# Kaon-antikaon nuclear optical potentials and the $\kappa$ meson in the nuclear medium

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We study the properties of the  $\kappa$  meson in the nuclear medium, starting from a chiral unitary model of S-wave,  $I=1/2 K\pi$  scattering, which describes the elastic  $K\pi$  phase shifts and generates a pole in the amplitude. Medium effects are considered by including pion and kaon self-energies. We explore the changes in the  $\kappa$  pole position and in the  $K\pi$  scattering amplitude at finite density, with two different models for the kaon self-energy.

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## I. INTRODUCTION

Kaon and antikaon properties in the nuclear medium are quite different. Whereas the kaon's self-energy is weak, repulsive, and can be well accounted for by a  $t\rho$  approximation, the situation is much more complicated for the antikaons, among other reasons because of the presence of the  $\Lambda(1405)$  resonance just below the  $\overline{KN}$  threshold. The antikaon self-energy is particularly interesting because if it is sufficiently attractive it could lead to the existence of a Kcondensed phase in the neutron stars, as it was pointed out in Ref. [1], (for a recent discussion see Ref. [2]).

There is already clear experimental evidence of strong medium effects that deviate from simple low density theorem predictions on the strong enhancement of the  $K^-$  production in heavy-ion collisions in the KaoS experiments at GSI [3]. Although this result could imply a very attractive potential, it is difficult to extrapolate it to the high density and low temperature relevant for the neutron stars.

Also, strong medium effects have been found in kaonic atoms. However, there is strong controversy on the theoretical interpretation of these data, with widely different potentials available in the literature which provide a reasonable agreement to the data. They could be classified in two basic kinds, some phenomenological potentials, usually deeply attractive [4-7], and other chirally motivated potentials [8-16], which, after the need of a self-consistent treatment of the kaon and the  $\Lambda(1405)$  in the medium was shown [11], are clearly less attractive.

Some other observables that could be sensitive to the kaonic nuclear potential have been suggested, like the study of the  $\phi$  mass in nuclear reactions. However, due to the cancellation between attraction and repulsion for the antikaon and kaon, respectively, little if some effect is expected [17].

On the other hand, some interesting developments have taken place in the study of the light scalar mesons, in particular the  $\sigma$  and the  $\kappa$ . The  $\sigma$  meson now appears to be quite well established both theoretically and experimentally [18] although some discussion on its nature still remains open. In particular, unitarized models consistent with chiral constraints describe the  $\pi\pi$  phase shifts and clearly predict a pole at masses of around 470 MeV [19,20]. Some works using this kind of chiral models have also studied the isospin I=1/2 channel and find a very wide  $\kappa$  meson at masses around 800 MeV. A compilation of results both experimental and theoretical can be found in Refs. [21-23].

In the unitarized chiral models the  $\sigma$  and  $\kappa$  mesons can be understood as  $\pi\pi$  and  $\pi K$  resonances and this has important consequences for their interaction with the nuclear medium, as their self-energy will be directly related to that of the  $\pi$ and K mesons. The  $\sigma$  medium properties have been studied using different approaches, finding always a strong reduction of its mass and a much narrower width at high baryonic densities [24–26]. There are some experimental signals that strongly suggest that this is indeed the case. A quite strong enhancement of the  $\pi\pi$  invariant mass spectrum at the low masses predicted for the in-medium  $\sigma$  has been found in both  $(\pi, \pi\pi)$  [27–30] and  $(\gamma, \pi\pi)$  reactions [31–33]. In the unitarized chiral models the mass reduction of the  $\sigma$  and its narrowing in the nuclear medium are mainly produced by the well known attractive P-wave pion-nucleus optical potential. A similar, although richer in complexity, situation could occur for the  $\kappa$  meson. Given the clearly different interaction of the kaons and antikaons with the medium, one could expect a splitting of the masses of  $\kappa$  and anti- $\kappa$ . The difference would be sensitive to the different optical potential suffered by kaons and antikaons. Furthermore, as for the  $\sigma$  case, the strong attraction over the pion could lead to a common mass reduction and a narrower width.

Our purpose in this paper is to investigate this possibility. We start by presenting a simple model that predicts a  $\kappa$  pole and is consistent with the meson-meson phenomenology at low and intermediate energies in the next section. Next, we incorporate the nuclear medium effects using two different potentials in order to study the sensitivity of the observables to these potentials.

