Surface tension in a compressible liquid-drop model: Effects on nuclear density and neutron skin thickness

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We examine whether or not the surface tension acts to increase the nucleon density in the nuclear interior within a compressible liquid-drop model. We find that it depends on the density dependence of the surface tension, which may in turn be deduced from the neutron skin thickness of stable nuclei.

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The saturation property of bulk nuclear matter is usually deduced from empirical data on the masses and radii of stable nuclei [1]. If nuclear matter is incompressible, the bulk saturation density is equal to the density in the nuclear interior. However, nuclear matter in nuclei is more or less compressible. The question of how the finite compressibility of nuclear matter makes the density in the nuclear interior deviate from the saturation density has yet to be clarified. In this paper, by using a compressible liquid-drop model, we show that a key feature in addressing this question is the density dependence of the surface tension. We then derive its relation with the neutron skin thickness.

The dependence of the surface energy on the density of the nucleon liquid naturally occurs. In the Fermi-gas model, the surface energy arises from a reduction of the available density of states of nucleons due to the presence of the surface, beyond which the nucleon flux vanishes. The total kinetic energy thus increases by an amount proportional to the surface area, and this increase depends on the nucleon density through the nucleon Fermi momentum. In addition to this kinetic contribution, the interaction between nucleons contributes to the surface energy more importantly. This is because a nucleon in the surface region does not perceive the same amount of attraction from the surrounding nucleons as that which it would if it were in the deeper region. This contribution is determined by the property of the nuclear medium, which is characterized by the nucleon density. We remark that for the same reasons, the surface energy depends also on the neutron excess of the liquid; this dependence has to be invariant under exchange between neutrons and protons.

As discussed by Yamada [2], the dependence of the surface energy on the inner liquid density controls the density deviation from the bulk saturation density. Usually, this dependence is not considered explicitly. This is because the surface tension is normally calculated for a planar interface between the saturated nucleon liquid and the vacuum, which are in mechanical equilibrium. However, this is not equivalent to mechanical equilibrium in a real nucleus, which is generally associated with additional pressures arising from the size and density dependence of the surface energy. These pressures in turn affect the equilibrium density of the compressible nucleon liquid.

In order to see this effect, we utilize a compressible liquid-drop model, which gives rise to a semiempirical mass formula in a way dependent on the density and neutron excess in the nuclear interior, $n_{\rm in}$ and $\delta_{\rm in}$. Generally, a liquid-drop model is advantageous to the description of various macroscopic properties of nuclei. During the past decade, it has been used in describing, e.g., neutron skin [3], nuclear fission [4], deformation of rapidly rotating nuclei [5], synthesis of superheavy nuclei [5], and nuclei in neutron star crusts [6].

Throughout this paper we consider nearly symmetric nuclei, for which we can set $R_n \simeq R_p$. We assume that the distribution of i nucleons $(i=n,p)$ is spherically symmetric, uniform at a number density n_i, and squared off at a radius R_i. For a nucleus of mass number A and charge number Z (neutron number $N=A-Z$), we thus obtain $n_{\rm in}=n_n+n_p$ and

$$\delta_{\rm in} = (n_n - n_p)/(n_n + n_p) \simeq (N-Z)/A. \quad (1)$$

We then write the binding energy E_B of the nucleus as

$$-E_B = E_{\rm vol} + E_{\rm surf} + E_{\rm Coul}. \quad (2)$$

Here

$$E_{\rm vol} = Aw(n_{\rm in}, \delta_{\rm in}), \quad (3)$$

with the bulk energy per nucleon w, is the volume energy,

$$E_{\rm surf} = 4\pi\sigma(n_{\rm in}, \delta_{\rm in}) R_p^2, \quad (4)$$

with the density-dependent surface tension σ, is the surface energy, and

$$E_{\rm Coul} = \frac{3Z^2e^2}{5R_p} \quad (5)$$

is the Coulomb energy. For w and σ, we adopt a form expanded with respect to the density and neutron excess around $n_{\rm in}=n_0$ and $\delta_{\rm in}=0$:

