Neutron capture reactions in strong magnetic fields of magnetars

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The statistical model is employed to investigate (n,γ) reactions in ultrastrong magnetic fields relevant for supernovae and neutron stars. The predominant mechanisms are argued to correspond to modifications of nuclear level densities and γ-transition energies due to interactions of the field with magnetic moments of nuclei. The density of states reflects the nuclear structure and results in oscillations of reaction cross sections as a function of the field strength, while magnetic interaction energy enhances radiative neutron capture process.

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Ultrastrong magnetic fields can develop in supernovae due to, e.g., the violent convective motion [1]. Recent theoretical studies, see, e.g., Ref. [2], and references therein, suggest enormous magnetic fields in nascent neutron star crusts with a strength ranging up to $B\sim 10^{17.5}$ G. During past decade the observations of soft γ repeaters (SGRs) and anomalous x-ray pulsars (AXPs) brought numerous evidences in support of such a "magnetar" concept. Such evidences appear as, e.g., short bright outbursts [3–5] and optical data [6] for SGRs, rapid braking of relatively slowly rotating neutron stars identified with SGRs [7,8] and AXPs [9,10]. Such pulsars are associated with supernova remnants (see, e.g., Ref. [5], and references therein).

Assertion of a possibility of ultramagnetized stellar media rises the question of the effect of magnetic field in nuclide transformations [11–13]. Incorporating magnetic field effects in an analysis of nuclear reaction network might provide more insights on supernovae and neutron stars, in particular, magnetodynamics at neutron star crusts formation. In this paper we consider an example of (n,γ) reactions.

The Hauser-Feshbach statistical approach constitutes useful framework for theoretical predictions of nuclear reaction cross sections for the vast number of medium and heavy nuclei which exhibit relatively high density of excited states already at neutron separation energies. At appropriate excitation energies the small level spacing in the compound nucleus allows us to make use of the statistical method calculations for compound nuclear reactions (see, e.g., Refs. [14,15]) with strongly overlapping resonances. The credibility of such a treatment for astrophysics has been extensively discussed recently, e.g., in Ref. [16]. The only necessary condition for model application is, in fact, large number of resonances at corresponding relative velocity v_{nI} of neutrons n and nuclei I, when the cross section can be described in terms of quantities averaged over resonances. In the most practical cases statistical model yields highly accurate cross sections, when the required ingredients are sufficiently reliable. Thus, an analysis of magnetic field dependence of respective inputs provides an information on the field effect in the reaction rates.

Within statistical theory the nuclear reaction cross sections are expressed in terms of smoothed transmission coefficients T which describe the absorption via an imaginary part of the (optical) nucleon-nucleus potential [17]. The high level density in the compound nucleus allows us to employ such an average treatment washing out resonance features. Since the projection m of a spin on magnetic field axis represents conserved quantum number (see Ref. [13], Eqs. (4) and (5), and discussion therein) we write the cross section of $I(n,\gamma)O$ reaction as

$$\sigma = \pi \lambda^2 \sum_{m_\pi} \frac{T_n(m_\pi)T_\gamma(m_\pi)}{T_{\mathrm{tot}}(m_\pi)}, \qquad (1)$$

where the wavelength $\lambda = \hbar/\mu_r v_{nI}$ with reduced mass μ_r, and the sum runs over the spin-projection parity m_π of compound nucleus.

The total transmission coefficient for ith channel $T_i(m_\pi)$ in Eq. (1) is given by an integration over excited states of I nucleus

$$T_i(m_\pi) = \int_{-\infty}^{E-S_{Ii}} \sum_{m_\pi^I} T_i(m_\pi, E_I, m_\pi^I)\mathcal{W}(E_I, m_\pi^I)dE_I, \qquad (2)$$

where the upper integration limit is determined by the total excitation energy E of the compound system and the channel separation energy S_{Ii}, while the level density $\mathcal{W}(E_I,m_\pi^I) = \sum_\nu \delta(E_I - E_I^\nu(m_\pi^I))$ includes the summation over nuclear states ν corresponding to spin projection m_π^I. The quantity $T_{\mathrm{tot}}(m_\pi)$ accumulates, in addition, all possible channels.

As demonstrated (see Refs. [11,12], and references therein) the nonrelativistic mean-field treatment provides realistic description of nuclei in magnetic fields of interest. In the following we make use of this approach to analyze constituents of statistical model.

