# Temperature dependence of spreading width of giant dipole resonance

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The quasiparticle-phonon nuclear model extended to finite temperature within the framework of thermofield dynamics is applied to calculate a temperature dependence of the spreading width $\Gamma^{\downarrow}$ of a giant dipole resonance. Numerical calculations are made for $^{120}$Sn and $^{208}$Pb nuclei. It is found that $\Gamma^{\downarrow}$ increases with $T$. The reason for this effect is discussed as well as a relation of the present approach to other ones existing in the literature.

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## I. INTRODUCTION

The present paper addresses the problem of a temperature dependence of the giant dipole resonance (GDR) width. Actually, our concern about is only with one part of the total GDR width, the spreading one.

GDR was found in a hot rotating nucleus formed in a collision of two heavy ions as early as 1981 [1]. As a result of quite sophisticated experiments performed during 20 years some integral characteristics of GDR were carefully studied. In particular, it is well proved that the energy of GDR and the exhaustion of the model independent energy weighted sum rule (EWSR) are quite stable against temperature increase. At the same time one observes a strongly increasing width of GDR with temperature ($T$) of a nucleus.

Several processes contribute to the GDR width at finite temperature [2–4]. Among them are quantum fluctuations which exist already in a cold nucleus: the Landau damping, the coupling with surface vibrations, the collisional damping (i.e., the coupling to incoherent two-particle–two-hole excitations) and the coupling to the single-particle continuum. At $T\neq 0$ the thermal fluctuations of a nuclear shape appear. Moreover, since a hot compound nucleus usually carries a large angular momentum, the rotation also affects the GDR width.

Extracting the GDR characteristics from the measured $\gamma$ spectra is not an absolutely unambiguous procedure. These spectra are in fact a weighted sum of the $\gamma$-ray yield emitted by many nuclei populated in the decay of the initial compound nucleus. The extracted GDR characteristics depend to some extent on assumptions about a shape of $E1$ strength function, and mass and temperature dependence of its parameters [5]. Also, the temperatures inferred from experimental excitation energy of a hot compound nucleus are sensitive to the level density parameter which is not known very accurately. In this respect, the impressive example is the fate of the so-called saturation of the GDR width at $T \geqslant 3.5-4$ MeV. This phenomenon was recognized as a nonexistent one after the appearance of new data and reanalysis of the previous ones [6,7]. Now it is widely accepted that the observed GDR width $\Gamma_{\rm GDR}$ continuously increases up to $T \sim 3.2$ MeV. The information about GDR at higher temperatures cannot be extracted reliably from the existing data.

Even a more ambiguous problem is the disentangling of different contributions to $\Gamma_{\rm GDR}$. Fortunately, due to the experiments with inelastically scattered $\alpha$ particles which yield a compound system with a small angular momentum [8] the effects of rotation and temperature on the GDR width were separated. However, in most cases conclusions can be drawn only by comparing the final results of theoretical calculations with the measured (extracted) experimental values. Sometimes conclusions appear to be controversial. For example, the adiabatic coupling model [9] reasonably describes the experimental data on the GDR width in $^{120}$Sn and $^{208}$Pb supposing the intrinsic GDR width $\Gamma_i$ almost independent of temperature. According to this model, the main effect, which explains increasing of $\Gamma_{\rm GDR}$, is the thermal nuclear shape fluctuations. On the other hand, in Ref. [10], a conclusion was reached that the behavior of the GDR parameters in the compound nucleus $^{86}$Mo cannot be explained by assuming $\Gamma_i$ be a constant. Moreover, the very recent measurement of the GDR width in $^{120}$Sn [11] reveals an overestimation of $\Gamma_{\rm GDR}$ by the adiabatic coupling model at relatively low temperature $T\approx 1$ MeV.

Different theoretical approaches also predict a quite different $T$ dependence for the GDR width. The first calculations of a thermal behavior of the spreading GDR width $\Gamma^{\downarrow}$ were performed by Bortignon *et al.* [12]. At that time, it was already well known that the coupling of a single-particle motion with collective surface vibrations is the main mechanism of damping of giant resonances in cold nuclei. In Ref. [12], a temperature dependence of this coupling was studied with the Matsubara thermal Green's function technique and it was found that the width was nearly constant when $T$ increased. The physical ground of these calculations was the nuclear field theory (NFT) [13] treating a nucleus as a system of interacting quasiparticles and vibrations (RPA phonons). In more recent studies [14], the very weak dependence of $\Gamma^{\downarrow}$ on $T$ was explained by the cancellation effect between self-energy and vertex contributions. However, several years ago in Ref. [15], where the problem was studied within the same formalism and under the same physical assumptions as in

 

Refs. [12,14], an increment of the spreading GDR width with $T$ was found.

The latter result is in correspondence with the numerous calculations of the so-called collisional damping of GDR, i.e., the coupling of collective dipole states with incoherent $2p-2h$ excitations [4]. The investigations of the collisional damping were performed within different approaches [16–22]. In most cases, calculations predict the increase of the GDR width with increasing in temperature, although the calculated width is smaller than the apparent one by a factor of 2–4 and exhibits a weaker temperature dependence. The only exception is the prediction of the extended time-dependent Hartree-Fock approach [20,21] for $^{208}$Pb. According to the calculations of [22], the collisional GDR width in this nucleus is quite stable against $T$ although in $^{120}$Sn it strongly increases with increasing in $T$.

Thus, the current situation with the temperature dependence of the GDR spreading width, as one can conclude from the above brief review, is not clear. That is why we present the results of calculations within one more approach. The approach was developed in Refs. [23–25] and is based on the two main ingredients: the quasiparticle-phonon nuclear model (QPM) [26–28] and the formalism of thermofield dynamics (TFD) [29,30]. For a long time QPM was successfully used in theoretical investigations of damping of various giant resonances in cold nuclei. The physical basis of QPM is very similar to that of the nuclear field theory, and both the models have produced quite close results as applied to nuclear structure calculations at $T=0$. In Refs. [23–25], the QPM was extended to finite temperatures by the use of the TFD formalism. Already at that formal stage interesting differences with Ref. [12] were noted. The main new scope of the present paper is the numerical calculations of the $T$ dependence of $\Gamma^{\downarrow}$ in the TFD-QPM approach. Moreover, based on the present results we discuss more carefully than before a relation of our approach to that of Refs. [12,14,15].

The paper is organized as follows. In Sec. II, the extension of the quasiparticle-phonon nuclear model to finite temperatures is presented. In Sec. III, the results of numerical calculations for $^{120}$Sn and $^{208}$Pb nuclei are presented. We discuss a physical background of our results and a comparison with other approaches in Sec. IV. A short conclusion is given in Sec. V.

## II. QPM AT FINITE TEMPERATURE

### A. Thermal RPA

First attempts to apply the TFD formalism to nuclear structure problems were made in Refs. [30–32] and [23–25]. Up to now the TFD formalism is not widely used in the nuclear structure studies. So it seems appropriate to outline how QPM can be extended to finite temperatures within the TFD thus repeating to some extent the results of Refs. [23–25].

