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Variational Monte Carlo calculation of ${}_{\Lambda\Lambda}^{6}$He and other *s*-shell hypernuclei

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A variational Monte Carlo analysis of recent binding energy data of the hypernucleus ${}_{\Lambda\Lambda}^{6}$He has been made treating this as a six-body problem. A phenomenological central Urbana-type $\Lambda\Lambda$ potential which fits the new data, predicts a bound state for the charge symmetric pair ${}_{\Lambda\Lambda}^{5}$H, ${}_{\Lambda\Lambda}^{5}$He and just or weakly bound state for ${}_{\Lambda\Lambda}^{4}$H is obtained. A three-range Gaussian $\Lambda\Lambda$ potential phase equivalent to the Nijmegen model D over estimates by 25–80 % the binding energy of ${}_{\Lambda\Lambda}^{6}$He and pair ${}_{\Lambda\Lambda}^{5}$H, ${}_{\Lambda\Lambda}^{5}$He compared to an Urbana potential. The simulated potential predicts bound ${}_{\Lambda\Lambda}^{4}$H. The incremental $\Delta B_{\Lambda\Lambda}$ value, leaving ${}_{\Lambda\Lambda}^{4}$H, for the above potentials is about half of that found in recent cluster model calculation which uses a $\Lambda\Lambda$ potential phase equivalent to the ND type in the Faddeev method.

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I. INTRODUCTION

Despite phenomenal growth in the production of Λ hypernuclei and measurement of their spectra over a wide mass number range, the data on $\Lambda\Lambda$ hypernuclear species [1–4] is limited to three, listed in the Table I along with the $\Lambda\Lambda$ separation energy $B_{\Lambda\Lambda}$. In one case excited state energy has also been measured. Since the last hypernuclear conference HYP2000, a revised event [1], considered to be reliable, is added to the list and a few more such events are likely to be announced in the forthcoming hypernucleus conference. The binding energy $\Delta B_{\Lambda\Lambda}$ $(=B_{\Lambda\Lambda}-2B_{\Lambda}$, where $B_{\Lambda\Lambda}$ and B_{Λ} are separation energies of both $\Lambda\Lambda$ and single Λ, respectively) is closely connected to the $\Lambda\Lambda$ potential. The $\Delta B_{\Lambda\Lambda}$ from the new $B_{\Lambda\Lambda}$ data of ${}_{\Lambda\Lambda}^{6}$He, called the NAGARA event, recently observed in the KEK hybrid experiment [1] E373, is approximately $\frac{1}{4}$th of the *p*-shell data and should not be taken as an indication of the mass number dependence until more data is accumulated.

The (K^{-},K^{+}) reaction on a ^{9}Be target in the Brookhaven alternating-gradient synchrotron experiment E906, has given evidence [4] for bound ${}_{\Lambda\Lambda}^{4}$H $(I=0,J=1^{+})$. This event along with NAGARA has renewed the interest in the theoretical studies of hypernuclei in the $S=-2$ sector. Recently Filikhin and Gal [5] (referred to as FG) and Filikhin, Gal, and Suslov [6] have carried out cluster model analyses of the $B_{\Lambda\Lambda}$ data of ${}_{\Lambda\Lambda}^{6}$He and ${}_{\Lambda\Lambda}^{10}$Be using the Faddeev method. A three-range Gaussian $\Lambda\Lambda$ potential phase equivalent to the Nijmegen model SC97 interaction agrees with the $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{6}$He and predicts a bound ${}_{\Lambda\Lambda}^{5}$H and ${}_{\Lambda\Lambda}^{5}$He. The new $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{6}$He is found to be incompatible [7] with ${}_{\Lambda\Lambda}^{10}$Be, even if a deexcitation of ${}_{\Lambda\Lambda}^{10}$Be* or ${}_{\Lambda}^{9}$Be* via an unobserved γ ray in the detection process is admitted; one such possibility [5,8] involving ${}_{\Lambda}^{9}$Be* at 3.1 MeV is recorded in Table I, thus again questioning the consistency of the existing data. Further Faddeev-Yakubosky calculation [9] for the four-body $\Lambda\Lambda pn$ system over a wide range of $\Lambda\Lambda$ interactions suggests unstable ${}_{\Lambda\Lambda}^{4}$H and while three-body $\Lambda\Lambda d$ model admits a bound state for weak $\Lambda\Lambda$ interaction required to fit ${}_{\Lambda\Lambda}^{6}$He. A very recent four-body stochastic variational calculation [10] of ${}_{\Lambda\Lambda}^{4}$H also predicts bound states.