# II. $K\pi$ SCATTERING IN A CHIRAL UNITARY APPROACH

We briefly revise in this chapter the model of  $K\pi$  scatter-

ing, which is based on the chiral unitary approach to meson-



FIG. 1. (Color online)  $K\pi \rightarrow K\pi$  phase shifts in the I=1/2 channel. The solid line is the result of the model described in the text. Data sets are taken from Refs. [6] (dots) and [37] (triangles).

meson scattering developed in Ref. [34]. In that work, a good agreement with experimental phase shifts for the *S*-wave meson-meson scattering in the I=0,1 channels was found. We solve the Bethe-Salpeter (BS) equation, namely T=V + VGT, in which the kernel *V* is taken as the  $I=1/2 K\pi$  tree level amplitude from the lowest order  $\chi PT$  Lagrangian. After projecting onto the *S* wave, this amplitude reads

$$V = \frac{1}{4f^2} \left[ -\frac{5}{2}s + m_\pi^2 + m_K^2 + \frac{3}{2} \frac{(m_K^2 - m_\pi^2)^2}{s} \right].$$
 (1)

In Eq. (1), *s* is the Mandelstam variable,  $m_{\pi}$  and  $m_K$  are the pion and kaon masses, respectively, and *f* is the meson decay constant, which we take to be  $f^2 = (100 \text{ MeV})^2 \simeq f_{\pi} f_K$  as in Ref. [19].

Despite the integral character of the BS equation, it was found in Ref. [34] that the V amplitude factorizes on-shell out of the integral in the VGT term, and so does T. Thus T can be algebraically solved in terms of the V and the G functions, the latter contains the integral of the two meson propagators,

$$G(\sqrt{s}) = \frac{1}{4\pi^2} \int_0^{q_{\text{max}}} dq \frac{q^2}{\omega_\pi(q)\omega_K(q)} \frac{\omega_\pi(q) + \omega_K(q)}{s - [\omega_\pi(q) + \omega_K(q)]^2 + i\epsilon},$$
(2)

with  $\omega_{\pi}(q) = \sqrt{q^2 + m_{\pi}^2}$  and  $\omega_K(q) = \sqrt{q^2 + m_K^2}$ . In Eq. (2) we regularize the *G* function with a cutoff in the momentum of the mesons in the loop. A value of  $q_{\text{max}} = 850$  MeV is used, what leads to a satisfactory fit to the I = 1/2  $K\pi$  phase shifts, as shown in Fig. 1. The scattering amplitude in this channel exhibits a pole in the second Riemann sheet (2ndRS) of the complex energy plane, which we identify with the  $\kappa$  meson. The pole position that we get is  $M_{\kappa} + i\Gamma_{\kappa}/2 = 825$ +460*i* (MeV). We have not considered in this calculation the contribution of the  $K\eta$  intermediate state. Its inclusion leads to a coupled channel calculation, as done in Ref. [34] for  $\pi\pi$ and  $\bar{K}K$  channels. However, it was found in Ref. [35] that this channel barely mixes with the  $K\pi$  channel. We have also checked that accounting for it barely modifies the phase shifts in the region beyond 1 GeV, and produces no visible effect at lower energies. Since we are mainly interested in the medium effects on the  $K\pi$  channel, we shall ignore the  $K\eta$  contribution from now on.

#### III. $K\pi$ SCATTERING IN THE NUCLEAR MEDIUM

We consider medium corrections to the  $K\pi$  scattering amplitude by dressing the meson propagators with appropriate self-energies regarding the interactions of the mesons with the surrounding nucleons of the nuclear medium. These selfenergies have been worked out elsewhere and we include here a brief description. Then we explain our method to search for the  $\kappa$  pole at finite baryon density.