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$$w(n_{\rm in},\delta_{\rm in}) = w_0 + \frac{K_0}{18n_0^2}(n_{\rm in}-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n_{\rm in}-n_0)\right]\delta_{\rm in}^2, \tag{6}$$

where n_0 and w_0 are the saturation density and energy of symmetric nuclear matter, K_0 is the incompressibility of symmetric nuclear matter, S_0 is the symmetry energy coefficient, and L is the density symmetry coefficient, and

$$\sigma(n_{\rm in},\delta_{\rm in}) = \sigma_0\left[1 - C_{\rm sym}\delta_{\rm in}^2 + \chi\left(\frac{n_{\rm in}-n_0}{n_0}\right)\right], \tag{7}$$

where $\sigma_0=\sigma(n_0,0)$, $C_{\rm sym}$ is the surface symmetry energy coefficient, and $\chi=(n_0/\sigma_0)\,\partial\sigma/\partial n_{\rm in}|_{n_{\rm in}=n_0,\delta_{\rm in}=0}$. In Eq. (2) we have ignored the energy contribution of the neutron skin thickness R_n-R_p, which will be considered later, and curvature corrections. We have also ignored pairing and shell corrections since we will confine ourselves to macroscopic properties of the nuclear ground state. We remark that in equilibrium $n_{\rm in}$ is related to $\delta_{\rm in}$, as we shall see just below.

Some of the coefficients characterizing the bulk energy (6) can be deduced from empirical data for the masses and root-mean-square charge radii of stable nuclei. The saturation density n_0, the saturation energy w_0, and the symmetry energy coefficient S_0 typically take on a value ranging 0.14–0.17 fm^{-3}, -16 ± 1 MeV, and 25–40 MeV, whereas the incompressibility K_0 and the density symmetry coefficient L, which control the density dependence of bulk nuclear matter, are not well constrained. Using a simplified version of the Thomas-Fermi model [7] we found that various sets of the values of K_0 and L ranging 180–360 MeV and 0–200 MeV reasonably reproduce the empirical masses and radii and that future systematic measurements of the matter radii of unstable neutron-rich nuclei would give a good constraint on the value of L.

We turn to the coefficients in the surface tension (7). The primary coefficient σ_0 and the surface symmetry coefficient $C_{\rm sym}$ can be estimated from the empirical mass data as $\sigma_0 \simeq 1$ MeV fm^{-2} and $C_{\rm sym}=1.5$–2.5. The parameter χ characterizing the density dependence of the surface tension is poorly known and hence the quantity of interest in this work. Myers and Swiatecki [8] simply set $\chi=0$, while the Fermi-gas model predicts $\chi=4/3$. We will see that a precise determination of n_0 from the values of $n_{\rm in}$ deduced, e.g., from electron-nucleus elastic scattering data requires reliable information about χ.

Let us now estimate the equilibrium value of $n_{\rm in}$ from pressure equilibrium and compare it to the bulk saturation density n_s at fixed $\delta_{\rm in}$. Within the present compressible liquid-drop model, the pressure equilibrium condition can be obtained from optimization of the binding energy (2) with respect to the size under fixed A and Z as

$$0 = P_{\rm vol} + P_{\rm surf} + P_{\rm Coul}. \tag{8}$$

Here

$$P_{\rm vol} = \frac{K_0}{9}(n_{\rm in}-n_0) + \frac{L}{3}n_0\delta_{\rm in}^2 \equiv \frac{K_0}{9}(n_{\rm in}-n_s) \tag{9}$$

is the volume pressure,

$$P_{\rm surf} = -\frac{2\sigma_0}{R_p}\left[1 - \frac{3}{2}\chi - C_{\rm sym}\delta_{\rm in}^2 + \chi\left(\frac{n_{\rm in}-n_0}{n_0}\right)\right] \tag{10}$$

is the surface pressure, and

$$P_{\rm Coul} = \frac{3Z^2e^2}{20\pi R_p^4} \tag{11}$$

is the Coulomb pressure. The bulk pressure vanishes at the saturation density,

$$n_s = n_0 - \frac{3Ln_0}{K_0}\delta_{\rm in}^2, \tag{12}$$

which generally decreases with increasing $\delta_{\rm in}$ as has already been discussed in Refs. [7,9]. The Coulomb pressure acts to increase the nuclear size, whereas the surface pressure tends to enlarge or reduce the nuclear size according to whether χ is larger or smaller than $\sim 2/3$.

