

Properties of the ω meson at finite temperature and density

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The mass shift, width broadening, and spectral density for the ω meson in a heat bath of nucleons and pions is calculated using a general formula which relates the self-energy to the forward scattering amplitude. We use experimental data to saturate the scattering amplitude at low energies with resonances and include a background Pomeron term, while at high energies a Regge parametrization is used. The peak of the spectral density is little shifted from its vacuum position, but the width is considerably increased due to collisional broadening. At normal nuclear matter density and a temperature of 150 MeV the spectral density of the ω meson has a width of 140 MeV. Zero temperature nuclear matter is also discussed.

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The modification of the free space properties of a vector meson in hadronic or nuclear matter is an important problem which has attracted much attention, see the reviews of Ref. [1]. Many works have relied on effective Lagrangians, however, in Ref. [2] (hereafter paper I) we adopted as model independent an approach as possible by using experimental data to construct the amplitude for vector mesons scattering from pions and nucleons. The low-energy region was described in terms of resonances plus background, while at high energies a Regge model was employed. Using this amplitude the in-medium self-energy of the ρ and ω mesons was calculated at finite temperature and density using the leading term of the exact self-energy expansion [3]. This requires that only single scatterings be important which was found to be justified in paper I by comparison with results from ultrarelativistic molecular dynamics (UrQMD) calculations. This procedure was satisfactory for the ρ meson and gave results consistent with those obtained by Rapp [4] by considering medium modifications of the pions comprising the meson. Less satisfactory was the ω meson case because little data was available for the decay of nucleon resonances in the ωN channel. Therefore in paper I two extreme models were adopted: a two-resonance model using the data of Manley and Saleski [5] and an $\omega \rightarrow \rho$ model which assumed that the resonance decays in the ωN and ρN channels were essentially the same. In the meantime much better resonance data in the ωN channel has become available with the analysis of Shklyar, Penner, and Mosel [6]. The purpose of the present paper is to use this new data to provide a more reliable ω self-energy than the two extreme models adopted in paper I.

First we consider the low-energy regime. We assume that the ω self-energy is dominated by scattering from the pions and nucleons present in the heat bath, as was justified in paper I by comparison with UrQMD results. Focussing on the latter we briefly outline the formalism for constructing the ωN amplitude, more details are to be found in paper I. We adopt the two-component duality approach due to Harari [7] (see also Collins [8]) which states that while ordinary Reggeons are dual to s -channel resonances, the Pomeron is dual to the background upon which the resonances are superimposed. We write the forward scattering amplitude in the rest frame of the heat bath

$$f_{\omega N}(E_\omega) = \frac{\sqrt{s}}{2q_{c.m.}m_N} \sum_R W_{\omega N}^R \frac{\Gamma_{R \rightarrow \omega N}}{M_R - \sqrt{s} - \frac{1}{2}i\Gamma_R} - \frac{q_{c.m.}r_P^{\omega N}}{4\pi m_N \sqrt{s}} \frac{(1 + \exp^{-i\pi\alpha_P})}{\sin \pi\alpha_P} s^{\alpha_P}. \quad (1)$$

Here the first term involves a sum over a series of Breit-Wigner resonances of mass M_R and total width Γ_R (replacement of this nonrelativistic form by the relativistic expression has a negligible effect on the results). The second term is the Pomeron background contribution discussed below. In the usual notation \sqrt{s} is the total energy which is related to the energy of the ω meson by $E_\omega - m_\omega = [s - (m_\omega + m_N)^2]/(2m_N)$, where m_ω and m_N are the omega and nucleon masses. In Eq. (1) $q_{c.m.}$ denotes the magnitude of the c.m. momentum and the statistical averaging factor $W_{\omega N}^R = (2s_R + 1)/6$, where s_R is the spin of the resonance. Since we are averaging over all spin directions we shall not distinguish longitudinal and transverse polarizations. Also $\Gamma_{R \rightarrow \omega N}$ in Eq. (1) represents the partial width for the resonance decay into the ωN channel. If we denote the c.m. momentum at resonance by $q_{c.m.}^R$, then for $q_{c.m.} \geq q_{c.m.}^R$ we use the value obtained from the total width and the branching ratio on resonance. However the threshold behavior of the partial width is known and we incorporate this for $q_{c.m.} \leq q_{c.m.}^R$ by replacing $\Gamma_{R \rightarrow \omega N}$ by $\Gamma_{R \rightarrow \omega N} (q_{c.m.}/q_{c.m.}^R)^{2l+1}$, where l is the relative angular momentum between the ω and the nucleon. Since the total width is the sum of the partial widths in principle this dependence should be incorporated in Γ_R , but this is impractical as there are many decay channels open so we simply take Γ_R to be a constant.