The QPM Hamiltonian in a cold nucleus consists of phenomenological mean fields for protons and neutrons, pairing interaction of the BCS type and separable multipole particle-hole interactions with the isoscalar and isovector items

$$H = H_{\text{sp}} + H_{\text{pair}} + H_{\text{ph}}, \tag{1}$$

where

$$H_{\text{sp}} = \sum_{jm}\sum_{\tau} (E_j - \lambda_\tau) c^+_{jm} c_{jm}, \tag{2}$$

$$H_{\text{pair}} = -\sum_{\tau} \frac{G_\tau}{4} \sum^{\tau}_{\substack{j_1 m_1 \\ j_2 m_2}} c^+_{j_1 m_1} c^+_{\overline{j_1 m_1}} c_{\overline{j_2 m_2}} c_{j_2 m_2}, \tag{3}$$

$$H_{\text{ph}} = -\frac{1}{2}\sum_{\lambda\mu}\sum_{\tau,\rho=\pm1} (\kappa_0^{(\lambda)} + \rho\kappa_1^{(\lambda)}) M^+_{\lambda\mu}(\tau) M_{\lambda\mu}(\rho\tau). \tag{4}$$

The operator $M^+_{\lambda\mu}(\tau)$ is the single-particle multipole operator

$$M^+_{\lambda\mu}(\tau) = \sum^{\tau}_{\substack{j_1 m_1 \\ j_2 m_2}} \langle j_1 m_1 | R(r) Y_{\lambda\mu}(\vec{r}/r) | j_2 m_2 \rangle c^+_{j_1 m_1} c_{j_2 m_2}$$

and $c^+_{jm}, c_{jm}$ are the creation and annihilation operators of particle with quantum numbers $n,l,j,m \equiv j,m$ and the energy $E_j$. The notation $\overline{jm}$ means the time-reversed state. The index $\tau$ is isotopic one. It takes two values, $\tau=n,p$. The symbol $\Sigma^\tau$ means that the summation is taken only over neutron or proton single-particle (hole) states and changing the sign of $\tau$ means changing $n \leftrightarrow p$. The parameters $G_n, G_p$ are constants of neutron-neutron and proton-proton BCS-pairing interactions and $\kappa_0^{(\lambda)}, \kappa_1^{(\lambda)}$ are coupling constants of isoscalar and isovector multipole-multipole (with multipolarity $\lambda$) interactions, respectively.

The first step in treating nuclear dynamics governed by the Hamiltonian (1) at finite temperature is formal doubling of the Hilbert space of a nucleus. To this aim, we introduce a fictitious (tilde) system which is of exactly the same structure as the initial one. For any operator $A$ acting in the initial Hilbert space there exists its tilde counterpart $\widetilde{A}$ acting in the space of tilde states. The tilde system is governed by the tilde Hamiltonian $\widetilde{H}$ which has the same structure as $H$, only the operators $c^+_{jm}, c_{jm}$ are substituted by their tilde counterparts $\widetilde{c}^{\,+}_{jm}$ and $\widetilde{c}_{jm}$.

The thermal Hamiltonian of the QPM is by definition

$$\mathcal{H} = H - \widetilde{H}. \tag{5}$$

An excitation spectrum of a hot nucleus is obtained by diagonalization of $\mathcal{H}$. At the same time, the thermal behavior of the nucleus is controlled by the thermal vacuum state $|0(T)\rangle$, which is the eigenstate of $\mathcal{H}$ with the zero eigenvalue.

To construct the thermal vacuum state $|0(T)\rangle$ we made two Bogoliubov transformations. The first one is the standard

$(u,v)$ Bogoliubov transformation from the particle operators to the quasiparticle ones $\alpha^+_{jm}$ and $\alpha_{jm}$,

$$c^+_{jm} = u_j\alpha^+_{jm} + (-1)^{j-m}v_j\alpha_{j-m},$$
$$c_{jm} = u_j\alpha_{jm} + (-1)^{j-m}v_j\alpha^+_{j-m}. \tag{6}$$

The same transformation (with the same $u_j, v_j$ coefficients) is made with the tilde operators thus producing tilde quasiparticle operators $\tilde{\alpha}^+_{jm}, \tilde{\alpha}_{jm}$. The second transformation is a unitary thermal Bogoliubov transformation [29] from ordinary and tilde quasiparticle operators to thermal quasiparticle operators $\beta, \beta^+, \tilde{\beta}, \tilde{\beta}^+$,

$$\beta_{jm} = x_j\alpha_{jm} - y_j\tilde{\alpha}^+_{jm},$$
$$\tilde{\beta}_{jm} = x_j\tilde{\alpha}_{jm} + y_j\alpha^+_{jm}, \tag{7}$$

where

$$x_j^2 + y_j^2 = 1.$$

The coefficients of the Bogoliubov rotations (6) and (7) are determined simultaneously by minimization of the free energy $F^{(\tau)}$ (separately for neutron and proton subsystems)

$$F^{(\tau)} = \langle 0(T)|H^{(\tau)}_{\rm sp} + H^{(\tau)}_{\rm pair}|0(T)\rangle - TS^{(\tau)} - \lambda_\tau\langle 0(T)|\hat{N}^{(\tau)}|0(T)\rangle, \tag{8}$$

where $\hat{N}^{(\tau)}$ is the operator of a number of neutrons (protons) in the nucleus,

$$\hat{N}^{(\tau)} = \sum_{jm}^{(\tau)} c^+_{jm}c_{jm}.$$

The entropy $S^{(\tau)}$ reads

$$S^{(\tau)} = -\sum_j^{\tau}(2j+1)[x_j^2 \ln x_j^2 + y_j^2 \ln y_j^2]. \tag{9}$$

Expectation values in (8) are taken with respect to the thermal ground state $|0(T)\rangle$ which at this stage is supposed to be the vacuum state for the thermal quasiparticle operators

$$\beta_{jm}|0(T)\rangle = \tilde{\beta}_{jm}|0(T)\rangle = 0. \tag{10}$$

In terms of the operators $\alpha^+, \tilde{\alpha}^+$ the vacuum $|0(T)\rangle$ is nothing but a coherent, or squeezed, state

$$|0(T)\rangle = \exp\left[\sum_{jm}\frac{y_j}{x_j}\alpha^+_{jm}\tilde{\alpha}^+_{jm}\right]|0\rangle,$$

where $|0\rangle$ is the direct product of the BCS vacuum and its tilde counterpart.

After variation of (8) over the coefficients $u_j, v_j, x_j, y_j$ we obtain the BCS equations at finite temperature [23,32],

$$N_\tau = \frac{1}{2}\sum_j^{\tau}(2j+1)\left(1 - \frac{(E_j-\lambda_\tau)(1-2n_j)}{\sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}}\right), \tag{11}$$

$$\frac{4}{G_\tau} = \sum_j^{\tau}(2j+1)\frac{1-2n_j}{\sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}}, \tag{12}$$

The expressions for the coefficients $u_j, v_j$ and the quasiparticle energy $\varepsilon_j$ are the following:

$$u_j^2 = \frac{1}{2}\left(1 + \frac{E_j-\lambda_\tau}{\varepsilon_j}\right), \quad v_j^2 = \frac{1}{2}\left(1 - \frac{E_j-\lambda_\tau}{\varepsilon_j}\right),$$
$$\varepsilon_j = \sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}, \tag{13}$$

and for the coefficients $x_j, y_j$ one gets

$$y_j^2 = n_j, \quad x_j^2 = 1 - n_j, \tag{14}$$

where $n_j$ is the Fermi-Dirac thermal occupation number for the quasiparticle with the energy $\varepsilon_j$,

$$n_j = \frac{1}{1+\exp(\varepsilon_j/T)}. \tag{15}$$

With the coefficients $u_j, v_j, x_j, y_j$ determined by (13)–(15) the part of the thermal Hamiltonian which consists of the single-particle and pairing terms and their tilde counterparts takes the form

$$\mathcal{H}_{\rm TSQP} = \sum_{jm\tau}\varepsilon_j(\beta^+_{jm}\beta_{jm} - \tilde{\beta}^+_{jm}\tilde{\beta}_{jm}).$$

The Hamiltonian $\mathcal{H}_{\rm TSQP}$ describes a system of independent thermal quasiparticles with temperature dependent energies $\varepsilon_j$ (and -$\varepsilon_j$ for the tilde thermal quasiparticles). The ground state of this system is the thermal vacuum state $|0(T)\rangle$ defined by (10).

