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Instability of nuclear wobbling motion and tilted axis rotation

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We study a possible correspondence between the softening of the wobbling mode and the "phase transition" of the one-dimensionally rotating mean field to a three-dimensionally rotating one by comparing the properties of the wobbling mode obtained by the one-dimensional cranking model + random phase approximation with the total Routhian surface obtained by the three-dimensional tilted-axis cranking model. The potential surface for the observed wobbling mode excited on the triaxial superdeformed states in ^{163}Lu is also analyzed.

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I. INTRODUCTION

The concept of the phase transition of the mean field is useful for describing structure changes in the atomic nucleus although it is a quantum system composed of finite number of fermions. A typical example is that a spherical mean field becomes unstable as the quadrupole vibration excited on top of it softens with changing particle numbers, then an axially symmetric mean field substitutes. This can rotate about one of the axes perpendicular to the symmetry axis. Consecutively, the axial symmetric mean field can become unstable as the γ vibration softens, then a triaxially deformed mean field substitutes. This can rotate about all the three principal axes. Usually, however, a rotation about one axis dominates because the rotation about the axis with the largest moment of inertia is energetically favorable. When some excitation energy is supplied, small rotations about other two axes become possible. Consequently this produces a kind of vibrational motion of the rotational axis, that is, the wobbling motion.

The small amplitude wobbling motion at high spins was first discussed by Bohr and Mottelson [1] in terms of a macroscopic rotor model. Then it was studied microscopically by Janssen and Mikhailov [2] and Marshalek [3] in terms of the random phase approximation (RPA). Since the small amplitude wobbling mode has the same quantum number, parity $\pi=+$ and signature $\alpha=1$, as the odd-spin member of the γ vibrational band, Mikhailov and Janssen [4] anticipated that it would appear as a high-spin continuation of the odd-spin γ band. But it has not been clear in which nuclei, at what spins, and with what shapes it would appear. Using the RPA, Shimizu and Matsuyanagi [5] studied Er isotopes with small $|\gamma|$, Matsuzaki [6] and Shimizu and Matsuzaki [7] studied ^{182}Os with a rather large negative γ but their correspondence to the experimental data was not very clear. In 2001, Ødegård *et al.* [8] found an excited triaxial superdeformed band in ^{163}Lu and identified it firmly as a wobbling band by comparing the observed and theoretical interband $E2$ transition rates. These data were investigated in terms of a particle-rotor model by Hamamoto [9] and in terms of the RPA by Matsuzaki *et al.* [10]. In 2002, two-phonon wobbling excitations were also observed by Jensen *et al.* [11] and their excitation energies show some anharmonicity.

The one-dimensionally rotating triaxial mean field may become unstable as the wobbling mode softens with changing some parameters. One of the present authors (M.M.) pointed out its theoretical possibility in Ref. [10]. The possibility of this phase transition was discussed in terms of the harmonic oscillator model by Cuypers [12] and Heiss and Nazmitdinov [13] but their conclusions are controversial. A theoretical framework to describe three-dimensional rotations, possibly with large amplitude, was first devised by Kerman and Onishi [14] within a time-dependent variational formalism. Onishi [15] and Horibata and Onishi [16] applied it to ^{166}Er and ^{182}Os, respectively. See Ref. [17] for recent applications. The three-dimensional cranking model was first used by Frisk and Bengtsson [18]. The term, "tilted (axis) cranking (TAC)" was, to our knowledge, first used by Frauendorf [19] and it was applied to a kind of two-dimensional rotation—the so-called shears band, observed, for example, in the $A\sim200$ region [20–22]. Applications to multiquasiparticle high-K bands were also extensively done; see Ref. [23], and references therein. When the rotation becomes fully three dimensional, a new concept, chirality, emerges [24,25]. The tilted axis cranking was also applied to this [26]. At finite temperature, the degree of freedom of spin orientation was studied macroscopically [27] and microscopically [28]. A relativistic formulation of the three-dimensional cranking was given by Kaneko *et al.* [29] as an extension of the one-dimensional one given by the Munich group [30]. Madokoro *et al.* [31] studied the shears band in ^{84}Rb starting from the meson exchange interaction although the pairing field was neglected.

