Isoscaling in the lattice gas model

Y. G. Ma,[1,2,*] K. Wang,[1] Y. B. Wei,[1] G. L. Ma,[1] X. Z. Cai,[1] J. G. Chen,[1] D. Q Fang,[1] W. Guo,[1] W. Q. Shen,[1,2] W. D. Tian,[1] and C. Zhong[1]
[1]*Shanghai Institute of Applied Physics, Chinese Academy of Sciences, P.O. Box 800-204, Shanghai 201800, China*
[2]*China Center of Advanced Sciences and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China*
(Received 4 March 2004; published 16 June 2004)

Isoscaling behavior is investigated using the isotopic and isobaric yields from an equilibrated thermal source which is prepared by the lattice gas model (LGM) for lighter systems with $A=36$. The isoscaling parameters α and β are observed to drop with temperature for the LGM with the asymmetric nucleon-nucleon potential. However, the isoscaling parameters do not show a temperature dependence for the LGM with the symmetric nucleon-nucleon potential. The relative neutron or proton density shows a nearly linear relation with the N/Z (neutron to proton ratio) of the system.

DOI: 10.1103/PhysRevC.69.064610 PACS number(s): 25.70.Pq, 24.10.Pa, 05.70.Jk

Isoscaling has been observed in a variety of reactions under the conditions of statistical emission and equal temperature recently by Tsang *et al.* [1–3]. This kind of scaling means that the ratio $R_{21}(N,Z)$ of the yields of a given fragment (N,Z) exhibits an exponential dependence on N and Z when these fragments are produced in two reactions with different isospin asymmetry, but at the same temperature. Experimentally isoscaling has been explored in various reaction mechanisms, ranging from evaporation [1], fission [4,5], and deep inelastic reactions at low energies to projectile fragmentation [6,7] and multifragmentation at intermediate energy [1,8,9]. While, isoscaling has been extensively examined in different theoretical frameworks, ranging from dynamical models, such as the antisymmetrical molecular dynamics model [10] and Boltzmann-Uehling-Uhlenback (BUU) model [8], to statistical models, such as the expansion emission source model and statistical multifragmentation model [2,3,11,12]. From all these reaction mechanisms and models, it looks that isoscaling is a robust probe to relate with the symmetrical term of the nuclear equation of state.

Typically, the investigations of isoscaling focused on yields of light fragments with $Z=2-8$ originating from deexcitation of massive hot systems produced using reactions of mass symmetric projectiles and targets at intermediate energies, such as ^{112,124}Sn+^{112,124}Sn in Michigan State University (MSU) data [1–3] or by reactions of high-energy light particles with a massive target nucleus [11,13]. In a recent article [6], the isoscaling using the heavy projectile residue from the reactions of 25 MeV/nucleon ^{86}Kr projectiles with ^{124}Sn, ^{112}Sn and ^{64}Ni, ^{58}Ni targets which was performed at Texas A&M University (TAMU) and the isoscaling phenomenon on a full sample of fragments emitted by hot thermally equilibrated quasiprojectiles with mass $A=20-30$ are also reported [7].

In this study, we present an isoscaling analysis for light fragments from thermal sources which are produced by the lattice gas model. Instead of a fixed charge number of the reaction system in MSU data or TAMU data, here we fixed the source mass for a lighter system, but changing the charge number and neutron number. Isospin fractionation is also observed via the relative free neutron and free proton density which is obtained by the isoscaling parameters.

A thermally equilibrated system undergoing statistical decay can be, within the grand-canonical approach, characterized by a yield of fragments with neutron and proton numbers N and Z [14,15]:

$$Y(N,Z) = F(N,Z)\exp\frac{B(N,Z)}{T}\exp\left(\frac{N\mu_n}{T}+\frac{Z\mu_p}{T}\right), \qquad (1)$$

where $F(N,Z)$ represents the contribution due to secondary decay from particle stable and unstable states to the ground state, μ_n and μ_p are free neutron and proton chemical potentials, $B(N,Z)$ is the ground-state binding energy of the fragment, and T is the temperature.