The above-threshold resonances included in our calculation are listed in Table I. The first five entries are taken from the fit labeled C- p - π +(5/2) by Shklyar, Penner, and Mosel [6] which extends to 2 GeV. We also include the $N(2190)$ strength taken from Manley and Saleski [5]; Vrana, Dytman, and Lee [9] report a roughly similar width and branching ratio for this resonance. It is also necessary to include sub-threshold resonances since they make a significant contribution. We include the $N(1520)D_{13}$ and the $N(1535)S_{11}$, as well as the smaller contributions from the $N(1440)P_{11}$, the

TABLE I. Baryon resonances included in the ωN amplitude.

Resonance	Mass (GeV)	Width (GeV)	ωN branching ratio (%)
$N(1710)P_{11}$	1.753	0.534	19.9
$N(1720)P_{13}$	1.725	0.267	0.8
$N(1900)P_{13}$	1.962	0.700	9.6
$N(1950)D_{13}$	1.927	0.855	47.0
$N(2000)F_{15}$	1.981	0.361	2.2
$N(2190)G_{17}$	2.127	0.547	49.0

$N(1650)S_{11}$, and the $N(1680)F_{15}$ resonances. In order to estimate the widths we assume that the vector dominance model is valid, even though it is better suited to high energies. This allows us to relate the photon and ω widths. Specifically, since these resonances are close to the ωN threshold, we can write for each of them $\Gamma_{\omega N} = q_{c.m.} \gamma_{\omega N}$ and $\Gamma_{\gamma N} = k_{c.m.} \gamma_{\gamma N}$, where $k_{c.m.}$ is the γN c.m. momentum. Then vector dominance gives

$$\gamma_{\gamma N} = 4\pi\alpha \frac{1}{g_\rho^2} \left(1 + \frac{g_\rho^2}{g_\omega^2} \right) \gamma_{\omega N}, \quad (2)$$

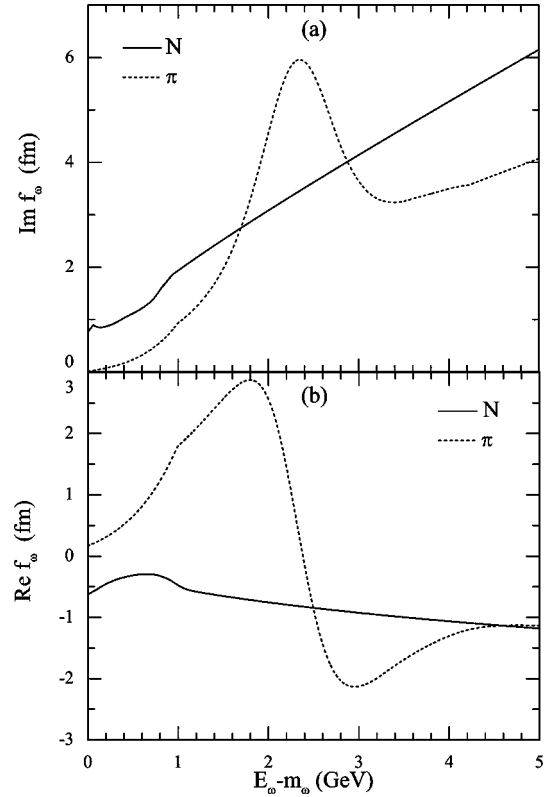
where α is the fine structure constant. For the coupling to the photon we take $g_\rho^2/4\pi = 2.54$ and $g_\rho^2/g_\omega^2 = 1/8$. The value of $\gamma_{\gamma N}$ can be deduced from the decay width and the photon branching ratio of the resonances [10].