The relation (8), if the Coulomb pressure $P_{\rm Coul}$ is ignored, can be reduced to Laplace's formula. This can be done by transforming Eq. (8) into

$$P_{\rm vol} + \frac{3\sigma_0\chi}{R_p} = \frac{2\sigma(n_{\rm in},\delta_{\rm in})}{R_p}. \tag{13}$$

Here the left side, arising from the energy derivative with respect to $n_{\rm in}$, corresponds to the pressure of the nucleon liquid, while the right side arises from the energy derivative with respect to R_p.

Deviation of the equilibrium value of $n_{\rm in}$ from the bulk saturation density n_s at fixed $\delta_{\rm in}$ can be estimated from condition (8) as

$$n_{\rm in} - n_s \simeq 0.016\left(\frac{230\ {\rm MeV}}{K_0}\right)\left(\frac{\sigma_0}{1\ {\rm MeV\ fm^{-2}}}\right)\left(\frac{5\ {\rm fm}}{R_p}\right) \times\left(1 - \frac{3}{2}\chi - \frac{3Z^2e^2}{40\pi R_p^3\sigma_0}\right)\ {\rm fm}^{-3}. \tag{14}$$

In this estimate we have used $P_{\rm surf} \simeq -2\sigma_0(1-3\chi/2)/R_p$. The ratio of the Coulomb pressure to the surface pressure, $3Z^2e^2/40\pi R_p^3\sigma_0$, is typically 0.2–0.6. We find from this estimate that for $\chi=0$ and $4/3$, the surface pressure can induce about 10% change in $n_{\rm in}$ in different directions. We thus see the role played by χ in determining n_s from empirical information about $n_{\rm in}$.

We now proceed to show that the neutron skin thickness, which has been neglected so far, is a quantity that may be useful for deduction of the value of χ. In doing so, as considered by Pethick and Ravenhall [10], it is convenient to describe the nuclear surface in a thermodynamically consistent manner. In this description, the nuclear surface is in thermodynamic equilibrium with the bulk system composed of A nucleons, and the neutron skin arises from adsorption of N_s neutrons onto the nuclear surface. The interior region composed of $A-N_s$ nucleons acts as a reservoir of neutron chemical potential μ_n, and neutrons can go back and forth between the skin and interior regions. Consequently, the relevant thermodynamic quantity is the thermodynamic potential, $\Omega=\Omega_{\rm vol}+\Omega_{\rm surf}+\Omega_{\rm Coul}$, divided in a similar way to the binding energy (2). In equilibrium, $\Omega_{\rm surf}=\sigma\mathcal{A}$, where $\mathcal{A}$ is the surface area. A small quasistatic change in the neutron excess in the interior region with A and N fixed gives rise to a change in the thermodynamic potential of the surface, $\Delta\Omega_{\rm surf}$, and a change in the neutron chemical potential, $\Delta\mu_n$, which are related as

$$\Delta\Omega_{\rm surf} = -N_s\Delta\mu_n. \tag{15}$$

This relation indicates that the neutron skin can be described in terms of the bulk and surface properties. We remark that at $N=Z$, a balance between the induced changes $\Delta\Omega_{\rm vol}$ and $\Delta\Omega_{\rm Coul}$ in the volume and Coulomb energies allows the neutron excess in the interior region to deviate from zero and hence a proton skin to occur, as we shall see.