The purpose of the present paper is to elucidate the work in Ref. [10] by comparing two types of theoretical calculations, the one-dimensional cranking model + RPA and the three-dimensional (tilted axis) cranking model. The former gives the "mass parameters" for the motion of the angular momentum vector, that is, the moments of inertia, while the latter provides the surfaces on which the angular frequency vector moves around.

II. MODEL

We start from a one-body Hamiltonian in the rotating frame

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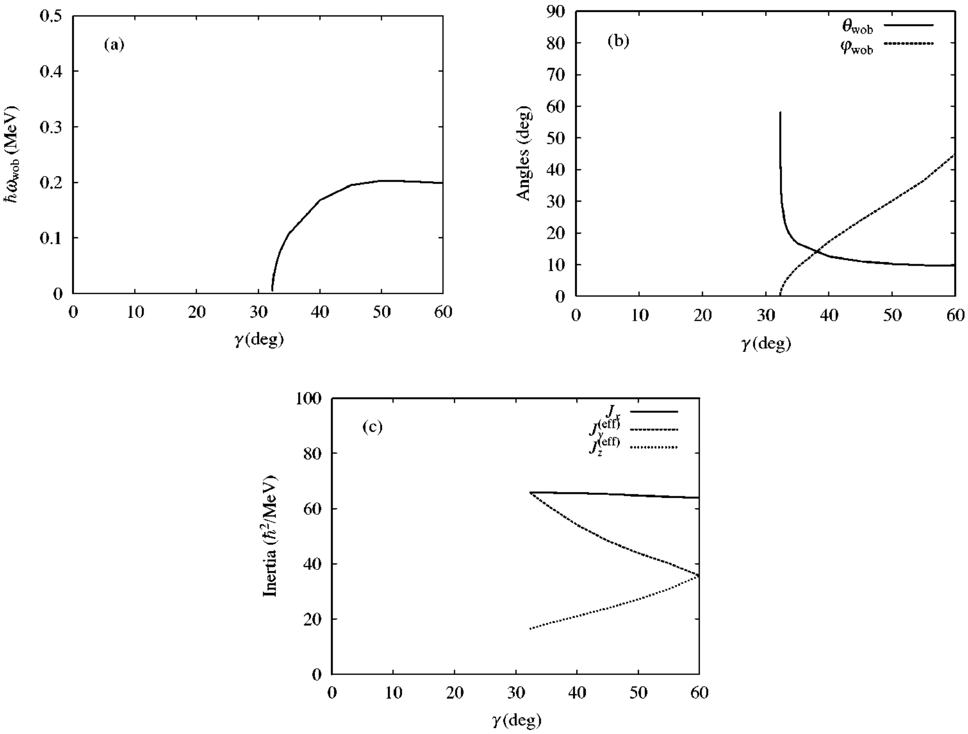

FIG. 1. Triaxiality dependence of (a) excitation energy of the wobbling motion, (b) wobbling angles, and (c) three moments of inertia associated with it in ^{146}Gd, calculated at $\hbar\omega_{\rm rot}=0.25$ MeV with $\epsilon_2=0.19$, $\Delta_n=0.8$ MeV, and $\Delta_p=0.6$ MeV.

$$h' = h - h_{\rm cr}, \tag{1}$$

$$h = h_{\rm nil} - \Delta_\tau(P_\tau^\dagger + P_\tau) - \lambda_\tau N_\tau, \tag{2}$$

$$h_{\rm nil} = \frac{\mathbf{p}^2}{2M} + \frac{1}{2}M(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + v_{ls}\mathbf{l}\cdot\mathbf{s} + v_{ll}(\mathbf{l}^2 - \langle \mathbf{l}^2\rangle_{N_{\rm osc}}). \tag{3}$$

($h_{\rm cr}$ is specified below.) In Eq. (2), $\tau=1$ and 2 stand for neutron and proton, respectively, and chemical potentials λ_τ are determined so as to give correct average particle numbers $\langle N_\tau\rangle$. The oscillator frequencies in Eq. (3) are related to the quadrupole deformation parameters ϵ_2 and γ in the usual way. (We adopt the so-called Lund convention.) They are treated as parameters as well as pairing gaps Δ_τ. The orbital angular momentum $\mathbf{l}$ in Eq. (3) is defined in the singly stretched coordinates $x_k=\sqrt{\omega_k/\omega_0}x_k$, with $k=1$–3 denoting x–z, and the corresponding momenta.

