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Competition between isoscalar and isovector pairing correlations in $N=Z$ nuclei

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We study the isoscalar ($T=0$) and isovector ($T=1$) pairing correlations in $N=Z$ nuclei. They are estimated from the double difference of binding energies for odd-odd $N=Z$ nuclei and the odd-even mass difference for the neighboring odd-mass nuclei, respectively. The empirical and BCS calculations based on a $T=0$ and $T=1$ pairing model reproduce well the almost degeneracy of the lowest $T=0$ and $T=1$ states over a wide range of even-even and odd-odd $N=Z$ nuclei. It is shown that this degeneracy is attributed to competition between the isoscalar and isovector pairing correlations in $N=Z$ nuclei. The calculations give an interesting prediction that the odd-odd $N=Z$ nucleus ^{82}Nb has possibly the ground state with $T=0$.

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There is a current topic with increasing interests in studying isovector ($T=1$) and isoscalar ($T=0$) proton-neutron (pn) pairing correlations in $N=Z$ nuclei [1]. At present, it is not clear whether pn pairing correlations are strong enough to form a static condensate. It is well known that an experimental signature of like-nucleon proton-proton (pp) and neutron-neutron (nn) $J=0$ pairing correlations in nuclei with neutron excess is the odd-even mass difference, which is extra binding energy of even-even nuclei relative to that of odd-mass nuclei. However, the odd-even mass differences for even-even $N=Z$ nuclei are larger than those of the neighboring even-even $N=Z+2$ nuclei, and it reflects the gain in pairing due to stronger pn correlations [2]. It has recently been shown [3,4] that the three-point odd-even mass difference for an odd-mass nucleus with neutron excess is an excellent measure of pp and nn pairing correlations in neighboring even-even nucleus, although it is still controversial [5]. This conclusion suggests that the pp and nn pairing correlations in $N=Z$ even-even nuclei also can be estimated from the odd-even mass difference of neighboring odd-mass nuclei with $N=Z+1$. On the other hand, the pn pairing can be estimated from the double difference of binding energies [2]. When we assume isospin symmetry in $N\approx Z$ nuclei, the $T=1$ pn pairing and like-nucleon (pp and nn) pairing are classified in the same $T=1$ pairing correlations, and the former correlation energy should be the same as the latter one.

Odd-odd $N=Z$ nuclei are an ideal experimental laboratory for the study of pn pairing correlations. It is well known that the lowest $T=0$ and $T=1$ states in odd-odd $N=Z$ nuclei are almost degenerate and exhibit the inversion of the sign of the energy difference $E_{T=1}-E_{T=0}$, while all even-even $N=Z$ nuclei have the $T=0$ ground states and the $T=1$ excited states with large excitation energies. Several authors [6–11] already pointed out that this degeneracy in odd-odd $N=Z$ nuclei reflects the delicate balance between the symmetry energy and the pairing correlations. The $T=0$ and $T=1$ ground-state binding energies of $N=Z$ nuclei were calculated by using an algebraic model based on IBM-4 [12]. In this paper, we study the $T=0$ and $T=1$ pairing correlations from a phenomenological point of view, and analyze them in the BCS calculations within a schematic model that includes $T=1$ and $T=0$ pairing interactions.

We begin with the estimation of $T=1$ pairing correlations in $N=Z$ nuclei. A typical indicator for $T=1$ pairing correlations is the following three-point odd-even mass difference:

