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Unusual features of β transition rates in heavy deformed nuclei

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A detailed examination of all the available log ft values for β transitions in heavy ($A > 228$) nuclei reveals a very similar distribution of these values for the allowed and the first-forbidden decays in this region, in sharp contrast with the observed trends over the whole periodic table. Data for over 500 transitions belonging to both the allowed and forbidden categories are presented in the form of histograms. Physical basis for this unusual behavior is sought by examining the interrelationships between the Nilsson model asymptotic quantum numbers of the β connected states. Explicit selection rules specific to this region, in terms of these quantum numbers, are proposed and their applicability is demonstrated by citing illustrative examples from the allowed decays of odd-A actinides.

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Following the spin-parity I^π selection rules for β decay, the transitions with $\Delta I=0$ or 1 and $\Delta\pi=$no are classified as allowed, and those with $\Delta I=0$ or 1 and $\Delta\pi=$yes as nonunique first forbidden ("$1f$") transitions. For well-deformed nuclei, whose wave functions are described in terms of the Nilsson model asymptotic quantum numbers $\Omega^\pi[Nn_3\Lambda\Sigma]$, additional selection rules involving $[Nn_3\Lambda]$ were enunciated by Alaga *et al.* [1,2] for odd-mass nuclei wherein $\Omega^\pi = I^\pi$. Application of these selection rules to β transitions in nuclei of the rare-earth region was examined by Mottelson and Nilsson [3]; they termed the allowed transitions, which obeyed Alaga selection rule

$$\Delta N = 0, \Delta n_3 = 0, \Delta\Lambda = 0, \qquad (1)$$

as allowed-unhindered "au." On examination of available data, they concluded that the au transitions have log ft values between 4.5 and 5.0. Noting that the β process is basically a single-particle transformation, Gallagher [4] extended the applicability of Alaga rules to even-mass deformed nuclei under the assumption that the nontransforming nucleon acts as a spectator. For deformed nuclei, the intrinsic state is defined by the band numbers K^π. Selection rules for allowed decays require $\Delta K \leqslant \Delta I$. For the cases wherein $\Delta K > \Delta I$, Alaga *et al.* [1] defined a K-forbiddenness number $\nu = \Delta K - \Delta I$ to explain larger log ft values for large ν. It is customary to correlate the various categories (allowed/forbidden) and their subcategories (superallowed, au, isospin forbidden, $0^+ \leftrightarrow 1^+$, etc.) with the observed range of log ft values in each case [3,5–7]. In turn, these correlations are frequently used to assign spin parities, and Nilsson orbitals in the case of deformed nuclei, for β-connected nuclear levels [5–9]. For instance, the Nuclear Data Sheets (NDS) evaluators [6] use, among others, the following "strong" rules as bases for spin-parity assignments.

(i) If log $ft < 5.9$, the transition is allowed: $\Delta I=0$ or 1, $\Delta\pi=$no. Only at, or very near to, closed shells (specifically around $Z=82$), the upper limit of 5.9 could be 5.1.

(ii) For deformed nuclei, if log $ft < 5.0$, transition is au; observation of au transition is definitive evidence for the presence of the particular pair of Nilsson orbitals.

A recent global review of log ft values by Singh *et al.* [9] includes a listing of all the 1997 ENSDF based log ft values (3900 cases out of about 20 000 known β transitions); this review also includes categorywise analysis of the listed values and points out several systematic features.

A detailed examination of allowed β transitions in deformed nuclei of the rare-earth region ($152 \leqslant A \leqslant 190$) by Sood and Sheline [5] identified 122 au transitions in this region. However, their similar survey of heavy ($A \geqslant 228$) nuclei [10] did not find unambiguous evidence for any au transitions therein; detailed analysis of all known β branches suggested just two possible au candidates. This study [10] also proposed extended selection rules for $[Nn_3\Lambda]$ to describe fast (log $ft \leqslant 6.0$) β transitions in heavy nuclei. Our recent study [11] covering very heavy [$(A=250\pm5)$ nuclei revealed that the dominant mode of β decay in this "frontier" region is through parity-changing $1f$ transitions; we had also compared the operative $[Nn_3\Lambda]$ selection rules for the fast allowed and $1f$ transitions. These studies [10,11], taken together with the global review [9], indicated certain features unique to heavy mass region and pointed to the need for a side-by-side exhaustive study of allowed and $1f$ β transitions in heavy nuclei. A few significant results of such a study are being reported herein.