A. One-dimensional cranking model + random phase approximation

Equations (1)–(3) with

$$h_{\rm cr} = \hbar\omega_{\rm rot} J_x \tag{4}$$

generate the system rotating one dimensionally. Then, since h' conserves parity π and signature α, nuclear states can be labeled by them. Nuclear states with quasiparticle (QP) excitations are obtained by exchanging the QP energy and wave functions such as

$$(-e'_\mu, \mathbf{V}_\mu, \mathbf{U}_\mu) \to (e'_{\bar\mu}, \mathbf{U}_{\bar\mu}, \mathbf{V}_{\bar\mu}), \tag{5}$$

where $\bar\mu$ denotes the signature partner of μ. We perform the RPA to the residual pairing plus doubly stretched quadrupole-quadrupole ($Q''\cdot Q''$) interaction between QP's. Since we are interested in the wobbling motion that has a definite quantum number, $\alpha=1$, only two components out of five of the $Q''\cdot Q''$ interaction are relevant. They are given by

$$H_{\rm int}^{(-)} = -\frac{1}{2}\sum_{K=1,2}\kappa_K^{(-)} Q_K''^{(-)\dagger} Q_K''^{(-)}, \tag{6}$$

where the doubly stretched quadrupole operators are defined by

$$Q_K'' = Q_K\left(x_k \to x_k'' = \frac{\omega_k}{\omega_0}x_k\right), \tag{7}$$

and those with good signature are

$$Q_K^{(\pm)} = \frac{1}{\sqrt{2(1+\delta_{K0})}}(Q_K \pm Q_{-K}). \tag{8}$$

The residual pairing interaction does not contribute because P_τ is an operator with $\alpha=0$. The equation of motion

$$[h' + H_{\rm int}^{(-)}, X_n^\dagger]_{\rm RPA} = \hbar\omega_n X_n^\dagger \tag{9}$$

for the eigenmode

$$X_n^\dagger = \sum_{\mu<\nu}^{(\alpha=\pm 1/2)} [\psi_n(\mu\nu) a_\mu^\dagger a_\nu^\dagger + \varphi_n(\mu\nu) a_\nu a_\mu] \tag{10}$$