The term $\mathcal{H}_{\rm ph}$ is the interaction of thermal quasiparticles. After the transformations (6) and (7) the multipole operator $M^+_{\lambda\mu}(\tau)$ takes the form

$$M^+_{\lambda\mu}(\tau) = \frac{(-)^{\lambda-\mu}}{\sqrt{2\lambda+1}}\sum_{j_1j_2}^{\tau} f^{(\lambda)}_{j_1j_2}[A^+_\beta(j_1j_2;\lambda\mu) + (-)^{\lambda-\mu}A_\beta(j_1j_2;\lambda-\mu)] + B_\beta(j_1j_2;\lambda\mu). \tag{16}$$

The value $f^{(\lambda)}_{j_1j_2}$ is a reduced single-particle matrix element of the one-body multipole operator $M^+_{\lambda\mu}$. The operators $A^+_\beta(j_1j_2;\lambda\mu)$ and $B_\beta(j_1j_2;\lambda\mu)$ are defined as follows:

$$A^+_\beta(j_1j_2;\lambda\mu) = \frac{1}{2}u^{(+)}_{j_1j_2}\Big(\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}[\beta^+_{j_1}\beta^+_{j_2}]_{\lambda\mu} - \sqrt{n_{j_1}}\sqrt{n_{j_2}}[\tilde{\beta}^+_{\overline{j_1}}\tilde{\beta}^+_{\overline{j_2}}]_{\lambda\mu}\Big) - v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}[\beta^+_{j_1}\tilde{\beta}^+_{\overline{j_2}}]_{\lambda\mu},$$

$$B_\beta(j_1j_2;\lambda\mu) = u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}\Big([\beta^+_{j_1}\tilde{\beta}_{j_2}]_{\lambda\mu} + (-)^{\lambda-\mu}[\beta_{j_1}\tilde{\beta}^+_{j_2}]_{\lambda-\mu}\Big) - v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}\Big([\beta^+_{j_1}\beta_{\overline{j_2}}]_{\lambda\mu}$$

$$+ \sqrt{n_{j_1}}\sqrt{n_{j_2}}[\tilde{\beta}_{j_1}\tilde{\beta}^+_{\overline{j_2}}]_{\lambda\mu}\Big),$$

where

$$u^{(+)}_{j_1j_2} = u_{j_1}v_{j_2} + u_{j_2}v_{j_1}, \quad v^{(-)}_{j_1j_2} = u_{j_1}u_{j_2} - v_{j_2}v_{j_1}.$$

The operator $A_\beta(j_1j_2;\lambda\mu)$ is the Hermitian conjugate of $A^+_\beta(j_1j_2;\lambda\mu)$. The square brackets $[\quad]_{\lambda\mu}$ stand for the coupling of single-particle angular momenta $j_1, j_2$ to the sum angular momentum $\lambda$.

At the next step we take into account the TRPA correlations due to interaction of thermal quasiparticles [24,30]. To proceed, we introduce the following thermal phonon operator:

$$\begin{aligned} Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{j_1j_2} & \psi^{\lambda i}_{j_1j_2}[\beta^+_{j_1}\beta^+_{j_2}]_{\lambda\mu} + 2\eta^{\lambda i}_{j_1j_2}[\beta^+_{j_1}\tilde{\beta}^+_{\overline{j_2}}]_{\lambda\mu} \\ & + \tilde{\psi}^{\lambda i}_{j_1j_2}[\tilde{\beta}^+_{\overline{j_1}}\tilde{\beta}^+_{\overline{j_2}}]_{\lambda\mu} - (-1)^{\lambda-\mu}\Big(\phi^{\lambda i}_{j_1j_2}[\beta_{j_2}\beta_{j_1}]_{\lambda-\mu} \\ & + 2\zeta^{\lambda i}_{j_1j_2}[\tilde{\beta}_{\overline{j_2}}\beta_{j_1}]_{\lambda-\mu} + \tilde{\phi}^{j_1j_2}_{\lambda i}[\tilde{\beta}_{\overline{j_2}}\tilde{\beta}_{\overline{j_1}}]_{\lambda-\mu}\Big). \end{aligned} \tag{17}$$

Further, we assume that these phonons are bosons and redefine the ground state of a hot nucleus. Hereafter it is a vacuum state for the thermal phonon operator $|\Psi_0(T)\rangle$, i.e., $Q_{\lambda\mu}|\Psi_0(T)\rangle = 0$. Thus the function $|\Psi_0(T)\rangle$ is a temperature dependent wave function of the compound state. With an assumption on the bosonic nature of the phonon operator (17) the norm of a thermal one-phonon wave function is

$$\begin{aligned} \frac{1}{2}\sum_{j_1j_2} & (\psi^{\lambda i}_{j_1j_2})^2 - (\phi^{\lambda i}_{j_1j_2})^2 + (\tilde{\psi}^{\lambda i}_{j_1j_2})^2 - (\tilde{\phi}^{\lambda i}_{j_1j_2})^2 + 2(\eta^{\lambda i}_{j_1j_2})^2 \\ & - 2(\zeta^{\lambda i}_{j_1j_2})^2 = 1. \end{aligned} \tag{18}$$

Then the thermal RPA equations can be obtained by applying either the variational principle or the equation of motion method. Here we show only the secular equation for energies $\omega_{\lambda i}$ of thermal one-phonon states $|\lambda i\rangle$ and expressions for amplitudes of a thermal phonon wave function. The secular equation reads

$$[X_n(\omega) + X_p(\omega)](\kappa_0^{(\lambda)} + \kappa_1^{(\lambda)}) - 4\kappa_0^{(\lambda)}\kappa_1^{(\lambda)}X_n(\omega)X_p(\omega) = 1, \tag{19}$$

where

$$\begin{aligned} X_\tau(\omega) = \frac{1}{2\lambda+1}\sum^{\tau}_{j_1j_2}(f^{(\lambda)}_{j_1j_2})^2 & \left[\frac{(u^{(+)}_{j_1j_2})^2(1-n_{j_1}-n_{j_2})(\varepsilon_{j_1}+\varepsilon_{j_2})}{(\varepsilon_{j_1}+\varepsilon_{j_2})^2-\omega^2}\right. \\ & \left. - \frac{(v^{(-)}_{j_1j_2})^2(n_{j_1}-n_{j_2})(\varepsilon_{j_1}-\varepsilon_{j_2})}{(\varepsilon_{j_1}-\varepsilon_{j_2})^2-\omega^2}\right]. \end{aligned} \tag{20}$$

The amplitudes are

$$\psi^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})-\omega_{\lambda i}},$$

$$\phi^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})+\omega_{\lambda i}},$$

$$\eta^{\lambda i}_{j_1j_2} = -\sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}-\varepsilon_{j_2})-\omega_{\lambda i}},$$

$$\zeta^{\lambda i}_{j_1j_2} = -\sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}-\varepsilon_{j_2})+\omega_{\lambda i}},$$

$$\tilde{\psi}^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})+\omega_{\lambda i}},$$

$$\tilde{\phi}^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})-\omega_{\lambda i}},$$

where the factor $\mathcal{N}^{\lambda i}_\tau$ is

$$\begin{aligned} \mathcal{N}^{\lambda i}_\tau = \frac{2\lambda+1}{2} & \left[\left.\frac{\partial}{\partial\omega}X^{\lambda i}_\tau(\omega)\right|_{\omega=\omega_{\lambda i}}\right. \\ & \left. + \left(\frac{1-X^{\lambda i}_\tau(\omega_{\lambda i})(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})}{X^{\lambda i}_{-\tau}(\omega_{\lambda i})(\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)})}\right)^2 \left.\frac{\partial}{\partial\omega}X^{\lambda i}_{-\tau}(\omega)\right|_{\omega=\omega_{\lambda i}}\right]. \end{aligned} \tag{21}$$

It is worthwhile to note that in contrast with RPA at $T=0$ the solutions of (19) with negative energies have physical meaning (see also Ref. [15]). They correspond to the tilde-phonon states $\tilde{Q}^+_{\lambda\mu i}|\Psi_0(T)\rangle$

$$\langle\Psi_0(T)|[\mathcal{H},Q^+_{\lambda\mu i}]|\Psi_0(T)\rangle = -\langle\Psi_0(T)|[\mathcal{H},\tilde{Q}^+_{\lambda\mu i}]|\Psi_0(T)\rangle = \omega_{\lambda i}.$$

