Causal theories of dissipative relativistic fluid dynamics for nuclear collisions

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Nonequilibrium fluid dynamics derived from the extended irreversible thermodynamics of the causal Müller-Israel-Stewart theory of dissipative processes in relativistic fluids based on Grad's moment method is applied to the study of the dynamics of hot matter produced in ultrarelativistic heavy ion collisions. The temperature, energy density, and entropy evolution are investigated in the framework of the Bjorken boost-invariant scaling limit. The results of these second order theories are compared to those of first order theories due to Eckart and to Landau and Lifshitz and those of zeroth order (perfect fluid) due to Euler. In the presence of dissipation perfect fluid dynamics is no longer valid in describing the evolution of the matter. First order theories fail in the early stages of evolution. Second order theories give a better description in good agreement with transport models. It is shown in which region the Navier-Stokes-Fourier laws (first order theories) are a reasonable limiting case of the more general extended thermodynamics (second order theories).

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I. INTRODUCTION

The study of space-time evolution and nonequilibrium properties of matter produced in high energy heavy ion collisions, such as those at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, USA and the Large Hadron Collider (LHC) at CERN, Geneva using relativistic dissipative fluid dynamics are of importance in understanding the observables. RHIC has already provided us with some interesting results [1]. One of the most important reasons for colliding heavy nuclei at high energies is the possibility of creating the quark-gluon plasma (QGP) [2].

High energy heavy ion collisions offer the opportunity to study the properties of hot and dense matter. To do so we must follow its space-time evolution, which is affected not only by the equation of state but also by dissipative, nonequilibrium processes. Thus we need to know the transport coefficients such as viscosities, conductivities, and diffusivities. We also need to know the relaxation times for various dissipative processes under consideration. Knowledge of the various time and length scales is of central importance to help us decide whether to apply fluid dynamics (macroscopic) or kinetic theory (microscopic) or a combination of the two. The use of fluid dynamics as one of the approaches in modeling the dynamic evolution of nuclear collisions has been successful in describing many of the observables [3,4]. The assumptions and approximations of the fluid dynamical models are another source for uncertainties in predicting the observables. So far most work have focused on the ideal or perfect fluid and/or multifluid dynamics. In this work we apply the relativistic dissipative fluid dynamical approach. It is known even from nonrelativistic studies [5] that dissipation might affect the observables.

The first theories of relativistic dissipative fluid dynamics are due to Eckart [6] and to Landau and Lifshitz [7]. The difference in formal appearance stems from different choices for the definition of the hydrodynamical four-velocity. These conventional theories of dissipative fluid dynamics are based on the assumption that the entropy four-current contains terms up to linear order in dissipative quantities and hence they are referred to as *first order theories* of dissipative fluids. The resulting equations for the dissipative fluxes are linearly related to the thermodynamic forces, and the resulting equations of motion are parabolic in structure, from which we get the Fourier-Navier-Stokes equations. They have the undesirable feature that causality may not be satisfied. That is, they may propagate viscous and thermal signals with speeds exceeding that of light.

Extended theories of dissipative fluids due to Grad [8], Müller [9], and Israel and Stewart [10] were introduced to remedy some of these undesirable features. These causal theories are based on the assumption that the entropy four-current should include terms quadratic in the dissipative fluxes and hence they are referred to as *second order theories* of dissipative fluids. The resulting equations for the dissipative fluxes are hyperbolic and they lead to causal propagation of signals [10,11]. In second order theories the space of thermodynamic quantities is expanded to include the dissipative quantities for the particular system under consideration. These dissipative quantities are treated as thermodynamic variables in their own right.

A qualitative study of relativistic dissipative fluids for applications to relativistic heavy ions collisions has been done using these first order theories [12–17]. The application of second order theories to nuclear collisions has just begun [18–20], and the results of relativistic fluid dynamics can also be compared to the prediction of spontaneous symmetry breaking results [21].

The rest of the paper is outlined as follows. In Sec. II the basic formulation of relativistic dissipative fluid dynamics will be briefly introduced. In Sec. III we discuss the role of dissipation in relativistic nuclear collisions. In Sec. IV we summarize the results and discuss the need for hyperbolic theories for relativistic dissipative fluids.

Throughout this paper we adopt the units $\hbar = c = k_B = 1$. The sign convention used follows the timelike convention with the signature $(+,-,-,-)$, and if u^α is a timelike vector, $u^\alpha u_\alpha > 0$. The metric tensor is always taken to be $g^{\mu\nu}$

$=\mathrm{diag}(+1,-1,-1,-1)$, the Minkowski tensor. Upper greek indices are contravariant and lower greek indices covariant. The greek indices used in four-vectors go from 0 to 3 (t,x,y,z) and the roman indices used in three-vectors go from 1 to 3 (x,y,z). The scalar product of two four-vectors a^μ, b^μ is denoted by $a^\mu g_{\mu\nu} b^\nu \equiv a^\mu b_\mu$. The scalar product of two three-vectors is denoted by boldface type, namely, $\mathbf{a},\mathbf{b}$, $\mathbf{a}\cdot\mathbf{b}$. The notations $A^{(\alpha\beta)} \equiv (A^{\alpha\beta}+A^{\beta\alpha})/2$ and $A^{[\alpha\beta]} \equiv (A^{\alpha\beta} - A^{\beta\alpha})/2$ denote symmetrization and antisymmetrization, respectively. The four-derivative is denoted by $\partial_\alpha \equiv \partial/\partial x^\alpha$. Contravariant components of a tensor are found from covariant components by $g_{\alpha\beta}A^\alpha = A_\beta$, $g_{\mu\alpha}g_{\nu\beta}F^{\alpha\beta}=F_{\mu\nu}$, and so on.

II. NONEQUILIBRIUM/DISSIPATIVE RELATIVISTIC FLUID DYNAMICS

In this section we give a brief review of the basics of nonequilibrium fluid dynamics. The central role of entropy is highlighted. Nonequilibrium effects are introduced by enlarging the space of basic independent variables through the introduction of nonequilibrium variables, such as dissipative fluxes appearing in the conservation equations. The next step is to find evolution equations for these extra variables. Whereas the evolution equations for the equilibrium variables are given by the usual conservation laws, no general criteria exist concerning the evolution equations of the dissipative fluxes, with the exception of the restriction imposed on them by the second law of thermodynamics.

The entropy is conserved in ideal fluid dynamics. Thus perfect fluids in equilibrium generate no entropy and no frictional-type heating because their dynamics is reversible and without dissipation. For many processes in nuclear collisions a perfect fluid model is adequate. However, real fluids behave irreversibly, and some processes in heavy ion reactions may not be understood except as dissipative processes, requiring a relativistic theory of dissipative fluids. An equilibrium state is characterized by the absence of viscous stresses, heat flow and diffusion, and maximum entropy principle, while a nonequilibrium state is characterized by the principle of nondecreasing entropy which arises due to the presence of dissipative fluxes.

Perfect fluid dynamics has been successful in describing most of the observables [3,4,22]. The current status of ideal hydrodynamics in describing observables can be found in Refs. [1,23,24]. Already at the level of ideal fluid approximation constructing numerical solution scheme to the equations is not an easy task. This is due to the nonlinearity of the system of conservation equations. Much work has been done in ideal hydrodynamics for heavy ion collision simulations (see, e.g., Ref. [25]). In this work the results are based on a simple one-dimensional consideration.

A natural way to obtain the evolution equations for the fluxes from a macroscopic basis is to generalize the equilibrium thermodynamic theories. That is, we assume the existence of a generalized entropy which depends on the dissipative fluxes and on the equilibrium variables as well. Restrictions on the form of the evolution equations are then imposed by the laws of thermodynamics. From the expression for generalized entropy one can then derive the generalized equations of state, which are of interest in the description of system under consideration. The phenomenological formulation of the transport equations for the first order and second order theories is accomplished by combining the conservation of energy-momentum and particle number with the Gibbs equation. One then obtains an expression for the entropy four-current, and its divergence leads to entropy production. Because of the enlargement of the space of variables the expressions for the energy-momentum tensor $T^{\mu\nu}$, particle four-current N^μ, entropy four-current S^μ, and the Gibbs equation contain extra terms. Transport equations for dissipative fluxes are obtained by imposing the second law of thermodynamics, that is, the principle of nondecreasing entropy. The kinetic approach is based on Grad's 14-moment method [8]. For a review on generalization of the 14-moment method to a mixture of several particle species see Ref. [26]. For applications and discussions of the moment method in kinetic and transport theory of gases see, e.g., Ref. [27] and for applications in astrophysics and cosmology see, e.g., Ref. [28]. The need for hyperbolic theory in relativistic and nonrelativistic systems is also emphasized in Ref. [29].