leads to a pair of coupled equations for the transition amplitudes

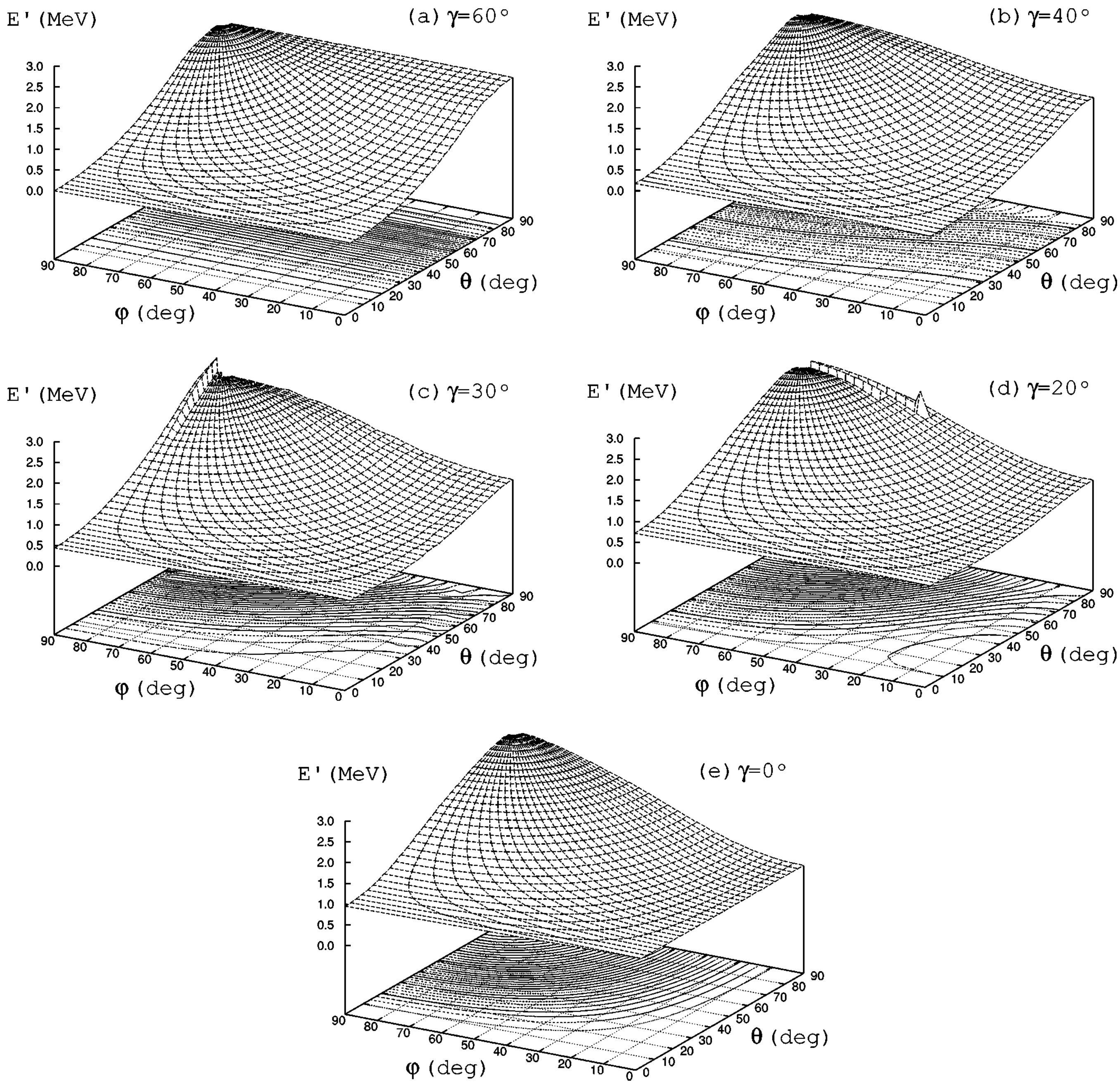

FIG. 2. Energy surfaces of the $[(\nu h_{9/2},f_{7/2})^2(\pi h_{11/2})^2]_{16^+}$ configuration in ^{146}Gd as functions of the tilting angle (θ,φ) calculated with the same parameters as Fig. 1, (a) $\gamma=60°$, (b) $\gamma=40°$, (c) $\gamma=30°$, (d) $\gamma=20°$, and (e) $\gamma=0°$. The interval of contours is 50 keV. Discontinuities in the surfaces are due to quasiparticle crossings.

$$T_{K,n} = \langle [Q_K^{(-)}, X_n^\dagger] \rangle. \tag{11}$$

Then, by assuming $\gamma \neq 0$, this can be cast [3] into the form

$$(\omega_n^2 - \omega_{\rm rot}^2)\left[\omega_n^2 - \omega_{\rm rot}^2 \frac{[\mathcal{J}_x - \mathcal{J}_y^{(\rm eff)}(\omega_n)][\mathcal{J}_x - \mathcal{J}_z^{(\rm eff)}(\omega_n)]}{\mathcal{J}_y^{(\rm eff)}(\omega_n)\mathcal{J}_z^{(\rm eff)}(\omega_n)}\right] = 0, \tag{12}$$

which is independent of $\kappa_K^{(-)}$s. This expression proves that the spurious mode ($\omega_n=\omega_{\rm rot}$; not a real intrinsic excitation but a rotation as a whole) given by the first factor and all normal modes given by the second are decoupled from each other. Here $\mathcal{J}_x=\hbar\langle \mathcal{J}_x\rangle/\omega_{\rm rot}$ as usual and the detailed expressions of $\mathcal{J}_{y,z}^{(\rm eff)}(\omega_n)$ are given in Refs. [3,6,7]. Among normal modes, one obtains

$$\omega_{\rm wob} = \omega_{\rm rot}\sqrt{\frac{[\mathcal{J}_x - \mathcal{J}_y^{(\rm eff)}(\omega_{\rm wob})][\mathcal{J}_x - \mathcal{J}_z^{(\rm eff)}(\omega_{\rm wob})]}{\mathcal{J}_y^{(\rm eff)}(\omega_{\rm wob})\mathcal{J}_z^{(\rm eff)}(\omega_{\rm wob})}}, \tag{13}$$

by putting $\omega_n=\omega_{\rm wob}$. Note that this gives a real excitation only when the argument of the square root is positive and it is nontrivial whether a collective solution appears or not. Evidently this coincides with the form derived by Bohr and Mottelson in a rotor model [1] and known in classical mechanics [32].

