Is the Θ^+ a $K\pi N$ bound state?

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Following a recent suggestion that the Θ^+ could be a $K\pi N$ bound state we perform an investigation under the light of the meson-meson and meson-baryon dynamics provided by the chiral Lagrangians and using methods currently employed to dynamically generate meson and baryon resonances by means of unitary extensions of chiral perturbation theory. We consider two-body and three-body forces and examine the possibility of a bound state below the three-particle pion-kaon-nucleon and above the kaon-nucleon thresholds. Although we find indeed an attractive interaction in the case of isospin $I=0$ and spin-parity $1/2^+$, the interaction is too weak to bind the system. If we arbitrarily add to the physically motivated potential the needed strength to bind the system and with such strong attraction evaluate the decay width into KN, this turns out to be small. A discussion on further work in this direction is done.

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I. INTRODUCTION

A recent experiment at SPring-8/Osaka [1] has found a clear signal for an $S=+1$, positive charge resonance around 1540 MeV, confirmed by the DIANA Collaboration at ITEP [2], CLAS at Jefferson Lab [3], and SAPHIR at ELSA [4]. The resonance has explicit exotic flavor quantum numbers, given the decay final states K^0p and K^+n. Its width is also intriguingly narrow, less than 20 MeV by present experimental bounds. A state with these characteristics was originally predicted by Diakonov *et al.* in Ref. [5], and since the experimental observation a large number of theoretical papers have appeared with different suggestions as to the nature of the state and possible partners [6–21]. Most of the works look at the quark structure of what is being called the pentaquark, since a standard three quark Fock space assignment is not allowed. The parity of this candidate state is as yet undetermined [22], and whereas quark model calculations in the ground state [23–25] assign to it negative parity, positive parity is predicted in the Skyrme model [5] requiring a p wave in the quark model [7,26].

Yet, at a time when many low energy baryonic resonances are being dynamically generated as meson-baryon quasibound states within chiral unitary approaches [27–34], it looks tempting to investigate the possibility of this state being a quasibound state of a meson and a baryon or two mesons and a baryon. Its nature as a KN s-wave state is easily ruled out since the interaction is repulsive. This is in general the case for scattering with exotic quantum numbers (not attainable with three quarks) which also explains the repulsive core nucleon-nucleon interaction [35]. Indeed the known kaon-nucleon phase shifts seem difficult to reconcile with the existence of a broad Θ^+ resonance, although a narrow one is not excluded [36]. KN in a p wave, which is attractive, is too weak to bind. The next logical possibility is to consider a quasibound state of $K\pi N$, which in s wave would naturally correspond to spin-parity $1/2^+$, the quantum numbers suggested in Ref. [5]. Such an idea has already been put forward in Ref. [37] where a study of the interaction of the three-body system is conducted in the context of chiral quark models, which suggests that it is not easy to bind the system although one cannot rule it out completely. Similar ideas have been exploited in the past [38] to describe the $f_1(1420)$ meson, then named E(1420), as a $KK\pi$ molecule bound by color singlet exchanges.

In the present work we further investigate in this direction and for this we use the meson-meson and meson-baryon interactions generated by the chiral Lagrangians and apply techniques of unitarized chiral perturbation theory which have been used in the dynamical generation of the low lying baryonic resonances.

II. A κN STATE?

Upon considering the possible structure of Θ^+ we are guided by the experimental observation [3] that the state is not produced in the K^+p final state. This would rule out the possibility of the Θ state having isospin $I=1$. Then we accept the Θ^+ to be an $I=0$ state. As we couple a pion and a kaon to the nucleon to form such state, a consequence is that the $K\pi$ substate must combine to $I=1/2$ and not $I=3/2$. This is also welcome dynamically since the s-wave $K\pi$ interaction in $I=1/2$ is attractive (in $I=3/2$ repulsive) [39]. The attractive interaction in $I=1/2$ is very strong and gives rise to the dynamical generation of the scalar κ resonance around 850 MeV and with a large width [39].