Another motivation of this study is an early remark of Mottelson and Nilsson [3] that "a selection rule associated with N should be somewhat stronger than the rules connected with other asymptotic quantum numbers." This question has remained unexplored sofar. The heavy mass region, with several neutron-proton orbitals around Fermi surface having N differing by 2 units, provides an excellent, and possibly the only, field for examination of this question. A preliminary report on N-forbiddenness [12] has been recently presented by us.

Our database is taken from the 1999 update of the table of isotopes [13] supplemented by more recent Nuclear Data

TABLE I. Central log *ft* values (with standard deviation indicated in parentheses) for all the known allowed ($\Delta I=0$ or 1; $\Delta\pi$=no) and the first-forbidden ($\Delta I=0$ or 1; $\Delta\pi$=yes) β transitions in $A>228$ nuclei are listed categorywise in comparison with the corresponding global values from Singh *et al.* [9].

	$\Delta\pi$=no			$\Delta\pi$=yes		
Category	$A>228$		Global	$A>228$		Global
	Number	log *ft*		Number	log *ft*	
Odd mass	89	7.2(1.1)	6.0(1.0)	123	7.0(1.0)	7.2(1.0)
Even mass	163	7.6(1.2)	6.2(1.2)	141	7.9(1.2)	7.6(1.2)
All *A*	252	7.5(1.2)	6.1(1.1)	264	7.5(1.2)	7.4(1.1)
$0^+\leftrightarrow 1^+$	27	6.9(0.8)	5.3(2.7)			

Sheets for $A\geqslant 248$ nuclei [14,15]. For configurations of individual levels, we have used the relevant reviews and other databases [13,16,17]. Our compilation includes 252 cases for allowed and 264 cases for 1*f* transitions in $A>228$ nuclei. In contrast, the recent global review [9], based on the NDS adopted sets, included only 36 allowed and 32 1*f* transitions for this region.

For a side-by-side comparison of the allowed and the 1*f* transitions of this region, we undertook the categorywise analysis of the log *ft* data following the procedure adopted in the recent global review [9] and likewise prepared histograms for each category and subcategory. As an illustration, we list in Table I the central log *ft* values, along with the standard deviation in each case, for all the data for $A>228$ nuclei, in comparison with the corresponding results from the global review [9], for the allowed and the 1*f* transitions. The results for the odd-*A* and the even-*A* decays, and for the $0^+\leftrightarrow 1^+$ Gamow-Teller (GT) transitions, are also listed. Side-by-side histograms for our data set for the allowed and 1*f* transitions in all the $A>228$ nuclei are presented in Figs. 1(a) and 1(b). Some of the significant conclusions from this comparative study are described below.

(a) A surprising, and quite unexpected, result from our analysis is the identical central log *ft* value=7.5 obtained for the allowed as well as the first-forbidden transitions of this region, with the same width of 1.2 in each case. This feature is clearly demonstrated in the histograms of Fig. 1. An examination of the much smaller database (36 allowed and 32 1*f*) of "evaluated" log *ft* values for the $A>228$ nuclei listed in the global review [9] also reveals identical central log *ft*, with similar width, for each of these categories. One is thus led to the conclusion that it is just not possible to distinguish between the allowed and the 1*f* β transitions in heavy nuclei on the basis of log *ft* values. There is no exclusive domain of log *ft* values for either category.

(b) The NDS [6] strong rule mentioned earlier specifies that, excepting at or very near to the $Z=82$ closed shell, the transition is allowed if log $ft<5.9$. We find that this rule certainly does not hold for β transitions in heavy nuclei. In fact, as seen in our Fig. 1, the number of cases with log $ft \leqslant 5.9$ for allowed transitions is 19, which exactly matches the corresponding number 19 of 1*f* transitions with log $ft \leqslant 5.9$.(c) Whereas the central log *ft* values for 1*f* transitions of heavy region match the corresponding global values, as seen in Table I, those for the allowed transitions are almost 1.4 units higher in comparison with the global results for this category.(d) As seen in the bottom row of Table I, the central log ft=6.9 for the $0^+\leftrightarrow 1^+$ GT transitions in heavy nuclei is much higher than the corresponding global value of 5.3.

