (1) becomes increasingly valid, when the value of $|q^2|$ increases. On the other hand, the "pole+continuum" model becomes more accurate when $|q^2|$ decreases. The important assumption is that the two presentations are close in a certain intermediate region of the values of q^2. To improve the overlap of the QCD and the phenomenological descriptions, one usually applies certain mathematical tools, i.e., the Borel transform. The Borel transformed dispersion relations (1) are known as QCD sum rules [1,2].

For example, the QCD sum rules for the nucleon provided a connection between the nucleon mass and the scalar quark condensate $\langle 0|\bar{q}q|0\rangle$ [2]. Similar relations have been obtained for the magnetic moments of the nucleons [6], etc.

Later the QCD sum rules were applied for the investigation of modified nucleon parameters in nuclear matter [7,8]. They were based on the Borel-transformed dispersion relation for the function $\Pi_m(q)$ describing the propagation of the system with the quantum numbers of the nucleon (the proton) in nuclear matter. Considering nuclear matter as a system of A nucleons with momenta p_i, one introduces the vector

$$p = \frac{\Sigma p_i}{A}, \tag{2}$$

which is thus $p \approx (m,0)$ in the rest frame of the matter. The function $\Pi_m(q)$ can be presented as $\Pi_m(q) = \Pi_m(q^2, \varphi(p,q))$ with the arbitrary function $\varphi(p,q)$ being kept constant in the dispersion relations in q^2.

The spectrum of the function $\Pi_m(q)$ is much more complicated than that of the function $\Pi_0(q^2)$. The choice of the function $\varphi(p,q)$ is dictated by attempts to separate the singularities connected with the nucleon in the matter from those connected with the properties of the matter itself. Since the latter manifest themselves as singularities in the variable $s=(p+q)^2$, the separation can be done by setting $\varphi(p,q)=(p+q)^2$ and by fixing [7–9]

$$\varphi(p,q) = (p+q)^2 \equiv s = 4E_{0F}^2 \tag{3}$$

with E_{0F} being the relativistic value of the nucleon energy at the Fermi surface.

The general form of the function Π_m can thus be presented as

$$\Pi_m(q) = q_\mu \gamma^\mu \Pi_m^q(q^2,s) + p_\mu \gamma^\mu \Pi_m^p(q^2,s) + I\Pi_m^I(q^2,s). \tag{4}$$

The in-medium QCD sum rules are the Borel-transformed dispersion relations for the components $\Pi_m^j(q^2,s)$ ($j=q,p,I$),

$$\Pi_m^j(q^2,s) = \frac{1}{\pi}\int \frac{\mathrm{Im}\,\Pi_m^j(k^2,s)}{k^2-q^2}dk^2. \tag{5}$$

It was shown in Refs. [7–10] that the "pole+continuum" model for the rhs of Eq. (5) can be used at least until we do not include the higher order terms of the density expansions of the functions $\Pi_m^j(q^2,s)$. Thus, one can expect that the characteristics of the nucleon in nuclear matter can be expressed through the in-medium values of QCD condensate.

In the lowest order of OPE the problem was approached in Refs. [7–10]. It was noticed that the condensates of the lowest dimensions ($d=3,4$) can either be calculated or expressed through the observables. The vector condensate $v_\mu(\rho)=\langle M|\Sigma_i \bar{q}^i(0)\gamma_\mu q^i(0)|M\rangle$ is proportional to the density of the matter ρ, being

$$v_\mu(\rho) = v_{N\mu}\rho, \quad v_{N\mu} = \langle N|\sum_i \bar{q}^i(0)\gamma_\mu q^i(0)|N\rangle. \tag{6}$$

Here the upper index i denotes the quark flavor. In the rest frame of the matter we get $v_\mu(\rho)=v(\rho)\delta_{\mu 0}$, $v_{N\mu}=v_N\delta_{\mu 0}$ with

$$v(\rho) = v_N\rho, \quad v_N = 3$$

being just the number of the valence quarks in the nucleon. The scalar condensate is

$$\kappa_m(\rho) = \langle M|\sum_i \bar{q}^i(0)q^i(0)|M\rangle = \kappa_0 + \kappa(\rho),$$

$$\kappa_0 = \kappa_m(0), \quad \kappa(\rho) = \kappa_N\rho + \cdots, \quad \kappa_N = \langle N|\sum_i \bar{q}^i(0)q^i(0)|N\rangle. \tag{7}$$

Here the dots denote the terms which are nonlinear in ρ. The expectation value $\langle N|\Sigma_i \bar{q}^i q^i|N\rangle$ is related to the πN sigma term $\sigma_{\pi N}$, i.e. [11],

$$\kappa_N = \langle N|\bar{u}u + \bar{d}d|N\rangle = \frac{2\sigma_{\pi N}}{m_u + m_d} \tag{8}$$

with $m_{u,d}$ standing for the current masses of the light quarks.

Turning to the condensates of dimension $d=4$, we find for the gluon condensate [7]

$$g_m(\rho) = \langle M|\frac{\alpha_s}{\pi}G^2(0)|M\rangle = g_0 + g(\rho),$$

$$g_0 = g_m(0), \quad g(\rho) = g_N\rho + \ldots \tag{9}$$

with the nucleon expectation value

$$g_N = \langle N|\frac{\alpha_s}{\pi}G^2(0)|N\rangle \approx -\frac{8}{9}m, \tag{10}$$

obtained in [12] in a model-independent way.

Also, the nonlocal condensate $\langle N|\bar{q}(0)\gamma_\mu q(x)|N\rangle$ provides the contributions of $d=4$ and those of the higher dimension. The term of the dimension $d=4$ is

$$\langle N|\bar{q}^i(0)\gamma_\mu D_\nu q^i(0)|N\rangle = \left(g_{\mu\nu} - \frac{4p_\mu p_\nu}{m^2}\right)mx_2 \tag{11}$$

with x_2 standing for the second moment of the nucleon structure function [9]. The nonlocality of the scalar operator $\bar{q}(0)q(x)$ manifests itself in the higher terms of the operator expansion. The nonlocality of the product of the gluon operators is not expected to be important because of the minor contribution of the gluon expectation value to the nucleon parameters.

The shift of the position of the nucleon pole, which in the linear approximation can be identified with the single-particle potential energy of the nucleon, was expressed as a linear combination of the condensates of the lowest dimension [7,8]. The vector and scalar expectation values appeared to be the most important ingredients. Their contributions cancelled to large extent, reproducing the familiar features of the Walecka model [13]. An alternative approach was developed in Refs. [14–16] with the dispersion relations in the time component q_0 at three-dimensional momentum $|\mathbf{q}|$ being fixed. It provided a similar result.

The lack of knowledge about the in-medium expectation values of the higher dimension became the obstacle for the development of both approaches. One of such expectation values is the scalar four-quark condensate $\langle M|\bar{q}q\bar{q}q|M\rangle$. It was noticed in Refs. [8,16] that the configuration $2\langle 0|\bar{q}q|0\rangle \times\langle N|\bar{q}q|N\rangle\rho$ (with ρ standing for the baryon density) is one of those, which composed the in-medium expectation value of the operator $\bar{q}q\bar{q}q$. In the gas approximation the expectation value of the color-singlet operator is

$$\langle M|\bar{q}q\bar{q}q|M\rangle = \langle 0|\bar{q}q\bar{q}q|0\rangle + 2\rho\langle 0|\bar{q}q|0\rangle\langle N|\bar{q}q|N\rangle + \rho\langle N|(\bar{q}q\bar{q}q)_{\mathrm{int}}|N\rangle \tag{12}$$

with the last term describing the "internal" action of the operators inside the nucleon. In the "ground-state saturation approximation" (also called "factorization approximation") formulated in Ref. [15] the last term of the rhs of Eq. (12) vanishes. This would lead to the change $\langle M|\bar{q}q\bar{q}q|M\rangle -\langle 0|\bar{q}q\bar{q}q|0\rangle=2\rho\langle 0|\bar{q}q|0\rangle\langle N|\bar{q}q|N\rangle$ of the value of the scalar four-quark condensate. Assuming this approximation one would be forced to conclude that the four-quark scalar condensate plays the crucial role in QCD sum rules, causing doubts on the convergence of OPE. The numerical results would contradict the known nuclear phenomenology [16].

There have been some attempts to get rid of the contribution of the four-quark condensates, applying the differential operators [9] or by choosing the form of the in-medium function $\Pi_m(q)$ which does not couple to the four-quark condensates [17]. However, some of the information appeared to be lost in the former case, while there still remained some unknown condensates in the latter case. Anyway, there was no consistent analysis of sum rules with the inclusion of the four-quark condensates until now. On the other hand, there are some indications that the second term of the rhs of Eq. (12) does not provide the true scale for the in-medium modification of the value of the scalar four-quark condensates. The calculations, carried out in Ref. [18] predicted strong cancellation between the second and the third terms in the rhs of Eq. (12). Also, the arguments based on chiral counting and supporting the violation of the in-medium factorization have been given in Ref. [19].

In the present paper we build and solve the QCD sum rules in nuclear matter in the gas approximation with the account of the condensates up to the dimension $d=6$. This means that we include the terms of the order $1/q^2$ of the OPE (recall that the leading OPE terms are of the order $q^4 \ln q^2$). This requires the inclusion of the four-quark condensates $\bar{u}\Gamma^X u\bar{u}\Gamma^Y u$, $\bar{d}\Gamma^X d\bar{d}\Gamma^Y d$ and $\bar{u}\Gamma^X u\bar{d}\Gamma^Y d$ with $\Gamma^{X,Y}$ standing for the basic 4×4 matrices, corresponding to the scalar, pseudoscalar, vector, pseudovector (axial), and tensor structures.

In the gas approximation the in-medium expectation value of any operator $\hat{A}$ is

$$\langle M|\hat{A}|M\rangle = \langle 0|\hat{A}|0\rangle + \rho\langle N|\hat{A}|N\rangle \tag{13}$$

with $|N\rangle$ standing for the state vector of the free unpolarized nucleon. Since we include only terms linear in ρ, we can neglect the Fermi motion of the nucleons of the matter. Thus we set

$$s = 4m^2 \tag{14}$$

in Eq. (3). Having in mind the future extension of the approach, we shall keep the dependence on s, using Eq. (14) for the specific computations.

We consider symmetric nuclear matter with an equal density of the protons and neutrons $\rho_p=\rho_n=\rho/2$.

The nucleon expectation values of the lowest dimensions can either be calculated in a model-independent way or expressed through the observables. The calculations of the four-quark condensates require model assumptions on the structure of the nucleon. The complete set of the four-quark condensates was obtained in Ref. [20] by using features of the perturbative chiral quark model (PCQM). The chiral quark model, originally suggested in Ref. [21], was developed further in Ref. [22]. In the PCQM the nucleon is treated as a system of relativistic valence quarks moving in an effective static field. The valence quarks are supplemented by a perturbative cloud of pseudoscalar mesons, in agreement with the requirements of the chiral symmetry. In [20] a simple version close to the SU(2)) flavor PCQM, which includes only the pions, has been used.

There are three types of contributions to the four-quark condensate in the framework of this approach. All four operators can act on the valence quarks. Also, four operators can act on the pion. There is also a possibility that two of the operators act on the valence quarks while the other two act on the pions. Following [20] we speak of the "interference terms" in the latter case.

To obtain the contribution of the pion cloud, we need the expectation values of the four-quark operators in pions. The latter have been deduced in Ref. [23] by using the current algebra technique. We obtain a remarkable cancellation of the pion contributions in the function $\Pi_m(q)$. This cancellation takes place in any model of the nucleon which treats the pion cloud perturbatively. Thus, the contributions of the four-quark condensates come from the terms, determined by the valence quarks only and from the interference terms.