One might be tempted to consider the Θ^+ state as a quasibound κN state. However the Θ^+ state would then be bound

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by about 200 MeV, apparently too large an amount. But recall that the large width of the κ (around 400 MeV) allows κ strength at lower energies and the large binding becomes more relative. One might next question that, with such a large width of the κ, the Θ^+ could not be so narrow as experimentally reported. However, this large κ width is no problem since in our scenario it would arise from $K\pi$ decay, but now the $K\pi N$ decay of the Θ^+ is forbidden as the Θ^+ mass is below the $K\pi N$ threshold.

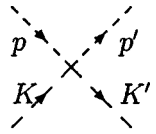
One might hesitate to call the possible theoretical Θ^+ state a κN quasibound state because of the large gap with the nominal κN mass. The name though is not relevant here and we can opt by calling it simply a $K\pi N$ state, but the fact is that the $K\pi$ system is strongly correlated even at these lower energies, and since this favors the binding of the $K\pi N$ state we shall take it into account.

A. $K\pi$ scattering matrix

We begin by refreshing how the κ can be generated in the Bethe-Salpeter approach used in Ref. [40] to generate the σ, $f_0(980)$, and $a_0(980)$ scalar resonances. From the lowest order ChPT Lagrangian [41] one takes the $K\pi$ amplitude which serves as kernel V of the Bethe-Salpeter equation (here the Lippman-Schwinger equation with relativistic meson propagator)

$$t_{K\pi} = V_{K\pi} + V_{K\pi} G_{mm} t_{K\pi}, \tag{1}$$

where $V_{K\pi}$ for $\mathrm{I}=1/2$ in s wave, which we call from now on t_{mm}, is given by

$t_{mm} =$ 

(2)

yielding (in an s wave)

$$\langle I=1/2\ I_3|t_{mm}|I=1/2\ I_3\rangle = \frac{4m_\pi^2 + 4m_K^2 - 4s + \frac{3\lambda}{2s}}{4f^2}, \tag{3}$$

where $f \simeq 100$ MeV is the meson decay constant, which we take as an average between f_π and f_K, s the Mandelstam variable, and $\lambda(m_\pi, m_K, \sqrt{s}) = [m_\pi^4 + m_K^4 + s^2 - 2(m_\pi^2 m_K^2 + m_\pi^2 s + m_K^2 s)]$ Källen's function.

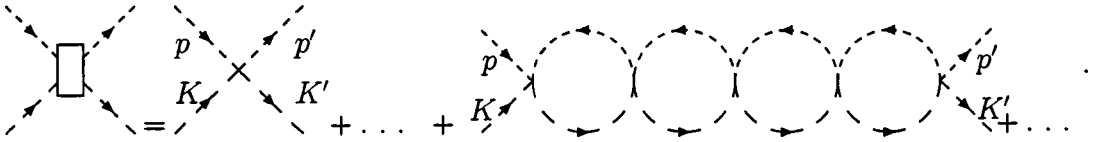
Also in Eq. (1) G_{mm} is the two-meson loop function defined in Ref. [40] and regularized with a three-momentum cutoff of 850 MeV (which produces satisfactory fits to the πK scattering phase shift in the κ and also in the $\pi\pi$ σ channels, not shown),

$$G_{mm} = \int_0^\Lambda \frac{q^2 dq}{4\pi^2} \frac{\omega_K + \omega_\pi}{\omega_K \omega_\pi} \frac{1}{(\sqrt{s} + \omega_\pi + \omega_K)(\sqrt{s} - \omega_\pi - \omega_K + i\epsilon)}, \tag{4}$$

$$\omega_K = \sqrt{m_K^2 + q^2}, \quad \omega_\pi = \sqrt{m_\pi^2 + q^2} \tag{5}$$

and V, t factorize in Eq. (1) with their on shell value as discussed in Ref. [40]. This simply means that one takes $p_i^2 = m_i^2$ in the expressions of the K_π kernel. Note that t_{mm} is attractive in the κ channel.

Equation (1), which we numerically solve, resums the πK scattering perturbation series



(6)

The κ state appears then as a pole of the $t_{K\pi}$ matrix in the complex plane.