It is straightforward to combine the above thermodynamic description of the nuclear surface with the compressible liquid-drop model adopted here. In this model, $\delta_{\rm in}$, N_s, $\mathcal{A}$, and μ_n read

$$\delta_{\rm in} = \frac{N-N_s-Z}{A-N_s} \simeq \frac{N-Z}{A} - \frac{3(R_n-R_p)}{2R_p}, \tag{16}$$

$$N_s \simeq 4\pi R_p^2 n_n(R_n-R_p), \quad \mathcal{A} \simeq 4\pi R_p^2, \tag{17}$$

and

$$\mu_n = w_0 + S_0\delta_{\rm in}(2-\delta_{\rm in}) + O(\delta_{\rm in}^3). \tag{18}$$

Here we have calculated μ_n at the bulk saturation density (12); the dependence of μ_n on the parameters L and K_0 characterizing the density dependence of the bulk energy does not appear up to second order in $\delta_{\rm in}$.

We can now obtain the expression for the neutron skin thickness in the absence of Coulomb energy. In this case the system is symmetric under exchange between neutrons and protons. Substitution of Eqs. (7) and (16)–(18) into Eq. (15) leads to

$$R_n - R_p = C\delta\left(1+\frac{3C}{2R_p}\right)^{-1} + O(\delta^2), \tag{19}$$

where

$$C \equiv \frac{2\sigma_0}{S_0 n_0}\left(C_{\rm sym} + \frac{3L\chi}{K_0}\right) \tag{20}$$

and $\delta\equiv(N-Z)/A$. The parameter C originates mainly from a change in the surface tension due to the small quasistatic change in $\delta_{\rm in}$. This change is characterized not only by the surface symmetry energy coefficient $C_{\rm sym}$, but also by the parameter χ through the dependence of the saturation density n_s given by Eq. (12) on $\delta_{\rm in}$. The term $3L\chi/K_0$ on the right side of Eq. (20) is associated with the finite compressibility and thus vanishes in the incompressible limit in which Eq. (19) reduces to the result for R_n-R_p obtained by Pethick and Ravenhall [10]. Note that this term does not exist in the result of Myers and Swiatecki [11] who presumed $\chi=0$ in a compressible liquid-drop picture, although it can be comparable with $C_{\rm sym}$. The fact that $3L\chi/K_0$ is poorly known suggests that one could not deduce the equation of state (EOS) parameters L and K_0 from experimental data for the neutron skin thickness without knowing χ. Consequently, it turns out that previous investigations that attempted to relate the neutron skin thickness with the EOS of nuclear matter [3,9,12] do not take full account of uncertainties in the parameters χ, L, and K_0. We can see from Eq. (19) that the neutron skin vanishes at $N=Z$, as it should in the absence of Coulomb energy.

Coulomb effects ignored in Eq. (19) induce a thin proton skin at $N=Z$ through a deviation of $\delta_{\rm in}$ from zero and a polarization of the nuclear interior, as discussed by Myers and Swiatecki [11]. First, in order to calculate the deviation of $\delta_{\rm in}$ at $N=Z$, one has only to consider the balance between $\Delta\Omega_{\rm vol}$ and $\Delta\Omega_{\rm Coul}$. In this case an increment in the proton radius R_p tends to reduce the Coulomb energy, while it leads to a cost of the symmetry energy in the nuclear interior. We may thus obtain

$$\delta_{\rm in} \simeq \frac{Ze^2}{20R_pS_0} \quad \text{at } N=Z, \tag{21}$$

which typically amounts to a small value of ~ 0.02. For the neutron skin thickness (19), this deviation effectively replaces δ by $\delta - Ze^2/20R_pS_0$ [11]. In the liquid-drop picture, the β stability condition that the neutron and proton chemical potentials are almost equal is given by

$$\delta_{\rm in} \simeq \frac{3Ze^2}{10R_pS_0}. \tag{22}$$

This $\delta_{\rm in}$ is much larger than $Ze^2/20R_pS_0$, which implies that generally stable nuclei, lying around the β stability line, have a neutron skin. Second, the polarization of the nuclear interior tends to reduce a difference between the root-mean-square radii of the neutron and proton density distributions, Δr_{np}, by redistributing nucleons in such a way as to deplete protons in the central region. This reduction amounts to $\sqrt{3/5}Ze^2/70S_0$ [11], which typically takes on a small value of 0.01–0.04 fm.