The ratio of the isotope yields from two different systems, having similar excitation energies and similar masses, but differing only in N/Z, cancels out the effect of secondary decay and provides information about the excited primary fragments [1]. Within the grand-canonical approximation [Eq. (1)], the ratio $Y_2(N,Z)/Y_1(N,Z)$ assumes the form

$$R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z) = C\exp(\alpha N+\beta Z), \qquad (2)$$

with $\alpha=\Delta\mu_n/T$ and $\beta=\Delta\mu_p/T$ with $\Delta\mu_n$ and $\Delta\mu_p$ being the differences in the free neutron and proton chemical potentials of the fragmenting systems. C is an overall normalization constant.

The tool we will use here is the isospin-dependent lattice gas model (LGM). The lattice gas model was developed to describe the liquid-gas phase transition for an atomic system by Lee and Yang [16]. The same model has already been applied to nuclear physics for isospin symmetrical systems in the grand-canonical ensemble [17] with a sampling of the canonical ensemble [18–24] and also for isospin asymmetrical nuclear matter in the mean field approximation [25]. Here we will make a brief description for the models.

In the lattice gas model, A $(=N+Z)$ nucleons with an occupation number s which is defined $s=1$ (-1) for a proton (neutron) or $s=0$ for a vacancy are placed on the L sites of

*Electronic address: ygma@sinr.ac.cn

lattice. Nucleons in the nearest-neighboring sites interact with an energy $\epsilon_{s_i s_j}$. The Hamiltonian is written as

$$E = \sum_{i=1}^{A} \frac{P_i^2}{2m} - \sum_{i<j} \epsilon_{s_i s_j} s_i s_j. \tag{3}$$

In order to investigate the symmetrical term of the nuclear potential in this model, we use two sets of parameters: one is an attractive potential constant $\varepsilon_{s_i s_j}$ between the neutron and protons but no interaction between like nucleon—i.e., proton and proton or neutron and neutron—namely,

$$\epsilon_{nn} = \epsilon_{pp} = 0 \text{ MeV},$$

$$\epsilon_{pn} = -5.33 \text{ MeV}. \tag{4}$$

This potential results in an asymmetrical potential among different kinds of nucleons; hence it is an isospin-dependent potential. For simplicity we call the calculation with this potential as isoLGM thereafter. Another set is the same interaction constant between like nucleons or unlike nucleons—i.e.,

$$\epsilon_{pn} = \epsilon_{nn} = \epsilon_{pp} = -5.33 \text{ MeV}. \tag{5}$$

In this case, the nucleon potential is symmetrical among all nucleons—i.e., isospin-independent potential. For simplicity, we call the calculation with Eq. (5) as noisoLGM thereafter. In this work, mostly we use isoLGM to explore isoscaling behavior, but we will also use noisoLGM to compare the isoscaling results.

In the LGM simulation, a three-dimensional cubic lattice with L sites is used. The freeze-out density of the disassembling system is assumed to be $\rho_f=(A/L)\rho_0$, where ρ_0 is the normal nuclear density. The disassembly of the system is to be calculated at ρ_f, beyond which nucleons are too far apart to interact. Nucleons are put into a lattice by Monte Carlo Metropolis sampling. Once the nucleons have been placed we also ascribe to each of them a momentum by Monte Carlo samplings of the Maxwell-Boltzmann distribution.

Once this is done the LGM immediately gives a cluster distribution using the rule that two nucleons are part of the same cluster if $P_r^2/2\mu - \epsilon_{s_i s_j} s_i s_j < 0$. This method is similar to the Coniglio-Klein's prescription [26] in condensed matter physics.

