Warm stellar matter with neutrino trapping

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The properties of hybrid stars formed by hadronic and quark matter in β equilibrium at fixed entropies are described by appropriate equations of state (EOSs) in the framework of relativistic mean-field theory. In this work we include the possibility of neutrino trapped EOS and compare the star properties with the ones obtained after deleptonization, when neutrinos have already diffused out. We use the nonlinear Walecka model for the hadron matter with two different sets for the hyperon couplings and the MIT Bag and the Nambu-Jona-Lasinio (NJL) models for the quark matter. The phase transition to a deconfined quark phase is investigated. Depending on the model and the parameter set used, the mixed phase may or may not exist in the EOSs at high densities. The star properties are calculated for each equation of state. The maximum mass stellar configurations obtained within the NJL have larger masses than the ones obtained within the Bag model. The Bag model predicts a mixed phase in the interior of the most massive stable stars while, depending on the hyperon couplings, the NJL model predicts a mixed phase or pure quark matter. Comparing with neutrino free stars, the maximum allowed baryonic masses for protoneutron stars are $\sim 0.4 M_\odot$ larger for the Bag model and $\sim 0.1 M_\odot$ larger for the NJL model when neutrino trapping is imposed.

I. INTRODUCTION

Protoneutron stars appear as an outcome of the gravitational collapse of a massive star. During its early evolution a protoneutron star with an entropy per baryon of the order of 1 to 2 contains trapped neutrinos. After 10 to 20 s, the star stabilizes at practically zero temperature and no trapped neutrinos are left [1]. The structure of compact stars is characterized by its mass and radius, which are obtained from appropriate equations of state (EOSs) at densities about one order of magnitude higher than those observed in ordinary nuclei. EOSs can be derived either from relativistic or potential models. The last ones are normally developed within a nonrelativistic formalism [2] and most of them are only suited at low densities because the EOS becomes acausal, i.e., the speed of sound exceeds the speed of light at densities which are relevant for neutron and protoneutron stars. Moreover, these models may lead to symmetry energies that decrease much more than expected beyond three times saturation density and this is a serious deficiency for neutron stars, which are highly asymmetric systems. In relativistic models these problems are not present. In this work we investigate the properties of hybrid stars formed by hadronic matter at low densities, mixed matter at intermediate densities, and quark matter at high densities. We use the nonlinear Walecka model for the hadron phase [3,4] and the MIT Bag [5] and the Nambu-Jona-Lasinio (NJL) [6,7] models for the quark matter.

In a previous work [8] we have used the same formalism in order to study the properties of hybrid stars obtained from EOSs at different temperatures. We have checked that the maximum masses of hybrid stars obtained with the Bag model are of the order of $\sim 1.6 M_\odot$, smaller than the maxi-

mum masses obtained within the NJL model, $\sim 1.9 M_\odot$. Even with a transition to a deconfinement phase the masses predicted can be quite high. However, the effect of temperature in the maximum masses allowed is not strong. The central energy densities, though, decrease with temperature. Some of the features exhibited for the Bag model were quite different from the ones obtained with the NJL model, due to the chiral conservation mechanism implicit in the latter one.

In this work, we verify the importance of including trapped neutrinos and consider entropies from zero to two Boltzmann units. Most of the formulas used for the Lagrangian densities, energies, pressures, partition functions, etc., are not shown in this paper because they are standard equations and can be seen, among others, in Ref. [8]. We compare the properties of warm stars obtained within the NJL model and the MIT Bag model, namely, strangeness content, neutrino fraction, onset of hyperons, mixed phase and quark phase, maximum allowed mass, and interior composition.

The present paper is organized as follows: in Sec. II the Lagrangian densities of the models used in the hadronic and quark matter are described, important relations and parameter sets are displayed, and the mixed phase is implemented. The results are shown and discussed in Sec. III and in the last section some remarks are made.