By incorporating these Coulomb effects into Eq. (19), we finally obtain

$$\Delta r_{np} \simeq \sqrt{\frac{3}{5}}\left[C\left(\delta - \frac{Ze^2}{20R_pS_0}\right)\left(1+\frac{3C}{2R_p}\right)^{-1} - \frac{Ze^2}{70S_0}\right], \tag{23}$$

where a factor $\sqrt{3/5}$ arises from a difference between the root-mean-square and the half-density radius for the rectangular distribution. Here we have ignored corrections due to the difference in the surface diffuseness between protons and neutrons. These corrections are lower order in A than the terms in Eq. (23) [3] and hence expected to be negligibly small for heavy stable nuclei of interest here.

We now ask how one can obtain information about χ from empirically deduced values of Δr_{np} for stable nuclei such as Ni and Sn isotopes. Such values can be deduced, e.g., from

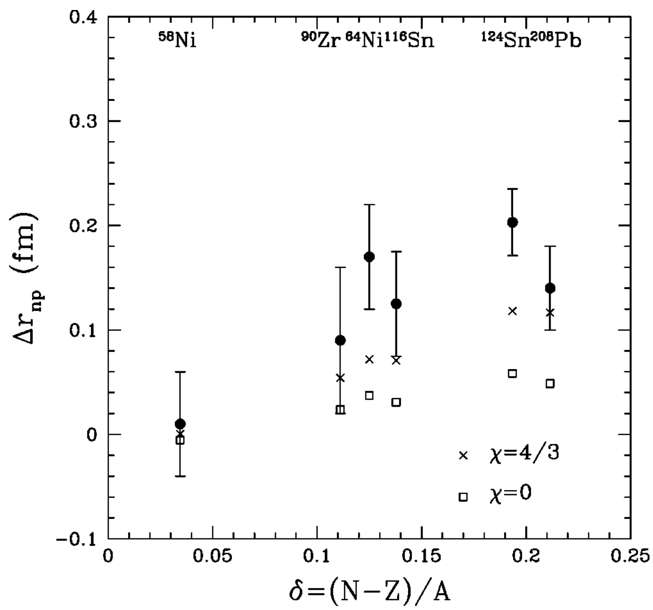

FIG. 1. Difference in the root-mean-square radius between neutrons and protons for six stable nuclei of $A>50$. The squares and crosses denote the results calculated from Eq. (23) for $\chi=0$ and $4/3$; the other parameters are set to be $n_0=0.16$ fm^{-3}, $S_0=30$ MeV, $K_0=230$ MeV, $L=100$ MeV, $\sigma_0=1$ MeV fm^{-2}, $C_{\rm sym}=1.8$, and $R_p=1.2A^{1/3}$ fm. The empirical data (dots) are taken from Refs. [13,14].

measurements of proton- and electron-nucleus elastic differential cross sections [13]. It is instructive to compare the deduced values with the prediction by the present liquid-drop model, which is given by Eq. (23). Figure 1 exhibits the deduced and predicted values as a function of δ. When $\chi=0$, the predicted values by setting the other parameters at typical values are appreciably smaller than the deduced ones. This suggests that χ is likely to be positive. However, the magnitude of χ remains to be clarified since χ is coupled with the uncertain parameters L and K_0 in Eq. (23). We remark that a staggering of the deduced values is far larger than that of the predicted values. This may be partly because the former values were deduced by various groups using different models for proton elastic scattering and nucleon density distribution, and partly because pairing and shell effects are ignored in the present prediction.

In summary we have found from a compressible liquid-drop model that whether or not the nucleon density in the nuclear interior is larger than the bulk saturation density depends on the density dependence of the surface tension, which in turn controls the neutron excess dependence of the neutron skin thickness. In order to deduce the density dependence of the surface tension and the bulk saturation density from the neutron skin thickness and the interior density, it would be useful to systematically analyze differential cross sections measured for proton and electron elastic scattering off stable nuclei. In such analysis of proton elastic scattering data obtained for incident energies above 500 MeV, one could relate the angle of diffraction maxima measured in the small momentum transfer region to the root-mean-square matter radius by using the Glauber theory in the optical limit approximation [15]. The research in this direction is under way.

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