$$\Delta_n^{(3)}(Z,N)=\frac{(-1)^N}{2}[B(Z,N+1)-2B(Z,N)+B(Z,N-1)], \tag{1}$$

where $B(Z,N)$ is the negative binding energy of a system. Since $B(Z,N\pm1)\approx B(Z,N)+\Delta\pm\lambda$ based on standard BCS theory with pairing gap Δ leads to $\Delta_n^{(3)}(Z,N)\approx\Delta$, the indicator $\Delta_n^{(3)}$ is often interpreted as a measure of the empirical pairing gap. However, it is well known that values of $\Delta_n^{(3)}(Z=\text{even},N)$ are large for even N and small for odd N. It was discussed [3] that $\Delta_n^{(3)}(Z=\text{even},N=\text{odd})$ is an excellent measure of $T=1$ pairing correlations, and the differences of $\Delta_n^{(3)}$ at adjacent even- and odd-N nuclei reflect the mean-field contributions. From a view point of the semiempirical mass formula, the above indicator is well known to be affected by the symmetry energy term in the liquid-drop model. In the macroscopic-microscopic shell model, however, the curvature contribution cancels out the symmetry energy contribution as pointed out by Satulta *et al.* [3]. What does the magnitude of the pairing gap in the $N=Z$ nuclei mean? We suggest that $\Delta_n^{(3)}(Z,Z+1)$ of odd-mass nucleus should be regarded as pure pairing gap in $N=Z$ adjacent even-even and odd-odd nuclei. For the $N=Z$ nuclei, the four and five point indicators cannot be adopted because they include large contributions from mean-field and pn correlations [2,5]. Figure 1 shows experimental values of $\Delta_n^{(3)}$ in odd-mass nuclei, where we plot $\Delta_n^{(3)}(Z,Z+1)$ for $16<A<60$. When there is no data of $\Delta_n^{(3)}(Z,Z+1)$ for $60<A<110$, we adopt $\Delta_n^{(3)}$ for nearest nuclei with $N=Z+1$. The expected quenching of neutron pairing at magic (or semimagic) particle number N or Z $=14$, 28, 40, and 50 is clearly seen in the figure.

The standard curve $12A^{-1/2}$ is also shown as a guide eye in Fig. 1. We can see that the average pairing gap is smaller than the values of the curve $12A^{-1/2}$. The global trend can be fitted by the curve $5.18A^{-1/3}$ MeV, as discussed in recent analyses [11,13], where $T=1$ pairing gap $\Delta_{T=1}$ obtained from

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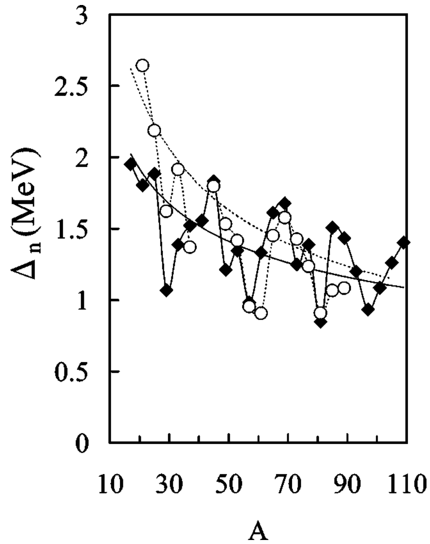

FIG. 1. The experimental odd-even mass differences $\Delta_n^{(3)}(Z=\text{even},Z+1)$ (solid diamonds) in odd-mass nuclei with $N=Z+1$, and the pairing gaps (open circles) obtained by the BCS calculations. The solid curve is $5.18A^{-1/3}$ and the dashed curve denotes $12A^{-1/2}$.

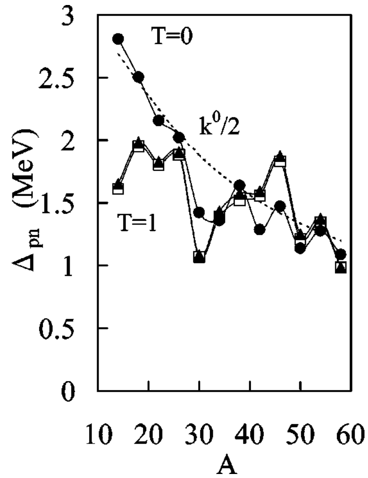

FIG. 2. The pn pairing gaps estimated from the double differences of experimental binding energies. The solid circles denote the $T=0$ pn pairing gap, and the solid triangles the $T=1$ pn pairing gap. The odd-even mass differences in odd-mass nuclei with $N=Z+1$ are shown by the open squares. The dashed curve is the half of the $T=0$ pairing force strength k^0.

some binding energy difference is fitted by the mass dependence $A^{-1/3}$ different from the standard one $12A^{-1/2}$. The difference between the two curves is quite large for light nuclei, while it is small for heavy nuclei. The average gap was recently analyzed [14] by $\Delta=\alpha+\beta A^{-1/3}$ which has theoretical foundation. This analysis also supports the weaker mass dependence. We now consider the following pairing Hamiltonian to describe the $T=1$ pairing correlations:

$$H = H_0 + H_P = \sum_{\alpha} \varepsilon_a c_\alpha^\dagger c_\alpha - \frac{1}{2} G \sum_{\kappa} P_\kappa^\dagger P_\kappa, \qquad (2)$$