Let us comment on the structure of a TRPA phonon. The components $\psi$ and $\phi$ are the same as in the standard quasiparticle RPA (QRPA) (see, e.g., Ref. [28]) and are only damped by the factor $(1-n_j)$. The components $\tilde{\psi}$ and $\tilde{\phi}$ are totally due to the tilde part of the Fock space of a heated nucleus. They vanish in a cold nucleus. Note that the $\omega$ dependence of the forward and backward tilde amplitudes is just opposite to that of the ordinary amplitudes. It means that, e.g., while $\psi$ is of a pole character $\tilde{\psi}$ is not and instead the amplitude $\tilde{\phi}$ is a pole amplitude. The most interesting amplitudes are $\eta$ and $\zeta$. They could be specified as crossover amplitudes containing both the ordinary and tilde thermal quasiparticles. Just due to them the poles $\varepsilon_{j_1}-\varepsilon_{j_2}$, which do not exist in QRPA at $T=0$, appear in (19). These poles can appear at quite low energies, thus enriching a low-energy part of the phonon spectrum in comparison with QRPA at $T=0$. The amplitudes $\eta$ and $\zeta$ depend on the superfluid factor $v^{(-)}_{j_1j_2}$ which is enhanced when both the states $j_1$ and $j_2$ are of a particle or a hole type. In contrast, the four other amplitudes are proportional to the superfluid particle-hole factor $u^{(+)}_{j_1j_2}$.

In nuclei with pairing correlations the amplitudes $\eta,\zeta$ vanish when $T\to 0$. However, in magic nuclei the thermal phonon operator (17) consists of only two types of components $\eta$ and $\zeta$,

$$Q^{+}_{\lambda\mu i}=\sum_{j_1j_2}\eta^{\lambda i}_{j_1j_2}[\beta^{+}_{j_1}\widetilde{\beta}^{+}_{\overline{j_2}}]_{\lambda\mu}+(-1)^{\lambda-\mu}\zeta^{\lambda i}_{j_1j_2}[\beta_{j_1}\widetilde{\beta}_{\overline{j_2}}]_{\lambda-\mu}.$$

The expressions for $\eta$ and $\zeta$ displayed above are valid in this case as well excepting that the value $v^{(-)}_{j_1j_2}$ equals to unity. The expression (20) also becomes simpler

$$X_{\tau}(\omega)=\frac{1}{2\lambda+1}\sum^{\tau}_{j_1j_2}\frac{(f^{(\lambda)}_{j_1j_2})^2(n_{j_1}-n_{j_2})(E_{j_1}-E_{j_2})}{(E_{j_1}-E_{j_2})^2-\omega^2}.$$

At the end of this section we display the expression for the matrix element $\Phi_{\lambda i}$ of the $E\lambda$-transition operator from the ground state of a hot nucleus to a thermal one-phonon state [i.e., for the transition $|\Psi_0(T)\rangle\to Q^{+}_{\lambda\mu i}|\Psi_0(T)\rangle$]. It reads [24]

$$\Phi_{\lambda i}=\sum_{j_1j_2}\langle j_1\|\mathcal{M}(E\lambda)\|j_2\rangle\Bigg\{\frac{1}{2}u^{(+)}_{j_1j_2}[\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(\psi^{\lambda i}_{j_1j_2}+\phi^{\lambda i}_{j_1j_2})-\sqrt{n_{j_1}}\sqrt{n_{j_2}}(\widetilde{\psi}^{\lambda i}_{j_1j_2}+\widetilde{\phi}^{\lambda i}_{j_1j_2})]-v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(\eta^{\lambda i}_{j_1j_2}+\zeta^{\lambda i}_{j_1j_2})\Bigg\},\qquad(22)$$

where $\langle j_1\|\mathcal{M}(E\lambda)\|j_2\rangle$ is a reduced single-particle matrix element of the $E\lambda$ transition operator.

### B. Interaction of TRPA phonons

Now the thermal Hamiltonian reads in terms of the TRPA phonons and thermal quasiparticles (note that the term $B^{+}_{\beta}B_{\beta}$ and its tilde counterpart are omitted)

$$\mathcal{H}=\sum_{\lambda\mu i}\omega_{\lambda i}(Q^{+}_{\lambda\mu i}Q_{\lambda\mu i}-\widetilde{Q}^{+}_{\lambda\mu i}\widetilde{Q}_{\lambda\mu i})-\frac{1}{2\sqrt{2}}\sum_{\lambda\mu i}\sum_{\tau}\sum^{\tau}_{j_1j_2}\frac{f^{(\lambda)}_{j_1j_2}}{\sqrt{N^{\lambda i}_{\tau}}}\{((-)^{\lambda-\mu}Q^{+}_{\lambda\mu i}+Q_{\lambda-\mu i})\times B_{\beta}(j_1j_2;\lambda-\mu)-((-)^{\lambda-\mu}\widetilde{Q}^{+}_{\lambda\mu i}+\widetilde{Q}_{\lambda-\mu i})\times\widetilde{B}_{\beta}(j_1j_2;\lambda-\mu)+\text{h.c.}\},\qquad(23)$$

The terms $\sim(Q^{+}+Q)B$, etc. (hereafter we denote their sum by $\mathcal{H}_{\text{qph}}$) couple a thermal one-phonon state with more complex thermal configurations, e.g., two-phonon ones. Due to this mixing the strength of a one-phonon state is fragmented over some energy interval. In other words, the term $\mathcal{H}_{\text{qph}}$ produces a spreading width of a thermal one-phonon state. To describe the fragmentation of thermal phonons, we use again the variational method with a trial wave function of the form

$$|\Psi_{\nu}(JM)\rangle=\Bigg\{\sum_i R_i(J\nu)Q^{+}_{JMi}+\sum_{\substack{\lambda_1i_1\\ \lambda_2i_2}}P^{\lambda_1i_1}_{\lambda_2i_2}(J\nu)\times[Q^{+}_{\lambda_1\mu_1i_1}Q^{+}_{\lambda_2\mu_2i_2}]_{JM}\Bigg\}|\Psi_0(T)\rangle.\qquad(24)$$

The equation for energies of states (24) is

$$\det\left|(\omega_{Ji}-\eta_{J\nu})\delta_{ii'}-\frac{1}{2}\sum_{\lambda_1i_1\lambda_2i_2}\frac{U^{\lambda_1i_1}_{\lambda_2i_2}(Ji)U^{\lambda_1i_1}_{\lambda_2i_2}(Ji')}{\omega_{\lambda_1i_1}+\omega_{\lambda_2i_2}-\eta_{J\nu}}\right|=0.\qquad(25)$$

The functions $U^{\lambda_1i_1}_{\lambda_2i_2}(Ji)$ are the coupling matrix elements between one- and two-phonon states. The expression for $U^{\lambda_1i_1}_{\lambda_2i_2}(Ji)$ is the following:

$$U^{\lambda_1i_1}_{\lambda_2i_2}(Ji,\tau)=-\frac{1}{\sqrt{2}}\sqrt{2\lambda_1+1}\sqrt{2\lambda_2+1}\times\sum^{\tau}_{j_1j_2j_3}\Bigg[(-)^{J}\Gamma^{\lambda_2i_2}_{j_1j_2}\begin{Bmatrix}\lambda_2&\lambda_1&J\\ j_3&j_2&j_1\end{Bmatrix}\mathcal{K}^{\lambda_1i_1Ji}_{j_3j_2j_1}+(-)^{\lambda_1-\lambda_2}\Gamma^{\lambda_1i_1}_{j_1j_2}\begin{Bmatrix}\lambda_1&\lambda_2&J\\ j_3&j_2&j_1\end{Bmatrix}\mathcal{K}^{\lambda_2i_2Ji}_{j_3j_2j_1}+(-)^{J-\lambda_1}\Gamma^{Ji}_{j_1j_2}\begin{Bmatrix}J&\lambda_1&\lambda_2\\ j_3&j_2&j_1\end{Bmatrix}\mathcal{L}^{\lambda_1i_1\lambda_2i_2}_{j_3j_2j_1}\Bigg],\qquad(26)$$