### A. Pion self-energy in the medium

The pion self-energy is driven by the excitation, in the *P* wave, of *ph* and  $\Delta h$  pairs. A derivation of the *P*-wave pion self-energy from the  $\pi N$  scattering amplitude can be found in Ref. [38], Chap. 5. Additionally, a resummation of short range correlation terms in terms of the Landau-Migdal parameter g' is done. The final expression reads

$$\Pi_{\pi}(q^{0}, \vec{q}; \rho) = \mathcal{F}^{2}(\vec{q}^{2})\vec{q}^{2} \frac{\left(\frac{D+F}{2f}\right)^{2} U(q^{0}, \vec{q}; \rho)}{1 - \left(\frac{D+F}{2f}\right)^{2} g' U(q^{0}, \vec{q}; \rho)}, \quad (3)$$

where (D+F)/(2f) is the  $\pi NN$  coupling from the lowest order chiral Lagrangian,  $U=U_N+U_{\Delta}$  is the Lindhard function for the *ph* and  $\Delta h$  excitations, and  $\mathcal{F}(\vec{q}^2)$  is a monopolar form factor. We follow the notation of Refs. [17,39].

#### B. Kaon and antikaon self-energies in the medium

We shall consider here two different potentials for the kaons, in order to test the sensitivity of the results to the models and their ability to discriminate between them. The first model is a chirally motivated potential [13] which provides a weak attraction for the antikaons. We shall refer to it as "model A." The second one is a phenomenological potential described in Ref. [4], leading to a very strong attraction for the antikaons. We shall call it "model B."

The KN interaction is smooth at low energies, and both models use a  $t\rho$  approximation. The kaon self-energy is given by

$$\Pi_K(\rho) = C m_K^2 \rho / \rho_0, \qquad (4)$$

where  $\rho$  is the nuclear density,  $\rho_0$  stands for the normal nuclear density, and *C* takes the value 0.13 for model A [40] and 0.114 for model B [4].

The  $\overline{K}N$  interaction, however, is much richer at low energies. Model A starts from Ref. [41], where the *S*-wave  $\overline{K}N$ scattering was studied in a chiral unitary model in coupled channels, leading to a successful description of many scattering observables, namely, threshold ratios of  $K^-p$  to several inelastic channels,  $K^-p$  and  $K^-n$  scattering lengths, and  $K^-p$  cross sections in the elastic and inelastic channels  $(\overline{K}^0 n, \pi^0 \Lambda, \pi^{\pm} \Sigma^{\mp}, \pi^0 \Sigma^0)$ . Medium effects were considered in Ref. [13] to obtain an effective  $\overline{K}N$  interaction in nuclear matter, from which the *S*-wave  $\overline{K}$  self-energy was obtained in a self-consistent way. Finally, the *P*-wave contribution to the  $\overline{K}$  self-energy, arising from the excitation of *Yh* pairs [ $Y = \Lambda, \Sigma, \Sigma^*(1385)$ ] was also included. Full details can be found in Refs. [13,17]. Model B, based on a dispersive calculation of the kaon potentials, which uses as input the  $K^{\pm}N$  scattering amplitudes, finds a strong  $\overline{K}$  potential that drops to -200 MeV at normal nuclear density and zero momentum, and shows a quite strong momentum dependence. We use the parametrization given in Ref [4],

$$\Pi_{\bar{K}}^{B}(q,\rho) = -\left[0.233 + 0.563 \exp(-1.242q/m_{K})\right] m_{K}^{2} \rho/\rho_{0}.$$
(5)

These two potentials have been chosen as representative examples of the kaon-nucleus potentials currently discussed in the literature. Some "deep" potentials (around 200 MeV depth at zero  $\overline{K}$  momentum) are found in phenomenological analysis like Refs. [5,6] and also in the dispersive calculation of model B [4] after some approximations. However, selfconsistent derivations of the  $\overline{K}$  optical potential starting from a good description of  $\overline{KN}$  scattering produce "shallow" potentials ( $\approx$ 40 to 60 MeV depth) like model A [13] and the potential obtained in Ref. [11]. Both kinds of potentials produce a good agreement with kaonic atom data, and the depth of the optical potential cannot be resolved by analysing these data as has been shown in Ref. [16].

Higher order effects in density in the kaon self-energy, like those induced by the short range correlations in the pion self-energy, have been studied in Refs. [42,43]. The effects have been found to be very small at the densities considered in this work, and we shall neglect any contribution from this source.