B. $N\kappa$ scattering

In order to determine the possible Θ^+ state we search for poles of the $K\pi N \to K\pi N$ scattering matrix. To such point we construct the series of diagrams

(7)

where we account explicitly for the $K\pi$ interaction by constructing correlated $K\pi$ pairs and letting the intermediate $K\pi$ and nucleon propagate. This requires a kernel for the two meson-nucleon interaction which we now address. The $K\pi$ correlation in the external legs is dispensable for the purpose of finding poles of the t matrix.

We formulate the meson-baryon Lagrangian in terms of the SU(3) matrices, B, Γ_μ, u, and the implicit meson matrix Φ standard in ChPT [42–45],

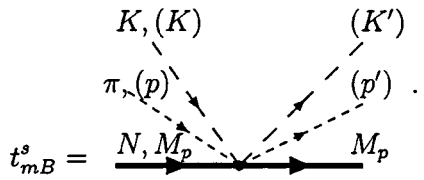
$$\mathcal{L} = \mathrm{Tr}(\bar{B} i\gamma^\mu \nabla_\mu B) - M_B \mathrm{Tr}(\bar{B}B) + \tfrac{1}{2} D\ \mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}) + \tfrac{1}{2} F\ \mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5[u_\mu, B]), \tag{8}$$

$$\nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B],$$

$$\Gamma_\mu = \tfrac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \tag{9}$$

with the definitions in Refs. [42–45].

First there is a contact three-body force simultaneously involving the pion, kaon, and nucleon, which can be derived from the meson-baryon Lagrangian (8) term containing Γ_μ:



$$t^s_{mB} = \tag{10}$$

We now show that a nucleon, kaon, and pion see an attractive interaction in an isospin zero state through this contact potential. By taking the isospin $I=1/2$ κ states

$$\kappa^0 = \frac{1}{\sqrt{3}}|\pi^0 K^0\rangle - \sqrt{\frac{2}{3}}|\pi^- K^+\rangle,$$

$$\kappa^+ = -\sqrt{\frac{2}{3}}|\pi^+ K^0\rangle - \frac{1}{\sqrt{3}}|\pi^0 K^+\rangle, \tag{11}$$

and combining them with the nucleon, also isospin $1/2$, we generate $I=0,1$ states

$$\Theta^0 = |I=0\ I_3=0\rangle = \frac{1}{\sqrt{2}}(|p\ \kappa^0\rangle - |n\ \kappa^+\rangle),$$

$$\Theta^1 = |I=1\ I_3=0\rangle = \frac{1}{\sqrt{2}}(|p\ \kappa^0\rangle + |n\ \kappa^+\rangle), \tag{12}$$

which diagonalize the scattering matrix associated to t_{mB},

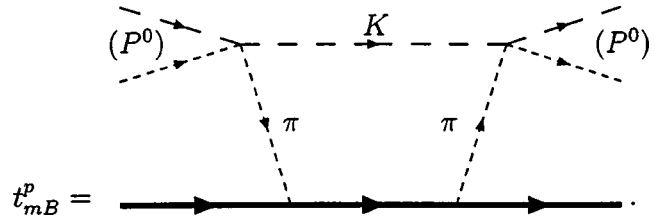
$$\langle\Theta^1|t^s_{mB}|\Theta^1\rangle = -\frac{1}{144 f^4}[-4(\not{K}+\not{K}') - 11(\not{p}+\not{p}')],$$

$$\langle\Theta^0|t^s_{mB}|\Theta^0\rangle = -\frac{21}{144\ f^4}[(\not{K}+\not{K}') - (\not{p}+\not{p}')], \tag{13}$$

where for a near-threshold study we will perform the usual nonrelativistic approximation $\bar{u}\gamma^\mu k_\mu u = k^0$. Since the $K\pi N$ system is bound by about 30 MeV, one can take for a first test k^0, p^0 as the masses of the K and π, respectively, and one sees that the interaction in the $I=0$ channel is attractive, while in the $I=1$ channel it is repulsive. This would give chances to the κN t matrix to develop a pole in the bound region, but rules out the $I=1$ state.

The series (7) might lead to a bound state of κN which would not decay since the only intermediate channel is made out of $K\pi N$ with mass above the available energy.