As evident from Eqs. (1) and (2) apart from neutron and γ-transmission coefficients T the level density of excited states $\mathcal{W}$ represents an important ingredient of statistical calculations. The mean-field treatment brings the noninteracting Fermi-gas model [15,18] for the nuclear level density. It is worthy to notice here that more sophisticated Monte Carlo shell model calculations (see, e.g., Ref. [19], and references therein) as well as combinatorial approaches [20] show ex-

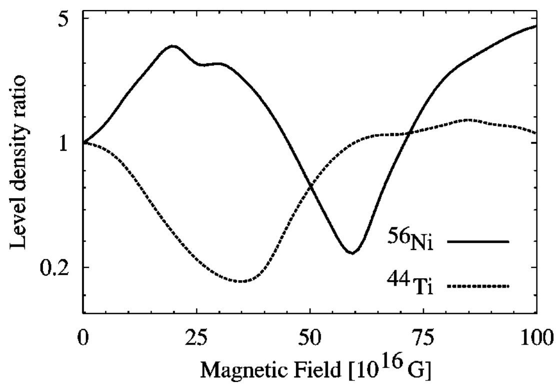

FIG. 1. Magnetic field dependence of nuclear level density at neutron separation energy for ^{56}Ni (solid line) and ^{44}Ti (dashed line).

cellent agreement with the back shifted Fermi-gas formula justifying thereby applicability of such mean-field description already at neutron separation energies. The phenomenological parametrization of excitation-energy E dependent density of states

$$\mathcal{W}(U) = \frac{\sqrt{\pi}}{12a^{1/4}}\frac{\exp(2\sqrt{aU})}{U^{5/4}}, \quad U = E - \delta \tag{3}$$

is defined by the level density parameter a and the backshift δ which gives an energy of the first excited state.

Making use of the excitation-energy dependent description by Ignatyuk *et al.* [21] the level density parameter a is expressed by $a(U,Z,N) = \tilde{a}(A)[1+C(Z,N)f(U)/U]$, where $\tilde{a}(A) = \alpha A + \beta A^{2/3}$, the quantity $C(Z,N)$ is identified with shell correction energy, and the function $f(U) = 1 - \exp(-\gamma U)$ accounts for washing out shell effects at high excitation energies. The values of parameters $\{\alpha,\beta,\gamma\} = \{0.1337, -0.06571, 0.04884\}$ are determined by fitting to experimental level density data [16]. At low energies $aU < 5$ we combine Eq. (3) with an expression $\mathcal{W} \propto \exp\{U/\tau\}/\tau$ obtaining the value of τ from proper tangential behavior.

As shown, e.g., in Refs. [11,12] the shell corrections dominate magnetic field effects in nuclei. Making use of the field dependent shell energy [11] and Eq. (3) we consider the level density of nuclei in magnetic fields at neutron separation energy (for more details see Ref. [13]). As illustrated in Fig. 1 the density of states oscillates as a function of magnetic field. The double magic in the laboratory ^{56}Ni displays increasing number of levels at weak fields, while the level spacing grows in case of slightly antimagic ^{44}Ti. Such a feature reflects the magic-antimagic switching [11,12] in nuclear structure at varying fields.

The individual transmission coefficient in Eq. (2) provides a measure for transition rate from the state $(E_I^{\nu}, m_{\pi}^{I\nu})$ of nucleus I with particle i in the continuum to an excited state in the compound nucleus (E, m_{π}), and can be expanded over particle partial waves l (see, e.g., Refs. [15–17]). The particular partial wave coefficients T_l are calculated by solving the Schrödinger equation with an optical potential for the particle-nucleus interaction. These quantities are predominantly determined by spatial motion and depend only slightly on nuclear moments [15]. Furthermore, at astrophysical energies the s-wave neutron scattering gives leading contribution to the total cross section. In this case the optical square well potential with the black nucleus approximation yields reasonable accuracy for s-wave neutron strength function (see, e.g., Refs. [16,22], and references therein). Such a limit implies negligible magnetic field effects in individual neutron transmission factors.