#### C. Search for the $\kappa$ pole

To look for the  $\kappa$  pole in the medium we follow the procedure described in Ref. [44] for the  $\sigma$  meson in  $\pi\pi$  scattering, conveniently adapted to the present problem. We need to evaluate the  $K\pi$  scattering amplitude, T, in the 2ndRS of the complex energy ( $\sqrt{s}$ ) plane. The analytical structure of T is driven by the  $G(\sqrt{s})$  function, which has a cut on the positive energy real axis. The rest of the functions in the BS equation are analytical and single-valued. To calculate  $G(\sqrt{s})$  at finite baryon density, we use the following spectral (Lehmann) representations of the meson propagators:

$$D_{\pi}(q^{0},\vec{q};\rho) = \int_{0}^{\infty} d\omega 2\omega \frac{S_{\pi}(\omega,\vec{q};\rho)}{(q^{0})^{2} - \omega^{2} + i\epsilon},$$
$$D_{\bar{K}(K)}(q^{0},\vec{q};\rho) = \int_{0}^{\infty} d\omega \left(\frac{S_{\bar{K}(K)}(\omega,\vec{q};\rho)}{q^{0} - \omega + i\eta} - \frac{S_{K(\bar{K})}(\omega,\vec{q};\rho)}{q^{0} + \omega - i\eta}\right),$$
(6)

where  $S_{\pi}$  and  $S_{\overline{K}(K)}$  are the spectral functions of the pion and the antikaon (kaon), respectively, that are directly related to

the self-energies. After some basic manipulations,  $G(\sqrt{s})$  can be written as

$$G(\sqrt{s}) = \frac{1}{2\pi} \int_0^\infty dW \left[ \frac{1}{\sqrt{s} - W + i\eta} F_P(W) - \frac{1}{\sqrt{s} + W} F_{NP}(W) \right],\tag{7}$$

and  $F_P(W)$ ,  $F_{NP}(W)$  (accompanying the "pole" and "nonpole" terms, respectively) are real, positively defined functions independent of  $\sqrt{s}$ . For  $\overline{K}\pi$  scattering they are given by

$$F_{P}(W) = \int \frac{d^{3}q}{(2\pi)^{3}} \int_{-W}^{W} du \, \pi S_{\pi} \left(\frac{W-u}{2}, \vec{q}; \rho\right) S_{\vec{k}} \left(\frac{W+u}{2}, \vec{q}; \rho\right),$$
  
$$F_{NP}(W) = \int \frac{d^{3}q}{(2\pi)^{3}} \int_{-W}^{W} du \, \pi S_{\pi} \left(\frac{W-u}{2}, \vec{q}; \rho\right) S_{K} \left(\frac{W+u}{2}, \vec{q}; \rho\right).$$
(8)

For  $K\pi$  scattering  $S_{\overline{K}}$  in Eq. (8) has to be replaced by  $S_K$ , and vice versa. The cutoff in the meson momentum is included in these functions (not explicitly shown). As an example, in vacuum  $F_P = F_{NP} \equiv F_{K\pi}$  factorizes out of the brackets in Eq. (7) and reads

$$F_{K\pi}(W) = \frac{1}{4\pi} \frac{q(W)}{W} \theta(W - (m_{\pi} + m_{K})), \qquad (9)$$

with  $q(W) = \lambda^{1/2}(W^2, m_{\pi}^2, m_K^2)/2W$  and  $\lambda$  the Källen function.

The first term in Eq. (7) is responsible for the cut in the real axis. It is easily shown that the G function in the 2ndRS can be written as

$$G^{2ndRS}(\sqrt{s}) = \frac{1}{2\pi} \int_0^\infty dW \Biggl[ \frac{1}{\sqrt{s} - W + i\eta} F_P(W) - \frac{1}{\sqrt{s} + W} F_{NP}(W) \Biggr] + iF_P(\operatorname{Re}\sqrt{s}). \quad (10)$$

#### **IV. RESULTS AND DISCUSSION**

We have calculated the  $K\pi$  and  $\bar{K}\pi$  scattering amplitudes and found the  $\kappa$  pole position for several nuclear densities from  $\rho=0$  to  $\rho=1.5 \rho_0$ . To discuss the results we shall distinguish between the following three cases: (1) free pion, in-medium kaons; (2) in-medium pion, free kaons; and (3) in-medium pions and kaons.