The decay into KN observed experimentally can be taken into account by explicitly allowing for an intermediate state provided by the p-wave interaction vertices from Eq. (8), through the diagram



$$t^p_{mB} = \tag{14}$$

The evaluation of this diagram requires the extra πNN Yukawa vertex, which one generates from the D, F terms of the Lagrangian (8) and to which we attach the commonly used πNN monopole form factor to account for the nucleon's finite size with a scale $\Lambda = 1$ GeV:

$$t^{I_j}_{\pi N} = i\left(\frac{G_A}{2f}\right)\vec{\sigma}\cdot\vec{q}\,F(|\vec{q}|^2)\langle N|\tau^{I_j}|N'\rangle, \tag{15}$$

with $G_A = D + F = 1.26$,

$$F(|\vec{q}|^2) = \frac{\Lambda^2}{\Lambda^2 + |\vec{q}|^2}.$$

The isospin factor for Eq. (14) turns out to be 3 for $I=0$ and $1/3$ for $I=1$. As we shall see, this diagram provides some attraction at low energies, but in the $I=1$ case the relative factor of $1/9$ makes it negligible compared with the repulsion generated by Eq. (10). The evaluation of the customary pole integrals over q^0 in Eq. (14) leads to

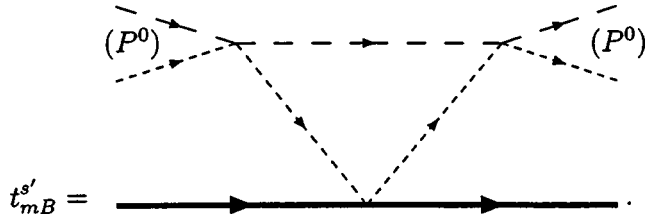
$$t^p_{mB} = 3t^2_{mm}\int \frac{d^3q}{(2\pi)^3}\left(\frac{G_A}{2f}\right)^2 |\vec{q}|^2 F(q)^2 \frac{M_N}{E_N}\frac{1}{p^0 + P^0 - \omega_K - E_N + i\epsilon}\frac{1}{4\omega_k\omega_\pi^3(E_N - p^0 + \omega_\pi)^2[P_0^2 - (\omega_\pi + \omega_K)^2]^2}$$
$$\times\{(E_N - p_0)^2[-P_0^2\omega_K + (\omega_K + \omega_\pi)^2(\omega_K + 2\omega_\pi)] + (E_N - p^0)[P^{0\,3}\omega_K - P_0^2\omega_K(\omega_K + 2\omega_\pi) + (\omega_K^2 + 3\omega_\pi\omega_K + 2\omega_\pi^2)^2$$
$$- P^0\omega_K(\omega_K^2 + 4\omega_\pi\omega_K + 3\omega_\pi^2)] + 2\omega_\pi[P^{0\,3}\omega_K - P_0^2\omega_K(\omega_K + \omega_\pi) + (\omega_K + \omega_\pi)^4 - P^0\omega_K(\omega_K^2 + 3\omega_\pi\omega_K + 2\omega_\pi^2)]\}, \tag{16}$$

with ω_K and ω_π as in Eq. (5), $E_N=\sqrt{M_N^2+q^2}, p^0=M_N$ the nucleon mass (incoming and outgoing energies), and P^0 the incoming pion-kaon system energy (masses minus possible binding energy).

Through the remaining *NK* propagator in the integral, Eq. (16) generates a real part from the principal value and an imaginary part corresponding to placing the intermediate *K* and *N* on shell. This would account for the decay of the Θ^+ state into *KN*.

C. Sequential two-body contributions

The existence of diagram (14) above can be interpreted as having πK interaction followed by πN interaction in p wave. One of course can also consider this latter interaction in s wave using the same Lagrangian (8) with two meson fields, as in



(17)

There is also a novelty with respect to diagram (14) since now the meson coupling to the nucleon can be either the π or the *K*, while in the case of the p wave, the requirement to include only ordinary baryons in the intermediate baryon state does not allow the *K* to be coupled to the nucleon.[1]

We need now the $mN\to mN$ amplitudes, which are easily obtained from the Lagrangian of Eq. (8) and give

$$t_{mN\to mN}=-\frac{1}{4f^2}C_{ij}(q^0+q^{0'}), \tag{18}$$

where q^0 and $q^{0'}$ are the initial and final meson energies and the C_{ij} coefficients are given in Table I.