where $\Gamma^{\lambda i}_{j_1j_2}=f^{(\lambda)}_{j_1j_2}/\sqrt{\mathcal{N}^{\lambda i}}$ and the functions $\mathcal{K}^{\lambda_2i_2Ji}_{j_3j_2j_1}$ and $\mathcal{L}^{\lambda_1i_1\lambda_2i_2}_{j_3j_2j_1}$ are

$$\mathcal{K}^{\lambda_1i_1Ji}_{j_3j_2j_1}=v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(-1)^{j_1+j_3+\lambda_1+J}(\psi^{\lambda_1i_1}_{j_1j_3}\psi^{Ji}_{j_2j_3}+\phi^{\lambda_1i_1}_{j_1j_3}\phi^{Ji}_{j_2j_3}+\eta^{\lambda_1i_1}_{j_1j_3}\eta^{Ji}_{j_2j_3}+\zeta^{\lambda_1i_1}_{j_1j_3}\zeta^{Ji}_{j_2j_3})+u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(-)^{j_1+j_2+\lambda_1}(\psi^{\lambda_1i_1}_{j_1j_3}\eta^{Ji}_{j_3j_2}+\phi^{\lambda_1i_1}_{j_1j_3}\zeta^{Ji}_{j_3j_2}+\eta^{\lambda_1i_1}_{j_1j_3}\widetilde{\psi}^{Ji}_{j_3j_2}+\zeta^{\lambda_1i_1}_{j_1j_3}\widetilde{\phi}^{Ji}_{j_3j_2})-u^{(+)}_{j_1j_2}\sqrt{n_{j_1}}\sqrt{1-n_{j_2}}(-1)^{J}(\eta^{\lambda_1i_1}_{j_3j_1}\psi^{Ji}_{j_2j_3}+\zeta^{\lambda_1i_1}_{j_3j_1}\phi^{Ji}_{j_2j_3}+\widetilde{\psi}^{\lambda_1i_1}_{j_3j_1}\eta^{Ji}_{j_2j_3}+\widetilde{\phi}^{\lambda_1i_1}_{j_3j}\zeta^{Ji}_{j_2j_3})-v^{(-)}_{j_1j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}(-)^{j_2+j_3}(\eta^{\lambda_1i_1}_{j_3j_1}\eta^{Ji}_{j_3j_2}+\zeta^{\lambda_1i_1}_{j_3j_1}\zeta^{Ji}_{j_3j_2}+\widetilde{\psi}^{\lambda_1i_1}_{j_3j_1}\widetilde{\psi}^{Ji}_{j_3j_2}+\widetilde{\phi}^{\lambda_1i_1}_{j_3j_1}\widetilde{\phi}^{j_3j_2}_{Ji}),$$

$$\mathcal{L}^{\lambda_1 i_1 \lambda_2 i_2}_{j_3 j_2 j_1} = v^{(-)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(-1)^{j_1+j_3+\lambda_1+\lambda_2}(\psi^{\lambda_1 i_1}_{j_1 j_3}\phi^{\lambda_2 i_2}_{j_2 j_3} + \phi^{\lambda_1 i_1}_{j_1 j_3}\psi^{\lambda_2 i_2}_{j_2 j_3} + \eta^{\lambda_1 i_1}_{j_1 j_3}\zeta^{\lambda_2 i_2}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\eta^{\lambda_2 i_2}_{j_2 j_3})$$
$$+ u^{(+)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(-)^{j_1+j_2+\lambda_1}(\psi^{\lambda_1 i_1}_{j_1 j_3}\zeta^{\lambda_2 i_2}_{j_3 j_2} + \phi^{\lambda_1 i_1}_{j_1 j_3}\eta^{\lambda_2 i_2}_{j_3 j_2} + \eta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2}) - u^{(+)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{1-n_{j_2}}(-)^{\lambda_2}(\zeta^{\lambda_1 i_1}_{j_3 j_1}\psi^{\lambda_2 i_2}_{j_2 j_1}$$
$$+ \eta^{\lambda_1 i_1}_{j_3 j_1}\phi^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\eta^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\zeta^{\lambda_2 i_2}_{j_2 j_3}) - v^{(-)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}(-1)^{j_2+j_3}(\eta^{\lambda_1 i_1}_{j_3 j_1}\zeta^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_3 j_1}\eta^{\lambda_2 i_2}_{j_3 j_2} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2}).$$

Let us note that in case the pairing correlations vanish, expression (26) completely agrees with that from Ref. [15] [see Eqs. (4.1)–(4.2) in the paper].

To calculate the $E1$-strength function in hot nuclei taking into account a fragmentation of thermal one-phonon dipole states, we explore the well-known strength function method [27,28]. Avoiding to solve Eq. (25); we directly calculate the function

$$b(E\lambda,\eta) = \sum_{\nu}\frac{1}{2\pi}\frac{\Delta}{(\eta-\eta_{\lambda\nu})^2+\frac{\Delta^2}{4}}|\Phi(J\nu)|^2, \qquad (27)$$

where the coefficients $\Phi(J\nu)$ are amplitudes of $E\lambda$ transitions from the ground state of a hot nucleus (a compound state) to states described by the wave functions (24). These amplitudes are superpositions of the matrix elements $\Phi_{Ji}$ (22) with the weight factors $R_i(J\nu)$ from (25),

$$\Phi(J\nu) = \sum_i R_i(J\nu)\Phi_{Ji},$$

and $\Delta$ is a smearing parameter.

## III. NUMERICAL RESULTS

We calculate energy centroids, variances and the exhaustion of the energy weighted sum rule (EWSR) of $E1$ strength distributions for $0 \leqslant T \leqslant 3$ MeV in $^{120}$Sn and $^{208}$Pb nuclei. All model parameters (mean field potentials, pairing constants, coupling constants of separable interactions, etc.) are fixed in accordance with the standard QPM procedure [26,28], i.e., by the use of experimental data on the energies of low-lying vibrational states and giant resonances at $T=0$. As a mean field the phenomenological Woods-Saxon potential is explored. The single-particle basis consists of all bound states and several quasibound states with relatively small escape width.

Pairing correlations that exist only in the neutron system of $^{120}$Sn are treated in the thermal BCS approximation. Since we do not make a particle number projection, a neutron energy gap in this nucleus vanishes at $T=T_c\approx 1$ MeV.

Only multipole-multipole particle-hole interactions with $1\leqslant\lambda\leqslant 7$ are included in the Hamiltonian (4). A radial form factor of the separable multipole interaction has the form $R(r)=dU/dr$, where $U$ is the central part of the Woods-Saxon potential. The coupling constant of the isoscalar dipole-dipole interaction is adjusted at every value of $T$ to make the energy of the spurious $1^-$ state zero in the TRPA calculations. The chemical potentials $\lambda_{n,p}$ are also adjusted at every $T$ value to keep the right average values of $N,Z$.

Energy centroids $\overline{E}$ and variances $\sigma$ of $E1$ strength functions ($E1$ SF) are calculated with the following formulas:

$$\overline{E} = \frac{m_1}{m_0}, \quad \sigma_{\text{th}} = \sqrt{\frac{m_2}{m_0}-\left(\frac{m_1}{m_0}\right)^2},$$

where $m_k$ is the $k$th energy moment of the corresponding function.

The model energy weighted dipole sum rule is calculated with the formula for a system of independent Bogoliubov quasiparticles at $T\neq 0$,

$$\text{EWSR} = \sum_{\tau}\sum_{j_1\geqslant j_2}(f^{(1)}_{j_1 j_2})^2[(\varepsilon_{j_1}+\varepsilon_{j_2})(u^{(+)}_{j_1 j_2})^2(1-n_{j_1}-n_{j_2}) - (\varepsilon_{j_1}-\varepsilon_{j_2})(v^{(-)}_{j_1 j_2})^2(n_{j_1}-n_{j_2})]. \qquad (28)$$

When a pairing gap vanishes (i.e., in magic nuclei or at $T>T_c$) Eq. (28) takes the form

$$\text{EWSR} = \sum_{\tau}\sum_{j_1\geqslant j_2}(f^{(1)}_{j_1 j_2})^2(E_{j_1}-E_{j_2})(n_{j_1}-n_{j_2}),$$

which coincides with that from Ref. [33].