The high-energy forward scattering amplitude is known [11] to be well approximated by the Regge form

$$f_{\omega N}(E_\omega) = - \frac{q_{c.m.}}{4\pi m_N \sqrt{s}} \sum_i \frac{1 + \exp^{-i\pi\alpha_i}}{\sin \pi\alpha_i} r_i^{\omega N} s^{\alpha_i}. \quad (3)$$

We shall consider a Pomeron term P and a Regge term P' . Since the different isospin structure of the ω and the ρ is expected to be insignificant at high energy, we adopt the same parametrization for the ωN and ρN scattering amplitudes as in paper I. Specifically the intercepts are $\alpha_P = 1.093$ and $\alpha_{P'} = 0.642$ with residues $r_P^{\omega N} = 11.88$ and $r_{P'}^{\omega N} = 28.59$. The units are such that with energies in GeV the total cross section is given in mb; specifically the optical theorem gives $\sigma = 4\pi \text{Im} f_{\omega N}/p$, where the momentum in the rest frame of the heat bath $p = q_{c.m.} \sqrt{s}/m_N$. The parameters for the Pomeron given here are also used for the background term in Eq. (1). Note that if the Pomeron intercept α_P were exactly 1, the Pomeron amplitude would be pure imaginary.

Because of the kinematics the resonance region ends at $E_\omega - m_\omega \sim 1$ GeV and the amplitude is smoothly matched onto the Regge part at approximately this point. The real and imaginary parts of $f_{\omega N}$ constructed in this manner are indicated by the solid curves in Fig. 1. Since the low-energy part contains a number of overlapping resonances the structure is washed out. We also indicate by dashed curves in Fig. 1 the corresponding results for $f_{\omega\pi}$ taken from paper I for which the single $b_1(1235)$ resonance employed is clearly visible (note that due to kinematics the resonance region ends at $E_\omega - m_\omega \sim 4$ GeV for the $\omega\pi$ system). Our result for the


FIG. 1. (a) The imaginary and (b) the real part of the amplitude for ωN scattering (solid line) and $\omega\pi$ scattering (dashed line).

imaginary part of $f_{\omega N}$ in Fig. 1(a) can be compared with that obtained by Sibirtsev, Elster, and Speth [12] using data for ω photoproduction from nuclei and employing the eikonal approximation and vector meson dominance. There is good qualitative agreement, while quantitatively our values are 20–30% lower than theirs. As these authors point out the real part at threshold is quite uncertain, even as regards sign. At the highest energy we consider the ratio of the real to the imaginary part agrees quite nicely with the πN data [12,13]. However this is to be expected since in paper I the high-energy Regge behavior was fixed by using the charge-averaged πN data.

For an ω meson scattering from a hadron a in the medium the retarded self-energy on shell can be written [2,3,14] as a single integral. For the case that a is a boson the result is

$$\Pi_{\omega a}(p) = - \frac{m_\omega m_a T}{\pi p} \int_{m_a}^{\infty} d\omega f_{\omega a} \left(\frac{m_\omega \omega}{m_a} \right) \times \ln \left[\frac{1 - \exp(-\omega_+/T)}{1 - \exp(-\omega_-/T)} \right]. \quad (4)$$

Here $\omega^2 = m_a^2 + k^2$ and $\omega_\pm = (E\omega \pm pk)/m_\omega$ with $E^2 = m_\omega^2 + p^2$. If a is a fermion ω_\pm has an additional chemical potential contribution $-\mu$ and the argument of the logarithm becomes $[1 + \exp(-\omega_-/T)]/[1 + \exp(-\omega_+/T)]$. The total self-energy is given by summing over all target species and including the vacuum contribution

$$\Pi_{\omega}^{\text{tot}}(E,p) = \Pi_{\omega}^{\text{vac}}(M) + \Pi_{\omega\pi}(p) + \Pi_{\omega N}(p). \quad (5)$$

Here the vacuum part of Π can only depend on the invariant mass $M = \sqrt{E^2 - p^2}$, whereas the matter parts can in principle depend on E and p separately. However, in the approximation we are using the scattering amplitudes are of necessity evaluated on the mass shell of the ω meson. This means that the matter parts only depend on p because M is fixed at m_{ω} . The dispersion relation is determined from the poles of the propagator with the self-energy evaluated on shell, i.e., $M = m_{\omega}$, giving

$$E^2 = m_{\omega}^2 + p^2 + \Pi_{\omega}^{\text{tot}}(p). \quad (6)$$

Since the self-energy has real and imaginary parts so does $E(p) = E_R(p) - i\Gamma(p)/2$. The width is given by

$$\Gamma(p) = -\text{Im}\Pi_{\omega}^{\text{tot}}(p)/E_R(p), \quad (7)$$

with

$$2E_R^2(p) = p^2 + m_{\omega}^2 + \text{Re}\Pi_{\omega}^{\text{tot}}(p) + \sqrt{[p^2 + m_{\omega}^2 + \text{Re}\Pi_{\omega}^{\text{tot}}(p)]^2 + [\text{Im}\Pi_{\omega}^{\text{tot}}(p)]^2}. \quad (8)$$

