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Movement of Efimov states in ^{20}C causing resonance in n-^{19}C scattering near scattering threshold

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Movement of Efimov states in ^{20}C as a result of increasing the n-^{18}C binding energy from about 100 to 500 keV is studied in a three body model for ^{20}C, assumed to be $n+n+^{18}$C system, employing separable potentials for the n-n and n-^{18}C binary subsystems. The computational analysis shows that for small values of energies ($<$ about 120 keV) which are just necessary to bind the two body (n-^{18}C) system, there could be more than one bound Efimov states. But the states are found to go on disappearing one by one as the binding energy is increased beyond 200 keV. As originally pointed out by Amado and Noble [Phys. Rev. D **5**, 1992 (1972)], the Efimov states move into the unphysical sheet associated with the two body unitarity cut on increasing the strength of the binary interaction to bind the two body system. By undertaking a detailed study of the scattering of neutron on a bound (n-^{18}C) system, we find that these states move over to the physical scattering region causing a resonance in n-^{19}C scattering around neutron incident energy of 1.6 keV, having a width of about 0.25 keV.

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With the realization that 2n-rich halo nuclei, characterized by their weak binding and large spatial extension, are the most suitable objects to search for Efimov states, several theoretical investigations [1–6] on this topic have been vigorously pursued in recent years. More than three decades ago, Efimov showed [7] that the number of bound states for three particles interacting through short range potentials may grow to infinity as the strength of a pair interaction approaches to just bind two particles and then decreases to a small finite number for stronger binding. Although the existence of Efimov states have been predicted in numerical model calculations, but so far, to the best of our knowledge, not a single state in a physical system has been identified experimentally as an Efimov state.

Amado and Noble studying the analytic properties of the Fredholm determinant in a three boson model [8] showed that with the increase of potential strength, the Efimov states move into the unphysical sheet associated with the two body unitarity cut. A detailed analysis was followed by Adhikari *et al.* [9] to study the movement of the Efimov states in the three boson model and in the s-wave spin doublet ($^2S_{1/2}$) three nucleon system. The key question addressed there was whether the Efimov states, with the increase in potential strength, move over to virtual states in the unphysical sheet or two of the Efimov states collide to produce a resonance pair, one of which may come close to the scattering region and produce an observable effect on the physical scattering process.

In the context of halo nuclei, the nucleus ^{20}C, considered as a three body system consisting of $n+n+^{18}$C(core), appears to be one of the most physical examples. Here the binary (n-^{18}C) system is known to be bound, and according to the earlier experimental data [10] has binding energy 160±100 keV. Sometimes back, we had set up a three body model employing separable potentials for the binary subsystems to study the Efimov effect in 2n halo nuclei, such as ^{14}Be [11] and ^{19}B, ^{22}C, and ^{20}C [12]. The results clearly showed the occurrence of Efimov state in ^{20}C near the three body threshold. Thus, for the n-^{18}C binding energy around 140 keV, an Efimov state at about 152 keV was predicted along with the ground state energy of ^{20}C to be 3.18 MeV—found to be in reasonable agreement with the experimental data. This result was also found to be in more or less quantitative agreement with the independent investigation carried out by Amorim *et al.* [4]. We also noticed that if the n-^{18}C binding energy is lowered to about 100 keV or even less than that, there could be, in conformity with the conclusions drawn by Efimov, more than one bound Efimov states.

Recently, Coulomb dissociation of ^{19}C into $n+^{18}$C has been studied at 67A [13], where the angular distribution of $n+^{18}$C in the c.m. led to a determination of neutron separation energy in ^{19}C to be 530±130 keV. This experimental result thus opened for us the doors to revisit our earlier analysis investigating the behavior of the movement of the Efimov states as a result of increasing the n-^{18}C binding energy from 200 keV to about 500 keV. To this end, we set up the three body scattering equations to study the scattering of neutrons by ^{19}C—a bound state of the n-^{18}C system—using two body separable potentials. By computing the integral equations for amplitude of n-^{19}C scattering near the elastic scattering but below the three body break-up threshold we investigate the behavior of elastic scattering amplitude as a function of incident neutron energy. The analysis clearly points out that with the increase of n-^{18}C binding energy, the Efimov states in ^{20}C have moved over to the physical scattering region, causing thereby a resonance near the scattering threshold at about 1.6 keV.