The results of our calculations within the TRPA are the same as in many previous studies (see, e.g., [20,33–35]). When temperature increases, only some minor redistribution of the $E1$ strength between different one-phonon $1^-$ states takes place. Nevertheless, in $^{120}$Sn the GDR energy centroid decreases by 1.5 MeV when $T$ increases from 0 to 3 MeV, whereas in $^{208}$Pb the value of $\overline{E}$ is practically independent of temperature (Fig. 1). The GDR variance calculated within TRPA characterizes the Landau width of the resonance. It weakly decreases with $T$ in $^{120}$Sn($\Delta\sigma\approx 0.7$ MeV) and does not change in $^{208}$Pb.

The values of EWSR (28) at different $T$ are shown in Figs. 2 and 3 for $^{120}$Sn and $^{208}$Pb, respectively. They are compared with the corresponding model independent values $S_1$,

$$S_1 = \frac{9}{8\pi}\frac{e^2\hbar^2}{m}\frac{NZ}{A} = 14.8\frac{NZ}{A}e^2\ \text{fm}^2\ \text{MeV}.$$

A difference between the calculated EWSR values and the model independent ones in the range $0<T<3$ MeV appears to be less than 10%. An excess of EWSR over $S_1$ in $^{120}$Sn at $T<T_c$ should be attributed to the effect of the BCS pairing. When the pairing correlations vanish, a value of EWSR becomes less than $S_1$ like it is in $^{208}$Pb in the whole temperature range. Thus, as it follows from Figs. 2 and 3, the model

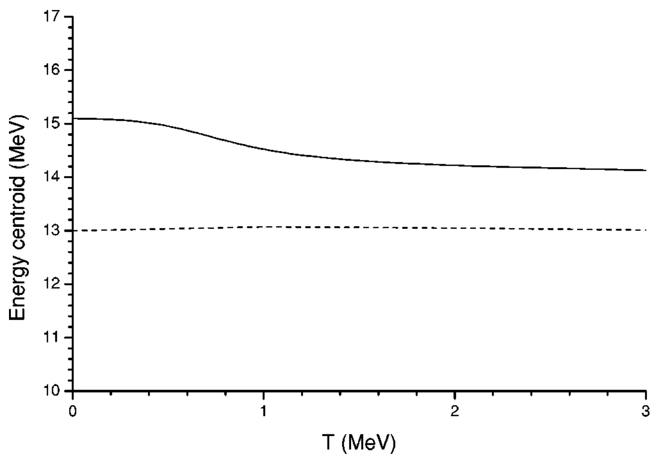


FIG. 1. Temperature dependence of the centroids of $E1$ strength functions in $^{120}$Sn (solid line) and $^{208}$Pb (dashed line).

EWSR is practically independent of temperature in $^{208}$Pb as well as in $^{120}$Sn at $T > T_c$. This remains valid for the TRPA calculations of the model dependent EWSR (i.e., after summation of the $E1$ strengths of all the TRPA roots).

In calculations taking into account the interaction of thermal phonons, the one-phonon part of a trial wave function (24) includes 21 dipole phonons with the largest $B(E1)$ values at given $T$ in $^{120}$Sn and 14 dipole states in $^{208}$Pb. These dipole TRPA-phonons exhaust more than 80% of the model EWSR. The two-phonon part of (24) consists of all possible thermal two-phonon $1^-$ states from the energy range 0–30 MeV constructed by combining normal parity phonons of different energies with angular momenta $1 < \lambda < 7$. Some additional limitations to the two-phonon space will be discussed in Sec. IV. The smearing parameter $\Delta$ in the Lorentzian weight function is taken to be equal to 1 MeV.

The phonon-phonon interaction $\mathcal{H}_{\text{qph}}$ (23) and the trial wave function (24) imply that only the cubic anharmonic terms of the interaction are taken into account and the phonon-phonon coupling does not affect the thermal ground state which is still treated as the thermal phonon vacuum $|\Psi_0(T)\rangle$. As it has been proved in Ref. [36], under the two

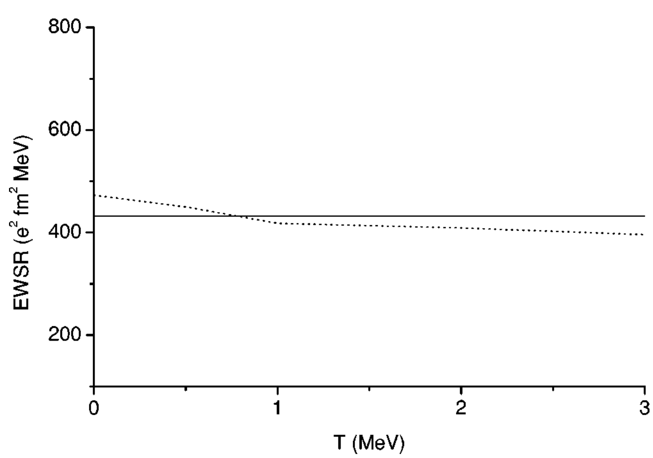


FIG. 2. The model EWSR in $^{120}$Sn at different $T$ (dashed line). Solid horizontal line, the value of the model independent energy weighted sum rule $S_1$.

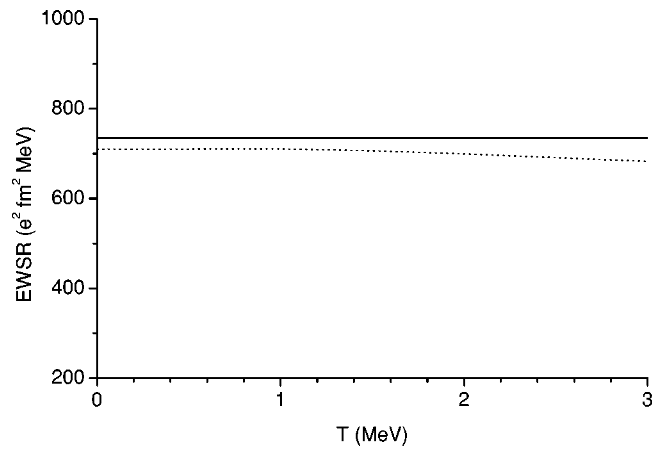


FIG. 3. The model EWSR in $^{208}$Pb at different $T$ (dashed line). Solid horizontal line, the value of the model independent energy weighted sum rule $S_1$.

assumptions a mixing of one- and two-phonon states does not influence an exhaustion of EWSR and a centroid of a one-phonon strength distribution calculated in a sufficiently wide energy range. It means that the TRPA results for EWSR and $\bar{E}$ discussed above and displayed in Figs. 1–3 remain valid for the calculations taking into account the coupling of one- and two-phonon configurations. We should note that in calculations taking into account a collisional damping within the quantal framework of the extended time-dependent Hartree-Fock theory [22], a sizable difference was found in thermal behavior of GDR energy centroids in $^{208}$Pb and $^{120}$Sn. Whereas in $^{208}$Pb $\bar{E}$ is more or less stable against temperature it sizably decreases with increasing $T$ in $^{120}$Sn. In our studies there is no noticeable difference between $^{208}$Pb and $^{120}$Sn in this respect.

In Fig. 4, we display a total photoabsorption cross-section in $^{208}$Pb calculated with our $E1$ SF's at $T=0$ and 1 MeV. The experimental data for $T=0$ taken from Ref. [37] are also shown. The latter is done to demonstrate that the model used with chosen parameters describes experimental data satisfactorily.

It is seen in Fig. 4 that the phonon-phonon interaction pushes a part of the $E1$ strength to lower excitation energy.