We find the contribution of the four-quark condensate to the in-medium modification $\Pi_m-\Pi_0$ of the function Π_0 to be much smaller, then one could expect by assuming the "in-medium factorization approximation." These terms are about 4–5 times smaller than the leading ones of the OPE series. This is consistent with the hypothesis of the convergence of OPE.

Thus, we obtain three sum rules equations for the functions $\Pi_m^q(q^2,s)$, $\Pi_m^p(q^2,s)$, and $\Pi_m^I(q^2,s)$ introduced in Eq. (4). The in-medium characteristics of the nucleon, i.e., the vector self-energy Σ_v and the effective mass m^* are the unknowns of these equations. There are two unknown parameters more, i.e., the residue at the nucleon pole λ_m^{*2} and the continuum threshold W_m^2. All these characteristics will be obtained from the QCD sum rules.

The dependence of the rhs of Eq. (5) on the parameters, which are expected to be determined, is not linear (except the dependence on λ_m^{*2}). Thus, even in the gas approximation the behavior of these parameters with ρ is linear only at sufficiently small values of the density. We consider two approaches to the problem. In the linearized case we determine only the linear parts of the in-medium modifications of the hadron (nucleon) and continuum parameters. We construct the combination of the sum rules for the function $\Pi_m-\Pi_0$ in such a way, that two of the equations determine the values of Σ_v and m^*-m separately. Note that it is possible to write the equation in which the parameter m^*-m is the only unknown, only because the proton has a definite space parity. The third equation enables to find the in-medium changes of the parameters $\delta\lambda^2=\lambda_m^{*2}-\lambda_0^2$ and $\delta W^2=W_m^2-W_0^2$. We express the nucleon characteristics Σ_v and m^*-m in terms of the vector, scalar, gluon, and four-quark condensates and of the moments of the structure functions.

In the nonlinearized version we do not assume the in-medium changes of the parameters to be small. The three equations for the Borel transformed function $\Pi_m(q)-\Pi_0(q^2)$ enable to obtain the values of Σ_v, λ_m^{*2}, and W_m^2. At densities ρ of the order of the saturation value ρ_0 the values of Σ_v and m^*-m appear to coincide within 25% and 10% accuracy with the values provided by the linear version. This causes somewhat larger difference in the values of the potential energy $U(\rho)$ which still has reasonable values.

Inclusion of the four-quark condensates and of the higher moments of the structure functions diminish the OPE value of the nucleon vector self-energy Σ_v by about 25% each. As to the scalar self-energy m^*-m, the four-quark condensates provide contribution of the same order as the leading OPE term. However, this contribution is almost totally compensated by the account of the higher moments of the structure functions. Thus, the value of the m^*-m is very close to that given by the leading order of OPE.

The structure of the paper is as follows. In Sec. II we present the sum rules in a form, which is convenient for our analysis. In Secs. III and IV we calculate the contribution of the dimension $d=6$, i.e., the expansion of the nucleon structure functions and the four-quark condensates. In Secs. V–VII we present the solutions in the linearized and nonlinearized forms. We discuss and summarize the results in Sec. VIII and IX.

II. GENERAL EQUATIONS

A. Sum rules in vacuum

To make the paper self-consistent, we recall the main points of the QCD sum rules approach in vacuum [1,2]. The function $\Pi_0(q^2)$ (often referred to as "polarization operator") is presented as

$$\Pi_0(q^2) = i\int d^4x\, e^{i(qx)}\langle 0|Tj(x)\overline{j}(0)|0\rangle \tag{15}$$

with j being the three-quark local operator (often referred to as "current") with the proton quantum numbers. The usual choice is [2]

$$j(x) = \varepsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu d^c(x), \tag{16}$$

where T denotes a transpose and C is the charge conjugation matrix. The upper indices denote the colors.

The lhs of Eq. (1) is approximated by several lowest terms of OPE, i.e., $\Pi_0(q^2)\approx\Pi_0^{\rm OPE}(q^2)$. The empirical data are used for the spectral function Im $\Pi_0(q^2)$ on the rhs of Eq. (1). Namely, it is known, that the lowest lying state is the bound state of three quarks, which manifests itself as a pole in the (unknown) point $k^2=m^2$. Since the next singularity is the branching point $k^2=W_{ph}^2=(m+m_\pi)^2$, one can present

$$\text{Im}\,\Pi_0(k^2) = \lambda_N^2\delta(k^2-m^2) + f(k^2)\theta(k^2-W_{ph}^2) \tag{17}$$

with λ_N^2 being the residue at the nucleon pole. Thus, Eq. (1) takes the form

$$\Pi_0^{\rm OPE}(q^2) = \frac{\lambda_N^2}{m^2-q^2} + \frac{1}{\pi}\int_{W_{ph}^2}^{\infty}\frac{f(k^2)}{k^2-q^2}dk^2. \tag{18}$$

Of course, the detailed structure of the spectral density $f(k^2)$ cannot be resolved in such an approach. The further approximations are based on the asymptotic behavior

$$f(k^2) = \frac{1}{2i}\Delta\Pi_0^{\rm OPE}(k^2) \tag{19}$$

at $k^2\gg|q^2|$ with Δ denoting the discontinuity. The discontinuity is caused by the logarithmic contributions of the perturbative OPE terms. The usual ansatz consists in extrapolation of Eq. (19) to all the values of k^2, replacing also the physical threshold W_{ph}^2 by the unknown effective threshold W_0^2, i.e.,

$$\frac{1}{\pi}\int_{W_{ph}^2}^{\infty}\frac{f(k^2)}{k^2-q^2}dk^2 = \frac{1}{2\pi i}\int_{W_0^2}^{\infty}\frac{\Delta\Pi_0^{\rm OPE}(k^2)}{k^2-q^2}dk^2. \tag{20}$$

Thus Eq. (1) takes the form

$$\Pi_0^{\rm OPE}(q^2) = \frac{\lambda_N^2}{m^2-q^2} + \frac{1}{2\pi i}\int_{W_0^2}^{\infty}\frac{\Delta\Pi_0^{\rm OPE}(k^2)}{k^2-q^2}dk^2. \tag{21}$$

The lhs of Eq. (21) contains QCD condensates. The rhs of Eq. (21) contains three unknown parameters: m,λ_N^2, and W_0^2. The OPE becomes increasingly true when the value $|q^2|$ increases. The "pole+continuum" model is more accurate at the smaller values of $|q^2|$. Thus one can expect Eq. (21) to be true in a certain limited interval of the values of $|q^2|$. To improve the overlap of the OPE and the phenomenological description one usually applies the Borel transform defined as

$$Bf(q^2) = \lim_{Q^2,n\to\infty}\frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n f(q^2)\equiv\tilde{f}(M^2),$$

$$Q^2=-q^2,\quad M^2=Q^2/n \tag{22}$$

with M called Borel mass. It is important in the applications to the sum rules that the Borel transform eliminates the polynomials and emphasizes the contribution of the lowest state in rhs of Eq. (21) due to the relation

$$B\frac{1}{m^2-q^2} = e^{-m^2/M^2}. \tag{23}$$

The Borel-transformed form of Eq. (21) reads

$$\tilde{\Pi}_0^{\rm OPE}(M^2) = \lambda_N^2 e^{-m^2/M^2} + \frac{1}{2\pi i}\int_{W_0^2}^{\infty}dk^2\, e^{-k^2/M^2}\Delta\,\Pi_0^{\rm OPE}(k^2) \tag{24}$$

and is known as QCD sum rules. Actually, there are two sum rules for the structures Π_0^q and Π_0^I of the function $\Pi_0(q)=q_\mu\gamma^\mu\Pi_0^q(q^2)+I\Pi_0^I(q^2)$ with I standing for the unit matrix.

It appeared to be more convenient to work with Eq. (24) multiplied by the numerical factor $32\pi^4$. The two sum rules for the nucleon in vacuum can be presented in the form [2]

$$L_0^q(M^2,W_0^2) = \Lambda_0(M^2), \tag{25}$$

$$L_0^I(M^2,W_0^2) = m\Lambda_0(M^2), \tag{26}$$

with

$$\Lambda_0(M^2)=\lambda_0^2 e^{-m^2/M^2}. \tag{27}$$

Here $\lambda_0^2=32\pi^4\lambda_N^2$,

$$L_0^q(M^2,W_0^2)=32\pi^4\left(\tilde{\Pi}_0^{q,\mathrm{OPE}}(M^2)-\frac{1}{2\pi i}\int_{W_0^2}^{\infty}dk^2 \times e^{-k^2/M^2}\Delta\,\Pi_0^{q,\mathrm{OPE}}(k^2)\right),$$

$$L_0^I(M^2,W_0^2)=32\pi^4\left(\tilde{\Pi}_0^{I,\mathrm{OPE}}(M^2)-\frac{1}{2\pi i}\int_{W_0^2}^{\infty}dk^2 \times e^{-k^2/M^2}\Delta\,\Pi_0^{I,\mathrm{OPE}}(k^2)\right). \tag{28}$$

The lhs of Eqs. (25) and (26) [2,6] have been obtained by including the condensates of dimension $d=8$, i.e., with the account of the terms of the order $1/q^4$ in OPE of the functions $\Pi_0^{q,\mathrm{OPE}}$,

$$L_0^q(M^2,W_0^2)=\frac{M^6E_2}{L^{4/9}}+\frac{bE_0M^2}{4L^{4/9}}+\frac{4}{3}a^2L^{4/9}-\frac{1}{3}\frac{\mu_0^2}{M^2}a^2, \tag{29}$$

$$L_0^I(M^2,W_0^2)=2aM^4E_1-\frac{ab}{12}+\frac{272}{81}\frac{\alpha_s}{\pi}a^3\frac{1}{M^2} \tag{30}$$

with the traditional notations $a=-(2\pi)^2\langle 0|\bar{q}q|0\rangle=-2\pi^2\kappa_0$ (we assumed the isotopic invariance $\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle=\langle 0|\bar{q}q|0\rangle$), $b=(2\pi)^2g_0$, $\mu_0^2=0.8\ \mathrm{GeV}^2$. Here E_i are the functions of the ratio W_0^2/M^2: $E_i=E_i(W_0^2/M^2)$. They are given by the formulas

$$E_0(x)=1-e^{-x},\quad E_1(x)=1-(1+x)e^{-x}, \tag{31}$$

$$E_2(x)=1-\left(1+x+\frac{x^2}{2}\right)e^{-x}.$$

The factor

$$L(M^2)=\frac{\ln M^2/\Lambda^2}{\ln \nu^2/\Lambda^2} \tag{32}$$

accounts for the anomalous dimension, i.e., the most important corrections of the order α_s enhanced by the "large logarithms." In Eq. (32) $\Lambda=\Lambda_{\mathrm{QCD}}=0.15\ \mathrm{GeV}$, while $\nu=0.5\ \mathrm{GeV}$ is the normalization point of the characteristic involved. Note that the two last terms on the rhs of Eq. (29) originate from the four-quark condensates $\langle 0|\bar{u}\Gamma^X u\bar{u}\Gamma^X u|0\rangle$ and can be expressed through the single term $(\langle 0|\bar{q}q|0\rangle)^2$ only in framework of the factorization hypothesis [1,2]. Also, the last term on the rhs of Eq. (30) is the six-quark condensate, evaluated in the same approximation.