The object of this short communication is to report the results of the numerical analysis on depicting the behavior of Efimov states, which in a three body bound system (^{20}C) appear as excited states, and then in the scattering sector (n-^{19}C elastic scattering) move to the unphysical sheet causing an observable effect in the form of a resonance. Recapitulating some of the basic steps, developed in Ref. [14] we write the wave function for the three body bound state in momentum space employing the separable potentials for the binary interactions, viz., n-n and n-c in the s state, as

$$\psi(\vec{p}_{12},\vec{p}_3;E) = D^{-1}(\vec{p}_{12},\vec{p}_3;E)[g(p_{12})F(\vec{p}_3) + f(p_{23})G(\vec{p}_1) + f(p_{31})G(\vec{p}_2)], \quad (1)$$

where the three body energy term $D(\vec{p}_{12},\vec{p}_3;E)=p_{12}^2/2_{\mu_{12}}+p_3^2/2_{\mu_{12-3}}-E$, in which the first two terms represent the kinetic energy and the third the total energy E in the three body c.m. system. Here p_{12} is the relative momentum of particles 1 and 2 labeling the two neutrons and μ_{12} is the reduced mass, $\vec{p}_3$ is the relative momentum of the third particle (i.e., the core) with respect to the c.m. of the two, and μ_{12-3} is the corresponding reduced mass. The two body structure functions $g(p_{ij})$ and $f(p_{ij})$ refer to the s-state separable interactions for n-n and n-c pairs, respectively. The details of these potentials along with the strength and range parameters are given in Ref. [14]. The spectator functions $F(p)$ and $G(p)$ appearing in Eq. (1) describe, respectively, the dynamics of the core (^{18}C) and of the halo neutrons and satisfy the homogeneous coupled integral equations (see Ref. [14])

$$[\Lambda_n^{-1} - h_n(p)]F(\vec{p}) = 2\int d\vec{q}K_1(\vec{p},\vec{q})G(\vec{q}), \quad (2)$$

$$[\Lambda_c^{-1} - h_c(p)]G(\vec{p}) = \int d\vec{q}K_2(\vec{p},\vec{q})F(\vec{q}) + \int d\vec{q}K_3(\vec{p},\vec{q})G(\vec{q}), \quad (3)$$

where the detailed expressions for the kernels K_1, K_2, and K_3 along with the symbols are given in Ref. [14]. These equations are then numerically computed as an eigenvalue problem following the procedure described in detail in Refs. [11,12] to determine the three body ground-state energy as well as the energy of the Efimov states. Table I depicts the three body energy for ^{20}C ground state and excited Efimov states for different two body (n-^{18}C) binding energies. As pointed out earlier, when the n-c pair interaction just binds the two body (n-^{18}C) system having binding energy around 60–100 keV, the three body system shows more than one Efimov state but for binding energy equal to or greater than 140 keV there appears only one Efimov state; the second one moving over to the unphysical region. When the two body binding increases further, say, beyond 220 keV, even the first Efimov state disappears and moves over to the second unphysical sheet. The main point of observation from Table I is that the difference between the three body energy of the Efimov state and the two body binding energy becomes narrower and narrower and eventually becomes negative with the increase in two body strength parameter.

TABLE I. ^{20}C ground and excited states three body energy for different two body input parameters. It can be noted that the three body energy becomes less than the two body energy beyond ϵ_2 =225 and 110 keV as discussed in the text. Hence the first and second Efimov states are not shown in the table beyond ϵ_2=240 and ϵ_2=240 and 140 keV, respectively. However, the results of the calculation are shown in Fig. 1.

n-^{18}C energy ϵ_2 keV	$\epsilon_3(0)$ MeV	$\epsilon_3(1)$ keV	$\epsilon_2(2)$ keV
60	3.00	79.5	66.95
100	3.10	116.6	101.4
140	3.18	152.0	137.5
180	3.25	186.6	
220	3.32	221.0	
240	3.35	238.1	
250	3.37		
300	3.44		
350	3.51		
400	3.57		
450	3.63		
500	3.69		
550	3.74		