The questions to be addressed now are: "What are the factors that make these allowed transitions to have comparative lifetimes (*ft*) very similar to those for the forbidden transitions and what are the operative selection rules to describe such occurrences?" In the following, we attempt to briefly answer these questions. For this purpose, we presently consider as illustrations only the odd mass nuclei, since these nuclei admit of relatively unmixed Nilsson single-particle orbitals as wave functions of the β connected states.

First we look for the physical characteristics unique to the heavy region. It is seen that the average difference between the neutron and the proton number, namely $(N-Z)$, for the nuclei under consideration here is about 50; it is in sharp contrast to this number being just around 35 in the medium-heavy ($A=150-190$) deformed nuclei. Thus, whereas in the rare-earth region a given up-spin proton orbital $(Nn_3\Lambda\uparrow)$ at the proton Fermi surface finds its spin-flip neutron counter-

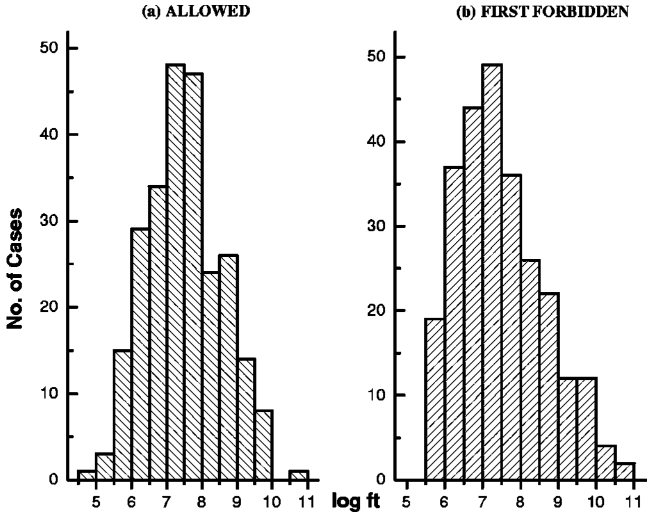

FIG. 1. Histograms corresponding to the experimentally deduced log *ft* values for (a) the allowed ($\Delta I=0,1$; $\Delta\pi$=no) and (b) the first-forbidden ($\Delta I=0,1$; $\Delta\pi$=yes) β transitions in $A>228$ nuclei. The total number of β transitions herein is 252 for the allowed and 264 for the first-forbidden categories, including (not shown here) an allowed log ft=14.7 and a first-forbidden log ft=13.5 cases.

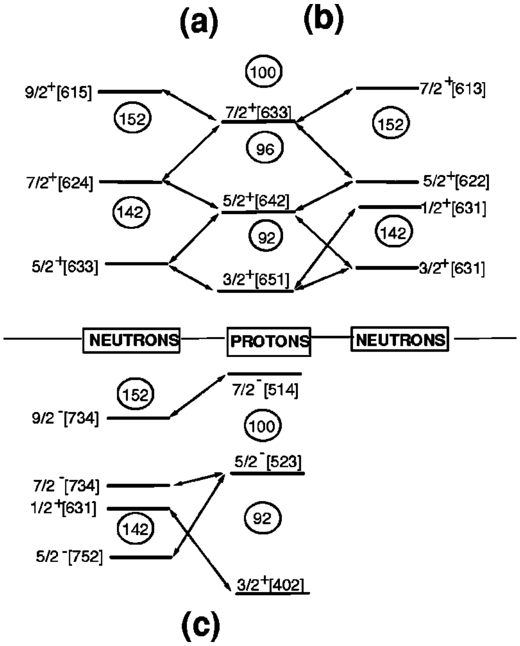

FIG. 2. The Nilsson single-particle orbitals, labeled by the asymptotic quantum number $\Omega^{\pi}[Nn_3\Lambda]$, of interest for the allowed β transitions in $A>228$ nuclei under specified categories: (a) $N=\Delta(n_3+\Lambda)=6$; $\Delta N=0(n_3+\Lambda)=0$; (b): $\Delta N=0,\Delta(n_3+\Lambda)=2$; (c) $\Delta N=2$, $\Delta(n_3+\Lambda)=2$. The allowed transition occurs between the states connected by arrow. The circled numbers denote the nucleon numbers where significant energy gaps are observed in the single-particle Nilsson level scheme.

