Dielectric function excited by ρ meson polarization in nuclear matter

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The dielectric function of nuclear matter has been studied in the framework of finite temperature field theory based on the quantum hadrodynamics II model. We find that it has two extrema and one singularity in terms of K_0/k. One of the extrema reflects Landau damping and the other is related to the resonance phenomenon.

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The property of hadronic matter at finite temperature and density is one of the interesting topics in recent research on relativistic heavy ion collision. The medium effect of hadronic matter, such as the dispersion relation, the dielectric function, the permeability, and the nuclear effective mass in hot/dense matter have excited a lot of investigations [1–8].

In the hot and dense environment, there are various excitation modes in medium, which lead to difference between the field in vacuum and in medium. Generally speaking, in order to understand this difference, two fundamental problems should be discussed. One is the dispersion relation of the excitation modes, namely, the relation between the three-momenta and frequencies of the modes, which is decided by the pole of in-medium propagators. It is a basic feature in dispersion relation that the timelike region is the normal dispersion area and the spacelike region is the Landau damping area. For the massive vector field like ρ meson, its field tensor involves both the electrolike and the magnetolike components. So another related problem is the dielectric and permeable property. Analogizing with electromagnetism theory we know that they can be described by the dielectric function and permeability, which mainly reflect the difference between the meson field in vacuum and in nuclear medium.

Some literatures have discussed the dielectric functions and dispersion relations of QED and QCD plasma under the Hard thermal loop approximation (HTLA) [9,10]. And the dispersion relation of nuclear matter which concerns the ω meson was also studied based on the QHD-I (Quantum hadrodynamics I) model in zero temperature but high density environment [5]. The ρ meson, whose decay width is larger than that of the ω and ϕ, has shorter lifetime compared with the central fireball formed in high-energy heavy ion collision. Therefore the ρ meson in medium has attracted much more attention [3,4,7,8,11]. The dispersion relation of ρ meson in medium has been investigated [3], but its dielectric property is not yet clear and needs further study. In this paper we shall start at QHD-II and discuss the dielectric function excited by the ρ meson at high temperature. In the following discussion we denote $K^2=K_0^2-\mathbf{k}^2$ and $|\mathbf{k}|=k$.

In electromagnetism theory, we know that the polarizations are the essence of dielectric property. The dielectric function including the polarization tensor of massless photon has been given in early works [9]. In the similar manner, one can obtain the formula of the dielectric function ε,

$$\varepsilon = 1 - \frac{\Pi_L}{K^2 - m_\rho^2}, \tag{1}$$

which relates to the polarizations of massive vector meson. The polarization tensor can be decomposed into transverse and longitudinal parts as: $\Pi^{\mu\nu}=P_L^{\mu\nu}\Pi_L+P_T^{\mu\nu}\Pi_T$. The transverse and longitudinal projects are defined by Kapusta [12].

The Lagrangian of QHD-II is [13]

$$\mathcal{L}_{QHD-II} = \mathcal{L}_I + \mathcal{L}_\pi + \mathcal{L}_{\rho\eta}, \tag{2}$$

$$\begin{aligned}\mathcal{L}_I = {}& \bar{\psi}[\gamma_\mu(i\partial_\mu - g_v V^\mu) - (M - g_s\phi)]\psi + \tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) \\ & - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_v^2 V_\mu V^\mu,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_\pi = {}& \tfrac{1}{2}(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - m_\pi^2\vec{\pi}\cdot\vec{\pi}) - ig_\pi\bar{\psi}\gamma_5\vec{\tau}\psi\cdot\vec{\pi} \\ & + \tfrac{1}{2}g_{\phi\pi}m_s\vec{\pi}\cdot\vec{\pi}\phi,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rho\eta} = {}& -\tfrac{1}{4}\vec{B}_{\mu\nu}\cdot\vec{B}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu - \tfrac{1}{2}g_\rho\bar{\psi}\gamma^\mu\vec{\tau}\psi\cdot\vec{b}_\mu \\ & + g_\rho(\partial^\mu\vec{\pi}\times\vec{\pi})\cdot\vec{b}_\mu + \tfrac{1}{2}g_\rho^2(\vec{\pi}\times\vec{b}_\mu)\cdot(\vec{\pi}\times\vec{b}^\mu) + \delta_{Higgs},\end{aligned}$$

where, $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$ and

$$\vec{B}_{\mu\nu} = \partial_\mu\vec{b}_\mu - \partial_\nu\vec{b}_\nu - g_\rho(\vec{b}_\mu\times\vec{b}_\nu),$$

where $\vec{B}_{\mu\nu}$ is the field tensor of ρ meson.