To get a further insight we plot in Fig. 1 the difference of three body and two body binding energy vs the two body energy for the first and second Efimov states thereby con-

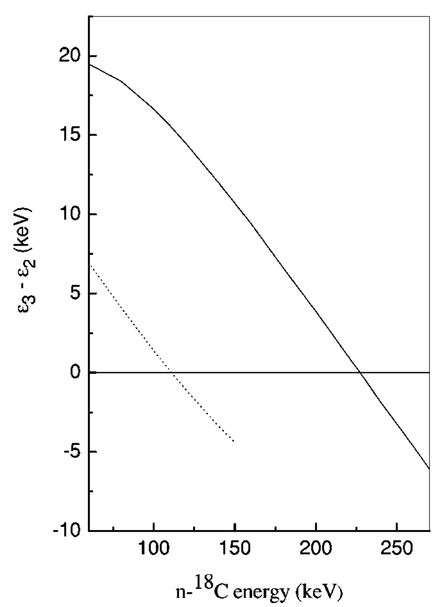

FIG. 1. Plot of the difference of three body and two body binding energy vs the two body energy for the first (solid line) and second (dashed line) Efimov states.

tinuously tracing the movement of these states as a function of the two body binding energy. We note that while the second Efimov state remains bound till the two body binding energy ϵ_2 is 110 keV, it moves over to the continuum [i.e., $\epsilon_3(2)<\epsilon_2$] for energy ϵ_2 greater than that, crossing over the horizontal line representing $\epsilon_3=\epsilon_2$. Similarly the first Efimov state is bound up to $\epsilon_2=225$ keV and then crosses over the horizontal line at about 230 keV thereby moving over to a continuum state. These "superfluous" solutions of the three body homogeneous integral equations, in fact, provided us a clue to look for the scattering sector to investigate the effect of these states in the continuum.

The feature noted above can, in fact, be attributed to the presence of the singularity in the two body propagator $[\Lambda_c^{-1}-h_c(p)]^{-1}$ which can be explicitly written as

$$[\Lambda_c^{-1}-h_c(p)]^{-1}=\left[\left(p^2+2d\alpha_3^2-\frac{d}{a}\alpha_2^2\right)h(p^2,\alpha_2^2;\alpha_3^2)\right]^{-1}, \tag{4}$$

where α_3^2 and α_2^2 are the parameters related respectively to the three body and two body binding energies, viz. $E=-\epsilon_3=-\alpha_3^2/2\mu_r$, $\epsilon_2=\alpha_2^2/2\mu_{23}$ and $d=(m+m_c)/(2m+m_c)$ and $a=m_c/(m+m_c)$. The function $h(p^2,\alpha_2^2;\alpha_3^2)$ represents the form of integral which can be easily worked out. The main point to note here is that the factor $(p^2+2d\alpha_3^2-\frac{d}{a}\alpha_2^2)^{-1}$ is essentially a two body propagator. As long as $\alpha_3^2>\alpha_2^2/2a$ i.e., the three body binding energy is numerically greater than that of the two body, this factor monotonically decreases as p increases. On the other hand, if α_3^2 approaches $\alpha_2^2/2a$ we then face a singularity and for $\alpha_3^2<\alpha_2^2/2a$ the singularity crosses over to the unphysical sheet. How does such a behavior affect in the scattering sector?

We want to analyze the effect of this singularity on the behavior of the scattering amplitude for n-^{19}C elastic scattering. To study the scattering process the function $G(p)$ in Eq. (3), describing the dynamics of the neutron in the presence of (n-^{18}C) system, must be subject to the boundary condition, viz.

$$G(\vec{p})=(2\pi)^3\delta(\vec{p}-\vec{k})+\frac{4\pi a_k(\vec{p})}{p^2-k^2-\iota\epsilon}, \tag{5}$$

where the first term represents the plane wave part and the second term is the outgoing spherical wave multiplied by an off-shell scattering amplitude in momentum space for the scattering of neutron by ^{19}C. The scattering amplitude is normalized such that, for the s-wave scattering,

$$a_k(\vec{p})_{|\vec{p}|=|\vec{k}|}\equiv f_k=\frac{e^{\iota\delta}\sin\delta}{k}. \tag{6}$$