where ε_a is the single-particle energy and P_κ is the $J=0$ pair operator with isospin $T=1, T_z=\kappa$. Implying isospin invariance to the above Hamiltonian, the pairing part H_P includes the isovector pn interactions. The standard BCS calculations with the pairing Hamiltonian (2) were performed in sd and fpg shells. We adopted single-particle energies from a spherical Woods-Saxon potential in the BCS calculations. The pairing force strength $G=24.5/A$ was chosen so as to fit the experimental odd-even mass difference $\Delta_n^{(3)}(Z=\text{even},Z+1)$ in odd-mass nuclei. The BCS results for $A>40$ almost agree with the experimental odd-even mass differences, and moreover reproduce the shell effects. The BCS calculations reproduce well the behavior of the observed odd-even mass difference over a wide range of $N=Z$ nuclei. Thus the $T=1$ pairing correlations can be estimated from the odd-even mass difference $\Delta_n^{(3)}(Z=\text{even},Z+1)$ in odd-mass nuclei.

To describe the pn pairing correlations in odd-odd $N=Z$ nuclei, let us estimate the following double difference of binding energies [2,15,16]:

$$\Delta_{pn}^T(Z,N) = \tfrac{1}{2}[B(Z,N)^T - B(Z,N-1) - B(Z-1,N) + B(Z-1,N-1)], \qquad (3)$$

where $B(Z,N)^T$ is the binding energy of lowest state with isospin T in odd-odd $N=Z$ nuclei. Figure 2 shows the double difference of binding energies calculated from the experimental binding energies. The odd-even mass differences for odd-mass nuclei are also displayed. Then we can see that the $\Delta_n^{(3)}(Z=\text{even},Z+1)$ agrees with the $\Delta_{pn}^{T=1}(Z+1,Z+1)$. This means that $T=1$ pn pairing for odd-odd $N=Z$ nuclei has the same correlation energy as the like-nucleon nn pairing, $\Delta_n=\Delta_{pn}^{T=1}$, when assuming isospin symmetry. Thus, the indicator $\Delta_{pn}^{T=1}$ gives the $T=1$ pn pairing gap in $N=Z$ nuclei. The $\Delta_{pn}^{T=0}$ can be regarded as the $T=0$ pn pairing gap as well. Figure 2 with these estimations indicates that the $T=0$ pn correlations are superior to the $T=1$ pn correlations in the ground states of sd shell nuclei, and the inversion occurs in the pf shell nuclei. The $T=0$ pn pairing gap $\Delta_{pn}^{T=0}$ cannot be explained by the $T=1$ pairing Hamiltonian (2).

In a previous paper [2], it has been shown that the $T=0$ matrix elements of the monopole field $V_m^T(a,b)$ are significantly larger than the $T=1$ ones, and are very important in determining the double differences of binding energies, where a,b are the single-particle orbitals. We can see that the matrix elements are quite large for isoscalar components but small for isovector components. In the USD interaction, the monopole matrix elements with $T=0$ have values around −3 MeV and are strongly attractive. If we assume that the $T=0$ monopole matrix elements are equal and independent of angular momentum J and the single-particle orbitals, $V_m^{T=0}$ is reduced to the J-independent isoscalar pn pairing interaction. Neglecting $T=1$ monopole components, let us add the J-independent $T=0$ pn pairing interaction [2,17] to the pairing Hamiltonian (2):

$$H = H_0 + H_P + H_{\pi\nu}^{\tau=0} = H_0 + H_P - k^0 \sum_{a\geqslant b} \sum_{J,M} A_{JM,00}^\dagger(ab) A_{JM,00}(ab), \qquad (4)$$

where $A_{JM,00}^\dagger(ab)$ is the pair operator with spin J and isospin

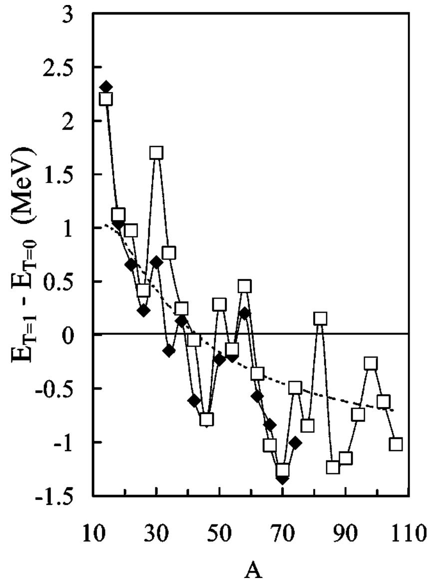

FIG. 3. The energy difference between the $T=0$ and $T=1$ states in odd-odd $N=Z$ nuclei. The experimental values of the differences are denoted by solid diamonds. The open squares present the values estimated from the experimental odd-even mass differences in Fig. 1 and the $T=0$ pairing force strength k^0. The dashed line is $k^0 - 10.4A^{-1/3}$.