B. Three-dimensional (tilted axis) cranking model

In this model the one-body Hamiltonian is given by Eqs. (1)–(3) with

$$h_{\text{cr}} = \hbar \mathbf{\Omega} \cdot \mathbf{J}, \tag{14}$$

$$\mathbf{\Omega} = \omega_{\text{rot}}(\cos\,\theta, \sin\,\theta\,\cos\,\varphi, \sin\,\theta\,\sin\,\varphi). \tag{15}$$

Pairing correlation is taken into the system by a simple BCS approximation with fixed gaps as in the case of the one-dimensional cranking. The expectation value $\langle \mathbf{J} \rangle$ calculated at each $(\omega_{\text{rot}}, \theta, \varphi)$ has three nonzero components in general; the stationary state that minimizes the total Routhian is obtained by requiring $\langle \mathbf{J} \rangle \| \mathbf{\Omega}$ (see Ref. [23] for details). Obtained tilted solutions do not possess the signature symmetry and therefore describe $\Delta I = 1$ rotational bands. In the present work, given a set of mean-field parameters, N_τ, ϵ_2, γ, and Δ_τ, a configuration is specified at $\theta = 0^\circ$ (principal axis cranking about the x axis). Then by changing θ and φ step by step, the most overlapped state is chased. This procedure gives an energy (total Routhian) surface for the angular frequency vector. Surfaces for QP excited configurations can also be calculated by adopting a procedure similar to Eq. (5).

III. RESULT AND DISCUSSION

For this first comparative calculation, we choose the $[(\nu h_{9/2}, f_{7/2})^2 (\pi h_{11/2})^2]_{16^+}$ four quasiparticle configuration in ^{146}Gd among this mass region in which many oblate isomers have been observed. This state is described by $\epsilon_2 = 0.19$, $\gamma = 60°$, $\Delta_n = 0.8$ MeV, $\Delta_p = 0.6$ MeV, and $\hbar\omega_{\text{rot}} = 0.25$ MeV. Calculations are performed in the model space of three major shells; $N_{\text{osc}} = 4-6$ for neutrons and $3-5$ for protons. The strengths of the $\mathbf{l} \cdot \mathbf{s}$ and $\mathbf{l}^2$ potentials are taken from Ref. [33].

In the present study we concentrate on the changes in the system with γ. Figure 1(a) reports the excitation energy $\hbar\omega_{\text{wob}}$ in the rotating frame. That in the laboratory frame in the case of $\gamma = 60°$ is given by $\hbar\omega_{\text{wob}} + \hbar\omega_{\text{rot}} = 0.198$ MeV $+ 0.25$ MeV. The excitation energy decreases steeply as γ decreases. In order to see its implication, we show in Fig. 1(b) the wobbling angles,

$$\theta_{\text{wob}} = \tan^{-1} \frac{\sqrt{|J_y^{(\text{PA})}(\omega_{\text{wob}})|^2 + |J_z^{(\text{PA})}(\omega_{\text{wob}})|^2}}{\langle J_x \rangle}, \tag{16}$$

$$\varphi_{\text{wob}} = \tan^{-1} \left| \frac{J_z^{(\text{PA})}(\omega_{\text{wob}})}{J_y^{(\text{PA})}(\omega_{\text{wob}})} \right|, \tag{17}$$

with (PA) denoting the principal axis. θ_{wob} clearly proves that the softening of the excitation energy is accompanied by a growth of the amplitude of the motion. φ_{wob} indicates that the fluctuation to the y direction grows. Corresponding to this, the three moments of inertia behave as in Fig. 1(c).

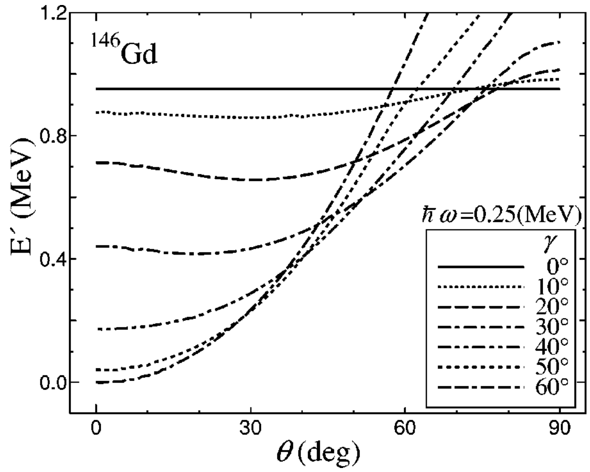

FIG. 3. Cross sections at $\varphi = 0°$ of the energy surfaces of ^{146}Gd.