TABLE II. Illustrative examples from odd-A actinides typifying the three divisions of allowed β decays in $A>228$ nuclei based on the proposed selection rules of the asymptotic quantum numbers $[Nn_3\Lambda]$ as shown in Fig. 2 and indicated below in subtitles as (a), (b), and (c). The numbers listed in the middle of each row are the log ft values for β transitions connecting the parent nucleus (on the left) ground state to the indicated ground or excited state (with E_x in keV) of the daughter nucleus (on the right). The odd nucleon configuration $\Omega^{\pi}[Nn_3\Lambda\Sigma]$ is specified in each case.

(a) $\Delta N=0,\Delta(n_3+\Lambda)=0$

${}^{229}_{88}\mathrm{Ra}_{141}:\frac{5}{2}^{+}[633\downarrow]-5.6\rightarrow{}^{229}_{89}\mathrm{Ac}_{140}:0.0,\frac{3}{2}^{+}[651\uparrow]$
${}^{233}_{93}\mathrm{Np}_{140}:\frac{5}{2}^{+}[642\uparrow]-5.2\rightarrow{}^{233}_{92}\mathrm{U}_{141}:0.0,\frac{5}{2}^{+}[633\downarrow]$
${}^{243}_{94}\mathrm{Pu}_{149}:\frac{7}{2}^{+}[624\downarrow]-5.6\rightarrow{}^{243}_{95}\mathrm{Am}_{148}:465.7,\frac{7}{2}^{+}[633\uparrow]$
${}^{255}_{98}\mathrm{Cf}_{157}:\frac{9}{2}^{+}[615\downarrow]-5.7\rightarrow{}^{255}_{99}\mathrm{Es}_{156}:0.0,\frac{7}{2}^{+}[633\uparrow]$

(b) $\Delta N=0,\ \Delta(n_3+\Lambda)=2$

${}^{233}_{90}\mathrm{Th}_{143}:\frac{1}{2}^{+}[631\uparrow]-6.5\rightarrow{}^{233}_{91}\mathrm{Pa}_{142}:94.7,\frac{3}{2}^{+}[651\uparrow]$
${}^{233}_{93}\mathrm{Np}_{140}:\frac{5}{2}^{+}[642\uparrow]-6.7\rightarrow{}^{233}_{92}\mathrm{U}_{141}:546.6,\frac{5}{2}^{+}[622\uparrow]$
${}^{245}_{95}\mathrm{Am}_{150}:\frac{5}{2}^{+}[642\uparrow]-6.4\rightarrow{}^{245}_{96}\mathrm{Cm}_{149}:252.8,\frac{5}{2}^{+}[622\uparrow]$
${}^{253}_{98}\mathrm{Cf}_{155}:\frac{7}{2}^{+}[613\uparrow]-7.2\rightarrow{}^{253}_{99}\mathrm{Es}_{154}:0.0,\frac{7}{2}^{+}[633\uparrow]$

(c) $\Delta N=2,\ \Delta(n_3+\Lambda)=2$

${}^{233}_{90}\mathrm{Th}_{143}:\frac{1}{2}^{+}[633\downarrow]-7.7\rightarrow{}^{233}_{91}\mathrm{Pa}_{142}:454.4,\frac{3}{2}^{+}[402\downarrow]$
${}^{233}_{90}\mathrm{Th}_{145}:\frac{1}{2}^{+}[633\downarrow]-8.2\rightarrow{}^{235}_{91}\mathrm{Pa}_{144}:344.6,\frac{3}{2}^{+}[402\downarrow]$
${}^{239}_{95}\mathrm{Am}:\frac{5}{2}^{-}[523\downarrow]-8.7\rightarrow{}^{239}_{94}\mathrm{Pu}_{145}:391.6,\frac{7}{2}^{-}[743\uparrow]$
${}^{251}_{100}\mathrm{Fm}:\frac{9}{2}^{-}[734\uparrow]-7.9\rightarrow{}^{251}_{99}\mathrm{Es}_{152}:461.4,\frac{7}{2}^{-}[514\downarrow]$