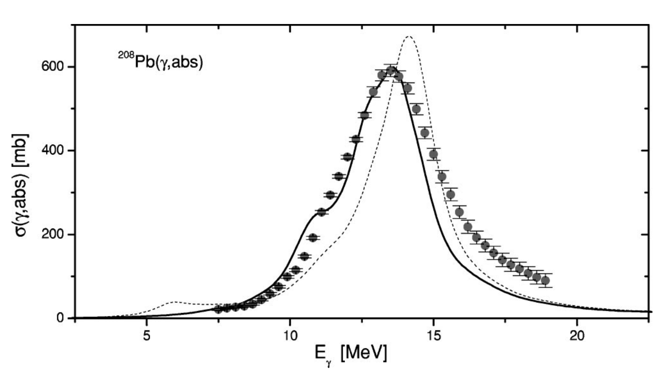


FIG. 4. The total photoabsorption cross section in $^{208}$Pb calculated with the theoretical $E1$ strength functions at $T=0$ (solid line) and $T=1$ MeV (dashed line). Experimental data (full circles with error bars) are taken from Ref. [37]).

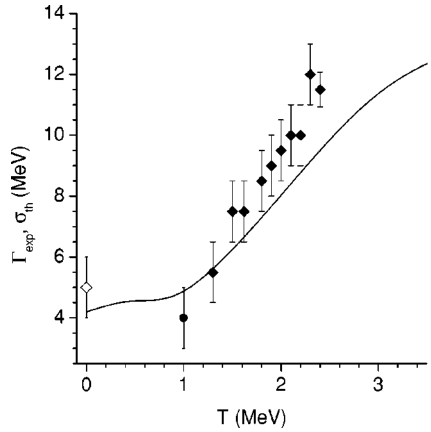


FIG. 5. Temperature dependence of the variance of the theoretical $E1$ strength function $\sigma_{th}$ and the experimental GDR width $\Gamma_{exp}$ in $^{120}$Sn. Full diamonds, the revised experimental data from Ref. [6]; full circle, the data from Ref. [11]; open diamond, the data from Ref. [38] (see also Ref. [39]).

Since the centroid of $E1$ SF changes only weakly with temperature, the main peak of the $E1$ SF is shifted to higher energies in comparison with the TRPA position.

In Figs. 5 and 6, a temperature dependence of the variance $\sigma_{th}$ of the calculated $E1$ SF in both the nuclei studied and the corresponding experimental data on the GDR width $\Gamma_{exp}$ are displayed. These figures should be commented before discussion of the results. In many of the papers, both theoretical or experimental, the width of GDR is defined quantitatively as a "full width at the half of the maximum" (FWHM) of the main bump of the calculated or measured $E1$ strength function. The FWHM seems to be an appropriate measure of the width if the strength function has a smooth shape. However, in our calculations the $E1$ SFs divide into several peaks and the FWHM cannot be determined in an appropriate way. That is why we use the variance as a quantitative measure of the GDR spreading width. However, in spite of FWHM and a variance characterize the same feature of a $E1$ SF (a fragmentation of $E1$ strength) a direct numerical comparison of both quantities is not justified. Thus, displaying of $\sigma_{th}$ and experimental GDR width $\Gamma_{exp}$ on the same figures has only demonstrative sense.

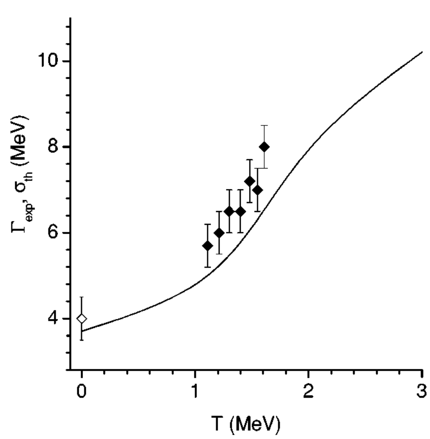


FIG. 6. Temperature dependence of the variance of the theoretical $E1$ strength function $\sigma_{th}$ and the experimental GDR width $\Gamma_{exp}$ in $^{208}$Pb. Full diamonds, the revised experimental data from Ref. [6]; open diamond, the data from Ref. [38] (see also Ref. [39]).

The most distinctive feature of the theoretical curves is the increase in the GDR spreading width with temperature. Moreover, the temperature dependence of $\Gamma_{exp}$ and $\sigma_{th}$ appears to be quite similar. One can see in Figs. 5 and 6 that $\Gamma_{exp} \approx \sigma_{th}$ at $T=0$. Assuming this relation conserved with increase in $T$ one can conclude that the spreading width forms a large part of the total GDR width $\Gamma_{exp}$.

As it was already mentioned in the Introduction, in Ref. [22] a strong difference in thermal behavior of the collisional GDR widths in $^{208}$Pb and $^{120}$Sn was found. Like it is for the GDR centroids in the same nuclei (see above) present calculations do not demonstrate such a difference.

## IV. DISCUSSION

To understand why in our approach the value $\Gamma^{\downarrow}$ increases with temperature, we analyzed the matrix elements of a phonon-phonon coupling $U^{\lambda_1 i_1}_{\lambda_2 i_2}(1^-_i)$ and found a strong effect of a few very low-lying thermal phonons appearing in the phonon spectrum only at $T\neq 0$ due to the nonvanishing thermal occupation factors. These states correspond to low-lying poles of the $(\varepsilon_{j_1}-\varepsilon_{j_2})$ type. Moreover, one amplitude $\eta_{j_1 j_2}$ dominates the phonon wave function, i.e., these phonons are noncollective and of the $p$-$p$ or $h$-$h$ type. Basing on the expression for the TRPA amplitudes in Sec. II A the following expression for the amplitude $\eta_{j_1 j_2}$ of such a phonon can be derived:

$$\eta^{\lambda i}_{j_1 j_2} \sim \sqrt{\frac{T}{\varepsilon_{j_1}-\varepsilon_{j_2}}} \approx \sqrt{\frac{T}{\omega_{\lambda i}}}.$$

If one of the phonons $|\lambda_1 i_1\rangle$ or $|\lambda_2 i_2\rangle$ in the matrix element $U^{\lambda_1 i_1}_{\lambda_2 i_2}(1^-_i)$ (26) is of the aforementioned type, the value $U$ appears to be also proportional to $\sqrt{T/\omega}$ and $\sigma \sim \Sigma U^2 \sim T/\omega$. Thus, a temperature dependence of $\Gamma^{\downarrow}$ arises. Certainly, this is only a qualitative estimation and we do not insist that the variance $\sigma$ has to be proportional to $T$ in our approach. As it is seen from the secular equation (25) and the expression (27) there is a strong interference between contributions of different TRPA dipole phonons to the resulting $E1$ strength function (27) which changes noticeably the temperature dependence of $\sigma$. The appearance of a small value $\omega$ in a denominator explains a strong influence of these noncollective phonons on the $\sigma$ value.

Thus, we conclude that the main reason for the increment of $\Gamma^{\downarrow}$ with $T$ is the interaction of GDR with the noncollective $p$-$p$ (or $h$-$h$) thermal phonons of the special type. On the whole this conclusion agrees with the results of Ref. [15] although in that paper a special role of the low-lying $p$-$p$ ($h$-$h$) phonons was not definitely pointed out. It seems that in Ref. [12] the noncollective thermal RPA excitations have been ignored (the same statement can be found in Ref. [15]). It follows from our consideration that if the thermal phonon space includes only those phonons which are of the $p$-$h$ type at $T=0$ $\Gamma^{\downarrow}$ will be quite stable against $T$.

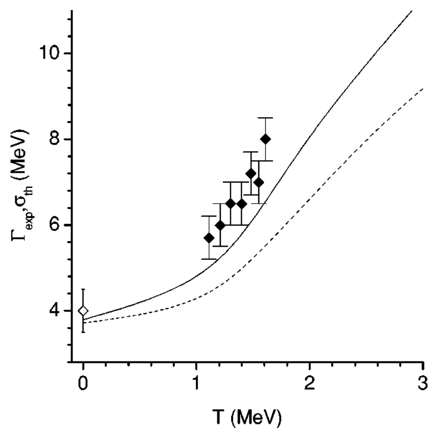


FIG. 7. Temperature dependence of $\sigma_{th}$ in $^{208}$Pb calculated with different two-phonon model spaces: dashed line, $B=50\%$; solid line, $B=60\%$ (see the text for comprehensive explanations).