In the early stages of relativistic nuclear collisions we want to describe phenomena at frequencies comparable to the inverse of the relaxation times of the fluxes. At such time scales, these fluxes must be included in the set of basic independent variables. In order to model dissipative processes we need nonequilibrium fluid dynamics or irreversible thermodynamics. A satisfactory approach to irreversible thermodynamics is via nonequilibrium kinetic theory. In this work we will, however, follow a phenomenological approach. Whenever necessary we will point out how kinetic theory supports many of the results and their generalization. A complete discussion of irreversible thermodynamics is given in the monographs [30–32], where most of the theory and applications are nonrelativistic but include relativistic thermodynamics. A relativistic, but more advanced, treatment may be found in Refs. [33–35]. In this work we will present a simple introduction to these features, leading up to a formulation of relativistic causal fluid dynamics that can be used for applications in nuclear collisions.

A. Basic features of irreversible thermodynamics and imperfect fluids

The basic formulation of relativistic hydrodynamics can be found in the literature (see, for example, Refs. [7,36–38]). We consider a simple fluid and no electromagnetic fields. This fluid is characterized by

$$N_A^\mu(x), \quad \text{particle 4-current}, \tag{1}$$

$$T^{\mu\nu}(x), \quad \text{energy-momentum tensor}, \tag{2}$$

$$S^\mu(x), \quad \text{entropy 4-current}, \tag{3}$$

where $A=1,\ldots,r$ for the r conserved net charge currents, such as electric charge, baryon number, and strangeness. N_A^μ and $T^{\mu\nu}$ represent conserved quantities:

$$\partial_\mu N_A^\mu = 0, \tag{4}$$

$$\partial_\mu T^{\mu\nu} = 0. \tag{5}$$

The above equations are the local conservation of net charge and energy-momentum. They are the equations of motion of a relativistic fluid. There are $4+r$ equations and $10+4r$ independent unknown functions. The second law of thermodynamics requires

$$\partial_\mu S^\mu \geqslant 0, \tag{6}$$

and it forms the basis for the development of the extended irreversible thermodynamics. The equality in Eq. (6) is for an equilibrium state, that is, for an ideal fluid.

We now perform a tensor decomposition of $N^\mu, T^{\mu\nu}$, and S^μ with respect to an arbitrary, timelike, four-vector u^μ, normalized as $u^\mu u_\mu = 1$, and the projection onto the three-space $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu \equiv \Delta^{\nu\mu}$ orthogonal to u^μ, that is, $\Delta^{\mu\nu} u_\nu = 0$. The tensor decomposition reads

$$N^\mu = n u^\mu + V^\mu, \tag{7}$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + 2W^{(\mu} u^{\nu)} + t^{\mu\nu}, \tag{8}$$

$$S^\mu = s u^\mu + \Phi^\mu, \tag{9}$$

where we have defined

$$W^\mu = q^\mu + hV^\mu, \tag{10}$$

$$t^{\mu\nu} = \pi^{\mu\nu} - \Pi\Delta^{\mu\nu}. \tag{11}$$

Here h is the enthalpy per particle defined by

$$h = \frac{(\varepsilon + p)}{n}. \tag{12}$$

The dissipative fluxes are orthogonal to u^μ and in addition the shear tensor is traceless:

$$u_\mu V^\mu = 0, \quad u_\mu q^\mu = 0, \quad u_\mu W^\mu = 0, \quad u_\mu t^{\mu\nu} = 0, \quad \pi^\nu_\nu = 0. \tag{13}$$

In the local rest frame (LRF) defined by $u^\mu = (1, \mathbf{0})$ the quantities appearing in the decomposed tensors have the following meanings: $n \equiv u_\mu N^\mu$ is the net density of charge, $V^\mu \equiv \Delta^\mu_\nu N^\nu$ is the net flow of charge, $\varepsilon \equiv u_\mu T^{\mu\nu} u_\nu$ is the energy density, $p + \Pi \equiv -\frac{1}{3}\Delta_{\mu\nu} T^{\mu\nu}$ is the local isotropic pressure plus bulk pressure, $W^\mu \equiv u_\nu T^{\nu\lambda}\Delta^\mu_\lambda$ is the energy flow, $q^\mu \equiv W^\mu - hV^\mu$ is the heat flow, $\pi^{\mu\nu} \equiv T^{\langle\mu\nu\rangle}$ is the stress tensor, $s \equiv u_\mu S^\mu$ is the entropy density, and $\Phi^\mu \equiv \Delta^\mu_\nu S^\nu$ is the entropy flux. The angular bracket notation, representing the symmetrized spatial and traceless part of the tensor, is defined by $A^{\langle\mu\nu\rangle} \equiv \left[\frac{1}{2}(\Delta^\mu_\sigma \Delta^\nu_\tau + \Delta^\mu_\tau \Delta^\nu_\sigma) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\sigma\tau}\right] A^{\sigma\tau}$. The space-time derivative decomposes into $\partial^\mu = u^\mu \mathrm{D} + \nabla^\mu$ with $u^\mu \nabla_\mu = 0$. In this space-time derivative decomposition $D \equiv u^\mu \partial_\mu$ is the convective time derivative and $\nabla^\mu \equiv \Delta^{\mu\nu}\partial_\nu$ is the gradient operator.

So far, u^μ is arbitrary. It has the following properties. Differentiating $u_\mu u^\mu = 1$ with respect to space-time coordinates, ∂_μ, yields

$$u_\mu \partial_\nu u^\mu = 0, \tag{14}$$

which is a useful relation. There are two choices for u^μ [33]. It can be taken parallel to the particle flux N^μ. This is known as the Eckart or particle frame, and in this frame $V^\mu = 0$. It can also be taken to be parallel to the energy flow. This is known as the Landau and Lifshitz or energy frame, and in this frame $W^\mu = 0$. This implies that $q^\mu = -hV^\mu$.

The two choices of velocity four-flow have different computational advantage of each of the formulations. The Landau-Lifshitz formalism is convenient to employ since it reduces the energy-momentum tensor to a simpler form. The price for this is the implicit definition of the four-velocity. The Eckart formalism has the advantage when one wants to have simple integration of particle conservation law. This choice is also more intuitive than that of Landau-Lifshitz. For a system with no net charge, the four-velocity in the Eckart formalism is not well defined, and therefore in general under this situation one should use the Landau-Lifshitz formalism. The Landau-Lifshitz formalism is also advantageous in the case of mixtures.

B. Conservation laws and the second law of thermodynamics

We will now consider one type of charge, namely, the net baryon number. We insert the expressions for the number four-current and the energy-momentum tensor in the conservation laws and project them onto the four-velocity and the projection tensor. Using the orthogonality properties (13 of dissipative fluxes) we obtain the following conservation laws. The equation of continuity (net charge conservation) $\partial_\mu N^\mu \equiv 0$, equation of motion (momentum conservation) $\Delta^\mu_\nu \partial_\lambda T^{\nu\lambda} \equiv 0$, and the equation of energy (energy conservation) $u_\mu \partial_\nu T^{\mu\nu} \equiv 0$ are, respectively,

$$\mathrm{D}n = -n\nabla_\mu u^\mu - \nabla_\mu V^\mu + V_\mu \mathrm{D}u^\mu, \tag{15}$$

$$(\varepsilon + p + \Pi)\mathrm{D}u^\mu = \nabla^\mu(p + \Pi) - \Delta^\mu_\nu \nabla_\sigma \pi^{\nu\sigma} + \pi^{\mu\nu}\mathrm{D}u_\nu - [\Delta^\mu_\nu \mathrm{D}W^\nu + 2W^{(\mu}\nabla_\nu u^{\nu)}], \tag{16}$$

$$\mathrm{D}\varepsilon = -(\varepsilon + p + \Pi)\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_{\langle\nu} u_{\mu\rangle} - \nabla_\mu W^\mu + 2W^\mu \mathrm{D}u_\mu. \tag{17}$$

The five conservation equations (15)–(17) contain 14 unknown functions, $n, \varepsilon, \Pi, W^\mu, \pi^{\mu\nu}$, and u^μ. To close the system of equations we need to obtain nine additional equations (for dissipative fluxes) in addition to the five conservation equations (for primary variables) we already know. In presenting the nine additional equations we will use the Eckart's definition of u^μ.