The matching of the lhs and rhs of Eqs. (25) and (26) have been achieved [2,6] in the domain

$$0.8\ \mathrm{GeV}^2<M^2<1.4\ \mathrm{GeV}^2$$

providing the values of the vacuum parameters

$$\lambda_0^2=1.9\ \mathrm{GeV}^6,\quad W_0^2=2.2\ \mathrm{GeV}^2 \tag{33}$$

if $m=0.94$ GeV.

B. Sum rules in nuclear matter

The OPE terms of the polarization operator in nuclear matter

$$\Pi_m(q)=i\int d^4x\, e^{i(qx)}\langle M|Tj(x)\overline{j}(0)|M\rangle \tag{34}$$

contains the in-medium values of QCD condensates. Some of these condensates vanish in the vacuum, obtaining nonzero values only in the medium. The other ones just change their values compared to the vacuum ones.

The spectrum of the function $\Pi_m(q)$ is much more complicated, than that of the vacuum function $\Pi_0(q^2)$. However, [7–10] the spectrum of the function $\Pi_m(q^2,s)$ at fixed value of s can be described by the "pole+continuum" model at least until we include the terms of the order ρ^2 in the OPE of $\Pi_m(q^2,s)$.

The description of the nucleon pole is based on the general expression for the propagator

$$G_N^{-1}=(G_N^0)^{-1}-\Sigma \tag{35}$$

with $G_N^0=(q_\mu\gamma^\mu-m)^{-1}$ being the propagator of the free nucleon, while

$$\Sigma=q_\mu\gamma^\mu\Sigma_q+\frac{1}{m}p_\mu\gamma^\mu\Sigma_p+\Sigma_s \tag{36}$$

is the general form of the self-energy of the nucleon in nuclear mater. In the kinematics, determined by Eq. (3) we obtain

$$G_N=Z\cdot\frac{q_\mu\gamma^\mu-p_\mu\gamma^\mu(\Sigma_v/m)+m^*}{q^2-m_m^2} \tag{37}$$

with

$$\Sigma_v=\frac{\Sigma_p}{1-\Sigma_q},\quad m^*=\frac{m+\Sigma_s}{1-\Sigma_q}. \tag{38}$$

The new position of the nucleon pole is

$$m_m^2=\frac{(s-m^2)\Sigma_v/m-\Sigma_v^2+m^{*2}}{1+\Sigma_v/m}, \tag{39}$$

while

$$Z=\frac{1}{(1-\Sigma_q)(1+\Sigma_v/m)}. \tag{40}$$

Thus, we shall present the dispersion relations for the functions $\Pi_m^i(q^2,s)(i=q,p,I)$ determined by Eq. (4) in the form

$$\Pi_m^{i,\mathrm{OPE}}(q^2,s) = \frac{Z\lambda_m^2 b_i}{m_m^2 - q^2} + \frac{1}{2\pi i}\int_{W_m^2}^{\infty} \frac{\Delta_{k^2}\Pi^{i,\mathrm{OPE}}(k^2,s)}{k^2 - q^2} \quad (41)$$

with $b_q=1$, $b_p=-\Sigma_v$, $b_I=m^*$. The Borel-transformed sum rules take the form

$$L_m^q(M^2,W_m^2) = \Lambda_m(M^2), \quad (42)$$

$$L_m^p(M^2,W_m^2) = -\Sigma_v \Lambda_m(M^2), \quad (43)$$

$$L_m^I(M^2,W_m^2) = m^* \Lambda_m(M^2), \quad (44)$$

with

$$\Lambda_m(M^2) = \lambda_m^{*2} e^{-m_m^2/M^2}. \quad (45)$$

Here

$$\lambda_m^{*2} = \lambda_m^2 \cdot Z \quad (46)$$

is the effective value of the residue in nuclear matter.

We present the lhs of Eqs. (42)–(44) as

$$L_m^i = \ell_m^i + u_m^i + \omega_m^i \quad (47)$$

with $\ell_m^i(M^2,W_m^2)$ standing for the lowest order OPE terms, $u_m^i(M^2,W_m^2)$ denoting the contribution of the higher moments of the structure functions, while $\omega_m^i(M^2)$ provides the contribution of the four-quark condensates. We write, correspondingly, $L_0^i=\ell_0^i+\omega_0^i$ for the lhs of the vacuum sum rules presented by Eqs. (25) and (26). We present also

$$\ell^i(M^2,W_m^2) = \ell_m^i(M^2,W_m^2) - \ell_0^i(M^2,W_m^2),$$

$$\omega^i(M^2) = \omega_m^i(M^2) - \omega_0^i(M^2). \quad (48)$$

In these notations the lowest order OPE terms are

$$\ell_0^q = \frac{M^6 E_{2m}}{L^{4/9}} + \frac{1}{4}\frac{bM^2 E_{0m}}{L^{4/9}}, \quad \ell_0^p = 0, \quad \ell_0^I = 2aM^4 E_{1m}, \quad (49)$$

and

$$\ell^q = f_v^q(M^2,W_m^2)v(\rho) + f_g^q(M^2,W_m^2)g(\rho),$$

$$\ell^p = f_v^p(M^2,W_m^2)v(\rho),$$

$$\ell^I = f_\kappa^I(M^2,W_m^2)\kappa(\rho), \quad (50)$$

with

$$f_v^q = -\frac{8\pi^2}{3}\frac{(s-m^2)M^2E_{0m} - M^4E_{1m}}{mL^{4/9}},$$

$$f_g^q = \frac{\pi^2 M^2 E_{0m}}{L^{4/9}},$$

$$f_v^p = -\frac{8\pi^2}{3}\frac{4M^4E_{1m}}{L^{4/9}},$$

$$f_\kappa^I = -4\pi^2 M^4 E_{1m}. \quad (51)$$

The functions $v(\rho)$, $\kappa(p)$, $g(\rho)$ are determined by Eqs. (6), (7), and (9). The notation $E_{km}(k=0,1,2)$ means that the functions depend on the ratio W_m^2/M^2. Actually, the higher moments of the structure functions of the nucleon have been neglected in Eqs. (50) and (51).

III. ACCOUNTING FOR *x*-DEPENDENCE OF THE OPERATORS. CONTRIBUTIONS OF THE HIGHER MOMENTS AND OF THE HIGHER TWISTS OF THE STRUCTURE FUNCTIONS

The calculation of the function $\Pi_m(q^2,s)$ defined by Eq. (34) is based on the presentation of the single-quark propagator in the medium

$$\langle M|Tq_\alpha^i(x)\bar{q}^i_\beta(0)|M\rangle = G_{\alpha\beta}(x) - \tfrac{1}{4}\langle M|\bar{q}^i(0)\gamma_\mu q^i(x)|M\rangle\gamma^\mu_{\alpha\beta} - \tfrac{1}{4}\langle M|\bar{q}^i(0)q^i(x)|M\rangle\delta_{\alpha\beta} \quad (52)$$

with $G(x)=(ix_\mu\gamma_\mu)/(2\pi^2x^4)$ being the free propagator of the quark in the chiral limit. Recall that i denotes the light quark flavor. In the lowest orders of OPE two of the quarks are described by the free propagators and only one of the quarks is presented by the second or the third term of the rhs of Eq. (52).

At $x=0$ the matrix elements in the second and third terms on the rhs are just the vector and scalar condensates defined by Eqs. (6) and (7). The contribution of the bilocal configurations can be expressed in terms of the higher moments and twists of the nucleon structure functions [9].

The bilocal operators on the rhs of Eq. (52) are not gauge invariant. The gauge invariant expression, achieved by substitution [24]

$$q^i(x) = q^i(0) + x_\alpha D_\alpha q^i(0) + \tfrac{1}{2}x_\alpha x_\beta D_\alpha D_\beta q^i(0) + \cdots \quad (53)$$

with D_α standing for the covariant derivatives, provides the infinite set of the local condensates. The expectation values depend on the variables (px) and x^2. In the gas approximation we only need the nucleon matrix elements Eq. (13). For the vector structure the general form is

$$\theta_\mu^i(x) = \langle N|\bar{q}^i(0)\gamma_\mu q^i(x)|N\rangle = \frac{p_\mu}{m}\phi_a^i((px),x^2) + ix_\mu m\phi_b^i((px),x^2) \quad (54)$$

with $q^i(x)$ defined by Eq. (53).

Expansion in powers of x^2 corresponds to the expansion of the function $\Pi_m(q)$ in powers of q^2. To obtain the terms of the order q^{-2} it is sufficient to include two lowest terms of the expansions in powers of x^2. One can present [9,25]

$$\phi_{a(b)}^i((px),x^2) = \int_0^1 d\alpha\, e^{-i\alpha(px)} f_{a(b)}^i(\alpha,x^2) \quad (55)$$

with

$$f^i_{a(b)}(\alpha,x^2) = \eta^i_{a(b)}(\alpha) + \tfrac{1}{8}x^2m^2\xi^i_{a(b)}(\alpha). \tag{56}$$

Here $\eta^i_a(\alpha)=f^i_a(\alpha,0)$ is the contribution of the quarks with the flavor i to the asymptotics of the nucleon structure function $\eta(\alpha)=\eta^u_a(\alpha)+\eta^d_a(\alpha)$, normalized by the condition

$$\int_0^1 d\alpha\ \eta(\alpha) = 3 \tag{57}$$

with the rhs presenting just the number of the valence quarks in the nucleon. Thus, expansion of the function $\varphi^i_a(px)=\phi^i_a((px),0)$ in powers of (px) is expressed through the moments of the distributions $\eta^i_a(\alpha)$. The moments are well known—at least, those, which are numerically important. Also, the first moment of the distribution $\xi_a(\alpha)=\xi^u_a(\alpha)+\xi^d_a(\alpha)$,

$$\xi = \int_0^1 (\xi^u_a(\alpha)+\xi^d_a(\alpha))d\alpha \approx -0.3, \tag{58}$$

was calculated in Ref. [26] by QCD sum rules method. The moments of the function $\eta^i_b(\alpha)$ can be obtained by using the equations of motion $D_\alpha\gamma^\alpha q^i(x)=m_i q^i(x)$. Thus, in the chiral limit [9]

$$\langle\varphi^i_b\rangle = \tfrac{1}{4}\langle\varphi^i_a\alpha\rangle,$$

$$\langle\varphi^i_b\alpha\rangle = \tfrac{1}{5}\langle\varphi^i_a\alpha^2\rangle - \tfrac{1}{4}\langle\xi^i\rangle,$$

$$\langle\xi^i_b\rangle = \tfrac{1}{6}\langle\xi^i_a\alpha\rangle. \tag{59}$$

Here we denoted

$$\langle f\rangle = \int_0^1 d\alpha\ f(\alpha) \tag{60}$$

for any function $f(\alpha)$.

Note that the nonlocality of the scalar condensate, i.e., of the last term on the rhs of Eq. (52) does not manifest itself in terms up to $1/q^2$. The first derivative in (px), as well as all the derivatives of the odd order vanish in the chiral limit due to QCD equation of motion. The next to leading order of the expansion in powers of x^2 vanishes due to certain cancellations [8] as well as in the case of vacuum [2] for the particular choice of the operator $j(x)$ presented by Eq. (16). We do not account for the nonlocality of the gluon operators, since the gluon expectation values play the minor role in our sum rules.