After performing the q^0 integration in the loop with three propagators with the explicit $(q^0+q^{0'})$ dependence of the vertex of Eq. (18), but taking $t_{K\pi}$ with the arguments of the external $K\pi$ system, we obtain for the case of a π coupling to the nucleon in Eq. (17),

$$t^{s'}_{mN}=-2t^2_{mm}\frac{P^0}{4f^2}\int\frac{d^3q}{(2\pi)^3}\frac{(\omega_\pi+\omega_K)}{\omega_\pi\omega_K}\left[\frac{1}{P^{02}-(\omega_\pi+\omega_K)^2}\right]^2. \tag{19}$$

It is worth noting that this expression is symmetric in π and *K*. Hence, the loops corresponding to having the K instead of the π coupling to the nucleon have the same expression up to some SU(3) flavor factors. A straightforward calculation shows that for $I=0$ the coefficient is the same, but opposite in sign, whether the pion or the kaon couple to the nucleon, implementing an exact cancellation of the two types of diagrams. It is also worth noting that in the case of $I=1$ there is no cancellation but instead one finds a repulsive contribution, obtained by changing the coefficient 2 of Eq. (19) by $-\frac{2}{3}$.

TABLE I. Flavor coefficients for meson-nucleon scattering C_{ij}.

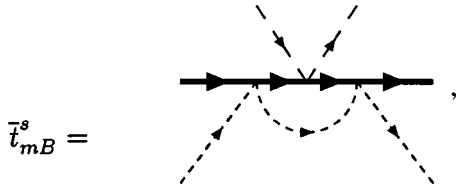
C_{ij}	π^0p	π^+n	π^-p	π^0n	K^0p	K^+n	K^+p	K^0n
π^0p	0	$\sqrt{2}$						
π^+n	$\sqrt{2}$	1						
π^-p			1	$-\sqrt{2}$				
π^0n			$-\sqrt{2}$	0				
K^0p					-1	-1		
K^+n					-1	-1		
K^+p							-2	
K^0n								-2

Diagram (17), when the meson exchange is iterated between the other meson and the nucleon, generates a subseries of the terms implicit in the Faddeev equations. For instance, the subseries of terms in the iterations of Eq. (17) with a pion generate the Faddeev series in the fixed center approximation, accounting for the interaction of the pion with the *KN* system (should it be bound by itself which is not the case) [46,47]. Yet, this subseries is inoperative, given the cancellation of the π and *K* contributions.

Thinking along the same lines we are led to the other subseries of the Faddeev equations in which the nucleon is the particle being exchanged between the mesons:

(20)

The basic vertex in this mechanism is



(21)

where once again the upper meson can be a pion or a kaon.

Following the same techniques as before we obtain for this term's contribution the result

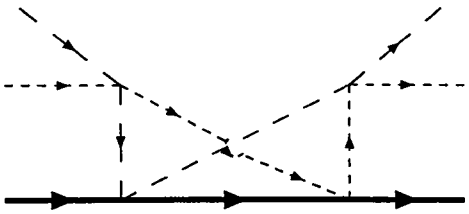
$$\bar{t}^s_{KN}+\bar{t}^s_{\pi N}=\frac{k^0p^{0'}}{f^6}[p^{0'}\tilde{G}_{K\pi}(P^{0'})-k^0\tilde{G}_{\pi K}(P^0)], \tag{22}$$

with P^0 and $P^{0'}$ the energy of the kaon/nucleon and pion/nucleon pair, respectively, the loop function

[1]Knowing the Θ^+ existence, we could exchange this particle there. Yet, the intermediate state would be off-shell by about 170 MeV, plus the small KN width of the Θ^+ makes the $KN\Theta^+$ coupling small, so this contribution can be safely neglected.