Recently Nogga, Kamada, and Glöckle [11] have solved the Faddeev equation for the mass number $A=4$ Λ hypernuclei based on a complete meson-theoretical YN interaction. The Nijmegen interaction model [12] SC89 and SC97 series do not reproduce perfectly the binding energy but NSC97f is quite close for $A=4$ binding energies of the 0^{+} and 1^{+} states and reproduces their separation rather well. They also observed that the potential SC97-sim simulated from the Nijmegen model SC97f overestimates by about 30% the B_{Λ} compared to the realistic one. Hence they concluded that the results based on the simulated forces should be taken with caution. Further it will be interesting to compare the results of VMC calculation of B_{Λ} of *s*-shell hypernuclei employing the realistic *NN* interaction, Argonne v_{18} by Sinha, Usmani, and Taib [13] with those of Shoeb *et al.* [14] (abbreviated as SNUK) who used the simplified Malfliet-Tjon *NN* potential in their analysis. The ΛN spin-dependent strength V_{σ} is almost the same and the contribution of dispersive forces to the spin-flip splitting 0^{+}-1^{+} of $A=4$ hypernuclei is consistent in two studies. In view of these findings we consider it appropriate to analyze the *s*-shell $\Lambda\Lambda$ hypenuclei as A-body systems using simplified baryon-baryon (BB) phenomenological forces following the work of SNUK for *s*-shell Λ-hypernuclei.

In this work we explore the nature of the phenomenological Urbana type $\Lambda\Lambda$ force from the study of the recent and

TABLE I. Binding energy of double lambda hypernuclei.

Hypernuclear species	${}_{\Lambda\Lambda}^{5}$He	${}_{\Lambda\Lambda}^{10}$Be	${}_{\Lambda\Lambda}^{13}$B
$B_{\Lambda\Lambda}$(MeV)	7.25 ± 0.19[a]	17.7 ± 0.4[b] (14.6 ± 0.4)[d]	27.6 ± 0.7[c]
$\Delta B_{\Lambda\Lambda}$(MeV)	1.01 ± 0.2	4.3 ± 0.4	4.8 ± 0.7

[a]Reference [1].
[b]Reference [2].
[c]Reference [3].
[d]Reference [8], assuming ${}_{\Lambda\Lambda}^{10}\mathrm{Be}\rightarrow\pi^{-}+p+{}_{\Lambda}^{9}\mathrm{Be}^{*}$.

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TABLE II. ${}^{6}_{\Lambda\Lambda}$He: variational parameter $\kappa_{\Lambda N}, s_{\Lambda N}, \kappa_{NN}, \kappa_{\Lambda\Lambda}, s_{\Lambda\Lambda}$ and kinetic energy $\langle T\rangle$, two-body energy V_{BN}, dispersive non-central three-body energy $\langle V^{NDS}_{\Lambda NN}\rangle$ and last column is total energy for 100 000 points (optimum variational parameters, potential parameters and the energy of subsystem ${}^{5}_{\Lambda}$He: $\kappa_{\Lambda N}=0.125$ fm^{-1}, $s_{\Lambda N}=1.0$, $C_{\Lambda N}=2.0$ fm^{-2}, $a_{\Lambda N}=0.8$ fm, $R_{\Lambda N}=1.0$ fm, $\kappa_{NN}=0.304$ fm^{-1}, $C_{NN}=1.0$ fm^{-2}, $a_{NN}=0.5$ fm, $R_{NN}=1.0$ fm, $V_5=-6.15$ MeV, $\varepsilon=0.25$, $W/6=0.09$ MeV, $E\pm\Delta E=-34.331\pm0.092$ MeV), $C_{\Lambda\Lambda}=1.0$ fm^{-2}, $a_{\Lambda\Lambda}=0.75$ fm, $R_{\Lambda\Lambda}=0.0$.