The γ-transmission coefficients are dominated by $E1$ transitions which are usually calculated on the basis of the Lorentzian representation of the giant dipole resonance (GDR). Within such a model $E1$ factors for emitting a photon of energy E_γ in a nucleus $O = {}^AZ$ are given by

$$T_{\mathrm{E1}}(E_\gamma) = \frac{8}{3}\frac{NZ}{A}\frac{e^2}{\hbar m_{\mathrm{N}}c^3}\frac{\Gamma_{\mathrm{GDR}}E_\gamma^4}{(E_\gamma^2 - E_{\mathrm{GDR}}^2)^2 + \Gamma_{\mathrm{GDR}}^2E_\gamma^2}. \tag{4}$$

The spatial motion of nucleons determines GDR energies E_{GDR} and widths Γ_{GDR} which can be well described with semiclassical accuracy (see, e.g., Refs. [23,24], and references therein). The Bohr–van Leeuwen theorem [25,26] suggests, indeed, that in classical limit the field effect can be omitted. This is also corroborated by more detail analysis [11,12] indicating only slight influence of magnetic fields of strengths considered here on nucleon spatial dynamics. Therefore, in calculation we adopt GDR energies and widths corresponding to laboratory conditions.

However, the energy difference of field interactions with magnetic moments in entrance and exit channels $E_{\mathrm{M}} = (\mathcal{M}_J - \mathcal{M}_O)B$ contributes noticeably to the γ-transition energy $E_\gamma = E - E_O - E_{\mathrm{M}}$, where the energy E_O and field projection of magnetic moment $\mathcal{M}_O$ correspond to final nucleus O, while in the initial channel $\mathcal{M}_J = \mathcal{M}_n + \mathcal{M}_I$, and the magnetic moments of neutrons $\mathcal{M}_n$ and target nuclei $\mathcal{M}_I$ can be assumed to be aligned along the magnetic field vector. We recall that $\mathcal{M}_i = g_i m_i$ with g factor g_i and spin projection m_i on the field axis of ith nuclear particle. The sensitivity of (n,γ) reaction to the field projection m of a spin requires to account for respective dependence in the level density. Within the mean-field treatment such additional distribution in Eq. (3) is given by Gaussian factor [15,27]

$$\mathcal{F}(m) = (\sqrt{\pi}\kappa)^{-1}\exp[-(m - m_{\mathrm{F}})^2/\kappa^2] \tag{5}$$

centered at the projection m_{F} associated with the Fermi energy. The spin cutoff parameter is evaluated under an assumption of spherical rigid nucleus of radius $R \approx 1.25A^{1/3}$ fm to be $\kappa^2 \approx 0.225A^{2/3}\sqrt{aU}$.

As argued above important magnetic field effects in radiative n-capture reaction are due to modifications of nuclear level densities and γ-transition energies. To illustrate the relationship between these mechanisms as well as the sensitivity to approximations we consider schematic example of neutron capture by neutron-odd nucleus with spin 1/2 which yields exit channel even-even nucleus corresponding to $m_{\mathrm{F}} = 0$. Then for s-wave neutrons the field projection of compound nucleus angular momentum $m_J = 1$, while selection rules for dipole γ emission imply $m_J - m_O = m_\gamma$ with the photon-angular-momentum projection $m_\gamma = 0, \pm 1$. This case results in magnetic energy $E_{\mathrm{M}} \approx g_{\mathrm{eff}}\mu_{\mathrm{N}}Bm_\gamma$, where the

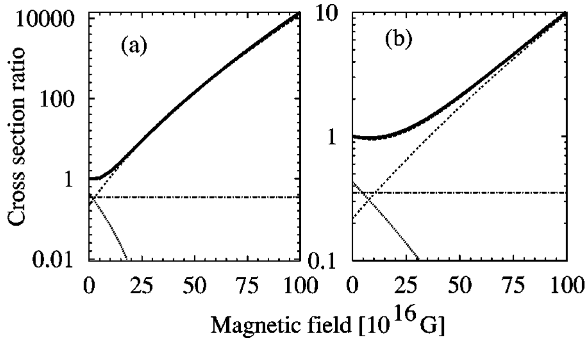

FIG. 2. Effect of magnetic energy in radiative neutron capture cross section for $|g_{\mathrm{eff}}|=3.8263$ (a) and 1 (b). The normalized at zero-field total cross section for ^{56}Ni and ^{44}Ti are indicated by thick solid and dashed lines, while thin dotted, dashed-dotted, and double-dotted lines represent the partial contributions of ^{56}Ni states with spins 0, 1, and 2, respectively.