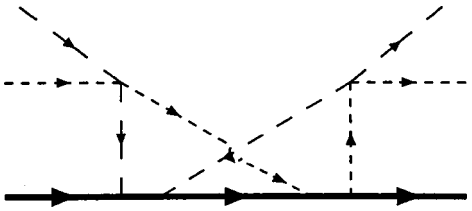
$$\tilde{G}_{\pi K}(P^0) = \int \frac{q^2 dq}{4\pi^2} \frac{1}{\omega_K} \left(\frac{1}{P^0 - \omega_K - E_N + i\epsilon} \right)^2 \left(\frac{m_N}{E_N} \right)^2 \tag{23}$$

and $\tilde{G}_{K\pi}$ having the same expression permuting π and K. The contribution of Eq. (22) vanishes in the SU(3) limit of equal meson masses, but for unequal meson masses there is a net attractive contribution which has about the same strength as that of the four meson contact term of Eq. (13). Other interaction terms where the meson lines cross each other are possible, but either vanish like



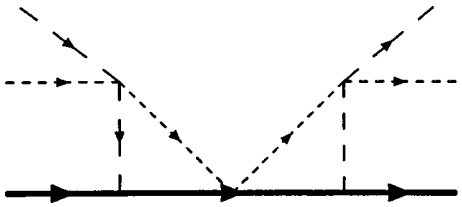
(24)

or are small since they involve baryons in the t channel which are very far off-shell such as



(25)

or involve one p-wave coupling inside a loop which makes it vanish for large baryon mass, for example,



(26)

III. BETHE-SALPETER ITERATION IN THE ($K\pi N$) SYSTEM

Now we turn our attention to the formulation of the three-body problem. We have implemented the correlation between π and K through multiple scattering, but we have not done so with the KN or πN interaction. In the case of the KN interaction this multiple scattering barely changes the lowest order t matrix $t_{mN\to mN}$ [28]. In the case of the πN system it generates attraction which is also weak at the low energies considered here and only becomes sizab1e around $\sqrt{s} = 1500$ MeV where it leads, together with other coupled channels, to the generation of the $N^*(1535)$ resonance [27,30,48].

The series of $K\pi$ loop diagrams of Eq. (6) is summed with the following equation:

$$G^\kappa(s) = \frac{G_{mm}(s)}{1 - t_{mm}(s) G_{mm}(s)} \tag{27}$$

which yields a κ propagator (that is, a propagator for a correlated spin $1/2$ pion-kaon state).

At last, if the Θ was going to exist as a three-body bound state, it should appear as a resonance of the κ-N scattering matrix which appears when summing the contribution of the diagrams of Eq. (7), given by

$$t_{\kappa N}(s) = \frac{t_{mB}(s)}{1 - t_{mB}(s) G_{mB}(s)}, \tag{28}$$

where t_{mB} sums the three nonvanishing contributions, Eqs. (10), (14), and (22),

$$t_{mB} = t^s_{mB} + t^p_{mB} + \overline{t^s_{mB}}. \tag{29}$$

The relevant loop function here, G_{mB} appearing as the big loop in Eq. (7), is made numerically more tractable by employing the Lehmann representation for G^κ,

$$G^\kappa(q^0, \vec{q}) = \frac{-1}{\pi} \int_{m_\pi + m_K}^{\infty} d\omega \frac{2\omega \ \mathrm{Im}\ G^\kappa(\omega^2 - |\vec{q}|^2)}{q_0^2 - \omega^2} \tag{30}$$

(although we have checked our codes also by direct computation). After factorizing the vertices with the on-shell prescription, we obtain

$$G_{mB}(s) = \frac{-1}{2\pi^3} \int_0^\Lambda q^2 dq \frac{M_N}{E_N(q)} \int_{\sqrt{(m_\pi + m_K)^2 + q^2}}^{\infty} \times d\omega \frac{\mathrm{Im}\ G^\kappa(\omega^2 - q^2)}{\sqrt{s} - \omega - E_N(q)}, \tag{31}$$

with

$$\sqrt{s} \in (m_N + m_K, m_N + m_K + m_\pi).$$

The algebraic formulation of the Bethe-Salpeter equation (28) is possible because we have factorized the $(k^0 + k'^0), (p^0 + p^{0\prime})$ dependence of Eqs. (13), (16), and (22) with its on-shell value given by the external variables. We have performed the loop integrals with the full off-shell part and found that the on-shell approximation induces errors of less than 20%, hence it is accurate enough for the exploratory purpose of the present work.