Before applying the boundary condition, we rewrite Eq. (3) substituting Eq. (2) for $F(p)$ and finally get the equation for the off-shell scattering amplitude as

$$4\pi\left(\frac{a}{d}\right)h(p^2,k^2;\alpha_2^2)a_k(\vec{p})=(2\pi)^3K_3(\vec{p},\vec{k})+4\pi\int\frac{d\vec{q}K_3(\vec{p},\vec{q})a_k(\vec{q})}{q^2-k^2-\iota\epsilon}+2(2\pi)^3\int d\vec{q}K_2(\vec{p},\vec{q})K_1(\vec{q},\vec{k})\tau_n(q)$$
$$+2(4\pi)\int d\vec{q}K_2(\vec{p},\vec{q})\tau_n(q)\int d\vec{q}'\frac{K_1(\vec{q},\vec{q}')a_k(\vec{q}')}{q'^2-k^2-\iota\epsilon}. \tag{7}$$

This integral equation needs to be solved numerically for the scattering amplitude. For s-wave scattering and in the limit when $k\to 0$, the singularity in the two body cut does not cause any problem; in fact the amplitude has only the real part. By employing Gauss quadrature and using a mesh size of 80×80 matrix, the off-shell amplitude $a_{k=0}(p)$ is computed by inverting the resultant matrix, which, in the limit, $a_0(p)_{p\to 0}\to -a$, gives the value of n-^{19}C scattering length. This value, although has a positive sign, turns out to be quite large. Nevertheless, the positive signature of the scattering length not only rules out the possibility of a virtual state in n-^{19}C system but also supports a bound state, which is quite consistent with the experimental finding.

To investigate the effect of two body binding energy on the three body scattering length, we study the zero energy n-^{19}C scattering for different values of the binding energy of the n-^{18}C system. Thus, for instance, choosing the binding energy $\epsilon_2=100$, 220, and 250 keV, we wish to scan the region where in the first two cases the second Efimov state and the first Efimov state are, respectively, just below the three body threshold whereas for the third binding energy both the Efimov states are in the continuum. We find that in all these cases the zero energy scattering length parameter has the value, though is quite sensitive to the n-^{18}C binding energy, yet it retains a positive sign all through. It thus rules out the possibility of the Efimov states turning over to the virtual states in the three body system. Here it is important to mention that the investigations [15] carried out so far in studying the behavior of Efimov states in a three body system for equal mass particles, both in nuclear and atomic systems, have all shown that the bound Efimov state turns into a virtual state rather than a resonance. As we shall see below the present study turns out to be the first of its kind to show that for the unequal mass particles, the Efimov states in the three body system move over to produce a resonance near the scattering threshold for the scattering of particle by a bound pair.