Now we are ready to calculate the contributions $\Pi_{nl}(q)$ of the nonlocal vector condensate to the polarization operator $\Pi_m(q)$. We express Π_{nl} in terms of the proton expectation values $\theta^i_\mu(x)=\langle p|\overline{q^i}(0)\gamma_\mu q^i(x)|p\rangle$. Employing the isotopic invariance we obtain

$$\Pi_{nl}(q) = \frac{4i}{\pi^4}\int \frac{d^4x}{x^8}\left(x^2\frac{\hat\theta^u+\hat\theta^d}{2} + \hat x(x,\theta^u+\theta^d)\right) \times e^{i(qx)}\cdot\rho, \tag{61}$$

contributing to the vector structures $\hat q$ and $\hat p$ of the polarization operator $\Pi_m(q)$. Here we denoted

$$\hat a = a_\mu\gamma^\mu. \tag{62}$$

Using Eq. (54) we obtain

$$\Pi_{nl}(q) = \Pi^a_{nl}(q) + \Pi^b_{nl}(q) \tag{63}$$

with

$$\Pi^a_{nl}(q) = \frac{4i}{\pi^4}\int\frac{d^4x}{x^8}\left(x^2\hat p\frac{\phi^u_a+\phi^d_a}{2} + \hat x(xp) \times(\phi^u_a+\phi^d_a)\right)e^{i(qx)}\cdot\rho,$$

$$\Pi^b_{nl}(q) = -\frac{6m}{\pi^4}\int\frac{d^4x}{x^6}\hat x(\phi^u_b+\phi^d_b)e^{i(qx)}\cdot\rho. \tag{64}$$

We present each of the terms $\Pi^{a(b)}_{nl}$ as the sum $\Pi^{1a(b)}_{nl}+\Pi^{2a(b)}_{nl}$, corresponding to the two terms of the expansion in powers of x^2 in Eq. (56). In particular, the contribution Π^{1a}_{nl}, which is numerically most important, can be presented as

$$\Pi^{1a}_{nl}(q) = \left[\frac{1}{6m\pi^2}\int_0^1 d\alpha\ \hat q'(pq')\ln\left(\frac{-q'^2}{\Lambda_c^2}\right)\eta_a(\alpha) + \frac{\hat p}{3m\pi^2}\int_0^1 d\alpha\ q'^2\ln\left(\frac{-q'^2}{\Lambda_c^2}\right)\eta_a(\alpha)\right]\rho \tag{65}$$

with $\eta_a(\alpha)=\eta^u_a(\alpha)+\eta^d_a(\alpha)$, $q'=q-p\alpha$ (see Appendix A). From Eqs. (3) and (14) one finds $(pq)=(s-m^2-q^2)/2$. The cutoff Λ_c will be eliminated by the Borel transform.

Presenting $q'^2=-(1+\alpha)(Q^2+A^2)$ where $Q^2=-q^2$, $A^2(\alpha)=\alpha(s-m^2-m^2\alpha)/(1+\alpha)$ we see that the second term of the expression $\ln q'^2=\ln q^2+\ln(q'^2/q^2)$ does not have a cut, running to infinity, but has a finite cut. This singularity requires a special treatment in QCD sum rules. On the other hand, it is the singularity in the u channel of the interaction of the baryon current with the quark of the nucleon of matter. It corresponds to the exchange terms on the rhs of the sum rules. In this paper we neglect the nonlocal singularities, thus claiming for the description of the nucleon in the Hartree approximation. However, we include the regular smooth dependence on the higher moments.

The contributions Π^{1b}_{nl} and $\Pi^{2a(b)}_{nl}$ can be expressed in terms of the moments of the functions η^i_b and $\xi^i_{a,b}$ (see Appendix A). Since the higher moments of the functions $\eta^i_a(\alpha)$, as well as the value of ξ are small, we include only the lowest moments of the functions $\eta^i_b(\alpha)$ and the first moments of the functions $\xi^i_a(\alpha)$—Eq. (59). The last of the equalities (59) enables us to neglect the contribution of the functions $\xi^i_b(\alpha)$.

Finally, the higher moments and higher twists of the nucleon structure functions provide the contributions u^i to the lhs L_m^i of the sum rules—Eq. (47),

$$u^q(M^2) = u_N^q(M^2)\rho,$$

$$u_N^q(M^2) = \frac{8\pi^2}{3L^{4/9}m}\left[-\frac{5}{2}m^2M^2E_{0m}\langle\eta\alpha\rangle + \frac{3}{2}m^2(s-m^2)\langle\xi\rangle\right],$$

$$u^p(M^2) = u_N^p(M^2)\rho,$$

$$u_N^p(M^2) = \frac{8\pi^2}{3L^{4/9}}\left[-5(M^4E_{1m} - (s-m^2)M^2E_{0m})\langle\eta\alpha\rangle - \frac{12}{5}m^2M^2E_{0m}\langle\eta\alpha^2\rangle + \frac{18}{5}m^2M^2E_{0m}\langle\xi\rangle\right],$$

$$u^I(M^2) = 0. \tag{66}$$

Here we denote $L = L(M^2)$. Parameter ξ is defined by Eq. (58).

IV. CONTRIBUTION OF THE FOUR-QUARK CONDENSATES

The four-quark expectation values contribute to the OPE terms $1/q^2$ of the function $\Pi_m(q)$. Now only one quark is determined by the free propagator $G_q(x)$. Two other quarks are described by the last term of the two-quark propagator,

$$\begin{aligned}\langle M|Tq_\alpha(x)\bar{q}_\beta(0)q_\rho(x)\bar{q}_\tau(0)|M\rangle &= [G_q(x)]^2 - \tfrac{1}{4}\langle M|\bar{q}\Gamma^X q|M\rangle G_q(x)\Gamma^X_{\alpha\beta} \\ &- \tfrac{1}{4}\langle M|\bar{q}\Gamma^X q|M\rangle G_q(x)\Gamma^X_{\rho\tau} + \tfrac{1}{16}\langle M|\bar{q}\Gamma^X q\bar{q}\Gamma^Y q|M\rangle\Gamma^X_{\alpha\beta}\Gamma^Y_{\rho\tau}\end{aligned} \tag{67}$$

with $\Gamma^{X,Y}$ being the basic 4×4 matrices

$$\Gamma^I = I, \quad \Gamma^{Ps} = \gamma_5, \quad \Gamma^V = \gamma_\mu, \tag{68}$$

$$\Gamma^A = \gamma_\mu\gamma_5, \quad \Gamma^T = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu),$$

acting on the Lorentz indices of the quark operators. Equation (67) is analogous to Eq. (52) for the single-quark propagator. We did not display the color indices in Eq. (67), keeping in mind that the quark operators are color antisymmetric—Eq. (16). One can write an equation similar to Eq. (67) for the quarks of different flavors.

Introducing the notations

$$H_m^{XY}(\rho) = \langle M|\bar{u}\Gamma^X u\bar{u}\Gamma^Y u|M\rangle, \quad R_m^{XY}(\rho) = \langle M|\bar{d}\Gamma^X d\bar{u}\Gamma^Y u|M\rangle \tag{69}$$

we write in the gas approximation

$$H_m^{XY}(\rho) = H_m^{XY}(0) + \rho h^{XY}, \quad R_m^{XY}(\rho) = R_m^{XY}(0) + \rho r^{XY}. \tag{70}$$

The characteristics h^{XY} and r^{XY} can be presented as

$$\begin{aligned}h^{XY} &= \tfrac{5}{6}(\langle 0|\bar{u}\Gamma^X u|0\rangle\langle N|\bar{u}\Gamma^Y u|N\rangle + \langle 0|\bar{u}\Gamma^Y u|0\rangle\langle N|\bar{u}\Gamma^X u|N\rangle) \\ &+ \langle N|(\bar{u}\Gamma^X u\cdot\bar{u}\Gamma^Y u)_{\text{int}}|N\rangle;\end{aligned} \tag{71}$$

$$\begin{aligned}r^{XY} &= \tfrac{2}{3}(\langle 0|\bar{d}\Gamma^X d|0\rangle\langle N|\bar{u}\Gamma^Y u|N\rangle + \langle 0|\bar{u}\Gamma^Y u|0\rangle\langle N|\bar{d}\Gamma^X d|N\rangle) \\ &+ \langle N|(\bar{d}\Gamma^X d\cdot\bar{u}\Gamma^Y u)_{\text{int}}|N\rangle.\end{aligned} \tag{72}$$

Here the lower index "int" means that all the four operators are acting inside the nucleon. The coefficients 5/6 and 2/3 on rhs of Eqs. (71) and (72) present the weights of the color-antisymmetric states—see Appendix B. These equations are consistent with Eq. (13) if we assume that some of the single-particle operators which compose the operator $\hat{A}$ can act on the vacuum state vector—see also [10].

The contribution of the four-quark expectation values to the in-medium change of the polarization operator can be written as

$$(\Pi)_{4q} = (\Pi_m)_{4q} - (\Pi_0)_{4q} = \frac{\rho}{q^2}\left(\sum_{X,Y}\mu_{XY}h^{XY} + \sum_{X,Y}\tau_{XY}r^{XY}\right). \tag{73}$$

Here μ_{XY} and τ_{XY} are certain matrices in Dirac space. They can be obtained by using the general expression for the function $\Pi_m(q)$ presented in [16]

$$\mu_{XY} = \frac{\theta_Y}{16}\text{Tr}(\gamma_\alpha\Gamma^X\gamma_\beta\Gamma^Y)\gamma_5\gamma^\alpha\hat{q}\gamma^\beta\gamma_5,$$

$$\tau_{XY} = \frac{\theta_Y}{4}\text{Tr}(\gamma_\alpha\hat{q}\gamma_\beta\Gamma^Y)\gamma_5\gamma^\alpha\Gamma^X\gamma^\beta\gamma_5, \quad \hat{q} = q_\mu\gamma^\mu. \tag{74}$$

Here $\theta_Y = 1$ if Γ^Y has a vector or tensor structure, while $\theta_Y = -1$ in the scalar, pseudoscalar and axial cases. The sign is determined by that of the commutator between matrix Γ^Y and the charge conjugation matrix C—Eq. (16).

The products $\mu_{XY}h^{XY}$ obtain nonzero values if the matrices Γ^X and Γ^Y have the same Lorentz structure. In this case all the structures presented by Eq. (68) contribute to $(\Pi_m)_{4q}$. The products $\tau_{XY}r^{XY}$ do not turn to zero only if Γ^Y has a vector or axial structure. In the latter case Γ^X should be an axial matrix as well. In the former case Γ^X can be either Lorentz scalar or Lorentz vector.

We denote $h^{XX} = h^X$, $\mu_{XX} = \mu_X$, $r^{XX} = r^X$, $\tau_{XX} = \tau_X$ for the similar Lorentz structures X and Y. The scalar and pseudoscalar expectation values are Lorentz scalars. Thus, their contributions can be expressed through single parameters. The latter is true also for the scalar-vector expectation value r^{SV}. We obtain

$$\mu_S = -\frac{\hat{q}}{2}, \quad \mu_{Ps} = \frac{\hat{q}}{2}, \quad (\tau_{SV})_\mu = -2q_\mu. \tag{75}$$

In the other channels the four-quark condensates have more complicated structure. In the vector and axial channels

$$h_{\mu\nu}^{V(A)} = a_h^{V(A)}g_{\mu\nu} + b_h^{V(A)}\frac{p_\mu p_\nu}{m^2},$$

$$r_{\mu\nu}^{V(A)} = a_r^{V(A)} g_{\mu\nu} + b_r^{V(A)} \frac{p_\mu p_\nu}{m^2}. \tag{76}$$

Using Eqs. (71) and (72) we obtain

$$\mu_V h^V = -a_h^V \hat{q} - b_h^V \frac{\hat{p}(pq)}{m^2}, \quad \mu_A h^A = a_h^A \hat{q} + b_h^A \frac{\hat{p}(pq)}{m^2}, \tag{77}$$

and

$$\tau_V r^V = (-10a_r^V - 2b_r^V)\hat{q} - 2b_r^V \frac{\hat{p}(pq)}{m^2},$$

$$\tau_A r^A = (-6a_r^A - 2b_r^A)\hat{q} + 2b_r^A \frac{\hat{p}(pq)}{m^2}. \tag{78}$$

In the tensor channel

$$h_{\mu\nu,\rho\tau}^T = a_h^T s_{\mu\nu,\rho\tau} + b_h^T t_{\mu\nu,\rho\tau} \tag{79}$$

with

$$s_{\mu\nu,\rho\tau} = g_{\mu\rho} g_{\nu\tau} - g_{\mu\tau} g_{\nu\rho},$$

$$t_{\mu\nu,\rho\tau} = \frac{1}{m^2}(p_\mu p_\rho g_{\nu\tau} + p_\nu p_\tau g_{\mu\rho} - p_\mu p_\tau g_{\nu\rho} - p_\nu p_\rho g_{\mu\tau}), \tag{80}$$

and

$$\mu_T h^T = b_h^T \left(-\frac{\hat{q}}{2} + \frac{2\hat{p}(pq)}{m^2} \right). \tag{81}$$

The complete set of the four-quark expectation values $a_r^X, b_r^X, a_h^X, b_h^X$ was obtained in Ref. [20] by using the approach motivated by the perturbative chiral quark model (PCQM) [21,22]. As explained in Introduction, the valence quarks are treated as the relativistic constituent quarks, while the sea quarks are approximated by those of perturbatively treated pions.