There is a technical detail worth mentioning. We have assumed in the calculations that the incoming and outgoing particles have zero momentum. This is certainly an approximation, but it simplifies the calculations since in the diagram (14) one has two identical pions propagating and in Eq. (21) one has two identical nucleon propagators and we evaluate these Feynman diagrams by partial derivation of a loop function with only one pion or one nucleon propagator, respectively. This causes no problem if one investigates the amplitudes at 30 MeV below threshold but the approximation induces an infrared divergence at threshold. We are not interested in this region but in any case we cure the divergence by assuming an average momentum of the particles in the three-body wave function. We take 100 MeV/c for this momentum and we should change $\omega_\pi(\vec{q})$ by $\omega_\pi(\vec{q} - \vec{p})$ which

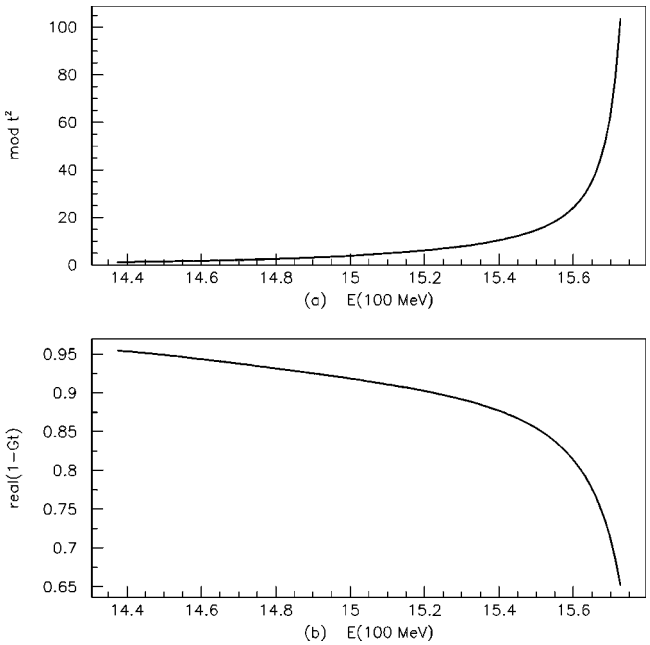


FIG. 1. Our final result: $K\pi N$ scattering matrix [modulus and denominator of Eq. (28)]. Energy units are 100 MeV.

close to threshold can be approximated by $\omega_\pi(q)+p^2/2m_\pi$. Similarly, for the diagram (21), the nucleon energy is changed to $E_N(q)+p^2/2m_N$. This cures the infrared divergence at threshold and has negligible influence away from it.

IV. NUMERICAL RESULTS AND CONCLUSION

We examine now the $t_{\kappa N}$ amplitude of Eq. (28) as a function of $\sqrt{s}$ of the three external particles (for simplicity we split the small binding energy between the pion and kaon in proportion to their masses). In Fig. 1(a) we show $|t|^2$ against $\sqrt{s}$.

We see that the function is monotonously increasing as a function of $\sqrt{s}$, but there is no trace of a pole or resonance. In order to see how far we are from a pole, we show in Fig. 1(b) the real part of the denominator of Eq. (28), $1-t_{mB}G_{mB}$. We see that in the region from $\sqrt{s}=1540$ MeV to 1570 MeV this value is bigger than 0.6, while it should be around zero to have a resonance. Typical values of t_{mB} and G_{mB} are $G_{mB}\simeq -0.05(100\ \text{MeV})^3$, $t_{mB}\simeq -(2-3)(100\ \text{MeV})^{-3}$ for a cutoff $\Lambda=1$ GeV. From these results we can conclude the following.

With the dynamics which we are considering we find no bound state around $\sqrt{s}=1540$ MeV.

The fact that $t_{mB}G_{mB}$ is far away from unity indicates that we are far away from having a pole of the κN scattering matrix.

In order to quantify this second statement we proceed as follows. We increase artificially the potential t_{mB} by adding to it a quantity which leads to a pole around $\sqrt{s}=1540$ MeV. This is reached by adding $-16(100\ \text{MeV})^{-3}$ to the already existing potential, which means we add an attractive potential around five or six times bigger than the existing one. If we do that we obtain the results for $|t|^2$ shown in Fig. 2.