II. FORMALISM

For the hadron phase we have used the nonlinear Walecka model with the inclusion of hyperons. The Lagrangian density of the model reads
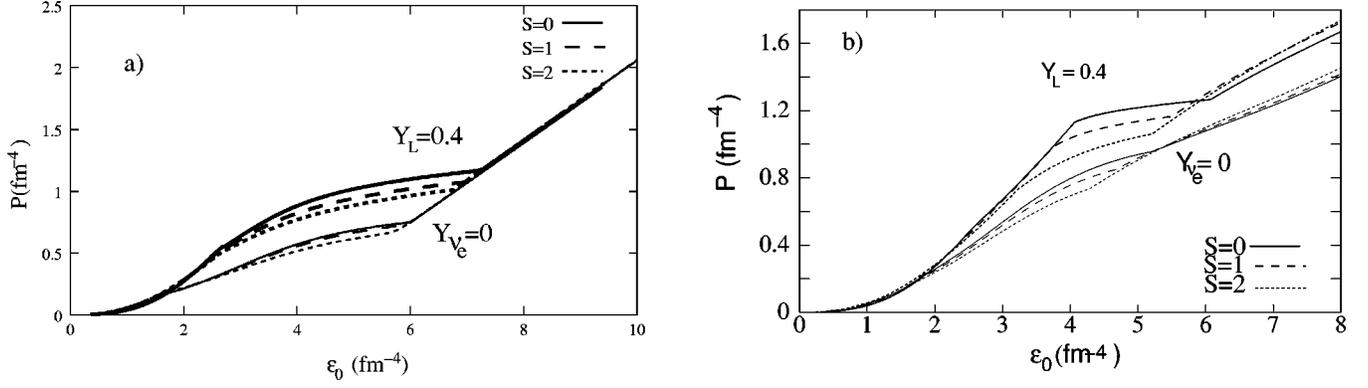
FIG. 1. EOS obtained with the GL force plus (a) Bag model and (b) NJL model.

$$\mathcal{L}_{NLWM} = \sum_B \bar{\psi}_B [\gamma_\mu (i\partial^\mu - g_{vB}V^\mu - g_{\rho B}\mathbf{t}\cdot\mathbf{b}^\mu)$$

$$- (M_B - g_{sB}\phi)]\psi_B + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2)$$

$$- \frac{1}{3!}\kappa\phi^3 - \frac{1}{4!}\lambda\phi^4 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu$$

$$+ \frac{1}{4!}\xi g_v^4 (V_\mu V^\mu)^2 - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu$$

$$+ \sum_l \bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l, \tag{1}$$

with Σ_B extending over the eight baryons, $g_{sB}=x_{sB}g_s$, $g_{vB}=x_{vB}g_v$, $g_{\rho B}=x_{\rho B}g_\rho$ and x_{sB}, x_{vB}, and $x_{\rho B}$ are equal to 1 for the nucleons and acquire different values in different parametrizations for the other baryons, $\Omega_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$, $\mathbf{B}_{\mu\nu}=\partial_\mu\mathbf{b}_\nu-\partial_\nu\mathbf{b}_\mu-g_\rho(\mathbf{b}_\mu\times\mathbf{b}_\nu)$ and $\mathbf{t}$ is the isospin operator.

We have chosen to work with a parametrization which describes the properties of saturating nuclear matter proposed in Ref. [9], since other common parameter sets, namely, TM1 [10] and NL3 [11] proved to be inadequate because, due to the inclusion of hyperons, the nucleon mass becomes negative at relatively low densities. The chosen parameters, in the sequel referred to as GL, are $g_s^2/m_s^2 = 11.79$ fm^2, $g_v^2/m_v^2 = 7.148$ fm^2, $g_\rho^2/m_\rho^2 = 4.41$ fm^2, $\kappa/M = 0.005\ 896$, and $\lambda = -0.000\ 642\ 6$, for which the binding energy is -16.3 MeV at the saturation density $\rho_0 = 0.153$ fm^{-1}, the symmetry coefficient is 32.5 MeV, the compression

modulus is 300 MeV, and the effective mass is $0.7M$. For the meson-hyperon coupling constants we have opted for two sets discussed in the literature. Set (a): According to Refs. [9,12] we choose the hyperon coupling constants constrained by the binding of the Λ hyperon in nuclear matter, hypernuclear levels, and neutron star masses ($x_\sigma=0.7$ and $x_\omega=x_\rho=0.783$) and assume that the couplings to the Σ and Ξ are equal to those of the Λ hyperon. Set (b): we take $x_{sB}=x_{vB}=x_{\rho B}=\sqrt{2/3}$ as in Refs. [13,14]. This choice is based on quark counting arguments.