From the Lagrangian, one can read out five ways of polarization of ρ meson (see Fig. 1).

These five diagrams can be classified as three types: the nucleon polarization, the pion polarization and the self-polarization.

The polarization tensor of ρ meson will be calculated in the real-time formalism of finite temperature field theory. First turn to the nucleon polarization,

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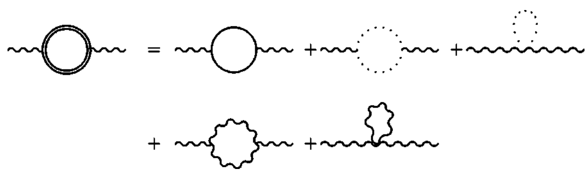
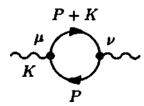


FIG. 1. Five ways of ρ meson polarizations. The plain line is for nucleon, dotted line for pion, and wiggly line for ρ meson.



$$\Pi^{ab(\rho N)}_{\mu\nu} = -\frac{i}{2} g_\rho^2 \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}[\tau^a \gamma_\mu G(P) \tau^b \gamma_\nu G(P+K)]$$
$$= -\frac{i}{2} g_\rho^2 \mathrm{Tr}(\tau^a \tau^b) \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu G(P) \gamma_\nu G(P+K)]$$
$$= \delta^{ab} \Pi^{(\rho N)}_{\mu\nu}, \tag{3}$$

G(P) is the thermal nuclear propagator $G(P) = G_F(P) + G_D(P)$, with

$$G_F(P) = \frac{\not{P} + m_N^*}{P^2 - m_N^{*2} + i\epsilon}, \tag{4}$$

$$G_D(P) = 2\pi i(\not{P} + m_N^*) n_F \delta(P^2 - m_N^{*2}), \tag{5}$$

where n_F is the Fermi-Dirac distribution function: $n_F = [\exp(\beta\omega_p) + 1]^{-1}$.

So,

$$\Pi_{\mu\nu}(K) = \Pi^F_{\mu\nu}(K) + \Pi^D_{\mu\nu}(K), \tag{6}$$

$$\Pi^F_{\mu\nu}(K) = -i g_\rho^2 \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu G_F(P) \gamma_\nu G_F(P+K)],$$

$$\Pi^D_{\mu\nu}(K) = -i g_\rho^2 \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu G_F(P) \gamma_\nu G_D(P+K)$$
$$+ \gamma_\mu G_D(P) \gamma_\nu G_F(P+K) + \gamma_\mu G_D(P) \gamma_\nu G_D(P+K)].$$

The Feynman part is divergent but can be renormalized [14]. At first we focus on the temperature-dependent part.

By using the definition of Kapusta, we get $\Pi^L = K^2/k^2 \Pi_{00}$, and

$$\Pi_L^{\rho N} = \frac{K^2}{k^2}\Pi_{00} = \frac{g_\rho^2}{\pi^2}\frac{K^2}{k^2} \int \frac{p^2 dp}{\omega_p} \left[\frac{4\omega_p^2 - 4\omega_p K_0 + K^2}{4pk} \ln A \right.$$
$$\left. + \frac{4\omega_p^2 + 4\omega_p K_0 + K^2}{4pk} \ln B - 2 \right] n_F, \tag{7}$$

where $\omega_p = \sqrt{p^2 + m_N^{*2}}$ and m_N^* is the effective mass of nucleon. And

$$A = \frac{-2\omega_p K_0 + K^2 + 2pk}{-2\omega_p K_0 + K^2 - 2pk}, \quad B = \frac{2\omega_p K_0 + K^2 + 2pk}{2\omega_p K_0 + K^2 - 2pk}. \tag{8}$$