There are three types of contributions to the expectation values in the approach of Ref. [20]. All four operators can act on the constituent quarks. Also, four operators can act on the pions. There are also the "interference terms" with two of the operators acting on the valence quarks while the other two act on the pions.

The contribution, corresponding to all four operators acting on pions is expressed in terms of the pion expectation values of the four-quark operators. The distribution of the pion field is determined by the PCQM. The contribution is

$$(\Pi_{4q})_{\text{pions}} = \frac{1}{16q^2}\left(\sum_{X,\alpha} \langle \pi^\alpha | \mu_X \bar{u}\Gamma^X u \bar{u}\Gamma^X u + 4\tau_X \bar{d}\Gamma^X d\bar{u}\Gamma^X u | \pi^\alpha \rangle \right) \frac{\partial \Sigma}{\partial m_\pi^2} \tag{82}$$

with Σ standing for the sum of the self-energy and pion-exchange contributions, while "α" denotes the pion isotopic states. Using the values of the four-quark operators averaged over pions [23], we find that

$$\sum_{X,\alpha} \mu_X \langle \pi^\alpha | \bar{u}\Gamma^X u \bar{u}\Gamma^X u | \pi^\alpha \rangle + 4\sum_{X,\alpha} \tau_X \langle \pi^\alpha | \bar{d}\Gamma^X d\bar{u}\Gamma^X u | \pi^\alpha \rangle = 0. \tag{83}$$

Due to Eq. (83) we can omit the contributions to the second terms of the rhs of Eqs. (71) and (72) which are caused by the pions only. Since the terms $\langle 0|\bar{q}q|0\rangle\langle \pi|\bar{q}q|\pi\rangle$ emerge as the ingredients of the expectation values $\langle \pi|\bar{q}q\bar{q}q|\pi\rangle$ [23], the cancellation (83) influences the first terms of rhs of Eqs. (71) and (72) as well. Thus, in order to calculate the rhs of Eq. (73) it is sufficient to substitute for the operators with the same flavor,

$$h^X = 2 \cdot \tfrac{5}{6}\langle 0|\bar{u}\Gamma^X u|0\rangle\langle N|(\bar{u}\Gamma^X u)_v|N\rangle + \langle N|(\bar{u}\Gamma^X u\bar{u}\Gamma^X u)_1|N\rangle. \tag{84}$$

Here the lower index "v" means that the operators act on the valence quarks only. The lower index "1" corresponds to the sum of the term in which all four operators act on the valence quarks and the term in which two of the operators act on the valence quarks while the other two act on pions. Of course, the first term on the rhs of Eq. (84) obtains a nonvanishing value only in the scalar case $\Gamma^X = I$.

The expectation values of the operators of different flavors, providing nonvanishing contributions to the rhs of Eq. (72) are the scalar-vector condensate,

$$r_\mu^{SV} = 2 \cdot \frac{2}{3}\langle 0|\bar{d}d|0\rangle\langle N|\bar{u}\gamma_\mu u|N\rangle + \langle N|(\bar{d}d\bar{u}\gamma_\mu u)_1|N\rangle \tag{85}$$

and

$$r_{\mu\nu}^X = \langle N|(\bar{d}\Gamma_\mu^X d\bar{u}\Gamma_\nu^X u)_1|N\rangle, \tag{86}$$

with X standing for vector or axial structures. In the first term on the rhs of Eq. (85) the nonlocality of the vector condensate is included.

The meaning of the lower index "1" is the same as in Eq. (84).

Using the complete set of the nucleon four-quark expectation values [20], we obtain

$$(\Pi)_{4q} = \left(A_{4q}^q \frac{\hat{q}}{q^2} + A_{4q}^p \frac{(pq)}{m^2}\frac{\hat{p}}{q^2} + A_{4q}^I m \frac{I}{q^2} \right) \frac{a}{(2\pi)^2}\rho \tag{87}$$

with the coefficients

$$A_{4q}^q = 0.25, \quad A_{4q}^p = -0.57, \quad A_{4q}^I = 1.90 \tag{88}$$

and with the conventional notation

$$a = -(2\pi)^2\langle 0|\bar{u}u|0\rangle. \tag{89}$$

We use the value $\langle 0|\bar{u}u|0\rangle = (-241 \text{ MeV})^3$, corresponding to $a = 0.55\ \text{GeV}^3$, employed in Ref. [6]. Note that a is just a convenient scale for presentation of the results. It does not reflect the chiral properties of Π_{4q}.

We can trace the structure of the three terms, composing Π_{4q} determined by Eq. (87)—see Appendix C. The $\hat{q}$ term results mainly as the sum of the expectation value of the product of the four u-quark operators, described by the first (factorized) term on the rhs of Eq. (84), and that of the product of two u and two d quark operators in the axial channel—Eq. (86). The $\hat{p}$ term is determined mostly by the expectation value (86) in the vector channel. The contribution proportional to the unit matrix I is determined by the scalar-vector expectation value (85). It is dominated by the first (factorized) term on the rhs, while the second term diminished the value by about 30%.

The contributions of the four-quark condensates to the lhs of the Borel transformed sum rules (42)–(44) are

$$\omega^i = \omega_N^i \rho, \quad \omega_N^i = A_{4q}^i f_{4q}^i,$$

$$f_{4q}^q = -8\pi^2 a, \quad f_{4q}^p = -8\pi^2 \frac{s-m^2}{2m} a, \quad f_{4q}^I = -8\pi^2 m a. \tag{90}$$

Note that we can modify our model approach by employing a more sophisticated model for the pion. Namely, among the interference terms contributing to the four-quark condensates, there is so-called "vertex interference," in which one of the vertices of the self-energy of the valence quark is replaced by the four-quark operator. Some of such terms contain the matrix elements $\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle$ and $\langle 0|\bar{d}\gamma_5 u|\pi^+\rangle$, contributing to the expectation values $\langle N|\bar{u}\gamma_5 d\bar{d}\gamma_5 u|N\rangle$, being connected with the matrix elements $\langle N|\bar{d}\Gamma^X d\bar{u}\Gamma^X u|N\rangle$ of all structures Γ^X by the Fierz transform. On the other hand, they depend on the values of the quark masses, since $\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle = [-i\sqrt{2}\,F_\pi M_\pi^2/(m_u+m_d)]$ with M_π (F_π) denoting the mass (decay constant) of pion. In a somewhat straightforward approach one substitutes the current quark masses. Following more sophisticated models of the pions [27] one should substitute the constituent quark masses, thus obtaining much smaller values. In the latter approach

$$A_{4q}^q = -0.11, \tag{91}$$

while the values of A_{4q}^p and A_{4q}^I remain unchanged. In this case we find the larger cancellation between the first term on the rhs of Eq. (84) and the contribution coming from the rhs of Eq. (86). The latter is dominated by the vector expectation values.

V. SUM RULES IN NUCLEAR MATTER

Actually, we shall solve the sum rules for the difference of the operators in nuclear matter and in vacuum,

$$L^q(M^2, W_m^2, W_0^2) = \Lambda_m(M^2) - \Lambda_0(M^2), \tag{92}$$

$$L^p(M^2, W_m^2) = -\Sigma_v \Lambda_m(M^2), \tag{93}$$

$$L^I(M^2, W_m^2, W_0^2) = m^* \Lambda_m(M^2) - m\Lambda_0(M^2) \tag{94}$$

with $L^i(M^2, W_m^2, W_0^2) = L_m^i(M^2, W_m^2) - L_0^i(M^2, W_0^2)$. The ingredients of Eqs. (92)–(94) are defined by Eqs. (25), (26), (42)–(44), and (47).

Note that we took into account the anomalous dimensions only for the leading OPE terms $q^2 \ln q^2$ and $\ln q^2$, neglecting the anomalous dimensions of the $1/q^2$ OPE terms.

Although the anomalous dimensions of the four-quark condensates are known [28], the anomalous dimension matrix is not diagonal in the basis determined by Eq. (68). The calculation of this matrix in our basis is a separate work which will be presented in further publications. We use the nucleon structure functions presented in Ref. [29], which include the anomalous dimensions of the structure functions.

We solve Eqs. (92)–(94) in the same interval of the values M^2 as it has been done in vacuum.

Since Eqs. (92)–(94) are not linear, the behavior of the in-medium parameters is not linear in ρ even if we limit ourselves to the gas approximation. However, if the density ρ is small enough, we can try the linear approximation, assuming the linear dependence of the nucleon characteristics on the density of matter.

VI. SUM RULES IN THE LINEAR APPROXIMATION

It is instructive to express the density in units of the observable saturation density

$$\rho_0 = 0.17 \text{ fm}^{-3} = 1.3 \times 10^{-3} \text{ GeV}^3. \tag{95}$$

The parameters which will be determined from the sum rules can be presented as

$$\Sigma_v = a_v \frac{\rho}{\rho_0}, \quad m^* = m + a_s \frac{\rho}{\rho_0}, \quad \delta\lambda^2 = \lambda_m^{*2} - \lambda_0^2 = a_\lambda \frac{\rho}{\rho_0},$$

$$\delta W^2 = W_m^2 - W_0^2 = a_W \frac{\rho}{\rho_0}. \tag{96}$$

To obtain the parameters in the linear approximation we set $Z=1$ and find

$$\Sigma_v = \Sigma_p, \quad m^* = m(1+\Sigma_q) + \Sigma_s, \quad \lambda_m^{*2} = \lambda_m^2,$$

$$m_m = m(1+\Sigma_q) + \Sigma_v + \Sigma_s = m^* + \Sigma_v \tag{97}$$

in Eqs. (38)–(40) and (46). We set $s=4m^2$ in Eq. (39).