At incident energies different from zero, i.e., $k\neq 0$, the

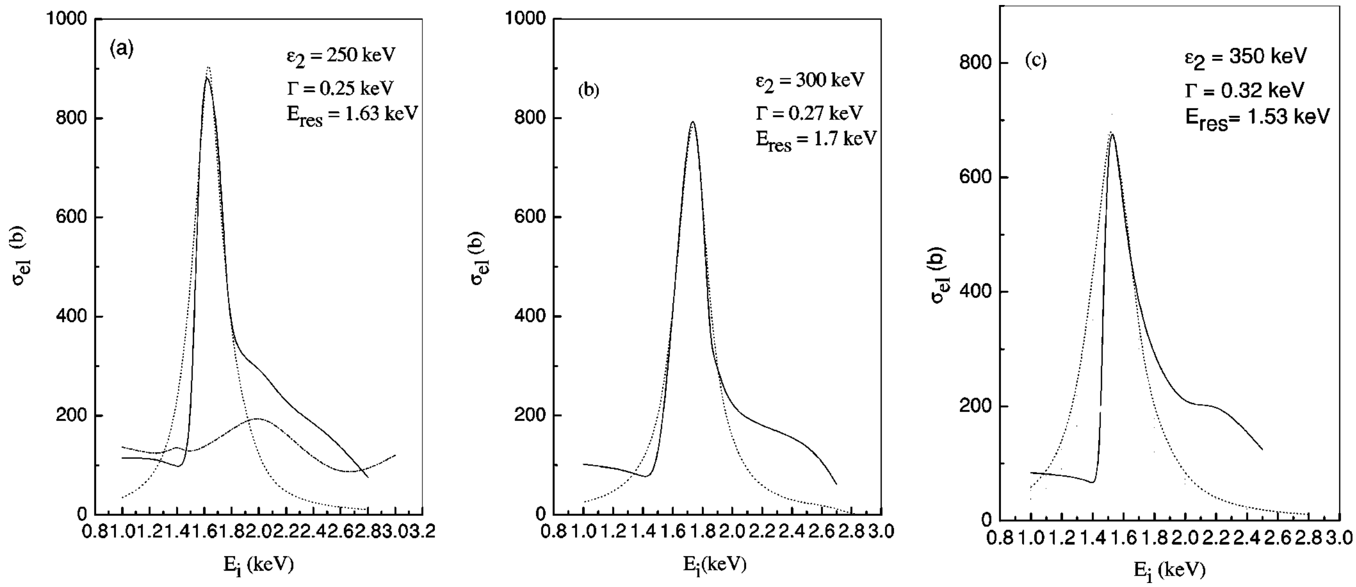

FIG. 2. Plot of elastic cross section of n-^{19}C scattering vs c.m. energy of neutron for n-^{18}C binding energies (ϵ_2) of (a) 250 keV, (b) 300 keV, and (c) 350 keV, respectively. The full curves represent the behavior of the cross section as obtained by computation whereas the dotted curves represent the Breit-Wigner fits as described in the text. The dashed curve in (a) shows the behavior of the cross section for n-^{18}C binding energy of 200 keV.

singularity in the two body propagator is tackled by following a very elegant technique of continuing the kernel onto a second sheet originally proposed by Balslev and Combes [16]. According to this, the integral contour is deformed from its original position along the real positive axis to the position rotated by a fixed angle. In other words, we let the variables p,q, etc., become complex by the transformation, $p \rightarrow p_1 e^{-\iota\phi}$ and $q \rightarrow q_1 e^{-\iota\phi}$. Here the angle ϕ is merely a parameter to be so chosen that the new contour lies as far away as possible from the singularities. By this operation, the kernel of the integral equation is analytically continued so as to become compact. It must, however, be added that the choice on the values of the angle ϕ is not completely free. These values are restricted by the requirement that the imaginary part of the scattering amplitude calculated from the integral equation must be unitary, i.e., it must satisfy the condition $\mathrm{Im}(f_k^{-1}) = -k$.

In Figs. 2(a), 2(b), and 2(c) we depict the behavior of elastic scattering cross section σ_{el} vs incident energy of the neutron on ^{19}C for three different binding energies, i.e., 250, 300, and 350 keV of the n-^{18}C system. We find that as the incident energy of the neutron is increased from 1.0 keV (c.m. energy) to about 1.4 keV the cross section stays more or less constant at about 100 b. Then suddenly it shows a sharp rise around 1.6–1.7 keV and falls to about 200 b around 1.8–1.9 keV. The full lines in the figures represent the behavior of the cross section as obtained by computing the integral equation whereas the dotted curve shows for comparison, the fit of the Breit-Wigner resonance shape by using the calculated value of the resonance energy and the width of the resonance. For binding energy of 250 keV of the n-^{18}C system, the resonance position is obtained at 1.63 keV while the full width Γ has the value 0.25 keV. However, for higher energies of, say, 300 and 350 keV, the position of the resonance shifts to 1.7 and 1.53 keV, respectively, while the resonance width has the corresponding values of 0.27 and 0.32 keV, respectively. Another feature which can be noticed from Fig. 2(c) is that for higher binding energy of the n-^{18}C system the behavior of the computed cross section deviates considerably as compared to the Breit-Wigner shape. Nevertheless, there is a definite prediction of the occurrence of a resonance in n-^{19}C scattering near threshold at 1.5 to 1.7 keV.