In this paper we choose small-size nuclei with $A=36$ as emission sources. Four isotonic sources—namely, ^{36}Ca, ^{36}Ar, ^{36}S, and ^{36}Si—corresponding to $N/Z=0.8$, 1.0, 1.25, and 1.57, respectively, are simulated. In all cases, the freeze-out density ρ_f is chosen to be $0.563\rho_0$, which corresponds to a 4^3 cubic lattice. Here 10 000 events were simulated for each T which ensures good statistics for results.

As an example, in Fig. 1, we present the isotopic scaling (the upper panel) and the isobaric scaling (the lower panel) from the isospin-dependent lattice gas model for ^{36}S to ^{36}Ca, respectively, at $T=5.0$ MeV. Charged particles with $Z \leqslant 8$ and $2 \leqslant N \leqslant 9$ have been accumulated to perform the ratios. There exists good linear behavior in the semilog plot. In order to extract the isoscaling parameters α and β, we use $R_{21}(N)=C\exp(\alpha N)$ to obtain α for a given Z [Fig. 1(a)] and use $R_{21}(Z)=C\exp(\beta Z)$ to obtain β for a given N [Fig. 1(b)], respectively. Figure 2 shows the extracted α or β versus Z or N. Obviously, α or β keeps the same value in the wide Z or N range and their absolute values are almost the same which is due to the absence of a Coulomb interaction in the lattice gas model.

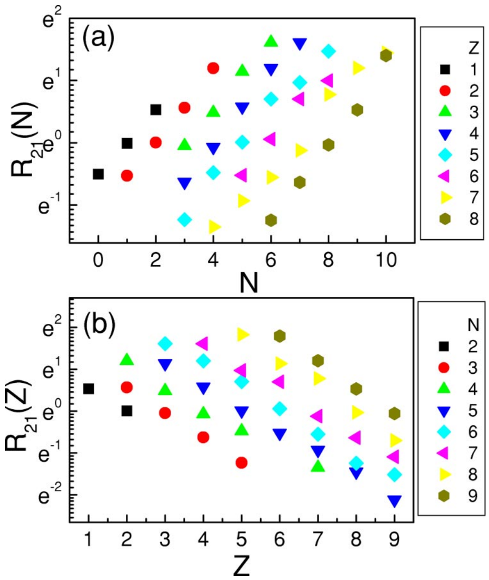

FIG. 1. (Color online) Isoscaling behavior of $R_{21}(N)$ (the upper panel) and $R_{21}(Z)$ (the lower panel) at $T=5.0$ MeV.

Figure 3 shows the temperature dependence of the absolute values of isoscaling parameters α and β. Here the scattering points are the calculation results for the LGM with asymmetrical nucleon-nucleon potential and the lines represent the results for the LGM with the same nucleon-nucleon potential for unlike and like nucleon pairs. A decreasing trend of the values is clearly seen when the asymmetrical nucleon-nucleon potential is used in the LGM, which indicates that the isospin dependence of the fragment yields becomes weak with increasing temperature. Considering that $\alpha=\Delta\mu_n/T$ and $\beta=\Delta\mu_p/T$, we can deduce the differences in free neutron and proton chemical potentials of the fragmenting systems which are shown in Fig. 4(a). Within the error bars, it looks like that $\Delta\mu_n$ and $\Delta\mu_p$ keep constant in the case of an asymmetrical potential [Eq. (4)]. However, a slight and

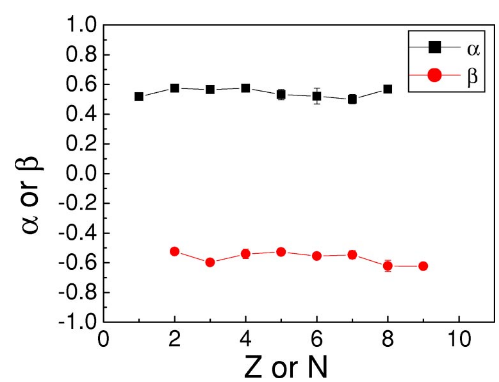

FIG. 2. (Color online) Isoscaling parameter α and β extracted in fixed Z from Fig. 1(a) and N from Fig. 1(b), respectively.