Expansion of the lhs of Eqs. (92)–(94) provides the equations

$$(f_v^q(M^2, W_0^2) v_N + f_g^q(M^2, W_0^2) g_N + u_N^q(M^2) + w_N^q)\rho_0 = \left(a_\lambda - (a_s + a_v)\frac{2m\lambda_0^2}{M^2}\right) e^{-m^2/M^2} - a_W \frac{\partial \ell_0^q(M^2, W_0^2)}{\partial W_0^2}, \tag{98}$$

$$(f_v^p(M, W_0^2) v_N + m u_N^p(M^2) + m\omega_N^p)\rho_0 = -a_v \lambda_0^2 e^{-m^2/M^2}, \tag{99}$$

$$(f^I_\kappa(M^2,W_0^2)\kappa_N+\omega^I_N(M^2))\rho_0 = \left(a_\lambda m-(a_s+a_v)\frac{2m^2\lambda_0^2}{M^2}\right)e^{-m^2/M^2} + a_s\lambda_0^2 e^{-m^2/M^2} - a_W\frac{\partial \ell_0^I(M^2,W_0^2)}{\partial W_0^2}. \qquad (100)$$

Note that in this form all three equations are tied. One can build up another set of equations with the functions L^p, L^I-mL^q, and L^q as the lhs. In this case the unknowns a_v and a_s are determined from the separate equations. The third equation determines the values of a_λ and a_W. We introduce

$$T^i_k(M^2,W_0^2)=\rho_0 f^i_k(M^2,W_0^2)\frac{e^{m^2/M^2}}{\lambda_0^2} \quad (k=v,g,\kappa),$$

$$T^i_u(M^2,W_0^2)=\rho_0 u^i_N(M^2,W_0^2)\frac{e^{m^2/M^2}}{\lambda_0^2},$$

$$T^i_\omega(M^2)=\rho_0 f^i_{4q}\frac{e^{m^2/M^2}}{\lambda_0^2} \qquad (101)$$

with the functions f^i_k and f^i_{4q} defined by Eqs. (51) and (90). We present

$$T^p_v(M^2,W_0^2)v_N+mT^p_u(M^2,W_0^2)+mT^p_\omega A^p_{4q}=-a_v, \quad (102)$$

$$T^I_\kappa(M^2,W_0^2)\kappa_N-mT^q_v(M^2,W_0^2)v_N-mT^q_g(M^2,W_0^2)g_N - mT^q_u(M^2,W_0^2)+T^I_\omega(M^2)A^I_{4q}-mT^q_\omega(M^2)A^q_{4q}=a_s, \qquad (103)$$

$$T^q_v(M^2,W_0^2)v_N+T^q_g(M^2,W_0^2)g_N+T^q_u(M^2,W_0^2)+T^q_\omega(M^2)A^q_{4q} + (a_s+a_v)\cdot\frac{2m}{M^2}=a_\lambda\frac{1}{\lambda_0^2}-a_W\frac{1}{\Lambda_0}\frac{\partial \ell_0^q(M^2,W_0^2)}{\partial W_0^2} \qquad (104)$$

with Λ_0 being defined by Eq. (27). The characteristics a_v and a_s are found from Eqs. (102) and (103) and are substituted into Eq. (104). The latter determines the values of a_λ and a_W.

Note that there is one more approximation on the rhs of Eq. (102). Namely, we neglected the value

$$a_W\left(\frac{\partial \ell_0^I}{\partial W_0^2}-m\frac{\partial \ell_0^q}{\partial W_0^2}\right)=a_W W_0^2\left(-2a+\frac{mW_0^2}{2L^{4/9}} + \frac{mb}{4W_0^2L^{4/9}}\right)e^{-W_0^2/M^2}$$

since there is about 80% cancellation between the two terms. This is due to the positive parity of the nucleon state—see Appendix D.

The values of the QCD parameters which enter the lhs of Eq. (102)–(104) are determined by Eqs. (6), (8), (10), (65), and (86). The expectation value $\kappa_N=\langle N|\bar{u}u+\bar{d}d|N\rangle$ is connected to the pion-nucleon sigma term $\sigma_{\pi N}$ by Eq. (8). The

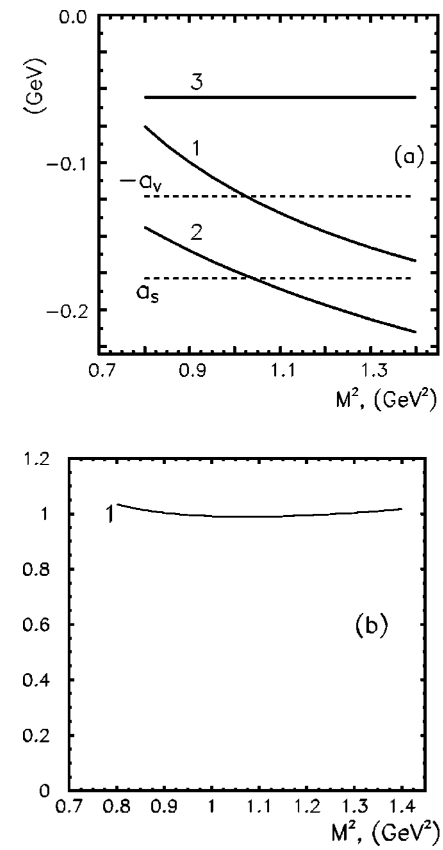

FIG. 1. Solution of Eqs. (102)–(104). In (a) the lines 1 and 2 show the lhs of Eqs. (102) and (103) for the self-energies. Line 3 shows the potential energy. The dashed lines show the constant values, corresponding to a_v and a_s on the rhs of these equations. The line in (b) shows the ratio of the lhs and rhs of Eq. (104).

value of $\sigma_{\pi N}$ can be extracted from the data on low-energy πN scattering, being expressed through the observable Σ term ($\Sigma_{\pi N}$) [30]. The value

$$\sigma_{\pi N}=(45\pm 7)\ \text{MeV} \qquad (105)$$

corresponds to $\Sigma_{\pi N}=64$ MeV [31]. We shall present the specific values, corresponding to $\kappa_N=8$. This value corresponds to $\sigma_{\pi N}=45$ MeV and the sum of the light quark masses $m_u+m_d=11$ MeV. There is an uncertainty in the value of κ_N due to the errors in determination of the values of $\sigma_{\pi N}$ and m_u+m_d. We also present the dependence of the characteristics of the nucleon on the value of κ_N.

The values of the parameters

$$a_v=0.108\ \text{GeV},\quad a_s=-0.178\ \text{GeV}, \quad a_\lambda=-1.29\ \text{GeV}^6,\quad a_W=-0.81\ \text{GeV}^2 \qquad (106)$$

are obtained by minimization of the relative difference of the rhs and lhs of Eqs. (102)–(104) by the chi-square method. The solution of these equations is illustrated in Fig. 1. Note, that if we construct the equation which is the difference of Eqs. (104) and (105), the function of M^2 in the lhs should be approximated by the constant value a_s+a_v, having the meaning of the potential energy. Such approximation holds with much better accuracy than the separate Eqs. (104) and (105) for the self-energies. The solution (106) corresponds to the values

$$\Sigma_v = 108 \text{ MeV}, \quad m^* - m = -178 \text{ MeV},$$
$$\frac{\delta\lambda^2}{\lambda_0^2} = -0.67, \quad \frac{\delta W^2}{W_0^2} = -0.37. \quad (107)$$

Although the sets of Eqs. (98)–(100) and (102)–(104) are mathematically identical, a procedure of matching of the two sides of the equations may lead to somewhat different solutions. Applying the same procedure of minimization to the set of Eqs. (98)–(100) we find $a_v=0.108$ GeV, $a_s=-0.254$ GeV, $a_\lambda=-1.61$ GeV^6, $a_W=-0.91$ GeV^2. Thus the parameters $\delta\lambda^2$ and δW^2 are determined with somewhat larger uncertainties than the self-energy Σ_v.

As we noted at the end of Sec. IV, our model approach to the calculation of the four-quark condensates can be modified by using more sophisticated models of the pions [27], i.e., by the account of the constituent quark masses. Using the value of A_{4q}^q given by Eq. (91), we obtain from Eqs. (102)–(104),

$$\Sigma_v = 108 \text{ MeV}, \quad m^* - m = -203 \text{ MeV},$$
$$\frac{\delta\lambda^2}{\lambda_0^2} = -0.71, \quad \frac{\delta W^2}{W_0^2} = -0.41 \quad (108)$$

at the saturation density $\rho=\rho_0$. Thus, this change of the value ω_N^q results in the change of the nucleon parameters by less than 15%.

Note that the functions $T_j^i(M^2)$ defined by Eq. (101) ($j=v,g,\kappa,u,\omega; i=q,p,I$) depend on M^2 rather weakly. Thus, approximating

$$T_j^i(M^2) = C_j^i, \quad (109)$$

we can replace in the lhs of Eqs. (102)–(104) the functions $T_j^i(M^2)$ by the constant coefficients C_j^i. Numerically the most important functions $T_v^p(M^2)$ and $T_\kappa^I(M^2)$ can be approximated by the constant values with the errors of about 4% and 8%. The largest errors of about 25% emerge in the averaging of the functions T_ω^i. This solves the problem of expressing the in-medium change of nucleon parameters through the values of the condensates

$$\Sigma_v = -(C_v^p v_N + mC_u^p + mC_\omega^p A_{4q}^p)\frac{\rho}{\rho_0}, \quad (110)$$

$$m^* - m = (C_\kappa^I \kappa_N - mC_v^q v_N - mC_g^q g_N - mC_u^q + C_\omega^I A_{4q}^I - mC_\omega^q A_{4q}^q)\frac{\rho}{\rho_0}. \quad (111)$$

The coefficients on the rhs of Eqs. (110) and (111) are

$$C_v^q = -0.062, \quad C_g^q = 0.011 \text{ GeV}^{-1}, \quad C_\omega^q = -0.067,$$
$$C_u^q = -0.074, \quad C_\kappa^I = -0.042 \text{ GeV},$$
$$C_\omega^I = -0.064 \text{ GeV}, \quad C_v^p = -0.090 \text{ GeV}, \quad (112)$$

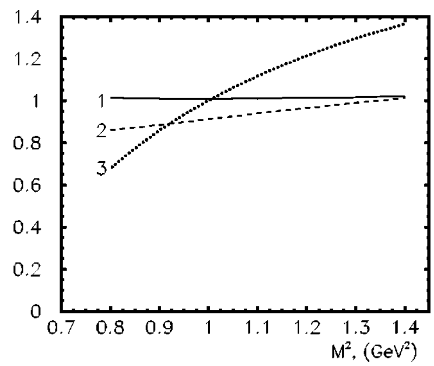

FIG. 2. Curves 1, 2, and 3 show the lhs to rhs ratios of Eqs. (92)–(94) correspondingly, at the values of the nucleon and continuum parameters given by Eq. (113).

$$C_\omega^p = -0.095, \quad C_u^p = 0.094.$$

Equations (110) and (111) reproduce the values of Σ_v and m^* provided by Eqs. (107) with the accuracy of 15% and 6% correspondingly.

VII. BEYOND THE LINEAR APPROXIMATION

Now we do not assume the linear dependence of the nucleon parameters on the density ρ. We find the values Σ_v, m^*, λ_m^{*2}, and W_m^2 which minimize the difference between the lhs and rhs of Eqs. (92)–(94). The consistency of the lhs and rhs is illustrated by Fig. 2. At the saturation density $\rho=\rho_0$ we obtain

$$\Sigma_v = 150 \text{ MeV}, \quad m^* - m = -200 \text{ MeV},$$
$$\lambda_m^{*2} = 1.25 \text{ GeV}^6, \quad W_m^2 = 2.11 \text{ GeV}^2. \quad (113)$$

The two last numbers correspond to the relative shifts $\delta\lambda^2/\lambda_0^2=-0.35$ and $\delta W^2/W_0^2=-0.03$. Thus the linear approximation is true at $\rho\approx\rho_0$ with the accuracy of about 25% for the vector self-energy and about 10% for the scalar one. The linear approximation overestimates the shift of the effective threshold.

Recall, that we presented the numerical results for $\kappa_N=8$. The dependence on the value of κ_N is shown in Fig. 3. The density dependence of the nucleon parameters at $\kappa_N=8$ is shown in Fig. 4.