The Lagrangian density for the MIT Bag model is identical to the one for the leptons, except for the degeneracy factor, which also accounts for the number of quark colors. In the energy density a factor $+B$ and in the pressure a factor $-B$ are inserted. This factor is responsible for the simulation of confinement. For the Bag model, we have taken $B^{1/4}=190$ MeV.

For the NJL model, the Lagrangian density is

$$L = \bar{q}(i\gamma^\mu\partial_\mu - m)q + g_S\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2]$$

$$+ g_D\{\det[\bar{q}_i(1+\gamma_5)q_j] + \det[\bar{q}_i(1-\gamma_5)q_j]\}, \tag{2}$$

where $q=(u,d,s)$ are the quark fields and λ_a ($0\leq a\leq 8$) are the U(3) flavor matrices. The model parameters are $m=\text{diag}(m_u,m_d,m_s)$, the current quark mass matrix ($m_d=m_u$), the coupling constants g_S and g_D, and the cutoff in three-momentum space, Λ.

TABLE I. Hybrid star properties for the EOS obtained with the GL force and the NJL model for fixed entropies and with neutrino trapping ($Y_L=0.4$).

	S	$M_{max}/M_\odot$	$M_{B\ max}/M_\odot$	ε_0(fm^{-4})	ε_{min}(fm^{-4})	ε_{max}(fm^{-4})
Set (a)	0	2.04	2.27	5.91	4.06	5.79
$x_s=0.7$	1	2.04	2.26	5.53	4.21	5.47
$x_\omega=0.783=x_\rho$	2	1.94	2.10	5.26	3.25	5.20
Set (b)	0	2.05	2.29	6.38	4.92	6.94
$x_H=\sqrt{2/3}$	1	1.98	2.19	6.08	4.72	6.46
	2	1.96	2.12	5.68	4.50	6.00

TABLE II. Hybrid star properties for the EOS obtained with the GL force and the NJL model for fixed entropies and no neutrino trapping ($Y_{\nu_e}=0$).

	S	$M_{max}/M_\odot$	$M_{B\ max}/M_\odot$	$\varepsilon_0(\text{fm}^{-4})$	$\varepsilon_{min}(\text{fm}^{-4})$	$\varepsilon_{max}(\text{fm}^{-4})$
Set (a)	0	1.91	2.20	4.90	1.30	5.21
$x_s=0.7$	1	1.88	2.13	4.80	1.09	4.66
$x_\omega=0.783=x_\rho$	2	1.82	2.01	4.66	1.01	4.28
Set (b)	0	1.84	2.09	6.26	4.60	7.25
$x_H=\sqrt{2/3}$	1	1.84	2.09	5.91	4.35	6.62
	2	1.82	2.02	5.33	3.18	5.66

We consider the set of parameters as [15,16]: $\Lambda=631.4$ MeV, $g_S\Lambda^2=1.824$, $g_D\Lambda^5=-9.4$, $m_u=m_d=5.6$ MeV, and $m_s=135.6$ MeV which were fitted to the following properties: $m_\pi=139$ MeV, $f_\pi=93.0$ MeV, $m_K=495.7$ MeV, $f_K=98.9$ MeV, $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=-(246.7\text{ MeV})^3$, and $\langle\bar{s}s\rangle=-(266.9\text{ MeV})^3$.