In vacuum the width $\Gamma_{\omega}^{\text{vac}} = -\text{Im}\Pi_{\omega}^{\text{vac}}/m_{\omega}$ is 8.4 MeV. We define the mass shift to be

$$\Delta m_{\omega}(p) = \sqrt{m_{\omega}^2 + \text{Re}\Pi_{\omega}^{\text{tot}}(p)} - m_{\omega}. \quad (9)$$

The ω meson mass shifts and widths are shown as a function of momentum in Fig. 2 for two temperatures and nucleon densities $n_N=0,1$, and 2 in units of equilibrium nuclear matter density n_0 (0.16 nucleons/fm³). At zero nucleon density only $f_{\omega\pi}$ is required so the results are the same as in paper I. In particular the mass shift Δm_{ω} in Fig. 2(a) is small and negative. It becomes positive when nucleons are introduced; at large momentum p the mass shifts are quite similar to the $\omega \rightarrow \rho$ model of paper I, however, for $p=0$ the shifts are larger than given by either model of paper I. Notice that higher temperatures lead to a smaller mass shift. However, the values of Δm_{ω} are at most a few tens of MeV, which is small in comparison to the vacuum mass. Rapp [4], who considers medium modifications of the pions comprising the omega, reports a negligible mass shift. For reference in zero-temperature nuclear matter, where pions do not contribute, the mass shift is approximately +30 MeV ($p=0$). There are a wide range of estimates in the literature. Post and Mosel [15] obtain 20 MeV with a resonance model, Lutz, Wolf, and Friman [16] find 70 MeV in a coupled channel approach and Mühlich, Falter, and Mosel [17] with their adopted $\text{Re}f_{\omega N}$ quote -35 MeV. A mass shift of much larger magnitude has been found in the chiral approach of Klingl, Waas, and Weise [18]. Large mass shifts have also been found using QCD sum rules [1,19], but these are tailored to the small distance behavior whereas, as Eletsky and Ioffe [20] have pointed out, the self-energy is determined by meson-nucleon scattering at relatively large distances of order 1 fm; see also Ref. [21].

Our results for the width Γ_{ω} are given in Fig. 2(b). Note that the widths given here are defined to be in the rest frame of the thermal system. As we have remarked, at $n_N=0$ the

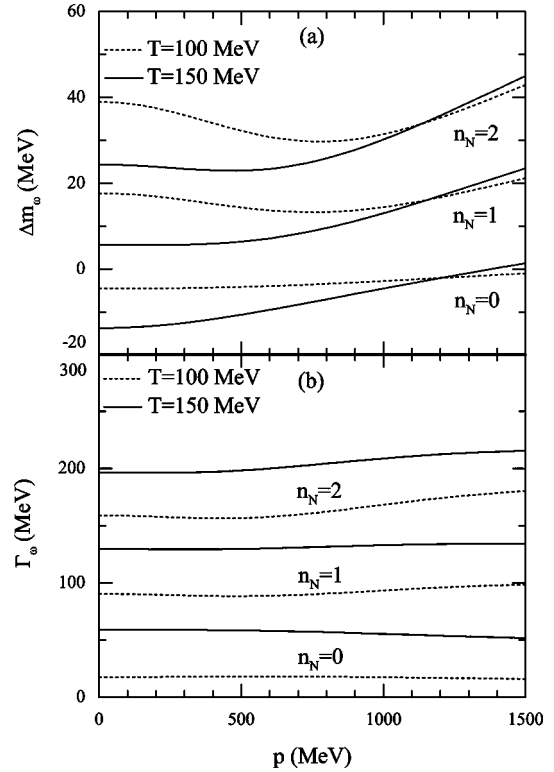


FIG. 2. (a) The mass shift and (b) the width of the ω meson as a function of momentum p . Results are shown for nucleon densities of 0, 1, and 2 in units of equilibrium nuclear matter density and temperatures of 100 and 150 MeV.