In Fig. 2 we show the  $\kappa$  pole position in the complex energy plane, using model A for the kaon self-energies. Every curve departs from a common point, which corresponds to the pole position in the vacuum case, and we increase nuclear density according to the following values:  $\rho/\rho_0$ =0,1/8,1/4,1/2,3/4,1,3/2, which correspond to the successive dots in each trajectory. The curves labeled 1 correspond to the case of in-medium kaons and free pions. The first interesting fact is that the pole trajectory splits up into two different branches corresponding to  $K\pi$  and  $\bar{K}\pi$  scattering. This was an expected result since the kaon self-energy is asymmetric for the particle compared to the antiparticle. We



FIG. 2. (Color online)  $\kappa$  pole trajectories at finite density. The labels correspond to the three cases discussed in the text.

can see that, in the  $K\pi$  branch, the  $\kappa$  mass from the pole,  $M_{\kappa}$ =Re  $\sqrt{s_{\text{pole}}}$ , moves to higher energies. This responds to the repulsion felt by the kaons in the medium. In the  $\bar{K}\pi$  branch one finds that the  $\kappa$  pole mass decreases slowly with the nuclear density. However, its decay width increases due to the opening of additional decay channels in the medium, as  $\kappa \rightarrow \pi MYh$  and  $\kappa \rightarrow \pi Yh$ . These channels are accounted for in the  $\bar{K}$  *S*- and *P*-wave self-energies.

The next case that we consider corresponds to the curve labeled as 2 in Fig. 2, i.e., free kaons and in-medium pions. Since the pion self-energy is the same for the three isospin components (in symmetric nuclear matter), we find a single trajectory for the  $\kappa$  pole. When the nuclear density increases, the pole position rapidly moves to lower energies. This is consistent to what happens for the  $\sigma$  meson and is due to the strong attraction experienced by the pion in the nuclear medium. From this we obtain that  $M_{\kappa}$  is strongly reduced, from 825 MeV in vacuum down to 650 MeV at  $\rho = 1.5\rho_0$ , getting very close to the  $K\pi$  threshold.  $\Gamma_{\kappa}$  also shows a noticeable reduction of 300 MeV at  $\rho = 1.5\rho_0$ . In fact, one may expect even a stronger reduction in the  $\kappa$  decay width, given the



FIG. 3. (Color online)  $\kappa$  pole evolution with density. Comparison between two models of anti-kaon self-energy.



FIG. 4. (Color online) Real part, imaginary part, and squared modulus of the kaon-pion scattering amplitude at several densities. Upper panels correspond to  $\bar{K}\pi$  channel, lower panels to  $K\pi$  channel.

proximity of the pole to the  $K\pi$  threshold and the consequent reduction of available phase space for  $\kappa \rightarrow K\pi$  decays. However, this reduction of phase space is partly compensated by



E(GeV)

FIG. 5. (Color online) Squared modulus of the kaon-pion scattering amplitude, in vacuum and at normal density, for the two models of kaon self-energy discussed in the text. Left:  $\overline{K}\pi$  channel; right:  $K\pi$  channel.

the simultaneous opening of pion-related in-medium channels, namely  $\kappa \rightarrow K ph$ ,  $K \Delta h$ . Such effect was also discussed in Refs. [25,44] for the  $\sigma$  meson in  $\pi\pi$  scattering.

E(GeV)

Eventually, the two curves labeled as 3 in Fig. 2 represent the  $\kappa$  pole evolution with density in the full model A. We find again separate  $K\pi$  and  $\bar{K}\pi$  branches, whose tendency to lower energies is a clear signal of the strong influence of the attractive pion self-energy. The  $K\pi$  branch shows a sudden curvature for densities of and beyond  $\rho_0$ .

In Fig. 3 we compare the results on the  $\kappa$  pole position for models A and B and densities up to  $\rho = \rho_0$ . We observe that the mass splitting between the  $K\pi$  and  $\bar{K}\pi$  branches is larger in model B, amounting to about 50 MeV at normal nuclear density. The anti- $\kappa$  pole trajectories, sensitive to the  $\bar{K}$  potential, show stronger differences, whereas the  $\kappa$  branches are quite similar since the kaon self-energy only differs at the level of 10% in the two models. Particularly, the  $\bar{K}$  selfenergy in model B does not include an imaginary part, leading to a further decrease of the  $\kappa$  decay width.