The condition of chemical equilibrium is imposed through the two independent chemical potentials for neutrons μ_n and electrons μ_e and it implies that the chemical potential of baryon B_i is $\mu_{B_i}=Q_i^B\mu_n-Q_i^e\mu_e$, where Q_i^e and Q_i^B are, respectively, the electric and baryonic charge of baryon or quark i. Charge neutrality implies $\Sigma_{B_i}Q_i^e\rho_{B_i}+\Sigma_l q_l\rho_l=0$ where q_l stands for the electric charges of leptons. In the mixed phase charge neutrality is imposed globally, $\chi\rho_c^{QP}+(1-\chi)\rho_c^{HP}+\rho_c^l=0$, where ρ_c^{iP} is the charge density of the phase i, χ is the volume fraction occupied by the quark phase, and ρ_c^l is the electric charge density of leptons. We consider a uniform background of leptons in the mixed phase since Coulomb interaction has not been taken into account. According to the Gibbs conditions for phase coexistence, the baryon chemical potentials, temperatures, and pressures have to be identical in both phases, i.e., $\mu_{HP,n}=\mu_{QP,n}=\mu_n$, $\mu_{HP,e}=\mu_{QP,e}=\mu_e$, $T_{HP}=T_{QP}$, $P_{HP}(\mu_n,\mu_e T)=P_{QP}(\mu_n,\mu_e,T)$, reflecting the needs of chemical, thermal, and mechanical equilibrium, respectively.

If neutrino trapping is imposed to the system, the beta equilibrium condition is altered to $\mu_{B_i}=Q_i^B\mu_n-Q_i^e(\mu_e-\mu_{\nu_e})$. In this work we have not included trapped muon neutrinos. Because of the imposition of trapping the total leptonic number is conserved, i.e., $Y_L=Y_e+Y_{\nu_e}=0.4$. As already mentioned, neutrino trapping is important during the cooling of the protoneutron star. Hence, at $S=0$ ($T=0$), it is not an

expected mechanism. Even though, we have chosen to show some results with neutrino trapping at $S=0$ only for the sake of comparison with results in the literature and with results for higher entropies.

In the following section the results obtained are shown and discussed.

III. RESULTS AND DISCUSSION

In all figures shown, unless stated otherwise, set (a) for the meson-hyperon coupling constants was used.

In Fig. 1, the EOSs obtained with both quark models are displayed for $S=0$, $S=1$, and $S=2$ with neutrino trapping ($Y_L=0.4$) and no neutrinos ($Y_{\nu_e}=0$). One can immediately see that the EOSs are harder and the mixed phase appears at higher densities if neutrino trapping is required independently of the model used. The energy density for the onset of the mixed phase can also be seen in Tables I–IV. In general, the effect of temperature both in neutrino rich and neutrino poor matter is to decrease the density for the onset of the mixed phase. The only exception corresponds to neutrino poor matter obtained using the Bag model for the quark phase. This, however, does not affect the maximum mass of a stable star which in all cases decreases with increasing entropy, more strongly when the Bag model is used. An existing difference between the EOSs constructed with the NJL or the Bag model is the behavior of the respective EOSs in the quark phase: contrary to the Bag model, the NJL model predicts an increase of the stiffness of the EOSs both with increasing entropy and with neutrino trapping. However, this behavior does not influence the properties of the compact stars because the calculated central densities of the most

TABLE III. Hybrid star properties for the EOS obtained with the GL force and the MIT Bag model for fixed entropies and with neutrino trapping ($Y_L=0.4$).

	S	$M_{max}/M_\odot$	$M_{B\ max}/M_\odot$	$\varepsilon_0(\text{fm}^{-4})$	$\varepsilon_{min}(\text{fm}^{-4})$	$\varepsilon_{max}(\text{fm}^{-4})$
Set (a)	0	2.00	2.22	5.06	2.73	7.38
$x_s=0.7$	1	1.91	2.07	4.95	2.52	6.95
$x_\omega=x_\rho=0.783$	2	1.83	1.92	4.76	2.53	6.75
Set (b)	0	1.98	2.19	5.26	3.47	7.38
$x_H=\sqrt{2/3}$	1	1.93	2.00	5.06	3.03	6.99
	2	1.81	2.19	5.35	2.98	6.82

TABLE IV. Hybrid star properties for the EOS obtained with the GL force and the MIT Bag model for fixed entropies and no neutrino trapping ($Y_{\nu_e}=0$).