From the phenomenological treatment of deriving the nine additional equations we need the expression for the out-of-equilibrium entropy four-current. The most general off-equilibrium entropy four-current $S^\mu(N^\mu, T^{\mu\nu})$ takes the form [10]

$$S^\mu = p(\alpha,\beta)\beta^\mu - \alpha N^\mu + \beta_\nu T^{\mu\nu} + Q^\mu(\delta N^\mu, \delta T^{\mu\nu}, \ldots), \tag{18}$$

where $\alpha \equiv \mu/T$ is the thermal potential, $\beta_\nu \equiv u_\nu/T$ is the inverse-temperature four-vector, and Q^μ is a function of deviations δN^μ and $\delta T^{\mu\nu}$ from local equilibrium,

$$\delta T^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{eq}, \quad \delta N^\mu = N^\mu - N^\mu_{eq}, \tag{19}$$

containing all the information about viscous stresses and heat flux in the off-equilibrium state.

Since the equilibrium pressure is only known as a function of the equilibrium energy density and equilibrium net charge density, we need to match/fix the equilibrium pressure to the actual state. We do this by requiring that the equilibrium energy density and the equilibrium net charge density be equal to the off-equilibrium energy density and off-equilibrium net charge density. This is equivalent to

$$\delta T^{\mu\nu} u_\mu u_\nu = \delta N^\mu u_\mu = 0. \tag{20}$$

With the help of the expression for the divergence of $p\beta^\mu$, that is,

$$\partial_\mu(p\beta^\mu) = N^\mu_{eq}\partial_\mu\alpha - T^{\mu\nu}_{eq}\partial_\mu\beta_\nu, \tag{21}$$

and the conservation laws for N^μ and for $T^{\mu\nu}$ the generalized second law of thermodynamics becomes

$$\partial_\mu S^\mu = -(\delta N^\mu)\partial_\mu\alpha + \delta T^{\mu\nu}\partial_\mu\beta_\nu + \partial_\mu Q^\mu. \tag{22}$$

Once a detailed form of Q^μ is specified, linear relations between irreversible fluxes $(\delta N^\mu, \delta T^{\mu\nu})$ and gradients $(\partial_{(\mu}\beta_{\nu)}, \partial_\mu\alpha)$ follow by imposing the second law of thermodynamics, namely, that the entropy production be positive. The key to a complete phenomenological theory thus lies in the specification of Q^μ.

C. Standard relativistic dissipative fluid dynamics

The standard Landau-Lifshitz and Eckart theories make the simplest possible assumption about Q^μ: that it is linear in the dissipative quantities $(\Pi, q^\mu, \pi^{\mu\nu})$. In kinetic theory this amounts to Taylor expanding the entropy four-current expression up to first order in deviations from equilibrium. This leads to an expression of entropy four-current which is just a linear function of the heat flux.

This can be understood as follows: Take a simple fluid with particle current N^μ. Let us choose $\beta^\mu = u^\mu/T$ parallel to the current N^μ of the given off-equilibrium state, so we are in the Eckart frame. Projecting Eq. (18) onto the three-space orthogonal to u^μ gives

$$\Phi^\mu \equiv \Delta^\mu_\nu S^\nu = \beta q^\mu + Q^\nu \Delta^\mu_\nu, \tag{23}$$

so that

$$\Phi^\mu = q^\mu/T + \text{second order terms}, \tag{24}$$

which, to linear order, is just the standard relation between entropy flux Φ^μ and heat flux q^μ. From Eq. (23) this implies that the entropy flux Φ^μ is a strictly linear function of heat flux q^μ, and depends on no other variables; also that the off-equilibrium entropy density depends only on the densities ε and n and is given precisely by the equation of state $s = s_{eq}(\varepsilon, n)$.

Alternatively we may begin with the ansatz for the entropy four-current S^μ: In the limit of vanishing Π, q^μ, and $\pi^{\mu\nu}$ the entropy four-current must reduce to the one of ideal fluid. The only nonvanishing four-vector which can be formed from the available tensors u^μ, q^μ, and $\pi^{\mu\nu}$ is βq^μ, where β is arbitrary but it turns out to be nothing else but the inverse temperature. Thus the first order expression for the entropy four-current in the Eckart frame is given by

$$S^\mu = s u^\mu + \frac{q^\mu}{T}, \tag{25}$$

and one immediately realizes that

$$\Phi^\mu = \frac{q^\mu}{T} \tag{26}$$

is the entropy flux. Using the expressions for N^μ, $T^{\mu\nu}$, and S^μ in the second law of thermodynamics $\partial_\mu S^\mu \geqslant 0$ and using the conservation laws $u_\nu \partial_\mu T^{\mu\nu} = 0$, $\partial_\mu N^\mu = 0$ and the Gibbs equation

$$\partial_\mu(p\beta^\mu) = N^\mu\partial_\mu\alpha - T^{\mu\nu}\partial_\mu\beta_\nu, \tag{27}$$

the divergence of Eq. (25) gives the following expression for entropy production:

$$T\partial_\mu S^\mu = q^\mu(\nabla_\mu\beta + Du_\mu) + \pi^{\mu\nu}\nabla_\mu u_\nu - \Pi\nabla_\mu u^\mu \geqslant 0. \tag{28}$$

Notice that the equilibrium conditions (i.e., the bulk free and shear free of the flow and the constancy of the thermal potential, i.e., no heat flow) lead to the vanishing of each factor multiplying the dissipative terms on the right, and therefore lead to $\partial_\alpha S^\alpha = 0$. The expression (28) splits into three independent, irreducible pieces:

$$\Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\langle\mu\nu\rangle} \geqslant 0, \tag{29}$$

where the thermodynamic forces are $X \equiv -\nabla_\mu u^\mu$, $X^\mu \equiv (\nabla^\mu T/T) - Du^\mu$, $X^{\langle\mu\nu\rangle} \equiv \left[\frac{1}{2}(\Delta^\mu_\sigma\Delta^\nu_\tau + \Delta^\nu_\sigma\Delta^\mu_\tau) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\sigma\tau}\right]\nabla^\sigma u^\tau$.

From the second law of thermodynamics, $\partial_\mu S^\mu \geqslant 0$, we see that the simplest way to satisfy the bilinear expression (28) is to impose the following linear relationships between the thermodynamic fluxes $\Pi, q^\mu, \pi^{\mu\nu}$, and the corresponding thermodynamic forces:

$$\Pi = -\zeta\nabla_\mu u^\mu, \tag{30}$$

$$q^\mu = \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right) = -\frac{\lambda n T^2}{\varepsilon + p}\nabla^\mu\left(\frac{\mu}{T}\right), \tag{31}$$

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}. \tag{32}$$

That is, we assume that the dissipative fluxes are linear and purely local functions of the gradients. We then obtain uniquely, if the equilibrium state is isotropic (Curie's principle), the above linear expressions.

These are the constitutive equations for dissipative fluxes in the standard Eckart theory of relativistic irreversible thermodynamics. They are relativistic generalizations of the corresponding Newtonian laws. The linear laws allow us to identify the thermodynamic coefficients, namely, the bulk viscosity $\zeta(\varepsilon,n)$, the thermal conductivity $\lambda(\varepsilon,n)$, and the shear viscosity $\eta(\varepsilon,n)$.

Given the linear constitutive equations (30)–(32), the entropy production rate (28) becomes

$$\partial_\mu S^\mu = \frac{\Pi^2}{\zeta T} - \frac{q_\mu q^\mu}{\lambda T^2} + \frac{\pi_{\mu\nu}\pi^{\mu\nu}}{2\eta T} \geqslant 0, \tag{33}$$

which is guaranteed to be non-negative provided that

$$\zeta \geqslant 0, \quad \lambda \geqslant 0, \quad \eta \geqslant 0. \tag{34}$$

Note that $q^\mu q_\mu < 0$ which can be most easily proven from $q^\mu u_\mu = 0$ in the LRF.