results are the same as in paper I. It was pointed out there that the calculated width of about 50 MeV at $T=150$ MeV is in agreement with the value of Schneider and Weise [22] and a little larger than given by Haglin [23]. Alam *et al.* [24] using the Walecka model achieve only half of our value, but they find that the magnitude increases very steeply with temperature. Turning to finite values of n_N , our results are rather insensitive to the momentum p and are intermediate between the two-resonance and $\omega \rightarrow \rho$ models discussed in paper I. For a temperature of 150 MeV and $n_N=1$, $\Gamma_{\omega}=130$ MeV, an enhancement of the vacuum width by a factor of 15. This is in line with Rapp's estimate [4] of a factor of 20 at a slightly higher temperature of 180 MeV. These values are somewhat smaller than the results Riek and Knoll [25] obtained with self-consistent coupled Dyson equations. For zero-temperature nuclear matter we find the width due to collisional broadening to be 75 MeV. A similar result was obtained by Riek and Knoll [25]. On the other hand, smaller values ~ 40 MeV were found by Post and Mosel [15], by Lutz, Wolf, and Friman [16] from a relativistic coupled channels approach, and by Mühlich, Falter, and Mosel [17] in a transport model.

The rate of dilepton production is directly proportional to the imaginary part of the photon self-energy [26,27] which is itself proportional to the imaginary part of the ω meson propagator because of vector meson dominance [28,29]:

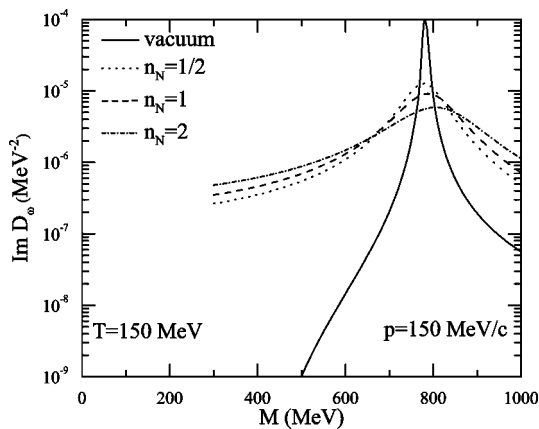


FIG. 3. The imaginary part of the ω meson propagator as a function of invariant mass for a momentum of 300 MeV/c and a temperature of 150 MeV. Results are shown for the vacuum and for nucleon densities of $\frac{1}{2}$, 1, and 2 in units of equilibrium nuclear matter density.

$$E_+ E_- \frac{dR}{d^3 p_+ d^3 p_-} \propto \frac{-\text{Im}\Pi_\omega^{\text{tot}}}{[M^2 - m_\omega^2 - \text{Re}\Pi_\omega^{\text{tot}}]^2 + [\text{Im}\Pi_\omega^{\text{tot}}]^2}, \quad (10)$$

where, as before, M is the invariant mass. Since the vacuum decay of the ω into three pions is complicated, while the width is tiny, we simply treat $\text{Im}\Pi_\omega^{\text{vac}}$ as a constant except for the application of a nonrelativistic phase space factor $[(M^2 - 9m_\pi^2)/(m_\omega^2 - 9m_\pi^2)]^2$ from threshold to $M=m_\omega$. A possible real vacuum contribution is ignored.

The imaginary part of the propagator, proportional to the spectral density, is plotted as a function of M in Fig. 3 for a temperature of 150 MeV. Pions alone have a small effect on the spectral density so we display results at $n_N = \frac{1}{2}, 1$, and $2n_0$. These parameters are characteristic of the final stages of a high-energy heavy ion collision. As seen from Fig. 3 there is little change in the position of the peak, but the spectral density is greatly broadened. For $n_N=1$ the width of the ω peak (full width, half maximum) is 140 MeV. This is quite similar to the $\omega \rightarrow \rho$ model in paper I and to the results of Rapp [4].

In summary, the in-medium properties of the ω meson found in paper I have been updated by employing the ωN resonance analysis of Ref. [6]. Taking as a reference point a temperature of 150 MeV, equilibrium nuclear matter density and zero momentum, the ω mass shift of 6 MeV was negligible. However, the width of 130 MeV represented a considerable increase from the vacuum value (8.4 MeV). Thus the spectral density, which determines dilepton production from this channel, was greatly broadened. These values are reasonably consistent with the bulk of the estimates in the literature. In particular the agreement with Rapp [4] is satisfying since he employed a many-body approach, while we attacked the same physics by using data to construct the scattering amplitudes. Nevertheless it should be borne in mind that there are effects that are not naturally included here. For example, it has been suggested, using effective Lagrangians, that a large, negative mass shift can arise from vacuum polarization [30,31] or quark structure [32] effects.