	S	$M_{max}/M_{\odot}$	$M_{B\ max}/M_{\odot}$	ε_0(fm^{-4})	ε_{min}(fm^{-4})	ε_{max}(fm^{-4})
Set (a)	0	1.64	1.83	4.50	1.53	6.00
$x_s=0.7$	1	1.50	1.64	4.82	1.59	5.90
$x_\omega=x_\rho=0.783$	2	1.50	1.62	4.60	1.67	5.73
Set (b)	0	1.64	1.83	4.50	1.81	6.06
$x_H=\sqrt{2/3}$	1	1.51	1.57	4.65	1.87	6.00
	2	1.51	1.63	4.64	1.98	5.81

massive stellar configurations usually lie within the range of energy densities of the mixed phase, as shown in Tables I–IV or are at most at the borderline of the quark phase.

As already discussed in Ref. [8], the presence of strangeness in the core and crust of neutron and protoneutron stars has important consequences in understanding some of their properties. In Fig. 2 we show, for the different EOSs, the strangeness fraction defined as $r_s=\chi r_s^{QP}+(1-\chi)r_s^{HP}$ with r_s^{QP} and r_s^{HP} the quark and hadronic strangeness fraction, respectively. For a quark phase described by the Bag model the strangeness fraction rises steadily and, at the onset of the pure quark phase it has almost reached $1/3$ of the baryonic matter if no trapping is imposed. However, it reaches a lower value once neutrino trapping is enforced. This behavior is independent of the hyperon-meson coupling constants used in this work. The effect of the temperature is to increase the strangeness fraction. The NJL model predicts a different behavior. In the mixed phase the strangeness fraction decreases with density. This behavior is due to the fact that for the densities at which the mixed phase occurs the mass of the strange quark is still very high. In the hadron and mixed phases the strangeness fraction depends on the parameter set employed and it is larger for set (b). In general, temperature increases the strangeness content and trapping decreases the strangeness fraction. It is interesting to compare the Bag model and NJL model results. At $10\rho_0$ we get with the NJL model a strangeness fraction of 0.25 for neutrino free matter and 0.18 with trapping. These numbers should be compared with the results obtained with the Bag model, respectively, 0.31 and 0.27. This trend, which is valid for all densities except for the mixed phase with set (a), is due to the large s-quark mass in

the NJL and has consequences on the difference of the maximum baryonic mass of the stars with and without neutrino trapping.

Analyzing Fig. 2 we can also discuss the model dependence of the hyperon and mixed phase onset. In general, there are larger fractions of hyperons if the NJL model is used. This is true for both the hadronic and the mixed phase and is a consequence of the large s-quark mass in this model. Trapping pushes the onset of hyperons, the mixed phase, and the pure quark phase to higher energies. The pure quark phase is only slightly affected but the mixed phase can occur at a density that is $1-2\rho_0$ higher. For neutrino rich matter, the onset of hyperons in the NJL model always occurs before the onset of the mixed phase even for $T=0$, contrary to neutrino free matter. The imposition of trapped neutrinos influences the threshold of hyperons and quarks through the conditions of charge conservation and chemical equilibrium.