Using the fundamental thermodynamic equation of Gibbs the entropy evolution equation can be written in the following convenient form:

$$T\partial_\mu S^\mu = \sigma_{\mu\nu}\pi^{\mu\nu} - \Pi\theta - \partial_\mu q^\mu + q^\mu a_\mu, \tag{35}$$

which can be found with the help of the *fluid kinematic identity*

$$\partial_\nu u_\mu = \sigma_{\nu\mu} + \omega_{\nu\mu} + \tfrac{1}{3}\theta\Delta_{\mu\nu} + a_\mu u_\nu, \tag{36}$$

where $a_\mu \equiv u^\nu \partial_\nu u_\mu$ is the four-acceleration of the fluid, $\omega_{\mu\nu} \equiv \Delta^\alpha_\mu \Delta^\beta_\nu \partial_{[\beta} u_{\alpha]}$ is the vorticity tensor, $\theta_{\mu\nu} \equiv \Delta^\alpha_\mu \Delta^\beta_\nu \partial_{(\beta} u_{\alpha)}$ is the expansion tensor, $\theta \equiv \Delta^{\mu\nu}\theta_{\mu\nu} = \partial_\mu u^\mu$ is the volume expansion, and $\sigma_{\mu\nu} \equiv \theta_{\mu\nu} - \frac{1}{3}\Delta_{\mu\nu}\theta$ is the shear tensor. The quantities defined here are the fluid kinematic variables.

The Navier-Stokes-Fourier equations comprise a set of nine equations. Together with the five conservation laws $\partial_\mu T^{\mu\nu} = \partial_\mu N^\mu = 0$, they should suffice, on the basis of naive counting, to determine the evolution of the 14 variables $T^{\mu\nu}$ and N^μ from suitable initial data. Unfortunately, this system of equations is of mixed parabolic-hyperbolic-elliptic type. Just like the nonrelativistic Fourier-Navier-Stokes theory, they predict infinite propagation speeds for thermal and viscous disturbances. Already at the nonrelativistic level, the parabolic character of the equations has been a source of concern [39]. One would expect signal velocities to be bounded by the mean molecular speed. However in the nonrelativistic case wave-front velocities can be infinite such as the case in the tail of Maxwell's distribution which has arbitrarily large velocities. However, a relativistic theory which predicts infinite speeds of propagation contradicts the foundation or the basic principles of relativity and must be a cause of concern especially when one has to use the theory to explain observables from relativistic phenomena or experiments such as ultrarelativistic heavy ion experiments. The other problem is that these first order theories possess instabilities: equilibrium states are unstable under small perturbations [11].

Most of the applications of dissipative fluid dynamics in relativistic nuclear collisions have used the Eckart/Landau-Lifshitz theory. However, the algebraic nature of the Eckart constitutive equations leads to severe problems. Qualitatively, it can be seen from Eqs. (30)–(32) that if a thermodynamic force is suddenly switched off/on, then the corresponding thermodynamic flux instantaneously vanishes/appears. This indicates that a signal propagates through the fluid at infinite speed, violating relativistic causality. This is known as a paradox since in special relativity the speed of light is finite and all maximum speeds should not be greater than this speed. This paradox was first addressed by Cattaneo [39] by introducing *ad hoc* relaxation terms in the phenomenological equations. The resulting equations conform with causality. The only problem was that a sound theory was needed. It is from these arguments that the causal extended theory of Müller, Israel, and Stewart was developed.

D. Causal relativistic dissipative fluid dynamics

Clearly the Eckart postulate (25) for Q^μ and hence S^μ is too simple. Kinetic theory indicates that in fact Q^μ is second order in the dissipative fluxes. The Eckart assumption, by truncating at first order, removes the terms that are necessary to provide causality, hyperbolicity, and stability.

The second order kinetic theory formulation of the entropy four-current, see Ref. [8], was the starting point for good work on extending the domain of validity of conventional thermodynamics to shorter space-time scale. The turning point was Müller's paper [9] which, for the first time, expressed Q^μ in terms of the off-equilibrium forces $(\Pi, \mathbf{q}, \pi^{ij})$ and thus linked phenomenology to the Grad expansion [8]. This marked the birth of what is now known as extended irreversible thermodynamics [30–32].

For small deviations, it will suffice to retain only the lowest-order, quadratic terms in the Taylor expansion of Q^μ, leading to linear phenomenological laws. The most general algebraic form for Q^μ that is at most second order in the dissipative fluxes gives [10]

$$S^\mu = su^\mu + \frac{q^\mu}{T} - (\beta_0\Pi^2 - \beta_1 q_\nu q^\nu + \beta_2 \pi_{\nu\lambda}\pi^{\nu\lambda})\frac{u^\mu}{2T} - \frac{\alpha_0 \Pi q^\mu}{T} + \frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T}, \tag{37}$$

where $\beta_A(\varepsilon,n) \geqslant 0$ are thermodynamic coefficients for scalar, vector, and tensor dissipative contributions to the entropy density, and $\alpha_A(\varepsilon,n)$ are thermodynamic viscous/ heat coupling coefficients. It follows from Eq. (37) that the effective entropy density measured by comoving observers is

$$u_\mu S^\mu = s - \frac{1}{2T}(\beta_0 \Pi^2 - \beta_1 q_\mu q^\mu + \beta_2 \pi_{\mu\nu}\pi^{\mu\nu}), \tag{38}$$

independent of α_A. Note that the entropy density is a maximum in equilibrium. The condition $u_\mu Q^\mu \leqslant 0$, which guarantees that entropy is maximized in equilibrium, requires that the β_A be non-negative. The entropy flux is

$$\Phi^\mu = \beta(q^\mu - \alpha_0 \Pi q^\mu + \alpha_1 \pi^{\mu\nu} q_\nu), \tag{39}$$

and is independent of the β_A.

The divergence of the extended current (37) together with the Gibbs equation (27) and the conservation equations (15)–(17) leads to

$$T\partial_\mu S^\mu = -\Pi\left[\theta + \beta_0\dot{\Pi} + \frac{1}{2}T\partial_\mu\left(\frac{\beta_0}{T}u^\mu\right)\Pi - \alpha_0\nabla_\mu q^\mu\right] - q^\mu\left[\nabla_\mu \ln T - \dot{u}_\mu - \beta_1\dot{q}_\mu - \frac{1}{2}T\partial_\nu\left(\frac{\beta_1}{T}u^\nu\right)q_\mu - \alpha_0\nabla_\nu\pi^\nu_\mu - \alpha_1\nabla_\mu\Pi\right] + \pi^{\mu\nu}\left[\sigma_{\mu\nu} - \beta_2\dot{\pi}_{\mu\nu} + \frac{1}{2}T\partial_\lambda\left(\frac{\beta_2}{T}u^\lambda\right)\pi_{\mu\nu} + \alpha_1\nabla_{\langle\nu}\, q_{\ \mu\rangle}\right]. \quad (40)$$

The simplest way to satisfy the second law of thermodynamics, $\partial_\mu S^\mu \geqslant 0$, is to impose, as in the standard theory, linear relationships between the thermodynamical fluxes and extended thermodynamic forces, leading to the following constitutive or transport equations:

$$\tau_\Pi\dot{\Pi} + \Pi = -\zeta\theta - \left[\frac{1}{2}\zeta T\partial_\mu\left(\frac{\tau_0}{\zeta T}u^\mu\right)\Pi\right] + l_{\Pi q}\nabla_\mu q^\mu, \quad (41)$$

$$\tau_q\Delta^\nu_\mu\dot{q}_\nu + q_\mu = \lambda(\nabla_\mu T - T\dot{u}_\mu) + \left[\frac{1}{2}\lambda T^2\partial_\nu\left(\frac{\tau_1}{\lambda T^2}u^\nu\right)q_\mu\right] - l_{q\Pi}\nabla_\mu\Pi - l_{q\pi}\nabla_\nu\pi^\nu_\mu, \quad (42)$$

$$\tau_\pi\Delta^\alpha_\mu\Delta^\beta_\nu\dot{\pi}_{\alpha\beta} + \pi_{\mu\nu} = 2\eta\sigma_{\mu\nu} - \left[\eta T\partial_\lambda\left(\frac{\tau_2}{2\eta T}u^\lambda\right)\pi_{\mu\nu}\right] + l_{\pi q}\nabla_{\langle\mu}\, q_{\ \nu\rangle}. \quad (43)$$