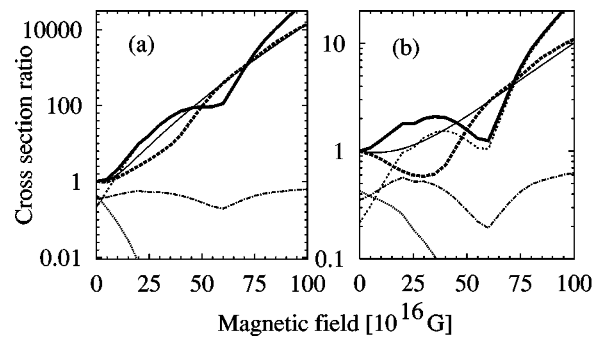

FIG. 3. The same as in Fig. 2 but including magnetic field effect in the level density.

nucleon magneton μ_{N}, and an effective g factor g_{eff} approaches the neutron value g_n for predominant contribution of active neutron in change of nuclear configuration or unity for spatial mechanisms.

It is worthy to notice that outer crusts of neutron stars as well as cores of supernova progenitors are expected to be composed of well-separated nuclides with the largest binding energy (see Refs. [11–13], and references therein). At laboratory conditions, i.e., vanishing magnetic field, such a bottom of fusion-fission valleys on the nuclear binding energy chart corresponds to transition metals of iron series due to pronounced shell closure. Magnetic field can shift the respective magic numbers towards smaller masses approaching titanium [11]. Thus in further calculations we employ the discussed above model ingredients for cases of ^{44}Ti and ^{56}Ni. Such a choice of symmetric nuclei with equal numbers of protons and neutrons yields transparent picture of the magnetic field effect with fundamental consequences on the nature of radiative neutron capture by ultramagnetized nuclei. Furthermore, taking into account predominant contribution of neutron channel to total transmission coefficients in Eq. (1) at small velocities v_{nI} we approximate the cross section normalized at zero field by respective normalized γ-transmission coefficient, $\sigma(B)/\sigma(0)\approx T_\gamma(B)/T_\gamma(0)$.

Incorporating the laboratory level density parameters we see from Fig. 2 that the contribution of magnetic energy E_{M} results in considerable enhancement of radiative n-capture process in strong magnetic fields. The largest contribution of zero-spin states in out channel at zero field sharply vanishes with increasing field strength, while the population of the highest allowed spin-states grows. This results in nearly constant cross section in weak field limit. In magnetic fields of large strengths such highest spin states of final nuclei give predominant contribution to the total cross section because of large extra energy in γ channel. The cross section ratio is almost the same for both nuclei. The comparison of panels (a) and (b) in Fig. 2 indicates that the enhancement is considerably stronger for larger values of g factor.

The mechanisms due to magnetic effects in level densities and γ energies are brought together in Fig. 3. The magnetic change in level spacing is seen to result in oscillations (cf. Fig. 1) of neutron capture cross sections around monotonic enhancement caused by the magnetic energy effect. Contribution of such oscillations is particularly pronounced for relatively small absolute values of g factors. Preferable occupation of higher spin states for reaction product remains. We note that the field dependence of the level density brings varying cross section also for an unchanged spin projection. The magnetic effects in level spacing gives rise to considerably different cross section ratios for ^{56}Ni and ^{44}Ti nuclei. For the case of product-nucleus ^{56}Ni, with closed shell at zero field, the neutron capture process displays an extra enhancement, while for ^{44}Ti the reaction can be suppressed at weak fields.

In summary, we have considered the radiative neutron capture nuclear reactions in ultramagnetized media relevant for supernovae and neutron stars. Employing the statistical model it is argued that the magnetic influence on the nuclear level densities and γ-transition energies dominates the field effect in the reactions. The nuclear structure is reflected by level densities which result in oscillations of radiative n-capture rate as a function of magnetic field strength. The interaction of the field with magnetic moments of nuclei modifies the photon energy, and enhances (n,γ) reaction cross sections as well as components of high spin states of reaction products at field strengths well in excess of 10^{16} G. Such an enhancement implies an acceleration of radiative neutron capture processes at increasing magnetic field and unchanged neutron density and temperature.

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