In order to better understand the importance of the neutrinos when neutrino trapping is imposed, in Fig. 3 the fraction of neutrinos is shown for increasing entropy. The behavior encountered for the Bag model (all parameter sets used) and the NJL model for set (a) is quite similar. It decreases at low densities when the electron fraction is still increasing with density and starts to increase in the hadron phase after the onset of hyperons, increases even more in the mixed and quark phases, where the fraction of electrons decreases continuously due to the onset of negative charged hyperons or the u and s quarks. In the mixed phase described within NJL for set (b), the neutrino fraction decreases, due to the decrease of hyperons and the very slow onset of the s quark. In general, the amount of neutrinos depends on the fraction of hyperons and quarks present in each phase, which are deter-
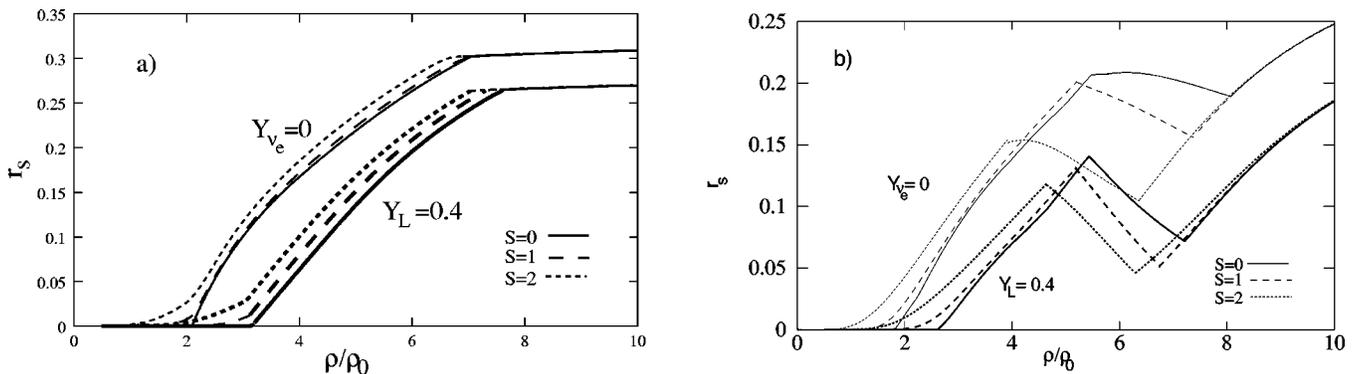
FIG. 2. Strangeness fraction r_s for the EOS obtained with the GL force plus (a) Bag model, (b) NJL model, and set (b).
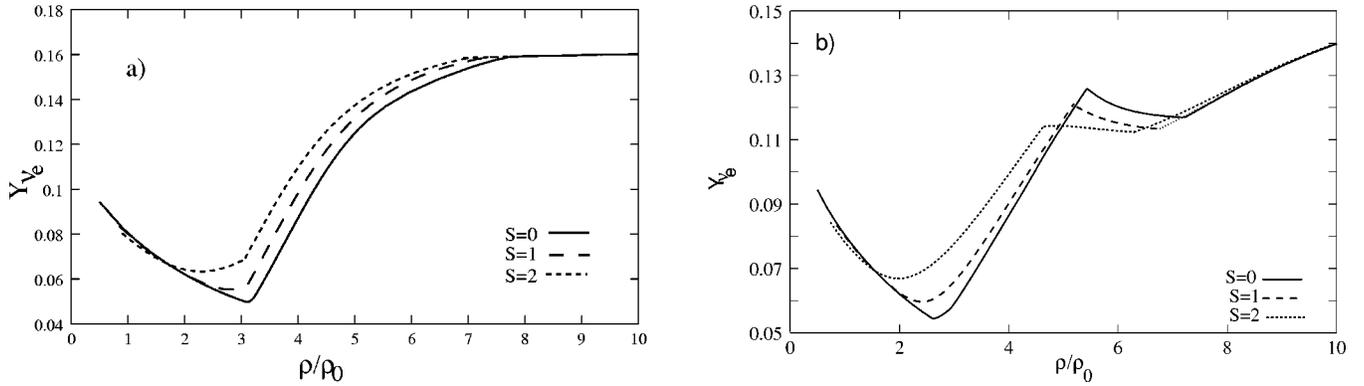
FIG. 3. Neutrino fraction for the EOS obtained with the GL force plus (a) Bag model, (b) NJL model, and set (b).

mined by the model used. For any density the Bag model predicts higher neutrino yields.