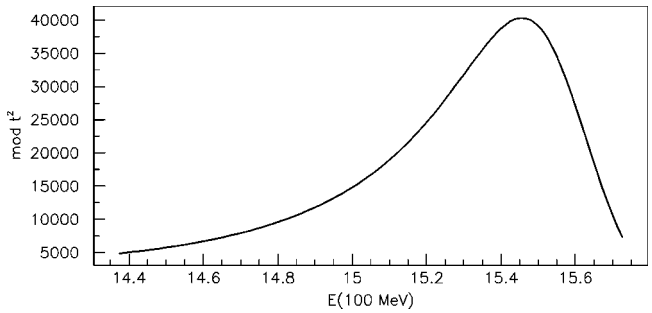


FIG. 2. We find a resonance with a reasonable width for a potential larger by a factor 6 (see text). Units are 100 MeV.

There is indeed a resonance around $\sqrt{s}=1540$ MeV with a width of around $\Gamma=40$ MeV, which is of the order of magnitude of the experimental one. Refinements of the theory, considering that in the generation of the resonance the external κ would be itself part of a loop, would lead according to our estimates to a smaller width, but for the order of magnitude the approximations performed are fair. This exercise gives a quantitative idea of how far one is from having a pole. We do not envisage at this stage a possible source of such a large attraction within our theoretical treatment. There is another exercise which we want to present here. We have regularized the πKN loop function with a cutoff in the three-momentum of 1 GeV. This is the natural scale for the problems we are dealing with. Yet, we could try to see how much Λ has to be increased to find a pole. The exercise conducted is the following: we have taken $\Lambda=4$ GeV and see how much more potential we have to add to get the resonance around 1540 MeV. This is done by adding a potential with a strength of $-2.5(100\ \text{MeV})^{-3}$, which amounts to about doubling the calculated one. The result for $|t|^2$ can be seen in Fig. 3.

What we see is that the width becomes much larger than before. This trend continues in the same manner and we can reduce the amount of extra potential as the cutoff Λ increases (although the dependence of G on the cutoff is by then logarithmic). The width also increases unrealistically for these larger values of Λ. Hence this does not seem to be the adequate path to follow in future searches.

As a positive output there are hopes, given by the trend of the results in Fig. 1 that a resonance could develop at higher

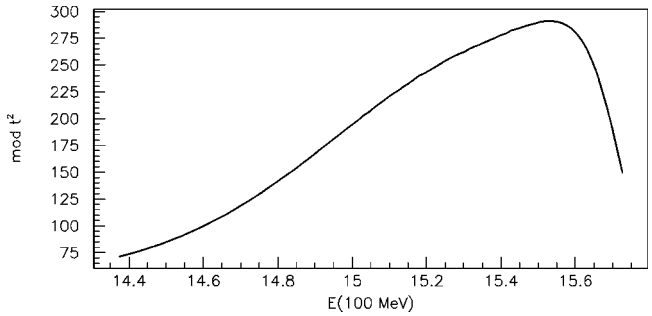


FIG. 3. We also find a resonance, this time too broad, by increasing the cutoff in the κN loop to 4 GeV and about doubling the potential. Units are 100 MeV.

energies above threshold. This would be a task worth following, which however would require to modify technically our approach which has relied on a below threshold situation avoiding the singularities of open physical channels above threshold.

Another point is that we have only partially solved the Faddeev equations, including therein a three-body potential with the basic units repeated in the Faddeev sequence of diagrams. A more standard three-body Faddeev approach would also be one of the tasks worth undertaking. The steps walked here and the dynamics used could be directly input to the full set of Faddeev equations.

In summary, we think our calculation is sufficiently accurate to claim that the nature of the Θ^+ as a bound πKN system is very unlikely, but this should be checked by other independent calculations and different technical approaches, given the importance of this resonance. At last, it would also be interesting to continue with the present study extrapolating the approach above $K\pi N$ threshold to explore the possibility of a resonance at not too high energies but beyond the scope of the present work.

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