part $(Nn_3\Lambda\downarrow)$ close to the neutron Fermi surface permitting an au β transition in accordance with the selection rule of Eq. (1), the much larger value of $(N-Z)$ in heavy nuclei presents no such matching conditions. This mismatch between the neutron and the proton wave functions results in lowering the overlap matrix elements M_{if}. Consequently, the comparative lifetime ft (which is proportional to inverse of $|M_{\mathrm{if}}|^2$) becomes much larger, leading to larger log ft values. To specifically illustrate this factor, we show, in Fig. 2, sets of neutron and proton orbitals which admit of allowed ($\Delta I=0$ or 1, $\Delta\pi$=no) β connections; these sets have however been separated into three groups, labeled (a), (b), and (c), depending on the degree of respective mismatch of $[Nn_3\Lambda]$ quantum numbers, as indicated in the figure caption and explained below. A few illustrative examples from odd A decays for each of the three groups are listed in Table II.

Group (a) of allowed transitions, illustrated in Fig. 2(a) with typical examples listed in Table II(a), corresponds to fast transitions with log $ft\leqslant 6.0$ which obey the extended selection rule of Ref. [10], namely,

$$N=n_3+\Lambda=6,\quad \Sigma_p=\uparrow,\quad \Sigma_n=\downarrow,\tag{2}$$

$$\Delta N=0,\quad \Delta\Lambda\neq 0,\quad \Delta(n_3+\Lambda)=0.$$

These transitions basically still connect an $i_{13/2}$ proton subshell and $i_{11/2}$ neutron subshell orbital; however, unlike au transitions (which separately obey $\Delta n_3=0$ and $\Delta\Lambda=0$), they connect states for which $\Delta\Lambda\neq 0$, but $\Delta(n_3+\Lambda)=0$. These allowed hindered (ah) transitions may thus be categorized as Λ-forbidden.

Group (b) transitions, shown in Fig. 2(b) and illustrated through a few representative cases listed in Table II(b), correspond to the transitions which obey the modified selection rule

$$\Delta N=0,\quad \Delta(n_3+\Lambda)=2.\tag{3}$$

These transitions are seen to have log $ft\geqslant 6.5$ and are thus significantly retarded. In this context, we note that, in the limit of no deformation, $(n_3+\Lambda)$ equals the 'ℓ' value of the originating spherical state. In the case of unique parity high spin orbitals, such a correlation is still justified. In the present case, the group (b) transitions essentially connect an $i_{13/2}(N=6,n_3+\Lambda=6)$ proton subshell state to a $g_{9/2}$ or $g_{7/2}$ neutron subshell state. The significant retardation of these transitions may be viewed as the reflection of the l-forbiddenness of β decays.

In group (c), we have placed the transitions obeying the modified selection rule

$$\Delta N=2,\ \Delta(n_3+\Lambda)=2.\tag{4}$$

The relevant orbital connections are indicated in Fig. 2(c); several illustrative examples for this group were reported earlier [12] and a few of them are listed in Table II(c). All these transitions are found to have log $ft>7.0$. This observation substantiates the earlier mentioned Mottelson-Nilsson [3] contention that "the selection rule associated with N is somewhat stronger than the rules for other asymptotic quantum numbers.” It may be noted that, since parity $\pi=(-)^N$, the corresponding $\Delta N=1$ parity-changing transitions are termed as the first-forbidden decays. The forbiddenness of $\Delta N=2$ allowed transitions is, as such, not surprising. We further note that, in the heavy mass region, all the possible allowed transitions between the negative parity levels are two oscillator shells apart transitions since they connect the $j_{15/2}(N=n_3+\Lambda=7)$ neutron subshell states and the $h_{9/2}(N=n_3+\Lambda=5)$ proton subshell states, as seen in Fig. 2(c). All these group (c) transitions are thus N-forbidden.