Finally, in Fig. 4 the temperature is shown as a function of the baryon density for increasing entropy and for $Y_{\nu_e}=0$ and $Y_L=0.4$. The opening of new degrees of freedom has an important effect on the variation of temperature with density [17]. Neutrino trapping makes the temperature vary more with baryonic density: temperature attains a higher value before the onset of the mixed phase because trapping hinders the onset of hyperons, e.g., of new degrees of freedom; on the other end temperature becomes lower in the quark phase because of the presence of more degrees of freedom, e.g., both leptons and quarks in opposition to the neutrino free matter which contains only quarks. Both with or without neutrino trapping the mixed phase is characterized by a decrease of the temperature. This behavior, a colder high density EOS, is due to deconfinement, and therefore to the appearance of a greater number of degrees of freedom. A similar behavior was obtained in Ref. [18].

Mixed protoneutron and neutron star profiles can be obtained from all the EOSs studied by solving the Tolman-Oppenheimer-Volkoff equations [19], resulting from a simplification of Einstein's general relativity equations for spherically symmetric and static stars. In Tables I–IV we show the values obtained for the maximum gravitational and baryonic masses of a neutron or protoneutron star as function of the central density for the EOSs studied in this work and

for three fixed entropies. The results are shown for the properties of the stars with and without neutrino trapping. In Tables I and II the GL force and the NJL model were used to derive the full EOS, while in Tables III and IV the EOS was obtained from the GL force and the MIT Bag model. Several conclusions can be drawn. For most EOSs studied, the central energy density ε_0 of the most massive stable stars falls inside the mixed phase, whose energy density limits are shown as ε_{min} and ε_{max}. This is always true for the stars described with the MIT Bag model. The results obtained within the NJL model depend on the hyperon couplings used: for set (a) the core of the most massive stars is a pure quark phase, both for neutrino rich or neutrino free matter. The maximum baryonic masses of the stars decrease with increasing entropy and are systematically larger if neutrino trapping is enforced. As a consequence, in the present description the most massive stars with neutrino trapping are unstable after cooling. Similar results have already been discussed for stars with strange matter [1,24]. Comparing Tables I–IV we conclude that the Bag model allows for smaller maximum gravitational masses, of the order $\sim 1.9 M_\odot$ if neutrino trapping is imposed and $\sim 0.4 M_\odot$ lower otherwise, than the NJL model, $\sim 2.0 M_\odot$ with neutrino trapping and $\sim 0.15 M_\odot$ lower without it. This has been checked also for other Bag constants. Comparing baryonic masses a similar conclusion is taken, i.e., within the Bag model the most massive stable stars with neutrino trapping are systematically $\sim 0.4 M_\odot$ higher than the corresponding neutrino free
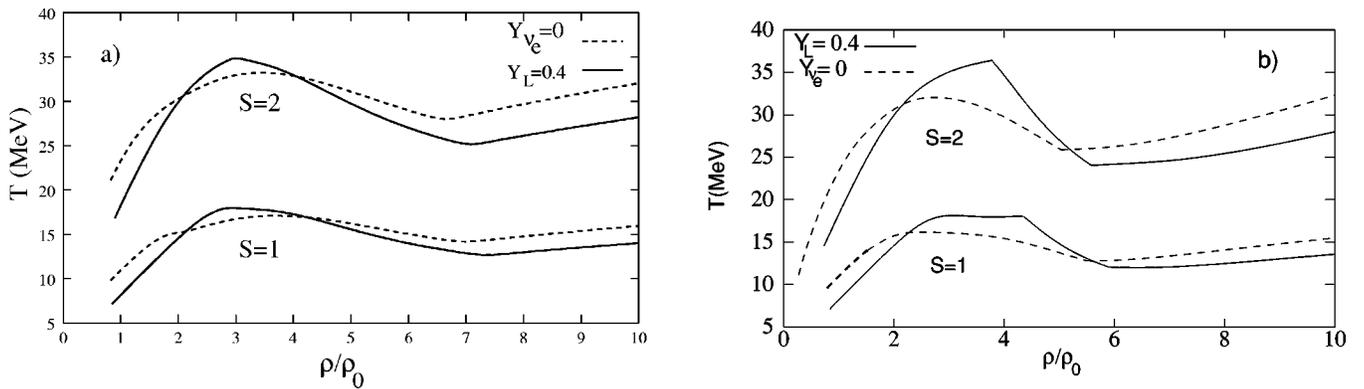
FIG. 4. Temperature range obtained with the GL force plus (a) Bag model and (b) NJL model. In both figures the solid lines stand for the case with neutrino trapping and the dashed line without neutrino trapping.