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- [1] C. Adami and G. E. Brown, *Phys. Rep.* **234**, 1 (1993); T. D. Cohen, R. J. Furnstahl, D. K. Griegel, and X. Jin, *Prog. Part. Nucl. Phys.* **35**, 221 (1995); T. Hatsuda, H. Shiomii, and H. Kuwabara, *Prog. Theor. Phys.* **95**, 1009 (1996); R. Rapp and J. Wambach, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 2000), Vol. 25, p. 1.
- [2] V. L. Eletsky, M. Belkacem, P. J. Ellis, and J. I. Kapusta, *Phys. Rev. C* **64**, 035202 (2001).
- [3] S. Jeon and P. J. Ellis, *Phys. Rev. D* **58**, 045013 (1998).
- [4] R. Rapp, *Phys. Rev. C* **63**, 054907 (2001).
- [5] D. M. Manley and E. M. Saleski, *Phys. Rev. D* **45**, 4002 (1992).
- [6] V. Shklyar, G. Penner, and U. Mosel, hep-ph/0301152.
- [7] H. Harari, *Phys. Rev. Lett.* **20**, 1395 (1968).
- [8] P. D. B. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press, Cambridge, UK, 1977).
- [9] T. P. Vrana, S. A. Dytman, and T.-S. H. Lee, *Phys. Rep.* **328**, 181 (2000).
- [10] Particle Data Group, K. Hagiwara *et al.*, *Phys. Rev. D* **66**, 010001 (2002).
- [11] A. Donnachie and P. V. Landshoff, *Phys. Lett. B* **296**, 227 (1992).
- [12] A. Sibirtsev, C. Elster, and J. Speth, nucl-th/0203044.
- [13] K. J. Foley *et al.*, *Phys. Rev.* **181**, 1775 (1969).
- [14] V. L. Eletsky, B. L. Ioffe, and J. I. Kapusta, *Eur. Phys. J. A* **3**, 381 (1998); V. L. Eletsky and J. I. Kapusta, *Phys. Rev. C* **59**, 2757 (1999).
- [15] M. Post and U. Mosel, *Nucl. Phys.* **A688**, 808 (2001).
- [16] M. F. M. Lutz, G. Wolf, and B. Friman, *Nucl. Phys.* **A706**, 431 (2002).
- [17] P. Mühlich, T. Falter, and U. Mosel, nucl-th/0402039.
- [18] F. Klingl, T. Waas, and W. Weise, *Nucl. Phys.* **A650**, 299 (1999).
- [19] S. Zschocke, O. P. Pavlenko, and B. Kämpfer, *Eur. Phys. J. A* **15**, 529 (2002).
- [20] V. L. Eletsky and B. L. Ioffe, *Phys. Rev. Lett.* **78**, 1010 (1997).
- [21] S. Mallik and A. Nyffeler, *Phys. Rev. C* **63**, 065204 (2001).
- [22] R. A. Schneider and W. Weise, *Phys. Lett. B* **515**, 89 (2001).
- [23] K. Haglin, *Nucl. Phys.* **A584**, 719 (1995).
- [24] J.-e. Alam, S. Sarkar, P. Roy, B. Dutta-Roy, and B. Sinha, *Phys. Rev. C* **59**, 905 (1999).
- [25] F. Riek and J. Knoll, nucl-th/0402090.
- [26] L. D. McLerran and T. Toimela, *Phys. Rev. D* **31**, 545 (1985).
- [27] H. A. Weldon, *Phys. Rev. D* **42**, 2384 (1990).
- [28] G. J. Gounaris and J. J. Sakurai, *Phys. Rev. Lett.* **21**, 244

- (1968).
- [29] C. Gale and J. I. Kapusta, Nucl. Phys. **B357**, 65 (1991).
- [30] H.-C. Jean, J. Piekarewicz, and A. G. Williams, Phys. Rev. C **49**, 1981 (1994).
- [31] A. Mishra, J. Reinhardt, H. Stöcker, and W. Greiner, Phys. Rev. C **66**, 064902 (2002); D. Zschiesche, A. Mishra, S. Schramm, H. Stöcker, and W. Greiner, nucl-th/0302073.
- [32] K. Saito, K. Tsushima, and A. W. Thomas, Phys. Rev. C **55**, 2637 (1997).