$\kappa_{\Lambda N}$ (fm^{-1})	$s_{\Lambda N}$	κ_{NN} (fm^{-1})	$\kappa_{\Lambda\Lambda}$ (fm^{-1})	$s_{\Lambda\Lambda}$	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (MeV)	$\langle V^{NDS}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
0.105	0.95	0.404	0.405	1.1	94.16	137.21	4.62	38.43±0.20
0.104	0.90	0.404	0.440	1.2	100.46	142.77	3.33	38.71±0.12

reliable $B_{\Lambda\Lambda}$ data of ${}^{6}_{\Lambda\Lambda}$He, called the NAGARA event. The simulated $\Lambda\Lambda$ potentials are being extensively used in the recent time. Therefore, we choose one of these to analyze the data so that a meaningful comparison with the cluster model could be made. The energy of A=5 and 4 systems is also predicted. The system is analyzed in the spirit of SNUK: freedom in choosing W, the strength of dispersive spin-dependent and noncentral ΛNN force [15] eliminates the need for three-body correlations in the calculation of B_Λ of *s*-shell hypernuclei. This was the case [16] even for the phenomenological central spin-dependent dispersive force proposed by Bodmer and Usmani [17]. From our earlier calculations [14] of *s*-shell Λ hypernuclei it seems that the effect of the two-pion exchanged three-body ΛNN force is being simulated by the variation of W and, therefore there is no need of introducing it in the present work. In the next section BB potentials and wave functions, Hamiltonians, the method of the calculation of $B_{\Lambda\Lambda}$ for the systems of baryon number A and the results are discussed. The conclusions are presented in the last section.

II. ENERGY CALCULATION FOR *A*-BODY *s*-SHELL ΛΛ HYPERNUCLEI AND RESULTS

${}^{6}_{\Lambda\Lambda}$He is treated as a six-body problem and to our knowledge this is the first such analysis. The study requires as input NN, ΛN and dispersive ΛNN potentials fitting ${}^{5}_{\Lambda}$He. We take these potentials to be a NN Malfliet-Tjon [18], a ΛN Urbana-type central spin-dependent potential [14,16,17] that fits the Λp scattering data and a dispersive spin-dependent noncentral ΛNN potential [15]. These potentials explain the B_Λ of ${}^{5}_{\Lambda}$He satisfactorily and we chose the relevant potential strength and variational parameters from the (second row) Table V of SNUK and which are listed in Table II. The only free-parameter is the singlet strength $V_0^{\Lambda\Lambda}$ of the $\Lambda\Lambda$ potential of Urbana type

$$V_{\Lambda\Lambda}(r) = V_c(r) - V_0^{\Lambda\Lambda}T_\pi^2(r), \tag{1}$$

where $V_c(r)=W_c[1+\exp(r-R)/d]^{-1}$ with $W_c=2137$ MeV, $R=0.5$ fm, and $d=0.2$ fm. $T_\pi(r)$ is the one-pion exchange tensor potential shape modified with a cutoff:

$$T_\pi(r) = (1 + 3/x + 3/x^2)[\exp(-x)/x][1-\exp(-cr^2)]^2,$$

with $x=0.7r$ and $c=2$ fm^{-2}. Our BB potentials are reasonable in the sense that their radial form factors agree with the general expectations of meson theoretical models.

Since three-body correlation are not crucial in view of the phenomenology [14] we adopt with regards to W, the strength of dispersive spin-dependent and noncentral ΛNN force, the trial wave function of ${}^{6}_{\Lambda\Lambda}$He is constructed from the product of only central two-body f_{BB} ($BB=\Lambda N$, NN, and $\Lambda\Lambda$ correlation functions as

$$\Psi^{(A)}_{\Lambda\Lambda} = \prod_{i<j} f_{NN}(r_{ij})\left[\prod_{j=1}^{A-2} f_{\Lambda_1 N}(r_{\Lambda_1 j}) \times\prod_{i=1}^{A-2} f_{\Lambda_2 N}(r_{\Lambda_2 i})\right] f_{\Lambda_1\Lambda_2}(r_{\Lambda_1\Lambda_2})\chi^{(A)}, \tag{2}$$

where $\chi^{(A)}$ is the appropriate spin function. Similarly wave functions for single Λ hypernucleus ($\Psi^{(A-1)}_{\Lambda}$) and corresponding core ($\Psi^{(A-2)}_{N}$) may be written. The correlation functions f_{BB} are obtained (vide Refs. [17,19]) from the procedure developed by the Urbana group, which involves the solution of two-body Schrödinger type equations with the appropriate two-body and an auxiliary potential. The latter one is such that the correlation functions have the asymptotic behaviour required by the full A-body equations:

$$f_{BB} \sim r^{-\nu_{BB}}\exp(-\kappa_{BB}r), \tag{3}$$

where the appropriate products of the f_{BB} in the wave function have the asymptotic behavior $\sim r^{-1}\exp(-\kappa_B r)$ if the appropriate choice of ν_{BB} ($\nu_{\Lambda N}=0.5$, $\nu_{\Lambda\Lambda}=-1.0$, $\nu_{NN}=0.0$) is made. However, for convenience we take $\nu_{\Lambda N}=-0.5$, $\nu_{\Lambda\Lambda}=-1.0$, and $\nu_{NN}=0.0$. This is reasonable since such a choice does not affect the result if the energy is optimized with respect to the variational parameters [17]. The parameter κ_B is related to the separation energy of baryon B. Our trial wave function has in total 14 variational parameters: $\kappa_{\Lambda N}$, $s_{\Lambda N}$, $C_{\Lambda N}$, $a_{\Lambda N}$, $R_{\Lambda N}$, $\kappa_{\Lambda\Lambda}$, $s_{\Lambda\Lambda}$, $C_{\Lambda\Lambda}$, $a_{\Lambda\Lambda}$, $R_{\Lambda\Lambda}$, κ_{NN}, C_{NN}, a_{NN}, and R_{NN}. $s_{\Lambda N}$and $s_{\Lambda\Lambda}$ take account the effect of the dispersive ΛNN potential on the $f_{\Lambda N}$ and $f_{\Lambda\Lambda}$ correlations.

The Hamiltonian of $\Lambda\Lambda$ hypernucleus of the baryon number A can be written as the sum of Hamiltonians $\mathrm{H}^{A-1}_{\Lambda k}$ of the subsystem of the Λ_k kth ($k=1,2$) particle and the $A-2$ nucleons of the core and of H^{A-2}_{N} for the $A-2$ nucleons of the core nucleus, and of the singlet $\Lambda\Lambda$ potential given in Eq. (1):

TABLE III. ${}^{5}_{\Lambda\Lambda}$He, ${}^{5}_{\Lambda\Lambda}$H: same as preceding table with $\nu_{\Lambda N}=0.5$, $\nu_{\Lambda\Lambda}=-0.5$, $\nu_{NN}=0.0$ [optimum variational parameters, potential parameters, and the energy of subsystem (${}^{4}_{\Lambda}$He, ${}^{4}_{\Lambda}$H): $\kappa_{\Lambda N}=0.130$ fm^{-1}, $s_{\Lambda N}=1.0$, $C_{\Lambda N}=0.2$ fm^{-2}, $a_{\Lambda N}=0.8$ fm, $R_{\Lambda N}=1.3$ fm, $\kappa_{NN}=0.31$ fm^{-1}, $C_{NN}=2.0$ fm^{-2}, $a_{NN}=0.6$ fm, $R_{NN}=1.3$ fm, $V_4=-6.188$ MeV, $\varepsilon=0.25$, $W/6=0.09$ MeV, $E\pm\Delta E=-10.483\pm0.020$ MeV], $C_{\Lambda\Lambda}=1.0$ fm^{-2}, $a_{\Lambda\Lambda}=0.75$ fm, $R_{\Lambda\Lambda}=0.0$

$\kappa_{\Lambda N}$ (fm^{-1})	$s_{\Lambda N}$	κ_{NN} (fm^{-1})	$\kappa_{\Lambda\Lambda}$ (fm^{-1})	$s_{\Lambda\Lambda}$	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (Mev)	$\langle V^{NDS}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
0.075	0.95	0.303	0.300	1.05	61.71	75.05	0.00	13.34±0.10
0.025	0.95	0.330	0.350	1.00	62.46	76.38	0.00	13.85±0.10

$$H^{A}_{\Lambda\Lambda}=H^{A-1}_{\Lambda_1}+H^{A-1}_{\Lambda_2}+H^{A-2}_{N}+V_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2}), \quad (4)$$

with the Hamiltonians

$$H^{A-1}_{\Lambda_k}=-\frac{\hbar^2}{2m_{\Lambda_k}}\nabla^2_{\Lambda_k}+\sum_{j=1}^{A-2}V_{\Lambda_k N}(r_{j\Lambda_k})+\sum_{i<j}^{A-2}V^{DSN}_{\Lambda_k NN}(r_{ij\Lambda_k})$$

$$H^{A-2}_{N}=-\sum_{i=1}^{A-2}\frac{\hbar^2}{2m_i}\nabla^2_i+\sum_{i<j}^{A-2}V_{NN}(r_{ij}). \quad (5)$$