In addition to these factors, the K-forbiddenness also plays a significant role in cases wherein $\Delta K > \Delta I$. This effect can be quite dramatic if the degree of forbiddenness $\nu=\Delta K-\Delta I$ is large. For example, the $I^\pi K=6^- 6$ isomer of ^{236}Np decays to the 843 keV 5^- level in ^{236}U with log $ft=14.7$, which is the largest log ft value among the 252 allowed decays under consideration. This 5^- ^{236}U level is a rotational member of $K^\pi=0^-$ octupole band, thus making the decay proceeding through a $\Delta K=6$ transition, with K forbiddenness $\nu=5$.

In summary, using a data base of 252 allowed and 264 nonunique first forbidden $(1f)$ β transitions in $A>228$ nuclei, we have shown that the log ft values for the allowed and the $1f$ transitions have exactly the same central value (7.5) and the same width (1.2). This result leads to the conclusion that the allowed and the $1f$ transitions of this region cannot be distinguished on the basis of log ft values. Further, a comparison of our results with the recent global review reveals that while our results for $1f$ transitions are in good agreement with the global values, those for the allowed transitions are much closer to the values for the forbidden decays from the global review. We have pointed out, and documented, four factors that can explain these observations. These factors are (a) Λ-forbiddenness, (b) ℓ-forbiddenness, (c) N-forbiddenness, and (d) K-forbiddenness, which individually and collectively make the log ft values for the allowed decays overlap almost completely with those for the first-forbidden decays. The role of these, and maybe other, factors in hindering or forbidding allowed and $1f$ transitions of this region, in deformed nuclei of other regions, needs to be investigated. Among other factors, the mismatch of neutron and proton orbitals in this region strongly inhibits the first-order allowed transition and hence the second-order matrix elements, largely unrelated to simple Nilsson structures, may bring in significant contribution for the observed log ft values. These, and other theoretical aspects, also need to be looked into.

[1] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **29**, 9 (1955).

[2] G. Alaga, Phys. Rev. **100**, 432 (1955); Nucl. Phys. **4**, 625 (1957).

[3] B. R. Mottelson and S. G. Nilsson, K. Dan. Vidensk. Selsk., Mat.-Fys. Skr. **1**, 8 (1959).

[4] C. J. Gallagher, Nucl. Phys. **16**, 215 (1960).

[5] P. C. Sood and R. K. Sheline, At. Data Nucl. Data Tables **43**, 259 (1989).

[6] Nuclear Data Sheets, first issue of any recent volume, General Policies—Summary of Bases for Spin and Parity Assignments, pp. vi–vii.

[7] S. Raman and N. B. Gove, Phys. Rev. C **7**, 1995 (1973).

[8] P. C. Sood and R. K. Sheline, Phys. Scr. **42**, 25 (1990).

[9] B. Singh, J. L. Rodriguez, S. S. M. Wong, and J. M. Tuli, Nucl. Data Sheets **84**, 487 (1998).

[10] P. C. Sood and R. K. Sheline, Phys. Rev. C **45**, 3006 (1992).

[11] P. C. Sood, R. K. Jain, and O. S. K. S. Sastri, J. Phys. G **29**, 1237 (2003).

[12] Raj Kumar Jain, O. S. K. S. Sastri, and P. C. Sood, Nucl. Phys. **B46**, 72 (2003).

[13] R. B. Firestone and V. S. Shirley, *Table of Isotopes*, 8th ed. (Wiley, New York, 1999).

[14] Y. A. Akovali, Nucl. Data Sheets **87**, 249 (1999); **94**, 131 (2001).

[15] A. Artna-Cohen, Nucl. Data Sheets **88**, 155 (1999).

[16] A. K. Jain, R. K. Sheline, P. C. Sood, and K. Jain, Rev. Mod. Phys. **62**, 393 (1990).

[17] P. C. Sood, D. M. Headly, and R. K. Sheline, At. Data Nucl. Data Tables **51**, 273 (1992); **58**, 167 (1994).