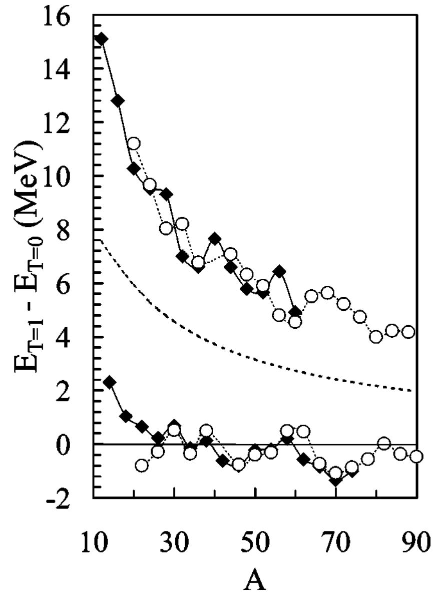

FIG. 4. The calculated energy differences between the lowest $T=0$ and $T=1$ states in even-even (upper plots) and odd-odd (lower plots) $N=Z$ nuclei. The solid diamonds are the same as Fig. 3. The open circles denote the energy differences obtained by the BCS calculations. The dashed curve is $2a(A)/A$.

$T=0$. The $T=1$ pairing interaction does not contribute to the double difference of binding energies $\Delta_{pn}^{T=0}$, and $\Delta_{pn}^{T=0} \approx k^0/2$. Then, the $T=0$ pairing force strength $k^0 = 244.5(1 - 1.67A^{-1/3})/A$ is chosen so as to fit the $T=0$ pn pairing gap as seen in Fig. 2. The isovector monopole components in USD are small, except for $V_m^{T=1}(s_{1/2}, s_{1/2})$. The deviations from the curve $k^0/2$ for ^{30}P and ^{34}Cl in Fig. 2 would be attributed to the large value of isovector component $V_m^{T=1}(s_{1/2}, s_{1/2})$. We recently introduced [17] monopole corrections to improve the energy levels of ^{48}Ca, etc. In this paper, we ignore these correction terms.

If we assume degenerate single-particle energies $\varepsilon_a = 0.0$, the above Hamiltonian has SO(5) symmetry [18] and the eigenenergy is assigned by the valence nucleon number n, seniority ν, $p = (n - \nu)/2$, and isospin T [2],

$$\langle H_{P_0} + H_{\pi\nu}^{\tau=0} \rangle_{\mathrm{SO}(5)} = -\frac{1}{2}Gp(2\Omega + 3 - \nu - p) - \frac{1}{2}k^0 \frac{n}{2}\left(\frac{n}{2} + 1\right) + \frac{1}{2}(G + k^0)T(T+1), \quad (5)$$

where $\Omega = \Sigma_\alpha$ is the degeneracy of shell orbits. Note that the above equation includes the so-called symmetry energy term with coefficient $a(A)/A = (G + k^0)/2$. The parameters G and k^0 used above give just the empirical symmetry energy formula $a(A) = 134.4(1 - 1.52A^{-1/3})$ determined by Duflo and Zuker [13].

We next consider energy difference between the lowest $T=0$ and $T=1$ states in odd-odd $N=Z$ nuclei. Odd-odd $N=Z$ nuclei with $A<40$ have the ground states with $T=0, J>0$ except for ^{34}Cl, while the ground states of odd-odd $N=Z$ nuclei with $40<A<74$ are $T=1$ and $J=0$ except for ^{58}Cu. Several authors discussed that this degeneracy is attributed to the delicate balance between the symmetry energy and pairing correlations, and that the energy difference between $T=1$ and $T=0$ states is well reproduced by $E_{T=1} - E_{T=0} = 2a(A)/A - 2\Delta_{T=1}$ using the value ~ 75 for $a(A)$ and the pairing gap $\Delta_{T=1} = 12A^{-1/2}$. However, if we substitute the odd-even mass difference $\Delta_n^{(3)}(Z=\mathrm{even}, Z+1)$ for $\Delta_{T=1}$, the energy difference $E_{T=1} - E_{T=0}$ becomes larger than the experimental value. The energy difference can be regarded as a measure of competition between the $T=0$ and $T=1$ pairing correlations as seen from the following identity,