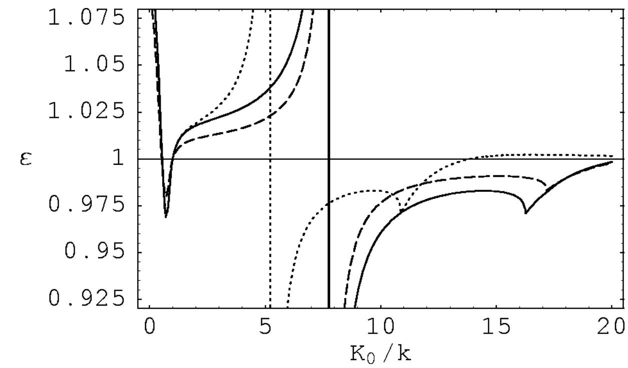


FIG. 2. The dielectric function of the nucleon polarization in terms of K_0/k. The solid line is for the case of $T = 200$ MeV, $k = 100$ MeV, the dashed line for $T = 190$ MeV, $k = 100$ MeV, and the dotted line for $T = 200$ MeV, $k = 150$ MeV.

Joining up Eqs. (1) and (7), and taking the real part of the dielectric function into account, one can draw out the curves in Fig. 2. In the numerical calculation, the effective mass of nucleon decreases with T which is decided by the self-consistent mass equation [13]. The other parameters are given in Table I.

In Fig. 2, the excitation mode k has been fixed so as to see the dielectric function varying with frequency. There is one singularity and two minimum values on the solid curve where their positions and magnitude change with k and T. When T rises, the magnitude of the two minimum values increase but the position of the singularity remains stable. The change of excitation mode moves the position of the singularity and has a tiny effect on the magnitude.

With more specific analysis to the mathematical structure of the dielectric function, one can find that the singularity appears when the mass shell condition is satisfied, which is obvious in Eq. (1). In QED, this effect can also be found. The only difference is that the singularity appears at $K_0/k = 1$ in QED or QCD because of the massless photon or gluon, while in QHD case it has been moved to right due to the mass of ρ meson.

The structures of the extrema are more complex than that of the singularity. Analyzing the polarization tensor mathematically, we discover that the extrema are firmly related to the singularities in ln A and ln B in Eq. (8). Setting the numerators and denominators of A and B to zero, one can obtain the positions of singularities.

$$\begin{cases} -2\omega_p K_0 + K_0^2 - k^2 + 2pk = 0, \\ -2\omega_p K_0 + K_0^2 - k^2 - 2pk = 0, \\ 2\omega_p K_0 + K_0^2 - k^2 + 2pk = 0, \\ 2\omega_p K_0 + K_0^2 - k^2 - 2pk = 0. \end{cases} \tag{9}$$

There are eight roots for these four equations

TABLE I. Parameters of QHD-II [13] with masses and the temperature in (MeV).

g_ρ^2	g_s^2	m_ρ	m_π	m_s	$m_N^*(T=190)$	$m_N^*(T=200)$	m_N
36.79	62.89	770	138	550	857.4	811.6	939

$$\begin{cases} n = \frac{1}{k}(\omega_p - \sqrt{k^2 - 2kp + \omega_p^2}), \\ n = \frac{1}{k}(\omega_p - \sqrt{k^2 + 2kp + \omega_p^2}), \\ n = \frac{1}{k}(-\omega_p + \sqrt{k^2 - 2kp + \omega_p^2}), \\ n = \frac{1}{k}(-\omega_p + \sqrt{k^2 + 2kp + \omega_p^2}), \end{cases} \tag{10}$$

$$\begin{cases} n = \frac{1}{k}(\omega_p + \sqrt{k^2 - 2kp + \omega_p^2}), \\ n = \frac{1}{k}(\omega_p + \sqrt{k^2 + 2kp + \omega_p^2}), \\ n = \frac{1}{k}(-\omega_p - \sqrt{k^2 - 2kp + \omega_p^2}), \\ n = \frac{1}{k}(-\omega_p - \sqrt{k^2 + 2kp + \omega_p^2}), \end{cases} \tag{11}$$

where $n = K_0/k$.

Obviously, the four roots in Eq. (10) are all in the region of $|n| < 1$. Note that $p > 0$ and $n > 0$; they must be linked with the left minimum value in Fig. 2. The other four roots in Eq. (11) are in the region of $|n| > 1$, so they are the counterparts of the right minimum value. Retrospect to QED case in HTLA, no such effect was presented as we know, because the vector field in QED is massless and the electronic mass has been neglected in HTLA. In Fig. 2 if the masses of fermion and meson vanished, the two minimum values would converge at the point of $n = 1$ which was the exact position of the mass-shell singularity of photon or gluon. In that case the extrema are hidden in the singularity effect. It is necessary to point out that whether it is QED or QCD, these extrema are always there. But only in QHD, due to the masses of meson and fermion, this effect can be obviously presented.