Some questions concerning a dependence of our numerical results on parameters still remain. An appearance of low-lying $p$-$p$ ($h$-$h$) states is dependent on the parameters of the mean field. In this respect, the use of a phenomenological Saxon-Woods potential gives the upper limit for the role of these low-lying $p$-$p$ ($h$-$h$) states because the density of single-particle states near the Fermi level is the largest one in this potential. The influence of these phonons on $\sigma$ will be weaker in calculations with a mean field obtained by the Hartree-Fock method.

There is one more ingredient directly affecting the calculated value of $\sigma$. The whole space of two-phonon states is overcomplete because thermal phonons are considered as bosons and no special projection of two- or four-fermion states into the bosonic ones is made. We partially reduce this over completeness by the special limitation in constructing the two-phonon part of the trial wave function (24). Namely, only two-phonon configurations combining two collective or one collective–one noncollective phonons are included in the wave function. However, there is no clear cut separation between collective and noncollective states, especially because the "true" collective states like low-lying quadrupole and octupole phonons dissolve with increasing temperature. Therefore, in practice one needs a quantitative measure of "collectivity of a phonon." We introduce this measure on the basis of the phonon structure. For example, a phonon is considered as a collective one if the largest two-fermion component in its wave function (17) exhausts less than $B\%$ of the total norm. Evidently, $B$ is a technical parameter. The larger is $B$ the larger is the thermal two-phonon space or the number of two-phonon configurations taken into account in the calculations. Enlargement of the two-phonon space means strengthening of fragmentation or damping and increasing in the variance of a distribution.

To estimate a possible effect of the two-phonon basis, we make the calculations with two different spaces of two-phonon states. In Fig. 7, we display the results of calculations of $\sigma(T)$ in Pb for $B=50\%$ and 60%. The variance decreases sizably with decreasing $B$. Moreover, a rate of the increase in $\sigma$ with $T$ also becomes slower at a smaller value of $B$. Nevertheless, a general trend of the thermal behavior of the spreading GDR width is saved.

Note, the effect of the size of the two-phonon subspace is much weaker at $T=0$ (this can be seen already in Fig. 7). In Ref. [27], a consistent procedure taking into account the Pauli principle corrections in the two-phonon components was developed. The procedure is based on the exact (from the point of view of their fermionic structure) commutator of phonon operators. Its influence on the strength functions of multipole giant resonances in cold nuclei was studied in Ref. [40]. The corresponding corrections were found to be small because at $T=0$ the main contribution to the spreading width of giant resonances is given by the coupling with the lowest collective quadrupole and octupole phonons and corresponding two-phonon configurations are weakly affected by the Pauli principle. However, with increasing $T$ the low-lying collective vibrations dissolve and their contribution to the damping of giant resonances diminishes. At the same time, the role of the Pauli principle acting between large number of weakly collective and noncollective states treated as bosons becomes more and more important. The results presented in Fig. 7 reflect this process.

The important role of noncollective thermal phonons raises a problem of the validity of a schematic multipole-multipole interaction in the present study. Indeed, calculations of the collisional damping [19,22] have clearly demonstrated a strong influence of the effective nucleon-nucleon interaction on both the absolute value of the GDR width at $T\neq 0$ and the slope of its temperature dependence. However, we do not claim to describe the GDR width quantitatively. Our aim is to elucidate the role of thermal effects in the GDR width behavior in view of conflicting results of Ref. [12] and Ref. [15]. In both the papers the same schematic multipole-multipole interaction was explored.

There is one more interesting and at the first glance principal difference between our approach and that of Refs. [12,14,15]. The difference has already been pointed out in Ref. [25] and now we would like to discuss it in more detail. In Refs. [12,15], the GDR width depends on thermal occupation numbers of two types—the Fermi-Dirac numbers for particle and holes and the Bose-Einstein occupation numbers for TRPA phonons. Bose occupation factors appear in a theory when the temperature-dependent Green's function of the single TRPA phonon treated as heated boson is introduced.

In the present paper, one cannot find the thermal bosonic occupation numbers, and it seems there is no room for them within the explored approach. We start with the model Hamiltonian written in terms of nucleonic (i.e., fermionic) variables. The thermal occupation numbers appear in the game when we make the thermal Bogoliubov rotation (7) and thus produce the thermal Fock space. All further manipulations explore these "heated" fermions and their combinations. Thus, the appearance of bosonic occupation numbers is quite questionable. Our thermal Hamiltonian in its final form (23) is the Hamiltonian of interacting phonons built from "heated" quasiparticles but the phonon system itself is not heated in the sense that there is no thermal smearing of phonons over their energy levels. This corresponds to a transparent phenomenological picture: when one heats a nucleus putting there a good piece of energy, a nucleonic motion is changed and due to this the properties of a nuclear surface are changed. As a consequence of the latter the properties of surface vibrations are changed. However, one cannot heat nuclear surface vibrations themselves.

In Ref. [15], the authors start just with the Hamiltonian of the interacting TRPA phonons implying, as an obvious fact, that the phonon system has the same temperature $T$ as the underlying fermions forming the thermal phonons. In our opinion this is an additional assumption which has to be justified. Similarly, in Ref. [14] from the beginning a nucleus is treated as a system of phonons and quasiparticles. But since phonons and quasiparticles are considered as some "initial" ingredients, the structure of phonons must be as it is in a cold nucleus and cannot be changed by heating the system. Thus, they cannot satisfy the thermal RPA equation.

The point is that quasiparticles and phonons are not independent variables in a nucleus. The phonon is a coherent superposition of bifermionic excitations. So, starting with the model Hamiltonian given in terms of nucleonic degrees of freedom one has to make a mapping of pure fermionic states to a subspace consisting of ideal "quasiparticle" and "bosonic" elementary modes.

In this regard, Hatsuda [30] discussed already two ways to consider a hot nucleus. The first is to make a mapping of the initial Hamiltonian and the initial pure fermionic Fock space of a cold system (nucleus) and only after this to thermalize a system in question. For the approach presented here it means that degrees of freedom should be doubled for the quasiparticle-phonon image of the Hamiltonian (1)–(4). Then one gets the thermal Hamiltonian with both the types of thermal occupation numbers and, consequently, also the GDR width should depend on them. However, Hatsuda [30] has also shown taking the Lipkin model as an example that "thermalizing" of the bosonic image of the initial fermionic Hamiltonian one cannot derive in the leading order the TRPA equations for these bosons.

The second way is just the way of the present paper: while heating we treat a nucleus as a system of fermions and only after this we project or transform the original nucleonic degrees of freedom to more convenient ones (bosonic or bosonic + fermionic).

We would like to stress that the problem how to treat a thermalized nucleus in terms of quasiparticles and phonons is not so trivial as it may seem at the first glance. It is in intimate correspondence with a proper choice of physically important degrees of freedom and their consistent mapping which has to comply with the particle statistic requirements. Some aspects of the problem were discussed also in Ref. [41].

The effect of the thermal phonon occupation numbers on the $T$ dependence of $\Gamma^{\downarrow}$ is not very significant, we guess. At least this is not the crucial point for increasing $\Gamma^{\downarrow}$ with temperature.

## V. CONCLUSIONS

A temperature dependence of the fragmentation of a giant dipole resonance has been studied within the quasiparticle -phonon model extended to finite temperature within the thermofield dynamics. According to the results of numerical calculations, the variance of the $E1$ strength function increases with $T$ in the temperature range $0 < T \leqslant 3$ MeV. In our opinion, this is the main result of the paper. Exploring very close physical ideas except for the other formalism we get qualitatively the same results as in Ref. [15] concerning the role of noncollective thermal excitations in thermal evolution of the GDR width.

We also draw attention to the problem of a proper choice of relevant nuclear degrees of freedom to describe a damping of giant resonances in a hot nucleus. To our knowledge, this aspect of a giant resonance theory was overlooked before.

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