To ensure that the appearance of the peak in the scattering cross section is an unambiguous signature of a resonance in n-^{19}C scattering, the present analysis has been further sub-

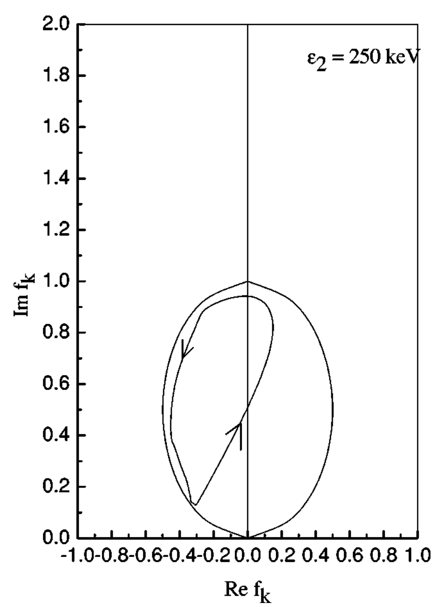

FIG. 3. Argand plot of $\mathrm{Im}(f_k)$ vs $\mathrm{Re}(f_k)$ within a unit circle in the n-^{19}C scattering when the binding energy of n-^{18}C system is taken as 250 keV.

jected to the following two criteria: first, by using the relation of the resonance energy $E_{\mathrm{res}}=(\hbar^2 k_{\mathrm{res}}^2)/(2\mu)$, where $k_{\mathrm{res}}^2 = k_R^2 - k_I^2$ and k_R and k_I are the real and imaginary parts in the complex k plane, viz. $k=k_R - \imath k_I$ and the width $\Gamma=(k_R k_I)/\mu$, we compute by using the values of resonance energy and the width, the corresponding values of k_R and k_I, which are found to be 3.75×10^{-2} fm^{-1} and 1.53×10^{-3} fm^{-1}, respectively. This ensures that not only does the resonance lie in the fourth quadrant in the k plane but also $k_R \gg k_I$. The second criterion is to study the behavior of Re(f_k) and Im(f_k) on the Argand diagram as a function of energy around the resonance position. In Fig. 3, we depict a plot of Im(f_k) vs Re(f_k) within a unit circle as we increase the incident energy of the neutron from 1.0 to 2.75 keV in steps of 0.25 keV, where the binding energy of the n-^{18}C system is taken to be 250 keV. We find that as the energy is increased the point described by (Re f_k, Im f_k) indicates an anticlockwise motion in the Argand plot, thereby satisfying an important condition to be obeyed for the occurrence of a resonance.

It may be interesting to ask whether the scattering cross section would still predict the resonance type structure if the pair energy of the bound system is decreased, to say, 200 keV or even less than that in which case the bound Efimov state in the three body system is found to occur. To answer this, we depict by a dashed curve the plot of scattering cross section for n-^{19}C scattering in Fig. 2(a) taking the (n-^{18}C) binding energy to be 200 keV. We find that the scattering cross section does not produce the resonancelike peak structure and remain more or less constant with the incident energy of the neutron.

To conclude, the present analysis clearly shows that on increasing the strength of the binary interaction to bind the two body system, the Efimov states move into the unphysical sheet associated with the two body unitarity cut. By studying the scattering of neutrons on ^{19}C(n+^{18}C) near the scattering threshold we here predict that for binding energies of (n-^{18}C) system greater than or equal to 250 keV, the disappearance of these Efimov states in ^{20}C gives rise to a resonance at the neutron c.m. energy around 1.6 keV with a full width of about 0.25 keV. This is, to the best of our knowledge, the first analysis of its kind where the effect of the occurrence of Efimov states in a three body bound system has been shown to appear in the form of a resonance in the scattering sector. It would therefore be interesting if the future experiments could be directed to confirm the occurrence of a resonance in the n-^{19}C system near the scattering threshold. This would indeed go a long way in our understanding to reveal a direct impact of the existence of Efimov states in bound systems on the scattering sector.

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