We have studied the real part of the dielectric function and given the fundamental characteristics of the function curve which contains two extrema and one singularity. But what do they mean in physics? Analyzing the imaginary part of the dielectric function which is related to the energy exchange between the excitation modes and the medium, we can also find two extrema, one is in the spacelike region and the other in the timelike region. Their positions are exactly the same with those in the real part respectively. This means that the extrema in Fig. 2 are related to some energy absorption phenomenon in the medium. Obviously, the extremum in the spacelike region reflects Landau damping mechanism because $K_0/k < 1$ belongs to the damping area in the dispersion relation. In this damping mechanism the excitation mode losses energy into the medium. The extremum in the timelike region may be excited by the resonant absorption in the medium. Detailed analysis to the numerical result in Fig. 2 demostrates that when the invariant mass of ρ meson is close to twice as large as the mass of nucleon, the real part of the dielectric function gives the timelike extremum. For example, on the solid curve in Fig. 2, the timelike extremum appears at $K_0/k \approx 16.2$. Notice that $k = 100$ MeV and $m_N^* = 811.6$ MeV, so the invariant mass of ρ meson $M_\rho^2 = K^2 \approx (16.2^2 - 1)k^2$ and $M_\rho \approx 1616.9$ MeV. Thus it can be seen that there is a resonance condition $M_\rho \sim 2m_N^*$ at this extremum.

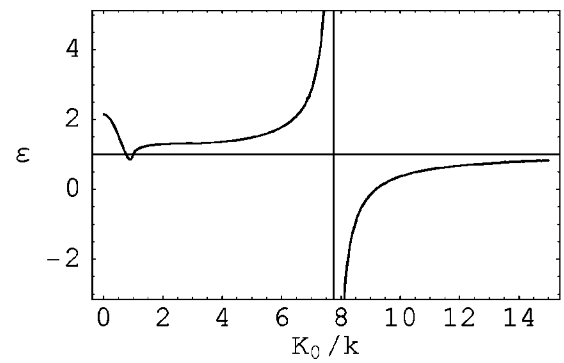


FIG. 3. Dielectric function contributed by the pion polarization with $k = 100$ MeV at $T = 200$ MeV.

Now we are in the position to give physical explanations to the dielectric curve: as what is shown in Fig. 2, in a certain excitation mode k, ε decreases with increasing K_0 at first and gives a minimum value in the spacelike region, which can be explained by the Landau damping effect. The nucleons will achieve the energy according to this damping mechanism and form the induced current to decide ε. Then with K_0 rising, the curve enters into the timelike region, namely, the normal dispersion area, and ε rises until the singularity appears when $K_0^2 - k^2 = m_\rho^2$, which is the mass-shell condition of physical particle. As we know, it is impossible for the in-medium meson to satisfy this condition. After passing the singularity, the excitation mode goes back to the normal dispersion area and ε is rising again with K_0. But when the invariant mass of ρ meson satisfies the resonance condition, the nucleons will form the induced current according to this resonant mechanism, ε again deviates far from one which is the value in vacuum, and the curve gives another minimum value. Moreover, as to different excitation modes, the positions of the singularities and extrema shift under the constraint of mass-shell and resonance conditions. The calculation also shows that in the timelike region when some mode

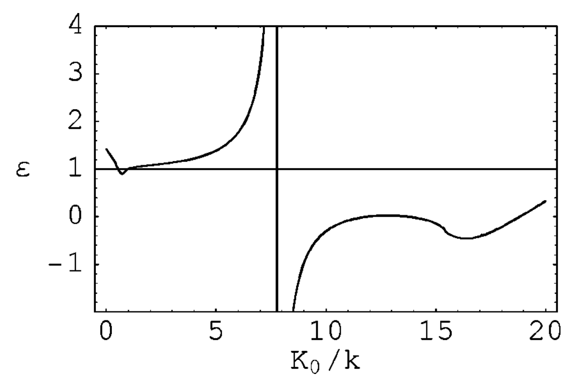


FIG. 4. Dielectric function contributed by the self-polarization with $k = 100$ MeV at $T = 200$ MeV.