Here the relaxational times $\tau_A(\varepsilon,n)$ are given by

$$\tau_\Pi = \zeta\beta_0, \quad \tau_q = \lambda T\beta_1, \quad \tau_\pi = 2\eta\beta_2, \quad (44)$$

and the heat-viscous coupling lengths coefficients $l_{AB}(\varepsilon,n)$ are given by

$$l_{\Pi q} = \zeta\alpha_0, \quad l_{q\Pi} = \lambda T\alpha_0, \quad l_{q\pi} = \lambda T\alpha_1, \quad l_{\pi q} = 2\eta\alpha_1. \quad (45)$$

A key quantity in these theories is the relaxation time τ of the corresponding dissipative process. It is a positive-definite quantity by the requirement of hyperbolicity. It is the time taken by the corresponding dissipative flux to relax to its steady-state value. It is connected to the mean collision time t_c of the particles responsible for the dissipative process, but the two are not the same. In principle they are different since τ is a macroscopic time, although in some instances it may correspond just to a few t_c. No general formula linking τ and t_c exists; their relationship depends in each case on the system under consideration.

Besides the fact that parabolic theories are necessarily noncausal, it is obvious that whenever the time scale of the problem under consideration becomes of the order of or smaller than the relaxation time, the latter cannot be ignored. Neglecting the relaxation time in this situation amounts to disregarding the whole problem under consideration.

Even in the steady-state regime the descriptions offered by parabolic and hyperbolic theories might not necessarily coincide. The differences between them in such a situation arise from the presence of τ in terms that couple the vorticity to the heat flux and shear stresses. These may be large even in steady states where vorticity is important. There are also other acceleration coupling terms to bulk and shear stresses and the heat flux. The coefficients for these vanish in parabolic theories, but they could be large even in the steady state. The convective part of the time derivative, which is not negligible in the presence of large spatial gradients, and modifications in the equations of state due to the presence of dissipative fluxes also differentiate hyperbolic theories from parabolic ones. However, it is precisely before the establishment of the steady-state regime that both types of theories differ more significantly. Therefore, if one wishes to study a dissipative process for times shorter than τ, it is mandatory to resort to a hyperbolic theory which is a more accurate macroscopic approximation to the underlying kinetic description.

Provided that the spatial gradients are not so large that the convective part of the time derivative becomes important, and that the fluxes and coupling terms remain safely small, then for times larger than τ it is sensible to resort to a parabolic theory. However, even in these cases, it should be kept in mind that the way a system approaches equilibrium may be very sensitive to the relaxation time. The future of the system at time scales much longer than the relaxation time, once the steady state is reached, may also critically depend on τ.

The crucial difference between the standard Eckart and the extended Israel-Stewart transport equations is that the latter are differential evolution equations, while the former are algebraic relations. The evolution terms, with the relaxational time coefficients τ_A, are needed for causality, as well as for modeling high energy heavy ion collisions relaxation effects are important. The price paid for the improvements that the extended causal thermodynamics brings is that new thermodynamic coefficients are introduced. However, as is the case with the coefficients ζ,λ, and η that occur also in standard theory, these new coefficients may be evaluated or at least estimated via kinetic theory. The relaxation times τ_A involve complicated collision integrals. They are usually estimated as mean collision times, of the form $\tau \approx 1/n\sigma v$, where σ is a collision cross section and v the mean particle speed.

The form of transport equations obtained here is justified by kinetic theory, which leads to the same form of the transport equations, but with extra terms and explicit expressions for transport, relaxation, and coupling coefficients. With these transport equations, the entropy production rate has the same non-negative form (33) as in the standard theory. In addition to viscous/heat couplings, kinetic theory shows that in general there will also be couplings of the heat flux and the anisotropic pressure to the vorticity. These couplings give rise to the following additions to the right-hand sides of Eqs. (42) and (43), respectively:

$$+\tau_q\omega_{\mu\nu}q^\nu \quad \text{and} \quad +\tau_\pi\pi^\lambda_{\langle\mu}\,\omega_{\ \nu\rangle\lambda}. \quad (46)$$

In the case of scaling solution assumption in nuclear collisions these additional terms do not contribute since they vanish. Also, the resulting expression for $\partial_\mu S^\mu$ in general contains terms involving gradients of α_A and β_A multiplying second order quantities such as the bilinear terms $(\partial_\mu \alpha_1) q_\lambda \pi^{\lambda\mu}$ and $(\partial_\mu \alpha_0) q^\mu \Pi$. In the present work where we will assume scaling solution these terms do not contribute to the overall analysis.

It is also important to remember that the derivation of the causal transport equations is based on the assumption that the fluid is close to equilibrium. Thus the dissipative fluxes are small:

$$|\Pi| \ll p, \quad (\pi_{\mu\nu}\pi^{\mu\nu})^{1/2} \ll p, \quad (-q_\mu q^\mu)^{1/2} \ll \varepsilon. \qquad (47)$$

These conditions will also be useful in guiding us when we discuss the initial conditions for the dissipative fluxes. Considering the evolution of entropy in the Israel-Stewart theory, Eq. (35) still holds.

It will be inconceivable if the more general theory does not conform to the principles of relativity. In order to check that the system of 14 equations conforms with causality one writes the five conservation equations and the nine evolution equations for dissipative quantities in one single linearized system as done in Ref. [11]. The system of 14 equations may be written as a quasilinear system of 14 equations in the form

$$M_B^{\alpha A}(U^c)\partial_\alpha U^B = f^A(U^c) \ (A,B = 1, \ldots, 14), \qquad (48)$$

where $M_B^{\alpha A}(U^c)$ and $f^A(U^c)$ can be taken to be components of 14×14 matrices and 14 vectors. The right-hand side contains all the collision terms, and the coefficients $M_B^{\alpha A}(U^c)$ are purely thermodynamical functions.

Let Σ be a characteristic hypersurface for the system (48) and let $\phi(x^\alpha)=0$ be the local equation for Σ. Then ϕ satisfies the characteristic equation

$$\det[M_B^{\alpha A}(\partial_\alpha \phi)] = 0. \qquad (49)$$

$\phi(x^\alpha)$ is a three-dimensional space across which the variables U^B are continuous but their first derivatives are allowed to present discontinuities $[\partial_\alpha U^B]$ normal to the surface $\Rightarrow [\partial_\alpha U^B] = U^B(\partial_\alpha \phi)$. The characteristic speeds are independent of the microscopic details such as cross sections. To solve the characteristic equation (49) we consider a coordinate system x^α chosen in such a way that at any point in the fluid the system of reference is orthogonal and comoving. If ϕ is a function of x^0 and x^1 only, the characteristic speeds can be determined from

$$\det(v M_B^{A0} - M_B^{A1}) = 0, \qquad (50)$$

where v is the characteristic speed defined by

$$v = -\partial_0 \phi / \partial_1 \phi. \qquad (51)$$

The 14-component vector U is split into a scalar-longitudinal six-vector $U_L = [\varepsilon, n, \Pi, u^x, q^x, \pi^{xx}]$; two longitudinal-transverse three-vectors (corresponding to the two transverse directions of polarization); $U_{LT_1} = [u^y, q^y, \pi^{xy}]$ and $U_{LT_2} = [u^z, q^z, \pi^{xz}]$ and purely transverse two-vector $U_T = [\pi^{yz}, \pi^{yy} - \pi^{zz}]$. Equation (50) for v accordingly splits into one sixth-degree and two third-degree equations. The purely transverse modes do not propagate. This general scheme, when applied to first order theories, always yields wave-front speeds v that are superluminal [11].

We will be studying the dynamics of a pion fluid in the hadronic regime and a quark-gluon plasma fluid in the partonic regime. It is therefore important to check if these systems conform with the principle of relativity under small perturbation of the equilibrium state. For a quark-gluon plasma we consider a gas of weakly interacting massless quarks and gluons. We also consider such a system to have a vanishing baryon chemical potential ($\mu_B = 0$). This implies also that the net baryon charge is zero ($n_B = 0$). The equation of state is given by $p = \varepsilon/3$. For massless particles or ultrarelativistic particles the bulk viscosity vanishes.