$$E_{T=1} - E_{T=0} = 2(\Delta_{pn}^{T=0} - \Delta_{pn}^{T=1}). \quad (6)$$

The relationships $\Delta_{pn}^{T=0} \approx k^0/2$ and $\Delta_{pn}^{T=1} \approx \Delta_n^{(3)}$ offer an alternative relation $E_{T=1} - E_{T=0} \approx k^0 - 2\Delta_n^{(3)}$ for the energy difference except for ^{30}P and ^{34}Cl. If we adopt the parameter $k^0 = 244.5(1 - 1.67A^{-1/3})/A$ and the average value of pairing gap $5.18A^{-1/3}$ for $\Delta_n^{(3)}$, we get the dashed curve in Fig. 3, which displays well the trend of the experimental values of energy difference $E_{T=1} - E_{T=0}$. Adopting the experimental odd-even mass differences for $\Delta_n^{(3)}$ and $k^0 = 244.5(1 - 1.67A^{-1/3})/A$, we obtain the energy difference $E_{T=1} - E_{T=0}$ denoted by the open squares. These values nicely reproduce the experimental values except for ^{30}P and ^{34}Cl as shown in Fig. 3. The disagreements in ^{30}P and ^{34}Cl are attributed to the large deviations of $T=0$ pairing gap from the curve $k^0/2$ due to the neglect of the shell effects in Fig. 2.

Moreover, we calculated the $T=0$ and $T=1$ energy differences for odd-odd $N=Z$ nuclei with $A \geqslant 78$, although there are no experimental data of the energy difference. The calculation predicts that ^{82}Nb has possibly the ground state with

$T=0$, while the other odd-odd $N=Z$ nuclei have the $T=1$ ground state. We call this isospin inversion hereafter. It is well known that a similar isospin inversion occurs at ^{58}Cu. The isospin inversion is due to characteristic situation, where the Fermi energy lies between large spin and small spin orbits with large energy gap, i.e., $1f_{7/2}$ and $2p_{3/2}$ for ^{58}Cu, and $1g_{9/2}$ and $2p_{1/2}$ for ^{82}Nb. In these cases, the $T=1$ pairing gap is quite small as seen in Fig. 1, and energy difference becomes large from the simple relation $E_{T=1}-E_{T=0}\approx k^0-2\Delta_n^{(3)}(Z=\text{even},Z+1)$.

Figure 4 shows the calculated energy differences $E_{T=1}-E_{T=0}$ in odd-odd and even-even $N=Z$ nuclei. The energy differences in the BCS approximations are calculated by $2a(A)/A+\Delta_{BCS}$ for even-even $N=Z$ nuclei and by $k^0-2\Delta_{BCS}$ for odd-odd $N=Z$ nuclei where $a(A)$ is the empirical symmetry energy coefficient and Δ_{BCS} is the BCS pairing gap. The BCS calculations well reproduce the experimental values of energy differences, except for odd-odd $N=Z$ nuclei with $A<40$. The BCS calculations show that the $T=0$ and $T=1$ states in ^{82}Nb are almost degenerate, while the ground states of adjacent odd-odd $N=Z$ nuclei have isospin $T=1$.

In conclusion, we investigated the $T=0$ and $T=1$ pairing correlations in $N=Z$ nuclei. The $T=1$ pairing correlations in $N=Z$ nuclei are extracted from the odd-even mass differences of the neighboring odd-mass nuclei, which can be fitted by the curve $5.18A^{-1/3}$. The *pn* pairing correlations are estimated from the double difference of binding energies. The $T=1$ *pn* pairing gap is the same as the *nn* pairing gap. The indicator $\Delta_{pn}^{T=0}$ presents the magnitude of $T=0$ *pn* pairing correlations. The energy differences between the $T=0$ and $T=1$ states are well described by the $T=1$ and $T=0$ pairing model. In odd-odd $N=Z$ nuclei, the $T=1$ pairing correlations compete with the $T=0$ pairing correlations, and the degeneracy of the $T=0$ and $T=1$ states occurs. The empirical values and BCS results reproduced the energy difference. In particular, our results predict that odd-odd $N=Z$ nucleus ^{82}Nb has the $T=0$ ground state or the $T=0$ and $T=1$ states are almost degenerate. The odd-even mass differences for even-even $N=Z$ nuclei are extremely larger than those of the neighboring even-even $N\neq Z$ nuclei. It would be affected by strong *pn* correlations. Further studies in this direction are in progress.

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