Qualitatively, this behavior can be understood as an irrotational-like moments of inertia

$$\mathcal{J}_k^{\text{irr}} \propto \sin^2\left(\gamma + \frac{2}{3}\pi k\right), \tag{18}$$

where $k = 1-3$ denoting the $x-z$ components, superimposed by the contribution from the alignment, $\Delta\mathcal{J}_x$. Alternatively, it can also be viewed as that, at large γ, multiple alignments lead to a rigid-body-like inertia

$$\mathcal{J}_k^{\text{rig}} \propto \left[1 - \sqrt{\frac{5}{4\pi}}\beta\,\cos\left(\gamma + \frac{2}{3}\pi k\right)\right], \tag{19}$$

with β being a deformation parameter defined by the mass distribution.

Now we proceed to three-dimensional calculations; we calculate energy surfaces as functions of the tilting angle (θ, φ) of $\mathbf{\Omega}$. Here we note that the (θ, φ) plane is represented as a rectangle although φ is meaningful for $\theta \neq 0°$. Figure 2(a) shows the $\gamma = 60°$ (symmetric about the x axis) case. Until down to $\gamma \sim 40°$, energy surfaces are qualitatively similar aside from becoming shallow gradually. But a further decrease of γ leads to an instability of the motion to the θ

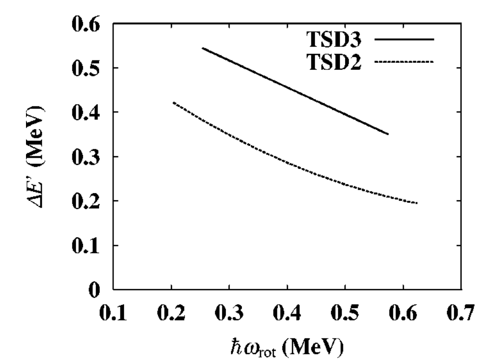

FIG. 4. Experimental excitation energies of the two- and one-phonon wobbling states relative to the yrast triaxial superdeformed states in ^{163}Lu. Data are taken from Ref. [11].

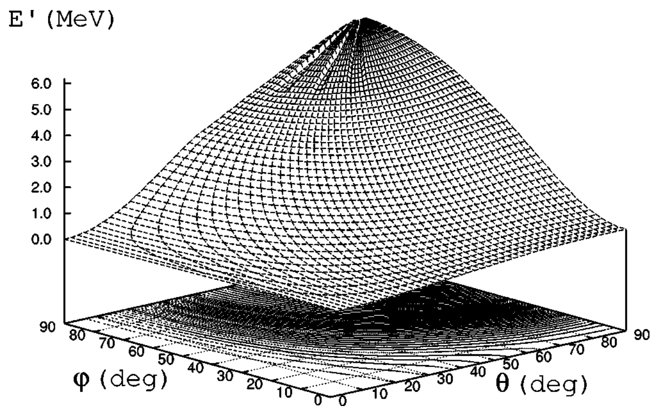

FIG. 5. Energy surface of the triaxial superdeformed one-quasiparticle configuration in ^{163}Lu as a function of the tilting angle (θ,φ) calculated at $\hbar\omega_{\mathrm{rot}}=0.5$ MeV with $\epsilon_2=0.43$, $\gamma=20°$, and $\Delta_n=\Delta_p=0.3$ MeV. The interval of contours is 100 keV.