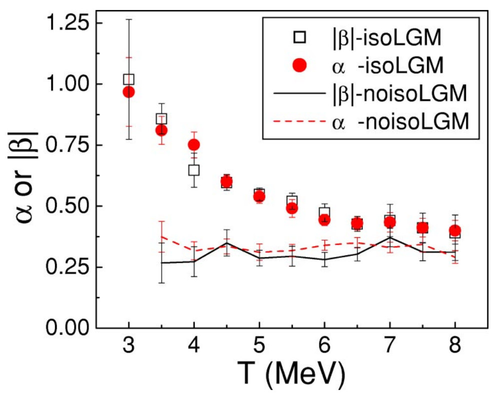

FIG. 3. (Color online) The absolute value of the isoscaling parameters α and β as a function of T from the yield ratios from the sources of ^{36}S and ^{36}Ca. The scattering points are the calculation results for the LGM with asymmetrical nucleon-nucleon potential [Eq. (4)] and the lines represent the results for the LGM with the same nucleon-nucleon potential for unlike and like nucleon pairs [Eq. (5)]. See text for details.

wide valley can be also identified around $T=5$ MeV as well as a slight kink showing in the SMM model for Sn systems [11]. For our system, this valley may be related to the liquid gas phase change for a similar system around 5 MeV in the same model calculation as well as the data [27]. This valley becomes more obvious if we plot the values of $(\Delta\mu_n - \Delta\mu_p)/2$ as a function of temperature as suggested in Refs. [7,11]. Figure 4(b) shows that a turning point seems to occur around 5 MeV. Of course, the error bars look larger for lower and higher temperatures, because of the poor statistics for diverse cluster species due to the evaporation mechanism in low T and the vaporization mechanism in higher T.

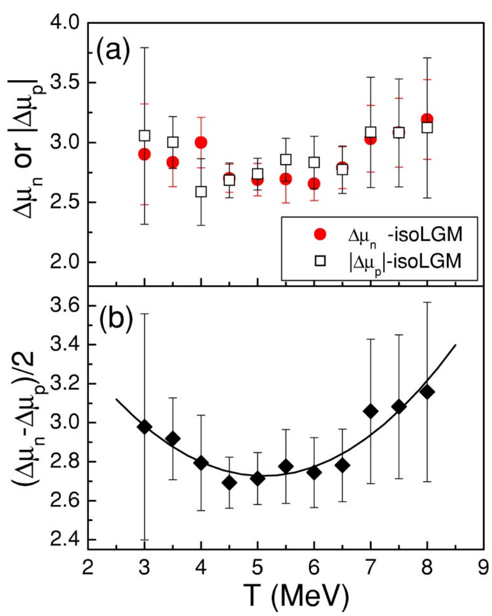

FIG. 4. (a) (Color online) The absolute value of the difference of neutron and proton chemical potential ($\Delta\mu_n$ and $|\Delta\mu_p|$) as a function of T. (b) $(\Delta\mu_n - \Delta\mu_p)/2$ as a function of temperature. Solid points are calculated results and line is its second polynomial fit. See text for details.

Around this point, an apparent critical behavior has been observed in the disassembly of hot nuclei with $A\sim 36$ in TAMU-NIMROD experimental data and model calculations [27] by a wide variety of observables such as the maximal fluctuations, critical exponent analysis, and fragment topological structure—namely, the nuclear Zipf law of Ma [28] and the heaviest and second heaviest correlation, etc. Hence, this turning point of the difference of neutron and proton chemical potentials might give additional evidence of the chemical phase separation when the liquid-gas phase transition occurs. Experimentally, this kind of turning point has been observed recently for a quasiprojectile from peripheral collisions of ^{28}Si+^{112,124}Sn at 30 and 50 MeV/nucleon in other TAMU data and can be understood as a signal of the onset of separation into isospin asymmetric dilute and isospin symmetric dense phases in a recent paper by Veselsky *et al.* [7].