$\Delta B_{\Lambda\Lambda}$ is written as

$$-\Delta B_{\Lambda\Lambda}=\frac{\langle\Psi^{(A)}_{\Lambda\Lambda}|H^{A}_{\Lambda\Lambda}|\Psi^{(A)}_{\Lambda\Lambda}\rangle}{\langle\Psi^{(A)}_{\Lambda\Lambda}|\Psi^{(A)}_{\Lambda\Lambda}\rangle}-2\frac{\langle\Psi^{(A-1)}_{\Lambda}|H^{A-1}_{\Lambda}|\Psi^{(A-1)}_{\Lambda}\rangle}{\langle\Psi^{(A-1)}_{\Lambda}|\Psi^{(A-1)}_{\Lambda}\rangle}+\frac{\langle\Psi^{(A-2)}_{N}|H^{A-2}_{N}|\Psi^{(A-2)}_{N}\rangle}{\langle\Psi^{(A-2)}_{N}|\Psi^{(A-2)}_{N}\rangle}. \quad (6)$$

The second and third terms on the right have already been evaluated for s-shell Λ-hypernuclei [14]. The first term needs to be optimized with respect to the variational parameters for values of $V_0^{\Lambda\Lambda}$, which give a bound state for ${}^{6}_{\Lambda\Lambda}$He. The variational parameters corresponding to the optimum energy need not be the same for the first and the last two terms. The estimates for the energy were made for 100 000 points. $V_0^{\Lambda\Lambda}$ is then chosen to reproduce the experimental $\Delta B_{\Lambda\Lambda}$ for ${}^{6}_{\Lambda\Lambda}$He. The $\Delta B_{\Lambda\Lambda}=0.99$ MeV for $V_0^{\Lambda\Lambda}=-6.1$ MeV is very close to the experimental value. The component energies: kinetic energy $\langle T\rangle$, two-body interaction energy $\langle V_{BN}\rangle$, and three-body dispersive energy $\langle V^{NDS}_{\Lambda NN}\rangle$ along with the total energy $-E\pm\Delta E$ of ${}^{6}_{\Lambda\Lambda}$He are listed in Table II. Having fixed $V_0^{\Lambda\Lambda}$ we calculate $\Delta B_{\Lambda\Lambda}$ for the $A=5$ charge symmetric pair ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He and for ${}^{4}_{\Lambda\Lambda}$H. For these hypernuclei the energy of the subsystems ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He, and ${}^{3}_{\Lambda}$H and the potential and variational parameters, taken from Table II and III of SNUK and are listed here in Tables III and IV. The hypernuclear pair ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He turns out to be particle stable with $\Delta B_{\Lambda\Lambda}=0.65$ MeV, about 30% less than for ${}^{6}_{\Lambda\Lambda}$He. $\Delta B_{\Lambda\Lambda}\approx 0.1$ to 0.01 MeV for ${}^{4}_{\Lambda\Lambda}$H (relative to deuteron binding 2.24 MeV) brings into question whether it is weakly or just bound.

Further, we perform a Variational Monte Carlo (VMC) calculation replacing the potential (1) with the three-range Gaussian potential (for ND)

$$V_{\Lambda\Lambda}(r)=9324\exp(-r^2/0.35^2)-379.1\exp(-r^2/0.777^2)-21.49\exp(-r^2/1.342^2), \quad (7)$$

given in Ref. [5] which is a simulated meson theoretical interaction of the Nijmegen group. This choice is because the results for this potential are expected to be intermediate to the models ESC00 and NSC97 and facilitates a comparison of $\Delta B_{\Lambda\Lambda}$ with that of FG. The results are listed in the third row of Tables II–IV. The $\Delta B_{\Lambda\Lambda}$ from simulated meson theoretical interaction of the Nijmegen group are 1.24 MeV ($A=6$), 1.14 MeV ($A=5$), and 0.48 MeV ($A=4$).

Our calculated values of $\Delta B_{\Lambda\Lambda}$ for the three-range Gaussian potential are about 50–60 % lower compared to 2.11, 2.27 MeV for ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He and 2.91 MeV for ${}^{6}_{\Lambda\Lambda}$He obtained with the cluster model by FG for the Nijmegen model D. Another interesting feature of our four-body calculation is that ${}^{4}_{\Lambda\Lambda}$H turns out to be bound for the ND potential whereas no clearcut result is obtained in Ref. [9]. A recent stochastic variational four-body calculation [10] of ${}^{4}_{\Lambda\Lambda}$H using the potentials given by FG also predicts a bound state. Thus results of two types of variational calculations for ${}^{4}_{\Lambda\Lambda}$H are consistent for the simulated potentials. The statement that simulated interaction over estimates [11] the energy in the s-shell Λ hypernuclei if extrapolated for the $\Lambda\Lambda$ s-shell hypernuclei