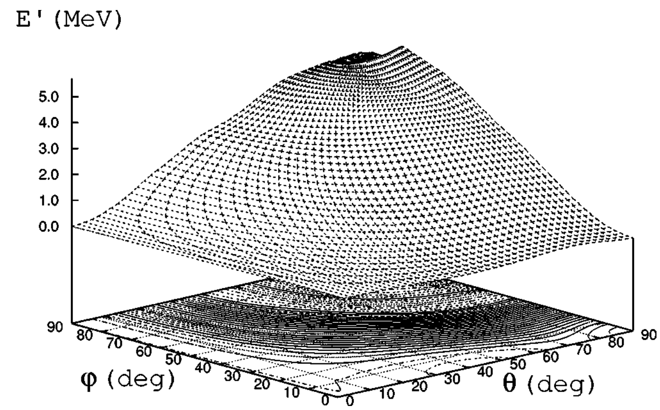

FIG. 6. The same as Fig. 5 but for the zero-quasiparticle configuration in ^{162}Yb.

direction with $\varphi=0°$, that is, the direction of the y axis. Together with the property that the surface is stable with respect to the direction of the z axis, the situation corresponds excellently to Fig. 1. The behavior of φ_{wob} in Fig. 1(b) can be interpreted as follows: when the system can fluctuate to the direction of the y axis without any energy cost, it does not fluctuate to the z axis.

To look at the energy surface more closely we gather their cross sections at $\varphi=0°$ (the x-y plane) in Fig. 3. This figure clearly shows that a tilted axis minimum appears at around $\gamma=30°$ although it is shallow. The correspondence to Fig. 1 in which the instability occurs at $\gamma=32°$ is excellent. Note that the reason why the wobbling angle seen from Fig. 3 is larger than θ_{wob} in Fig. 1(b) is that this is drawn for $\mathbf{\Omega}$.

IV. POTENTIAL SURFACE FOR THE WOBBLING MODE IN ^{163}Lu

The analyses above are purely theoretical. Then, is there any experimental signature of the softening of the wobbling motion? We think the answer is yes. Figure 4 shows the experimental [11] excitation energies (in the rotating frame) of the TSD3 (two-phonon wobbling) and the TSD2 (one-phonon wobbling) relative to the TSD1 (yrast 1QP TSD) in ^{163}Lu, where TSD is the abbreviation for triaxial superdeformation. $\Delta E'_{\text{two phonon}}<2\times\Delta E'_{\text{one phonon}}$ indicates a signature of softening of the energy surface. We obtained $\hbar\omega_{\mathrm{wob}}=0.185$ MeV, $\theta_{\mathrm{wob}}=14.2°$, and $\varphi_{\mathrm{wob}}=7.6°$ for the one-phonon wobbling state in the RPA (see also Refs. [10,34] for the RPA calculation). The small value of φ_{wob} looks to indicate a softening to the y direction. Calculated energy surface is shown in Fig. 5. Calculations were done in the model space of five major shells, $N_{\mathrm{osc}}=3-7$ for neutrons and 2–6 for protons, with $\epsilon_2=0.43$, $\gamma=20°$, $\Delta_n=\Delta_p=0.3$ MeV, and $\hbar\omega_{\mathrm{rot}}=0.5$ MeV where the calculated $\hbar\omega_{\mathrm{wob}}$ approaches the experimental one. This figure shows again the surface softens to the direction of the y axis.

In Refs. [10,34], it was shown that the alignment of the $\pi i_{13/2}$ quasiparticle was essential for the appearance of the wobbling motion. In order to see this fact from the viewpoint of the potential surface, we calculated the 0QP (nonyrast TSD at high spins) configuration in ^{162}Yb. Figure 6 clearly shows that a tilted axis minimum in the x-y plane is realized when the wobbling motion does not occur due to the lack of $\pi i_{13/2}$ QP's that make $\mathcal{J}_x$ larger than $\mathcal{J}_y^{(\mathrm{eff})}$ [10,34]. This result proves that the low-Ω high-j orbital favors the principal axis rotation on which the wobbling motion occurs.

For a deeper understanding of the two-phonon states, the application of more sophisticated many body theories such as the self-consistent collective coordinate (SCC) method [35] is desirable.

V. SUMMARY

To summarize, we have proved that a tilted axis rotation emerges when the wobbling mode becomes unstable as the triaxiality parameter changes in an oblate configuration in ^{146}Gd. Its instability is caused by the growth of the fluctuation of the motion of the angular momentum or frequency vector to the direction of the y axis. Having performed this theoretical calculation, we have argued that the signature of the softening of the wobbling motion can be seen in the observed spectra of the triaxial superdeformation in ^{163}Lu and shown that a tilted axis minimum would appear if it were not for the $\pi i_{13/2}$ quasiparticle.

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