Further, we shall investigate the effect of the asymmetrical nucleon-nucleon potential in the lattice gas model on the isoscalig behavior. We take the same potential between like nucleons and unlike nucleons in the lattice gas model [Eq. (5)] to compare with the LGM with isospin asymmetrical nucleon-nucleon potential [Eq. (4)]. In this case, we observe that the temperature dependence of α and β vanishes and their absolute values are smaller than the isoLGM case as shown by the lines in Fig. 3. The insensitivity of α and β to temperature means that the isoscaling of fragment yields only stems from the initial N/Z of the system. This is possible since the nucleon-nucleon potential is the same for like nucleons and unlike nucleons in this noisoLGM case. When temperature increases, the yield of light clusters also increases, but their isospin population does not change between the different cluster species, which results in no dependence for the isotopic or isobaric ratio on temperature. In this case, the system looks like a single-component fluid; the chemical phase separation cannot occur. However, for the isoLGM, the nucleon-nucleon potential is asymmetrical between like pair and unlike pair; it will give an additional isoscaling contribution due to the isospin-dependent equation of state except from the trivial contribution from the initial difference of the isospin of the source, which results in a bigger absolute value of α and β than the noisoLGM case. In this context, the information on the equation of state of the asymmetric nucleon-nucleon potential could be extracted by the temperature dependence of isoscaling parameters.

Finally, we test the relative free neutron density and proton density, which can be deduced by the equations

$$\rho_n = \exp(\alpha'), \qquad (6)$$

$$\rho_p = \exp(\beta'), \qquad (7)$$

in the lattice gas model. Here α' and β' are the scaling parameters where we take $Y_1(N,Z)$ from ^{36}Ar instead of ^{36}Ca as shown above. Principally, the association of a number density with $\exp(\Delta\mu/T)$ only is valid for a classical gas of free particles. The connection in this context of the LGM where the particles are interacting is not clear. However, as an attempt, we will still use the above equation to deduce the

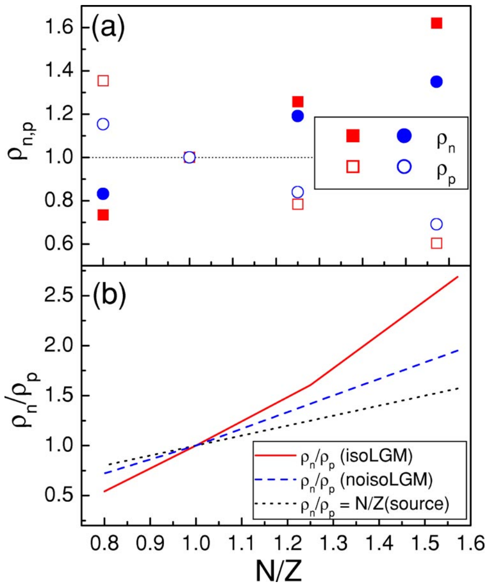

FIG. 5. (Color online) (a) The relative free neutron (solid symbols) or proton (open symbols) density as a function of N/Z of the system at $T=5$ MeV for the isoLGM case (squares) and noisoLGM case (circles). (b) The solid and dashed lines represent the ratio ρ_n/ρ_p for the isoLGM case and noisoLGM case, respectively. The dotted line shows the initial ρ_n/ρ_p assuming neutrons and protons homogeneously distributed in a volume proportional to the nucleon number.