Using Eq. (91) for the value of A_{4q}^q we obtain the results which are close to those presented by Eq. (113),

$$\Sigma_v = 142 \text{ MeV}, \quad m^* - m = -223 \text{ MeV},$$
$$\lambda_m^{*2} = 1.24 \text{ GeV}^6, \quad W_m^2 = 2.09 \text{ GeV}^2. \quad (114)$$

Note that the difference between the linear and nonlinear solutions has a strong effect on the value of the nucleon potential energy

$$U(\rho) = \Sigma_v(\rho) + m^*(\rho) - m, \quad (115)$$

which is about −40 MeV and −70 MeV for the solutions (113) and (114) at the phenomenological saturation point $\rho=\rho_0$.

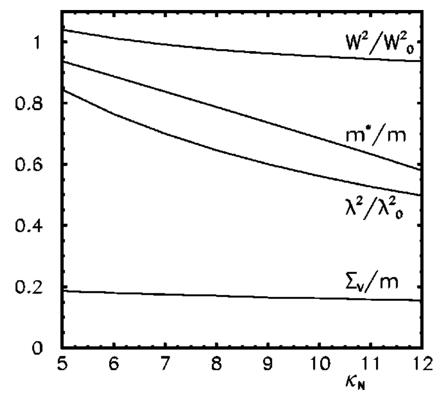

FIG. 3. Dependence of the solutions of Eqs. (92)–(94) on the value of the nucleon expectation value κ_N at $\rho=\rho_0$. The values of W_0^2, λ_0^2 are given by Eq. (33).

VIII. DISCUSSION

It is instructive to follow how the values of the nucleon self-energies change, while we include the various contributions of the lhs of the sum rules. The solutions of the general equations (92)–(94) are illustrated by Fig. 5. At the saturation density ρ_0 the vector self-energy Σ_v and the effective mass m^* are 340 MeV and 750 MeV correspondingly, if only the terms l^j—Eq. (50) are included in L_j^i of Eq. (47). One can see from Fig. 5 that the higher moments of the structure functions and the four-quark condensates subtract about 100 MeV each from the value of Σ_v. On the contrary, the two contributions to m^* cancel to large extent, with the four-quark condensate subtracting about 200 MeV, and the moments of the structure functions adding about this value.

We come to similar results in the linear approximation Sec. VI. The moments of the structure functions and the four-quark condensate subtract 60 MeV and 110 MeV from the lowest dimension value $\Sigma_v=270$ MeV. The OPE value of the scalar self-energy m^*-m is -140 MeV. The four-quark condensates and the moments of the structure functions add -100 MeV and $+100$ MeV, correspondingly.

Turning to the role of the anomalous dimensions, we note that their inclusion into the moments of the structure functions lead to minor changes of several MeV of the values of vector and scalar self energies. Neglecting the anomalous dimensions of all the in-medium contributions increases the values of the vector self-energy $\Sigma_v=230$ MeV, and of the scalar self-energy $m^*-m=-140$ MeV. Thus, we find for the potential energy $U>0$ if $\kappa=8$. However, the value of m^* decreases with κ, while the vector self-energy practically does not change. We find that $U<0$ if $\kappa>10$, i.e., $\sigma_{\pi N}>55$ MeV—Eq. (105).

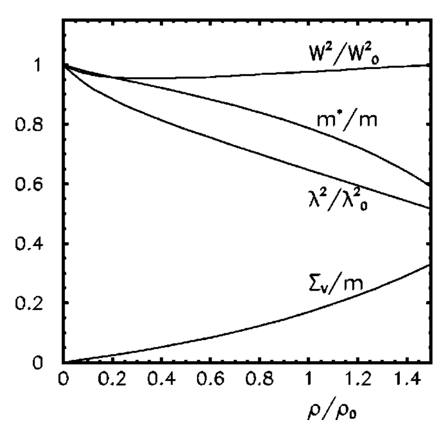

FIG. 4. Density dependence of the nucleon and continuum parameters beyond the linear approximation at $\kappa_N=8$. The horizontal axis corresponds to the density of the matter, related to the phenomenological saturation value.

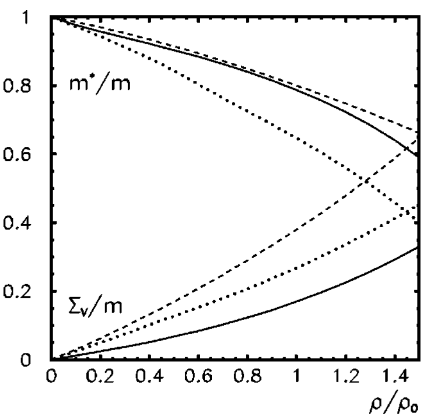

FIG. 5. Sum-rule predictions for the dependence of the nucleon parameters m^*/m and Σ_v/m on the ratio ρ/ρ_0 at $\kappa_N=8$. The curves correspond to the successive inclusion of more complicated condensates. Dashed lines, only expectation values of the operators of the lowest dimension $\bar{q}(0)\gamma_0 q(x=0)$ and $\bar{q}(0)q(x=0)$ and of the gluon operators $(\alpha_s/\pi)G^2(0)$ are included [see Eq. (50)]. Dotted lines, local four-quark condensates are added [Eqs. (85) and (86)]; solid lines, x dependence of the vector condensates (expressed in terms of the nucleon structure functions) is included.

The authors of Ref. [16] carried out the detailed analysis of the nucleon self-energies depending on the in-medium values of the four-quark condensates. They considered the QCD sum rules, based on the dispersion relations in energy q_0 at $|\mathbf{q}|$ being fixed. The authors of Ref. [16] found that the values of the self-energies depend strongly on the value of the scalar-scalar condensate, while the dependence on the values of the other four-quark expectation values appeared to be negligible small. Actually, they presented $\langle M|\bar{q}q\bar{q}q|M\rangle - \langle 0|\bar{q}q\bar{q}q|0\rangle = 2f\langle 0|\bar{q}q|0\rangle\langle N|\bar{q}q|N\rangle\rho$, and studied the dependence of the nucleon parameters on the value of f. Our model calculations correspond to $f=0.14$. It was found in Ref. [16], that the values $0<f<0.3$ provide the results, which are consistent with the nuclear phenomenology. We can deduce from Fig. 1 of Ref. [16], that there values are $m^*/m=0.65$ and $\Sigma_v/m=0.28$ for $f=0.14$. Neglecting all the other four-quark condensates, we find the close values $m^*/m=0.67$ and $\Sigma_v/m=0.25$. Note, however, that our approach is based on the dispersion relations in another variable, *i.e.*, in q^2, with the relativistic pair energy s being kept fixed. (This enables us to avoid the singularities, connected with the excitations of medium [8–10].) In our case the influence of the vector-scalar expectation value is stronger than in Ref. [16]. For example, if we assume factorization approximation for the vector-scalar condensate, our value of the nucleon effective mass is about twice smaller than the value obtained in Ref. [16]. The values of the vector self-energy are still close in the two approaches.

In the paper [17] the calculations of the four-quark condensate were avoided by a specific choice of the function $\Pi_m(q)$. The limits $160\ \text{MeV} < \Sigma_v < 310\ \text{MeV}$ and $0.62\ \text{GeV} < m^* < 0.83\ \text{GeV}$ have been obtained. In Ref. [9] the authors got rid of the four-quark condensates, applying the differential operators. They found the vector and scalar fields to be about 220 MeV and −350 MeV in the gas approximation.

These results are consistent with each other and with the results of nuclear physics calculations. Various approaches in the nuclear physics studies (see, e.g., Ref. [32]) provide the values between 180 MeV and 350 MeV for the vector fields, and between −200 MeV and −400 MeV for the scalar fields.

There is agreement with the earlier results in some other points. The 30% reduction of the vector field, caused by nonlocality of the vector condensate, was found in Refs. [8,9]. The strong reduction of the nucleon pole residue was obtained in Refs. [8,9,16]. Also, it was first noted in Ref. [16] that the shift of the continuum threshold is very small.

IX. SUMMARY

We analyzed QCD sum rules in nuclear matter by taking into account terms of the order of $q^2 \ln q^2$, $\ln q^2$, and $1/q^2$ of the operator product expansion. The consistency of the lowest OPE terms in QCD sum rules [7–10,14–16] with the nuclear phenomenology was known for a long time. However the lack of information on the four-quark condensates, contributing to the terms of the order $1/q^2$ was the main obstacle for the further development of the approach.

In this paper we studied the sum rules, treating the QCD condensates in the gas approximation and included the contribution of the four-quark condensates, expressed through the nucleon expectation values. The latter were obtained in Ref. [20] by employing results of the perturbative chiral quark model [21,22]. We included also the higher moments of the nucleon structure functions which contribute to the terms of the order $\ln q^2$ and $1/q^2$. Taking into account the four-quark condensate we included all Lorentz structures.

We took into account the nonlocal structure of the vector condensate, which manifests itself through the higher moments of the structure functions. We include corrections, which have the smooth dependence on these moments. However, we did not include the nonlocal singularities in the u channel. Such singularities correspond to the exchange interaction between the nucleon and the matter. Thus, our approach corresponds to Hartree description of the in-medium nucleon. The nonlocal structure of the scalar condensate manifests itself in the higher orders of OPE.

Considering only the linear changes of the nucleon parameters we obtained a linear combination of the QCD sum rules equations in which the nucleon effective mass m^* and the vector self-energy Σ_v are the only unknown parameters. A more detailed analysis going beyond the linear approximation shows that this approach works well at densities close to the saturation value $\rho = \rho_0$. In this approach we solved the problem of expressing the in-medium change of the nucleon parameters in terms of the in-medium values of QCD condensates—Eqs. (110) and (111).

The terms, containing the four-quark condensates provide the corrections of the order 20–25 % to the leading terms of the OPE of the function $\Pi_m - \Pi_0$, which are determined by the local vector and scalar condensates. This is consistent with the hypothesis about the convergence of the OPE series. The four-quark condensates diminish the OPE value of the vector self-energy Σ_v by about 25%. The scalar self-energy $m^* - m$ is more sensitive to the four-quark expectation values. Inclusion of these condensates makes the OPE value of $m^* - m$ about 80% larger. Inclusion of the nonlocality of the vector condensate, which manifests itself in terms of the higher moments of the structure functions subtracts 25% more from the value of Σ_v, and almost totally compensates the contribution of the four-quark condensates to the shift $m^* - m$. Thus the value of $m^* - m$ appears to be very close to the one, determined by the lowest orders of OPE.

The contribution of the four-quark condensate to the vector self-energy Σ_v is caused mainly by the vector-vector structure. The contribution to the scalar parameter $m^* - m$ is of more complicated origin, with the scalar-vector, scalar-scalar, vector-vector, and axial–axial terms being numerically important.

As it was noted earlier [9,10], the QCD sum rules can be viewed as a connection between the exchange of uncorrelated $\bar{q}q$ pairs and the exchange of strongly correlated pairs with the same quantum numbers (mesons). This results in a certain connection between the Lorentz structures of the in-medium expectation values and of the nucleon propagator. In the leading orders of OPE the vector (scalar) structure of the propagator is determined by the vector (scalar) expectation value. The scalar-vector four-quark condensate is determined mainly by the contribution which is proportional to the vector expectation value. On the other hand, it contributes to the scalar Lorentz structure of the nucleon propagator. In the meson-exchange picture such terms can be explained by the complicated structure of the nucleon-meson vertices. This can be instructive for model building of nuclear forces.