In the absence of any conserved charge the convenient choice of the four-velocity is the Landau-Lifshitz frame. In this case the characteristic equations for the wave-front speeds become very simple. For the longitudinal modes we get only the *fast* longitudinal mode (associated with the true acoustical wave). The absence of heat conduction has as a consequence of the disappearance of the *slow* longitudinal propagation mode (associated with thermal dissipation wave). The phase velocity of the fast longitudinal mode is given by

$$v_L^2 = \frac{1 + 2p\beta_2}{6p\beta_2} = \frac{5}{9}, \qquad (52)$$

where we have used $\beta_2 \approx (3/4)(1/p)$ (see Ref. [10] for the coefficients α_i and β_i). Thus if we are considering only the shear viscosity we will get the above result. The wave-front speed (signal speed) for the transverse plane wave is given by

$$v_T^2 = \frac{1}{8p\beta_2} = \frac{1}{6}. \qquad (53)$$

For a pion fluid with vanishing chemical potential $\mu_\pi = 0$ we have, for the fast longitudinal mode and the transverse mode, the following expressions for the wave-front speeds:

$$v_L^2 = \frac{\frac{2}{3}\beta_0 + \beta_2 + \beta_0\beta_2(\varepsilon + p)\,\partial p/\partial \varepsilon}{\beta_0\beta_2(\varepsilon + p)}, \qquad (54)$$

$$v_T^2 = \frac{1}{2\beta_2(\varepsilon + p)}. \qquad (55)$$

Note that for the pion system we are in the relativistic regime. Then the equation of state is taken to be that of a noninteracting gas of pions only. The bulk viscosity does not vanish. We show the dependence of the wave-front speeds and the adiabatic speed of sound on temperature in Fig. 1. Thus in all the systems we will consider here (see the following section) causality is obeyed.

III. THE ROLE OF DISSIPATION IN RELATIVISTIC NUCLEUS-NUCLEUS COLLISIONS

We now describe the evolution of the matter produced at high energy nuclear collisions such as those at RHIC and

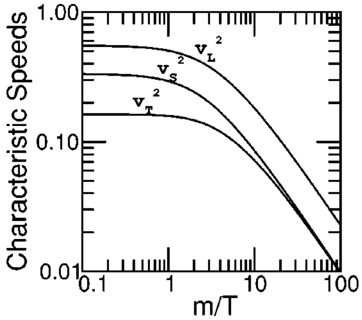

FIG. 1. The transverse (v_T), longitudinal (v_L), and sound (v_s) phase velocities in a pion gas, as a function of $z=m/T$, when m is the mass of pion.

LHC. Of interest is the midrapidity region with no net baryon number. Based on the observation that the rapidity distribution of the charged particle multiplicity dN_{ch}/dy is constant in the midrapidity region [40], that is, it is invariant under Lorentz transformation in the midrapidity region, it is reasonable to assume that all other quantities such as number density, energy density, and dissipative fluxes also have this symmetry. Thus these quantities depend on the proper time, $\tau=t/\gamma=\sqrt{t^2-z^2}$. The longitudinal component of the matter velocity is parametrized as $v=z/t=\tanh y$, with $t=\tau\cosh y$, $z=\tau\sinh y$, and the space-time rapidity is defined as $y=\frac{1}{2}\ln((t+z)/(t-z))$. This is the Bjorken [40] scaling solution assumption for high energy nuclear collisions. Then the four-velocity can be written as

$$u^\mu = (\cosh y, 0, 0, \sinh y). \tag{56}$$

We use the following transformation matrix of the derivatives to reduce the equations to simple forms:

$$\begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial \tau} \\ \frac{1}{\tau}\frac{\partial}{\partial y} \end{pmatrix}. \tag{57}$$

We also note that using the transformation of derivatives and the definition of the four-velocity we can write

$$\theta \equiv \partial_\mu u^\mu = \frac{1}{\tau}, \tag{58}$$

$$D \equiv u^\mu \partial_\mu = \frac{\partial}{\partial \tau}. \tag{59}$$

The recipe given here will be used to simplify the equations of relativistic fluid dynamics in the following sections. In this work we consider a (1+1)-dimensional scaling solution in which we have one nonvanishing spatial component of the four-velocity in a (3+1) space-time.

In order to solve the fluid dynamical equations one needs the initial and boundary conditions in addition to a realistic equation of state. The initial conditions can be taken in principle from transport calculations describing the approach to equilibrium, such as the parton cascade model commonly known as VNI [41], which treats the entire evolution of the parton gas from the first contact of the cold nuclei to hadronization or Heavy Ion Jet Interaction Generator (HIJING) [42]. For initializing a hadronic state one can use ultrarelativistic quantum molecular dynamics (UrQMD) [43].

Another frequently used relation between the initial temperature and the initial time is based on the uncertainty principle [44]. The formation time τ of a particle with an average energy $\langle E\rangle$ is given by $\tau\langle E\rangle \simeq 1$. The average energy of a thermal parton is about $3T$. Hence, we find $\tau_0 \simeq 1/(3T_0)$. However, if data for hadron production are available, such as at SPS, they can be used to determine or at least constrain the initial conditions for a hydrodynamical calculation of observables such as the photon spectra [45].

Under the simplifying assumption of an ideal fluid, the full hydrodynamical equations can be solved numerically using an equation of state and the initial conditions, such as initial time and temperature, as input. The final results depend strongly on the input parameters as well as on other details of the model, as in the simple one-dimensional case. For a system out of equilibrium the Euler equations should be replaced by the Navier-Stokes [16,17] or hyperbolic dissipative equations [18–20].

In dissipative fluid dynamics entropy is generated by dissipation. The dissipative quantities, namely, Π, q^μ, and $\pi^{\mu\nu}$ are not set *a priori* to zero. They are specified through additional equations. Since we will be working with a baryon-free system ($n=0$), a convenient choice of the reference frame is the Landau and Lifshitz frame. The number current, energy-momentum tensor, and the entropy four-current in this frame are obtained from Eqs. (7)–(9) with $W^\mu=0$. In the LRF the energy-momentum tensor is given by

$$T^{\mu\nu}_{LRF} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & (p+\Pi-\pi/2) & 0 & 0 \\ 0 & 0 & (p+\Pi-\pi/2) & 0 \\ 0 & 0 & 0 & (p+\Pi+\pi) \end{pmatrix}. \tag{60}$$

This satisfies $T^\nu_\nu=\varepsilon-3(p+\Pi)$, $\pi^\nu_\nu=\pi^{\mu\nu}u_\nu=0$. To study the dynamics of the system it is necessary to apply a boost in the longitudinal direction. Using Eq. (56) we have

$$T^{\mu\nu} = \begin{pmatrix} \mathcal{W}\cosh^2 y - \mathcal{P} & 0 & 0 & \mathcal{W}\cosh y \sinh y \\ 0 & \mathcal{P}_\perp & 0 & 0 \\ 0 & 0 & \mathcal{P}_\perp & 0 \\ \mathcal{W}\cosh y \sinh y & 0 & 0 & \mathcal{W}\sinh^2 y + \mathcal{P} \end{pmatrix}, \tag{61}$$

with $\mathcal{W}=\varepsilon+\mathcal{P}$ the effective enthalpy density, $\mathcal{P}=p+\Pi+\pi$ the effective longitudinal pressure, and $\mathcal{P}_\perp=p+\Pi-\pi/2$ the effective transverse pressure. For perfect fluids $\pi=\Pi=0$. It is clear that the effects of viscosity is to reduce pressure in the longitudinal direction and increase pressure in the transverse direction. The (1+1)-dimensional scaling solution implies that the thermodynamic quantities depend on τ only. Thus the scaling solution and the relations $\partial\tau/\partial x^\mu=u_\mu$ and

$\partial f(\tau)/\partial x^\mu = u_\mu(\partial f(\tau)/\partial\tau)$ [where $f(\tau)$ represent thermodynamic variables such as temperature, chemical potential, and dissipative fluxes] reduce the first order transport equations (30)–(32) to

$$\Pi = -\zeta\frac{1}{\tau}, \tag{62}$$

$$V^\mu = 0, \tag{63}$$

$$\pi = -\frac{4}{3}\eta\frac{1}{\tau}. \tag{64}$$

The current V^μ is induced by heat conduction, $V^\mu = -q^\mu/h$. Equation (63) implies that there is no heat conduction in the scaling solutions. This is independent of the fact that $n=0$, another condition that also makes V^μ vanish.