relative free neutron density and proton density. In Fig. 5(a) we show the ρ_n (solid symbols) and ρ_p (open symbols) as a function of N/Z of the systems at $T=5$ MeV for the isoLGM case (squares) and the noisoLGM case (circles). In both cases, nearly linear relations of ρ_n and ρ_p have been observed with the increasing of N/Z. Figure 5(b) shows the ratio ρ_n/ρ_p. The solid (dashed) line represents the deduced ρ_n/ρ_p from solid and open squares (circles) of Fig. 5(a) with isoLGM (noisoLGM) calculations and the dotted line is the initial value of ρ_n/ρ_p which is calculated assuming neutrons and protons homogeneously distributed in a volume proportional to the nucleon number. In the isoLGM case, the values of ρ_n/ρ_p for neutron-rich nuclei (^{36}S and ^{36}Si) is much larger than the initial value, while the values of ρ_n/ρ_p for proton-rich nuclei (^{36}Ca) are much less than the initial value. In the noisoLGM case, the values of ρ_n/ρ_p also increase with the initial N/Z value of sources, which basically originates from the initial difference of the isospin of hot emitters. The stronger enhancement of ρ_n/ρ_p in the isoLGM case may indicate a neutron enrichment while a proton depletion in the nuclear gas phase. In this context, it may be consistent with the isospin fractionation effect which is a signal expected for the liquid-gas phase transition in asymmetrical systems [9,29–32]. However, this interpretation is not unique, since a larger ρ_n/ρ_p can be also directly attributable to the interaction and thus probably not an increase in neutron density.

In summary, isoscaling is investigated using the fragment yield from an equilibrated thermal source with the same mass number but different N/Z which was prepared by the lattice gas model with an asymmetrical potential between like nucleons and unlike nucleons. Isotopic scaling and isobaric scaling are observed for light clusters and the isoscaling parameters α and β are extracted from the ratios of $R_{21}(N,Z)$ for fixed proton number or neutron number. It is found that α and β are almost the same and they drop with temperature. However, the difference of the neutron $(\Delta\mu_n)$ and proton $(\Delta\mu_p)$ chemical potentials does not change much with temperature and a slight and wide valley for $(\Delta\mu_n - \Delta\mu_p)$ is observed around $T=5$ MeV even though we have larger error bars for lower and higher temperatures, which may indicate the onset of a phase separation into isospin asymmetric dilute and isospin symmetric dense phases where the liquid-gas phase change occurs. The relative free neutron density or proton density is attempted to be deduced from the isoscaling parameter and they reveal a nearly linear relation to the N/Z of the initial system. However, the values for neutron-rich sources are much larger than the initial value of N/Z as well as the values for neutron-poor sources are much less than the initial value of N/Z. This may be from the isospin fractionation effect or can be directly attributable to the interaction.

In addition, in order to investigate the effect of the asymmetrical nucleon-nucleon potentials in the LGM, we also adopt the same potential between like nucleons and unlike nucleons in the LGM. In this case, isoscaling still remains but the temperature dependence of the isoscaling parameters (α and β) vanishes and their absolute values decrease. The insensitivity of the isoscaling parameter to temperature means that the isospin partition between different fragments is almost the same regardless the excitation extent of system and the chemical phase separation is absent in this case. However, for the isoLGM, the asymmetrical nucleon-nucleon potential between like pair and unlike pair gives an additional contribution from the isospin-dependent equation of state to the isoscaling behavior except from the trivial contribution from the initial difference of the isospin of the source, which results in a bigger absolute value of α and β than the noisoLGM case. In this context, information on the equation of state of the asymmetric nucleon-nucleon potential could be extracted by the temperature dependence of isoscaling parameters.

The authors appreciate Dr. Subal Das Gupta and Dr. Jicai Pan for providing the original code of the LGM and Dr. Betty Tsang and Dr. Martin Veselsky for communications and discussions. This work was supported partly by the Major State Basic Research Development Program under Contract No. G200077404, the National Natural Science Foundation of China under (NNSFC) Grant Nos. 10328259 and 10135030, the Chinese Academy Sciences for Distinguished Young Scholars of NNSFC under Grant No. 19725521, and the Shanghai Phosphor Program Under Contract No. 03 QA 14066.