stars. This difference reduces to $\sim 0.1 M_\odot$ within the NJL model.

It is also seen that the maximum mass does not depend much on the hyperon couplings. One can also observe that the mixed phases start at higher densities if neutrino trapping is considered and they tend to shrink with increasing entropy. In general, changes in the maximum mass due to neutrino trapping are larger than those due to variations in the entropy of the system. Our results for the Bag model are systematically larger than those shown in Ref. [1] where a Bag constant equal to $(197 \text{ MeV})^4$, corresponding to a harder quark EOS, was used. We do not discuss the radius of the maximum mass star because it is sensitive to the low density EOS and we did not describe properly this range of energy densities.

IV. FINAL REMARKS

In the present paper we have studied the EOSs for proto-neutron stars using both the Bag model and the NJL model for describing the quark phase and a relativistic mean-field description in which baryons interact via the exchange of $\sigma-, \omega-, \rho-$ mesons for the hadron phase. The EOSs were constructed with and without the imposition of neutrino trapping at three fixed entropies.

For the hadron part of the EOSs we have considered a parametrization which describes the properties of saturating nuclear matter proposed in Ref. [9]. For the hyperon-meson coupling constants we have used two choices and verified that for the NJL model the onset of the mixed phase and the properties of the corresponding compact star are sensitive to the hyperon couplings.

Some properties of the EOSs, such as the strangeness fraction and the amount of neutrinos, show different patterns depending on the quark model used, NJL or Bag model. Both strangeness and neutrino fractions are higher within the MIT Bag model. While the strangeness fraction increases monotonically with density for the MIT Bag model, the NJL model predicts a decrease of the strangeness fraction in the mixed phase, due to the large s-quark mass.

The EOSs with trapped neutrinos are harder, have smaller mixed phases which occur at higher densities. These proper-ties influence the properties of the corresponding compact stars: the maximum baryonic allowed mass of a stable star is higher when neutrinos are trapped. After cooling and delep-tonization these stars will become unstable and decay into low mass blackholes [1,24]. However, it should be pointed out that the mass difference is much smaller within the NJL model. In this model, due to the only partial chiral symmetry restoration the s-quark mass is still quite high at the densities of interest for compact stars and therefore, the strange content is smaller than the one obtained with the Bag model.

Another important characteristic of the EOSs studied is the decrease of the temperature with density which occurs in the mixed phase. This is true for both models, NJL and Bag model, and for neutrino trapped or neutrino free matter. The temperature decrease is much stronger for matter with trapped neutrinos.

Within the present formalism, the core of the most massive stable stars, with few exceptions, lies within the mixed phase, excluding the possibility of stars with a quark core. For the Bag model this fact is independent of the hyperon coupling, contrary to what is observed with the NJL model: within this model the existence of stars with a quark core depends on the hyperon couplings chosen.

For the quark phase we have chosen to use always un-paired quark matter. Recently, many authors [20–23] have discussed the possibility that the quark matter is in a color-superconducting phase, in which quarks near the Fermi surface are paired, forming Cooper pairs which condense and break the color gauge symmetry [25]. At sufficiently high density the favored phase is called color flavor locked phase, in which quarks of all three colors and all three flavors are allowed to pair. The consequences of using such paired quark phase in the construction of the EOSs for the mixed phase are being investigated.

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