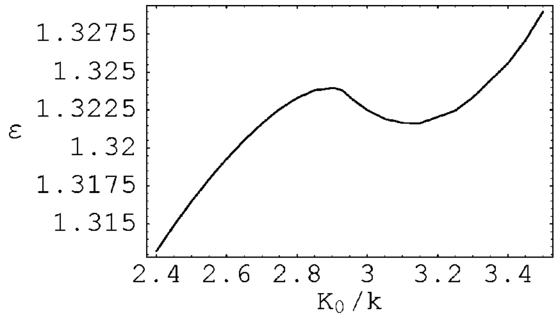


FIG. 5. Exaggerated region around the resonant point on the dielectric function curve contributed by the pion polarization, with $k=100$ MeV and $T=200$ MeV.

carries extremely large momentum (e.g., $K\to\infty$), $\varepsilon\to 1$, which is consistent with the related dispersion relation.

For the other two types of polarizations, the results are

$$\Pi_L^{(\rho\pi)} = \frac{g_\rho^2}{2\pi^2}\frac{K^2}{k^2}\int \frac{p^2dp}{\omega_p}\left[\frac{4\omega_p^2 - 4\omega_p K_0 + K_0^2}{2pk}\ln A + \frac{4\omega_p^2 + 4\omega_p K_0 + K_0^2}{2pk}\ln B - 4\right]n_B, \tag{12}$$

where $n_B=[\exp(\beta\omega_p)-1]^{-1}$ is the Bose-Einstein distribution function, and

$$\Pi_L^{(\rho\rho)} = \frac{g_\rho^2}{2\pi^2}\frac{K^2}{k^2}\int \frac{p^2dp}{\omega_p}\left[\frac{2K_0^2 + 12\omega_p^2 - 10\omega_p K_0 - 4k^2 - 2p^2}{2pk}\times\ln A + \frac{2K_0^2 + 12\omega_p^2 + 10\omega_p K_0 - 4k^2 - 2p^2}{2pk}\times\ln B - 8\right]n_B. \tag{13}$$

The other two types of polarizations give almost the same result as what is shown in Figs. 3 and 4, except that the timelike extremum in Fig. 3 seems to have disappeared. This may be due to the relatively small magnitude of the timelike extremum. In fact, zooming in the region around the possible resonant point, one can see the minimum value appearing on the curve (Fig. 5), where the resonance begins at $K^2\sim 4m_\pi^2$. This result strongly supports our explanation to the resonant mechanism.

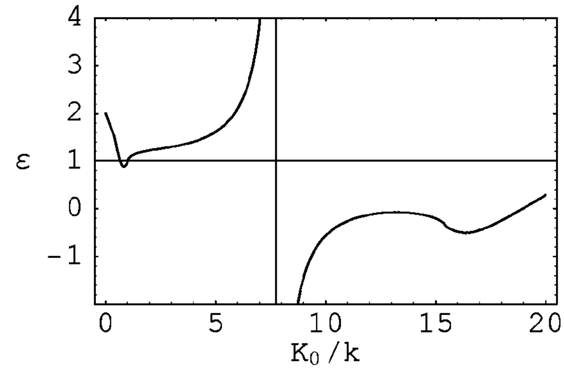


FIG. 6. The final dielectric function of ρ meson in QHD-II model with $k=100$ MeV at $T=200$ MeV.

In Figs. 2, 3, and 4, one can see that at the same temperature, the pion polarization has the greatest effect on the magnitude of the dielectric function. Self-polarization takes the second place, while the nucleon polarization has the least contribution. Adding up all the Feynman diagrams, we can obtain the final dielectric function which is induced by ρ meson. The numerical result is plotted in Fig. 6.

$$\varepsilon_\rho = 1 - \frac{1}{K^2 - m_\rho^2}(\Pi_L^{(\rho N)} + \Pi_L^{(\rho\pi)} + \Pi_L^{(\rho\rho)}). \tag{14}$$

Let us draw some conclusions on the dielectric function of nuclear matter excited by ρ meson in QHD-II. First, the dielectric function appears a singularity when the nuclear mass-shell condition is satisfied. Besides the singularity, it has an extremum in the spacelike and the timelike region respectively, which is the main difference compared with the case in QED or QCD. This new phenomenon may be owing to the masses of ρ meson and fermion, which separate the two extrema from the mass shell singularity. In QED or QCD, these extrema are hidden in the singularity at $n=1$, which is the mass shell of massless particle. Finally, the extremum in the spacelike region is the Landau damping effect and the one in the timelike region is related to the resonance caused by ρ mesons and nucleons in the medium.

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