[1] M. B. Tsang *et al.*, Phys. Rev. Lett. **86**, 5023 (2001).
[2] M. B. Tsang *et al.*, Phys. Rev. C **64**, 041603 (2001).
[3] M. B. Tsang *et al.*, Phys. Rev. C **64**, 054615 (2001).
[4] W. A. Friedman, Phys. Rev. C **69**, 031601(R) (2004).
[5] M. Veselsky, G. A. Souliotis, and M. Jandel, Phys. Rev. C **69**, 044607 (2004).
[6] G. A. Souliotis *et al.*, Phys. Rev. C **68**, 024605 (2003).
[7] M. Veselsky *et al.*, Phys. Rev. C **69**, 031602(R) (2004).
[8] T. X. Liu *et al.*, Phys. Rev. C **69**, 014603 (2004).
[9] E. Geraci *et al.*, Nucl. Phys. **A732**, 173 (2004).
[10] A. Ono *et al.*, Phys. Rev. C **68**, 051601 (2003).
[11] A. S. Botvina, O. V. Lozhkin, and W. Trautmann, Phys. Rev. C **65**, 044610 (2002).
[12] S. R. Souza *et al.*, Phys. Rev. C **69**, 031607(R) (2004).
[13] M. N. Andronenko, L. N. Andronenko, and W. Neubert, Prog. Theor. Phys. Suppl. **146**, 538 (2002).
[14] S. Albergo *et al.*, Nuovo Cimento Soc. Ital. Fis., A **89**, 1 (1985).
[15] J. Randrup and S. E. Koonin, Nucl. Phys. **A356**, 223 (1981).
[16] T. D. Lee and C. N. Yang, Phys. Rev. **87**, 410 (1952).
[17] T. S. Biro *et al.*, Nucl. Phys. **A459**, 692 (1986); S. K. Samaddar and J. Richert, Phys. Lett. B **218**, 381 (1989); Z. Phys. A **332**, 443 (1989); J. M. Carmona *et al.*, Nucl. Phys. **A643**, 115 (1998).
[18] W. F. J. Müller, Phys. Rev. C **56**, 2873 (1997).
[19] X. Campi and H. Krivine, Nucl. Phys. **A620**, 46 (1997).
[20] J. Pan and S. Das Gupta, Phys. Rev. C **53**, 1319 (1996).
[21] J. Pan and S. Das Gupta, Phys. Lett. B **344**, 29 (1995); Phys. Rev. C **51**, 1384 (1995); Phys. Rev. Lett. **80**, 1182 (1998); S. Das Gupta *et al.*, Nucl. Phys. **A621**, 897 (1997).
[22] J. Pan and S. Das Gupta, Phys. Rev. C **57**, 1839 (1998).
[23] F. Gullminelli and P. Chomaz, Phys. Rev. Lett. **82**, 1402 (1999).
[24] Y. G. Ma *et al.*, Chin. Phys. Lett. **16**, 256 (1999); Phys. Rev. C **60**, 24607 (1999).
[25] S. Ray *et al.*, Phys. Lett. B **392**, 7 (1997).
[26] A. Coniglio and E. Klein, J. Phys. A **13**, 2775 (1980).
[27] Y. G. Ma *et al.*, Phys. Rev. C **69**, 031604(R) (2004); also (unpublished).
[28] Y. G. Ma, Phys. Rev. Lett. **83**, 3617 (1999); Eur. Phys. J. A **6**, 367 (1999).
[29] H. Muller and B. D. Serot, Phys. Rev. C **52**, 2072 (1995).
[30] Ph. Chomaz and F. Gulminelli, Phys. Lett. B **447**, 221 (1999).
[31] H. S. Xu *et al.*, Phys. Rev. Lett. **85**, 716 (2000).
[32] Y. G. Ma, Acta Phys. Sin. **49**, 654 (2000).