TABLE IV. ${}^{4}_{\Lambda\Lambda}$H: same as preceding table with $\nu_{\Lambda N}=0.05$, $\nu_{\Lambda\Lambda}=0$, $\nu_{NN}=0$ (optimum variational parameters, potential parameters and the energy of subsystem ${}^{3}_{\Lambda}$H:, $\kappa_{\Lambda N}=0.07$ fm^{-1}, s$_{\Lambda N}=1.0$, $C_{\Lambda N}=3.70$ fm^{-2}, $a_{\Lambda N}=1.6$ fm, $R_{\Lambda N}=3.30$ fm, $\kappa_{NN}=0.27$ fm^{-1}, $C_{NN}=3.7$ fm^{-2}, $a_{NN}=1.6$ fm, $R_{NN}=3.30$ fm, $V_3=-6.220$ MeV, $\varepsilon=0.25$, $W/6=0.09$ MeV, $E\pm\Delta E=-2.343\pm0.026$ MeV), $C_{\Lambda\Lambda}=1.0$ fm^{-2}, $a_{\Lambda\Lambda}=0.75$ fm, $R_{\Lambda\Lambda}=0.0$.

$\kappa_{\Lambda N}$ (fm^{-1})	$s_{\Lambda N}$	κ_{NN} (fm^{-1})	$\kappa_{\Lambda\Lambda}$ (fm^{-1})	$s_{\Lambda\Lambda}$	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (MeV)	$\langle V^{NDS}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
0.025	0.94	0.280	0.010	0.85	29.48	32.76	0.396	2.513±0.058
0.045	1.0	0.280	0.010	1.00	36.03	39.55	0.560	2.966±0.073

analyzed here then $\Delta B_{\Lambda\Lambda}$ found here are expected to be slightly reduced for the use of realistic interactions. Further, energies of $\Lambda\Lambda$ *s*-shell hypernuclei are being over estimated in cluster model compared to microscopic VMC calculation as the two-body correlations in the former are masked while the latter one includes these explicitly.

Recently Nemura, Akaishi, and Suzuki [20] and earlier others (e.g., vide Ref. [21]) have made an observation from the study *s*-shell hypernuclei that the energy of the core is affected by the presence of the Λ, a characteristic of all the calculations [13,14,16,17,19] incorporating correlations in the wave functions. This is expected to be situation for $\Lambda\Lambda$ hypernuclei. However, the effect of rearrangement energy is likely to be largely cancelled in the $\Delta B_{\Lambda\Lambda}$, being a difference of energies of two hypernuclei. Therefore, the evaluation of behavior of $V_{\Lambda\Lambda}(r)$ from the $\Delta B_{\Lambda\Lambda}$ is likely to be least affected from the effect of core rearrangement energy.

III. CONCLUSIONS

In this work we studied $\Lambda\Lambda$ *s*-shell hypernuclei in VMC approach treating these *A*-body systems where central two-body correlations consistent with the simple BB interaction are properly taken care of. The Urbana $\Lambda\Lambda$ potential, which fits the $B_{\Lambda\Lambda}$ data of ${}^{6}_{\Lambda\Lambda}\text{He}$, predicts the particle stable systems ${}^{5}_{\Lambda\Lambda}\text{H}$, ${}^{5}_{\Lambda\Lambda}\text{He}$ but the question of stability of ${}^{4}_{\Lambda\Lambda}\text{H}$ cannot be clearly answered due to the large statistical error in our calculation. VMC calculation for the simulated three-range Gaussian potential (for ND) gives the bound state energies for $A=5$ and 6 hypernuclei much lower than the cluster model in Faddeev approach. Our result of four-body ${}^{4}_{\Lambda\Lambda}\text{H}$ calculations for the simulated potential is consistent with those of Ref. [10] but disagrees with that of FG. The inconsistency in the results of present work and the cluster model approach appears to be a reflection of not incorporating two-body *BB* correlations in the later one, an inherent limitation of cluster model methods. Of the many meson-theoretical interaction models and the phenomenological $\Lambda\Lambda$ potentials currently in use which one is the correct one could be decided by the future experiments confirming the existence ${}^{4}_{\Lambda\Lambda}\text{H}$. Further revising and improving the statistics of existing events and addition of new species with reliable $B_{\Lambda\Lambda}$ values will be useful in extracting the $\Lambda\Lambda$ interaction.

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