The values of the nucleon parameters Σ_v and $m^* - m$ are (at least qualitatively) consistent with those, obtained earlier in framework of nuclear physics [32] and of QCD sum rules approach [7–9,14–17]. The four-quark condensates, as well as the higher moments of the structure functions provide large contributions to the nucleon parameters. This future accounting for the main radiative corrections is expected to make the results more accurate.

Another direction of the development of the approach is to go beyond the gas approximation. The presentation of the results, especially Eq. (111) for m^* enables to make the next step, studying the self-consistent set of equations for the nucleon effective mass and the quark condensates, as suggested in Ref. [10].

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APPENDIX A

The contribution $X_{1a}(q)$ expressed by Eq. (65) can be obtained by direct substitution of Eq. (53) into Eq. (55) and by using the formula

$$\int \frac{d^4x}{x^8}(ax)(bx)e^{i(q'x)} = \frac{1}{6}\left[(ab) + \frac{2(aq')(bq')}{q'^2}\right]\int \frac{d^4x}{x^6}e^{i(q'x)} \tag{A1}$$

for any vectors "a" and "b." Thus, all the contributions to the function X_{1a} are expressed through the integral

$$\int \frac{d^4x}{x^6}e^{i(q'x)} = -\frac{i\pi^2}{8}q'^2\ln(-q'^2) + \cdots . \tag{A2}$$

Here the dots denote the terms which will be killed by the Borel transform. This leads to Eq. (65).

To establish the connection between Eq. (65) and the two terms on the rhs of Eq. (41) note, that the rhs of Eq. (65) consists of the terms of the form [see Eq. (61)]

$$\begin{aligned} X &= \int_0^1 d\alpha\,\ln(Q^2 + A^2(\alpha))f(\alpha) \\ &= \ln Q^2\int_0^1 d\alpha\, f(\alpha) \\ &\quad + \int_0^1 d\alpha\,\ln\frac{Q^2 + A^2(\alpha)}{Q^2}f(\alpha). \end{aligned} \tag{A3}$$

The first term on the rhs contains the standard logarithmic factor containing the cut, running to infinity. It is described by our "pole+continuum" model in a usual way. The second term contains a finite cut. Such terms need special treatment. The cut of the second term describes the singularities in the u channel, caused by the nonlocal structure of the vector condensate. They correspond to the exchange terms on the rhs of the sum rules. We neglect such contributions, thus coming to the Hartree description of the nucleon in nuclear matter.

APPENDIX B

To obtain the coefficients of the first (factorized) terms in the rhs of Eqs. (83) and (84), recall that we need the expectation values of the color-antisymmetric operators

$$T^{XY,f_1f_2} = (:\bar{q}^{f_1a}\Gamma^X\bar{q}^{f_1a'}\cdot\bar{q}^{f_2b}\Gamma^Y q^{f_2b'}:)(\delta_{aa'}\delta_{bb'} - \delta_{ab'}\delta_{ba'}) \tag{B1}$$

with f_1, f_2 standing for the quark flavors. The dots denote the normal ordering of the operators, a, a', b, b' represent the color indices. It is convenient to present

$$\delta_{aa'}\delta_{bb'} - \delta_{ab'}\delta_{ba'} = \frac{2}{3}\delta_{aa'}\delta_{bb'} - \frac{1}{2}\sum_\rho \lambda^\rho_{aa'}\lambda^\rho_{bb'} \tag{B2}$$

with λ^ρ standing for the SU(3) Gell-Mann matrices $\mathrm{Tr}\,\lambda^\rho\lambda^\sigma = 2\delta^{\rho\sigma}$.

The factorization approximation for the quarks of different flavors is

$$\langle M|\bar{u}^a_\alpha u^{a'}_\beta \bar{d}^b_\gamma d^{b'}_\delta|M\rangle = \langle M|\bar{u}^a_\alpha u^{a'}_\beta|M\rangle\langle M|\bar{d}^b_\gamma d^{b'}_\delta|M\rangle \tag{B3}$$

with $\alpha,\beta,\gamma,\delta$ being the Lorentz indices, and only the first term on the rhs of Eq. (B2) contributes. Using also Eq. (13) we come to Eqs. (71) and (84).

The factorization approximation formula for the quarks of the same flavor, e.g., $q^{f_1}=q^{f_2}=u$ is

$$\begin{aligned}\langle M|\bar{u}^a_\alpha u^{a'}_\beta\bar{u}^b_\gamma u^{b'}_\delta|M\rangle &= \langle M|\bar{u}^a_\alpha u^{a'}_\beta|M\rangle\langle M|\bar{u}^b_\gamma u^{b'}_\delta|M\rangle \\ &\quad - \langle M|\bar{u}^a_\alpha u^{b'}_\delta|M\rangle\langle M|\bar{u}^b_\delta u^{a'}_\beta|M\rangle. \end{aligned} \tag{B4}$$

Thus in the factorization approximation

$$\begin{aligned}\langle M|\bar{u}\Gamma^X u\bar{u}\Gamma^Y u|M\rangle &= \tfrac{1}{16}\left[\mathrm{Tr}\,\Gamma^X\cdot\mathrm{Tr}\,\Gamma^Y - \tfrac{1}{3}\mathrm{Tr}(\Gamma^X\Gamma^Y)\right] \\ &\quad\times(\langle M|\bar{u}u|M\rangle)^2 \end{aligned} \tag{B5}$$

and

$$\langle M|\sum_\rho \bar{u}\Gamma^X\lambda^\rho u\cdot\bar{u}\Gamma^Y\lambda^\rho u|M\rangle = -\tfrac{1}{9}\mathrm{Tr}(\Gamma^X\Gamma^Y)(\langle M|\bar{u}u|M\rangle)^2. \tag{B6}$$

Thus, for the factorized part of the expectation value of the color-antisymmetric operator $T^{II,uu}$ is $(\Gamma^X=\Gamma^Y=I)$

$$\langle M|\bar{u}^a u^{a'}\bar{u}^b u^{b'}|M\rangle = C(\langle M|\bar{u}u|M\rangle)^2 \tag{B7}$$

with

$$C = \tfrac{2}{3}\left(1 - \tfrac{1}{12}\right) - \tfrac{1}{2}\left(-\tfrac{4}{9}\right) = \tfrac{5}{6}. \tag{B8}$$

Employing also Eq. (13) we come to Eqs. (70) and (83).

APPENDIX C

Here we present for illustration the calculation of the most important contributions of the four-quark condensates to $\hat{q}$ structure. Using Eq. (73) we find for the contribution of the first term on the rhs of Eq. (84),

$$\begin{aligned}\Pi^1 &= \left(-\frac{1}{2}\right)2\cdot\frac{5}{6}\cdot\frac{\langle 0|\bar{u}u|0\rangle}{q^2}[\langle p|(\bar{u}u)_v|p\rangle\rho_p \\ &\quad + \langle n|(\bar{u}u)_v|n\rangle\rho_n]. \end{aligned} \tag{C1}$$

This is equivalent to

$$\Pi^1 = -\frac{5}{4}\frac{\langle 0|\bar{u}u|0\rangle}{q^2}\cdot J\rho \tag{C2}$$

with $J=\int\bar{\psi}(x)\psi(x)d^3x$, while $\psi(x)$ is the renormalized PCQM wave function of the constituent quark, normalized by the condition $\int\bar{\psi}(x)\gamma_0\psi(x)d^3x=1$. Using the value $J=0.54$ [22], one finds

$$\Pi^1 = -0.67\frac{\langle 0|\bar{u}u|0\rangle}{q^2}\rho. \qquad \text{(C3)}$$

The interval contribution is determined mostly by the expectation values of the operators $\bar{d}\Gamma^X d\bar{u}\Gamma^X u$. This happens just due to the large numerical coefficients on the rhs of Eq. (76). Using Eq. (76) we find the contribution to be

$$\Pi^2 = (-10a_r^V - 6a_r^A - 2b_r^V - 2b_r^A)\frac{\rho}{q^2}. \qquad \text{(C4)}$$

Substituting the values $a_r^V = -0.074\varepsilon_0^3$, $a_r^A = 0.084\varepsilon_0^3$, $b_r^V = 0.31\varepsilon_0^3$, $b_r^A = 0.06\varepsilon_0^3$ ($\varepsilon_0 = 241$ MeV) [20], we obtain $\Pi^2 = -0.50(\varepsilon_0^3/q^2)$ and

$$\Pi^1 + \Pi^2 = 0.17\frac{\varepsilon_0^3\rho}{q^2}. \qquad \text{(C5)}$$

A more accurate calculation, accounting for the internal contributions of the operators $\bar{u}\Gamma^X u\bar{u}\Gamma^X u$ leads to the first term on the rhs of Eq. (88).

The contribution to $\hat{p}$ structure is obtained in similar way. Turning to I structure, note that the contribution comes from the scalar-vector condensate $\bar{d}d\bar{u}\gamma_\mu u$—Eq. (85). The first ("factorized") term on the rhs provides the contribution

$$\Pi^3 = -\frac{2}{3}\int \frac{d^4x}{\pi^2 x^4}(x, \theta^\mu(x))e^{i(qx)}\langle 0|\bar{d}d|0\rangle\rho \qquad \text{(C6)}$$

with $\theta^q(x)$ defined by Eq. (54). If $\theta_\mu^\mu(x) = \theta_\mu^\mu(0)$, we obtain

$$\Pi^3 = -\frac{2(pq)}{q^2}\langle 0|\bar{d}d|0\rangle\frac{\rho}{m}.$$

Taking into account the dependence of θ_μ^μ on x we actually include the higher moments and twists of the nucleon structure functions. Proceeding in the same way as in Sec. III, we obtain for the Borel transform of Π^3

$$B\Pi^3 = -8\pi^2 mYa\rho. \qquad \text{(C7)}$$

Here

$$Y = \frac{1}{3}\left(\frac{s-m^2}{m^2}\langle\eta\rangle - \langle\alpha\eta\rangle - \frac{1}{2}\langle\xi\rangle - \frac{1}{4}\frac{m_0^2}{m^2}\langle\eta\rangle\right). \qquad \text{(C8)}$$

The first term, that is the pure local contribution, would give $Y = 3.0$, the higher order contributions subtract 0.32 from this value. Thus, the factorized term would provide $A_{4q}^I = 2.68$. Account of the second term on the rhs of (85) leads to $A_{4q}^I = 1.90$.

APPENDIX D

Present vacuum sum rules given by Eqs. (29) and (30) in the form

$$\ell_0^q(M^2, W_0^2) = \Lambda_0 + \int_{W_0^2}\frac{\partial \ell_0^q}{\partial W^2}dW^2,$$

$$\ell_0^I(M^2, W_0^2) = m\Lambda_0 + \int_{W_0^2}\frac{\partial \ell_0^I}{\partial W^2}dW^2 \qquad \text{(D1)}$$

with $\Lambda_0(M^2)$ determined by Eq. (27). In the combination $\ell_0^I - m\ell_0^q$ which is just the projection on the negative-parity component of the lowest state the contribution of the residue vanishes

$$\ell_0^I - m\ell_0^q = \int_{W_0^2}\left(\frac{\partial \ell_0^I}{\partial W^2} - m\frac{\partial \ell_0^q}{\partial W^2}\right)dW^2. \qquad \text{(D2)}$$

The condition

$$\left|\frac{\partial(\ell_0^I - m\ell_0^q)}{\partial W^2}\right| \ll \left|\frac{\partial \ell_0^{q,I}}{\partial W^2}\right|$$

at $W^2 = W_0^2$ means that we cannot imitate the contribution of the negative-parity pole of the order Λ_0^2 on the rhs of Eq. (D2) by changing the value of W_0^2.

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