In the second order theory Π and π have to be determined from the second order transport equations. In the Landau-Lifshitz frame the transport equations are still given by Eqs. (41)–(43) but with slightly different heat coupling coefficients in the bulk and shear viscous pressure equations. Under the scaling solution assumption those coupling terms do not contribute to the dynamics of the system. The (1+1)-scaling solution in (3+1) dimensions reduces the relaxation equations (41)–(43) to

$$\frac{\partial\Pi}{\partial\tau} = -\frac{\Pi}{\tau_\Pi} - \frac{1}{2}\frac{1}{\beta_0}\Pi\left[\beta_0\frac{1}{\tau} + T\frac{\partial}{\partial\tau}\left(\frac{\beta_0}{T}\right)\right] - \frac{1}{\beta_0}\frac{1}{\tau}, \tag{65}$$

$$\frac{\partial q^\mu}{\partial\tau} = -\frac{q^\mu}{\tau_q} + \frac{1}{2}\frac{1}{\beta_1}q^\mu\left[\beta_1\frac{1}{\tau} + T\frac{\partial}{\partial\tau}\left(\frac{\beta_1}{T}\right)\right], \tag{66}$$

$$\frac{\partial\pi^{\mu\nu}}{\partial\tau} = -\frac{\pi^{\mu\nu}}{\tau_\pi} - \frac{1}{2}\frac{1}{\beta_2}\pi^{\mu\nu}\left[\beta_2\frac{1}{\tau} + T\frac{\partial}{\partial\tau}\left(\frac{\beta_2}{T}\right)\right] + \frac{1}{\beta_2}\left[\tilde{\Delta}^{\mu\nu} - \frac{1}{3}\Delta^{\mu\nu}\right]\frac{1}{\tau}. \tag{67}$$

In the last of the above equations $\tilde{\Delta}^{\mu\nu} = \Delta^{\mu\nu}$ for $0 \leqslant \mu,\nu \leqslant 1$ and 0 otherwise (because of only one nonvanishing spatial component of the four-velocity).

For the (1+1)-dimensional Bjorken similarity fluid flow in (3+1) dimensions the energy equation (17) becomes

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} - \frac{1}{\tau}\pi - \Pi\frac{1}{\tau} = 0, \tag{68}$$

where $\pi \equiv \pi^{00} - \pi^{zz}$ is determined from the shear viscous tensor evolution equation (67)

$$\frac{d}{d\tau}\pi = -\frac{1}{\tau_\pi}\pi - \frac{1}{2}\pi\left[\frac{1}{\tau} + \frac{1}{\beta_2}T\frac{d}{d\tau}\left(\frac{\beta_2}{T}\right)\right] + \frac{2}{3}\frac{1}{\beta_2}\frac{1}{\tau}, \tag{69}$$

and Π is determined from Eq. (65). Note that in Eq. (68) π and Π are positive. In the case of massless particles the bulk pressure equation (65) does not contribute since the bulk viscosity is negligible or vanishes [37]. We will distinguish the perfect fluid, first order, and second order theories by the quantity π:

$$\pi \equiv 0, \quad \text{perfect fluid}, \tag{70}$$

$$\pi = \frac{4}{3}\eta/\tau, \quad \text{first order theory}, \tag{71}$$

$$\frac{d\pi}{d\tau} = -\frac{\pi}{\tau_\pi} - \frac{1}{2}\pi\left[\frac{1}{\tau} + \frac{1}{\beta_2}T\frac{d}{d\tau}\left(\frac{\beta_2}{T}\right)\right] + \frac{2}{3}\frac{1}{\beta_2}\frac{1}{\tau}, \quad \text{second order theory}. \tag{72}$$

Equation (68) can be written in terms of the ratios of nondissipative to dissipative terms as

$$\frac{\partial\varepsilon}{\partial\tau} + \frac{\varepsilon + p}{\tau} = \frac{\varepsilon + p}{R\tau}, \tag{73}$$

where the ratio R, associated with the Reynolds number in Refs. [13,50], is defined by

$$R = \frac{(\varepsilon + p)}{\pi}. \tag{74}$$

For this exploratory study a simple equation of state is used, namely, that of a weakly interacting plasma of massless u,d,s quarks and gluons. The pressure is given by $p=\varepsilon/3 = aT^4$ with zero baryon chemical potential. That is, $\mu=0$, $\varepsilon = 3p$ or $s=4aT^3$, $\eta = bT^3$, and $\zeta=0$, $a,b=$const. The energy equation (68) and the shear viscous pressure equation (69) reduce to

$$\frac{d}{d\tau}T = -\frac{T}{3\tau} + \frac{T^{-3}\pi}{12a\tau}, \tag{75}$$

$$\frac{d}{d\tau}\pi = -\frac{2aT\pi}{3b} - \frac{1}{2}\pi\left(\frac{1}{\tau} - 5\frac{1}{T}\frac{d}{d\tau}T\right) + \frac{8aT^4}{9\tau}. \tag{76}$$

For a perfect fluid and a first order theory Eq. (75) can be solved analytically. In this case the solution of Eq. (75) is

$$\frac{T}{T_0} = \left[\frac{\tau_0}{\tau}\right]^{1/3}\left\{1 + \frac{R_0^{-1}}{2}\left(1 - \left[\frac{\tau_0}{\tau}\right]^{2/3}\right)\right\}, \tag{77}$$

where T_0 and R_0 are the initial values of the temperature and the Reynolds number at the initial proper time $\tau=\tau_0$. Note that when $R_0^{-1}=0$ we obtain the familiar ideal fluid results while a nonvanishing R^{-1} makes the cooling rate smaller. Here

$$a = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2}{90} \tag{78}$$

is a constant determined by the number of quark flavors and the number of gluon colors. The only relaxation coefficient we need is β_2 which, for massless particles, is given by $\beta_2 = 3/(4p)$. The shear viscosity is given [51] by $\eta = bT^3$, where

$$b = (1 + 1.70N_f)\frac{0.342}{(1 + N_f/6)\alpha_s^2 \ln(\alpha_s^{-1})} \qquad (79)$$

is a constant determined by the number of quark flavors and the number of gluon colors. Here N_f is the number of quark flavors, taken to be 3, and α_s is the strong fine structure constant, taken to be 0.4–0.5.

The role of dissipation can be examined by rewriting energy equation as

$$\frac{\partial \varepsilon}{\partial \tau} = (R^{-1} - 1)\frac{\varepsilon + p}{\tau}. \qquad (80)$$

As is seen in this equation, the energy density increases with time if $R<1$ and decreases if $R>1$. When $R=1$, the critical value for Reynolds number, the thermodynamic quantities do not change with time. One of the mathematical advantages of the parabolic theories is the direct connection between the Reynolds number and the initial conditions (T_0, τ_0). This is because the first order theory does not have well-defined initial conditions for the dissipative fluxes, and the latter are related to the thermodynamic forces by linear algebraic expressions.

We now discuss the issue of initial condition for π. For an ideal fluid π vanishes since there are no dissipative fluxes. For the first order theories the initial condition for π is not well defined and is given by the initial conditions (T_0, τ_0). For the second order theories we have well-defined initial condition for π since the dissipative fluxes are found from their evolution equations.

In deriving the transport equations it is assumed that the dissipative fluxes are small compared to the primary variables (ε, n, p). For shear flux we require that

$$[\pi^{\mu\nu}\pi_{\mu\nu}]^{1/2} = \sqrt{\frac{3}{2}}\pi \ll p. \qquad (81)$$

In terms of π this condition can be written as

$$\pi \ll \sqrt{\frac{2}{3}}p. \qquad (82)$$

In first order theories the question of how much a particular dissipative flux is generated as a response to corresponding thermodynamic/kinematic forces in nuclear collisions is governed by the primary initial conditions (T_0, τ_0). That is, one just reads off the value of π_0 from the linear algebraic expression for π. We have seen that for values of the Reynolds number less than one, the thermodynamic quantities increase with time. This might be signaling the instability of the solution. Alternatively this might imply that we are using the first order theories beyond their domain of validity. The primary initial conditions can in principle be extracted from experiments. These in turn will give us the value of π_0. This value of π_0 will eventually determine how the thermodynamic variables evolve with time. This is clearly understood by looking at the ratio of the pressure term to viscous term, namely, R, as already discussed above.