We have also calculated the  $K\pi$  and  $\bar{K}\pi$  scattering amplitudes for several nuclear densities in model A. These amplitudes could be eventually tested experimentally in reactions with pions and kaons in the final state. The results are shown in Fig. 4. The  $\bar{K}\pi$  amplitude changes rather smoothly with increasing density. The real part changes little at low energies and flattens at higher energies. The imaginary part displays an increase of strength below 800 MeV as compared to the vacuum case, and the threshold is shifted to lower energies. Beyond 800 MeV, though, we find a progressive decrease of strength.

The  $K\pi$  channel shows a similar behavior for  $\sqrt{s} \geq 800$  MeV as the  $\overline{K}\pi$  channel. However, we observe a greater accumulation of strength below 800 MeV in the imaginary part, which peaks at about 700 MeV at  $\rho = 1.5\rho_0$ . The real part also reflects a rapidly changing structure in this energy region. This effect was already observed in Refs. [25] for the  $\pi\pi$  scattering amplitude in the  $\sigma$  channel, and it was found in Ref. [44] that this was correlated to a migration of the  $\sigma$  pole to lower energies at finite densities. A similar behavior is found here for the  $\kappa$  meson. In the  $\overline{K}\pi$  case some strength of Im *T* spreads well below the vacuum threshold mainly because of the attraction experienced by the antikaon, which moves the  $\overline{K}\pi$  threshold down, and the coupling to in-medium channels with the same quantum numbers occur-

ring at lower energies, for instance meson hyperon - hole excitations. However, for the  $K\pi$  case the strength of Im *T* is strongly accumulated above the in-medium threshold, which is basically determined by the repulsive potential affecting the kaon. The little strength below the vacuum threshold corresponds only to channels in which the pion is absorbed by a particle-hole excitation. We have also included in Fig. 4 the squared modulus of the amplitude. In the  $\overline{K}\pi$  channel the main visible effect is a strong decrease of strength beyond 700 MeV, whereas in the region around and below threshold little effect can be seen because of the lack of phase space, though it was clearly visible in the imaginary part of the amplitude. The  $K\pi$  channel clearly exhibits a prominent structure close to threshold as commented above.

The resulting scattering amplitudes for model B show a similar trend as for model A but there are some differences. In Fig. 5 we show the squared modulus of the amplitudes for model B, at normal nuclear density, together with model A and the free case for reference. The most visible effect, as commented above, corresponds to the  $K\pi$  channel where both models are quite similar and there is little theoretical discussion concerning the *K* potential in the medium. We find very similar results for this channel in the two approaches. A stronger medium effect is observed in the  $\bar{K}\pi$  amplitude for model B, as a consequence of the more attractive  $\bar{K}$  potential in this model.

In summary, we find a noticeable modification of the *S*-wave  $K\pi$  interaction in the  $\kappa$  channel at finite densities. The  $\overline{K}\pi$  channel is sensitive to the different  $\overline{K}$  potentials used in this work. The most visible medium effect, though, appears in the  $K\pi$  channel, where the  $\overline{K}$  potential has little influence, and might be observed in  $K\pi$  invariant mass distributions around 650 MeV.

### V. CONCLUSIONS

We have studied the  $\kappa$  meson properties in the nuclear medium. We follow a chiral unitary model of  $K\pi$  scattering that reproduces the elastic phase shifts in the I=1/2 channel and dynamically generates the  $\kappa$  meson, which appears as a pole in the scattering amplitude. The medium effects have been considered by dressing the pion and kaon propagators with self-energies that have been calculated elsewhere. When the density is increased, the  $\kappa$  pole moves to lower masses and decay widths. However, it does not become as narrow as to provide a clear signal to be observed experimentally. We have also studied the scattering amplitude at finite densities for the  $K\pi$  and  $\bar{K}\pi$  channels. We have found that the amplitude is strongly modified in the medium in both channels, and particularly it is sensitive to the differences between models of the antikaon potential. The most noticeable effect is the accumulation of strength found at lower invariant

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masses of the  $K\pi$  system. We suggest that this effect could be experimentally observed.

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