In the second order theories the question of how much a particular dissipative flux is generated as a response to corresponding thermodynamic/kinematic forces in the early stages of nuclear reactions is not trivial but interesting. In order to find the viscous contribution to the time evolution of thermodynamic quantities we need to solve the differential equation for π. Therefore one has to determine the initial conditions for π independently. Although we do not know the exact form of the initial value for π we will discuss the limiting cases. The first and most important limiting case is based on the assumption made when deriving the second order theory transport equations, namely, that the dissipative fluxes be small compared to the primary variables. For the shear viscous flux this means that the shear viscous stress tensor must be small compared to the pressure. The value of π will always be less than p, hence the initial value π_0 will always be less than p_0. This has an interesting consequence: the initial Reynolds number is always greater than one. Thus in second order theory under these conditions there will be no increase of thermodynamic variables with increasing time. In general the thermodynamic quantities will decrease with time for as long as the condition

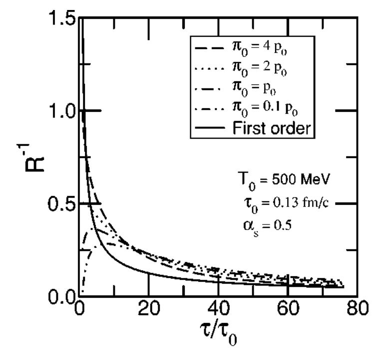

FIG. 2. Proper time evolution of the inverse Reynolds number for different values of π_0 for the given primary initial conditions.

$$\pi \leq \varepsilon + p \qquad (83)$$

is satisfied, which in the present case implies that $\pi_0 \leq 4p_0$. However, values of π greater than the pressure p leads to unphysical negative effective enthalpy. Unlike in the first order theories where it is not always possible to address this problem of negative effective enthalpy, in the second order theories we are guided by the limitations which are embedded in the valid application regimes of the theories, namely, the condition $(\pi^{\mu\nu}\pi_{\mu\nu})^{1/2} \ll p$. This condition guarantees that the effective enthalpy is always positive.

Under physical initial conditions the second order theory gives a Reynolds number that is always greater than one. This can be seen from Fig. 2 where for illustrative purposes we also include curves for unphysical initial conditions for π. Note that $\pi_0 = 4p_0$ is the maximum value before the solutions becomes unstable. This is a critical value that gives a Reynolds number $R_0 = 1$. As expected the first order theory gives $R<1$ at the same time. Throughout this work, unless otherwise stated so, we use the primary initial conditions based on the uncertainty principle as already discussed. Under this prescription of primary initial conditions, which might be relevant for RHIC and LHC, the first order theories

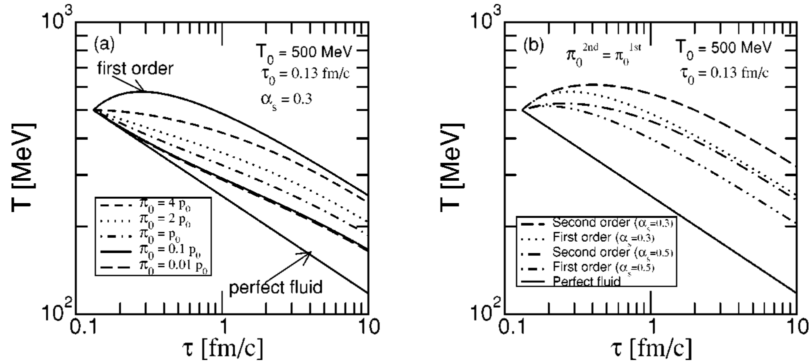

FIG. 3. The proper time evolution of temperature (a) for different choices of π_0 and (b) for $\pi_0^{2nd}=\pi_0^{1st}$.

are not suitable in describing the dynamics of thermodynamic quantities. On the other hand, the second order theories are suitable in describing the physical process happening at earlier times. An advantage of the extended irreversible thermodynamics, or second order theories, is their ability to be applicable over a wide range of regimes. However, for a different choice of initial conditions both theories might yield similar results, as we shall see [46].

There are other two ways of determining the initial conditions for π. The first one is by using the existing microscopic models such as VNI [41], (HIJING) [42], and (UrQMD) [43] to extract the various components of $\pi^{\mu\nu}$ from $T^{\mu\nu}$. Since we are dealing here with a partonic gas VNI seems to be a good choice for the present work. We will use the results from the improved version of VNI [47] to fit our calculation in order to extract the initial value for π. Another way of determining the initial value for π is to extract the initial value of the Reynolds number experimentally. Two of the most experimentally accessible quantities are the multiplicity per unit rapidity dN/dy and the transverse energy per unit rapidity dE_T/dy. A detailed study for the initial and boundary conditions for dissipative fluxes is needed to fully incorporate these fluxes into the dynamical equations for the thermodynamic quantities.

We use Eqs. (75)–(77) to study the proper time evolution of temperature. The other thermodynamic quantities, namely, energy density and entropy density, are related to the temperature by the equation of state. It is important to show the entropy results due to the importance of entropy in the theory of irreversible extended thermodynamics and due to the fact that entropy is related to multiplicity.

In Fig. 3(a) we start by showing the dependence of the temperature evolution on the initial value of π. One sees that there is a peak in T in the case of first order theory since $R_0<1$, and no peak in the second order theory since $R_0>1$. In studying the dependence of the results on the initial conditions for π we have also included even the critical value for illustrative purposes. For $0.1<\pi_0/p_0<1$ the choice of π_0 is important, but below $\pi_0=0.1p_0$ the equation for π gives the same contribution to the evolution of thermodynamic quantities.

It is also tempting to choose the initial conditions for the second order theory to match what the first order theory predicts to be the initial value of π. Note the order of the curves in Fig. 3(b). The second order theory predicts larger deviations than the first order theory. This should be exactly the same picture if both theories are synchronized in a regime where both are valid. Unfortunately, it is not trivial to make the reverse match of initial conditions. This situation will arise in natural way when both first order and second order theories are applied in the situation where they are both valid, as we will see later.

In what follows we will try to get close to the conditions that are realized in the laboratory. We will consider scenarios close to those at RHIC and LHC. But first, we have to estimate the initial value of π for these two scenarios. We will use the recent results from VNI calculations for the proper time evolution [47]. We will make a fit to the data points and extract the initial value of π. This is done in Fig. 4. Even though the motive is to extract an initial condition for π, there is something interesting in Fig. 4. In this figure a comparison between the perfect fluid approximation, the first order theory, and the second order theory is clear. The kinetic theory result, of course, differs significantly from the perfect fluid dynamics result. The first order theory obviously fails terribly. The essential point, however, is that the second order theory is in good agreement with the VNI results. Due to the preliminary nature of VNI results we cannot yet claim perfect agreement between the two approaches. However, the fact that both have similar power laws is striking. In the beginning it looks like τ^{-1} and then later on $\tau^{-4/3}$ for the VNI results. One expects that when the full three-dimensional problem is studied within the fluid dynamical approach we might have even better agreement. The fitted value of π_0 is found to be about $0.2p_0$ which is, of course, a physical value. The value of α_s used is about 0.5. For all RHIC results presented here we will use the expected primary initial conditions with $\pi=0.2p_0$ and $\alpha_s=0.5$. For the LHC scenario we

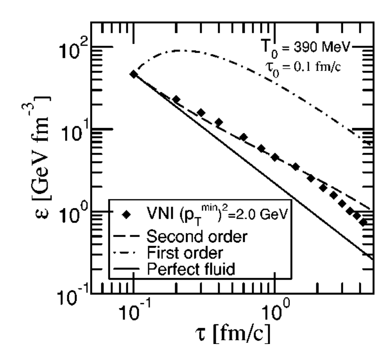

FIG. 4. Proper time evolution of the energy density. The data points are from VNI simulations and the curves are fluid dynamical results.

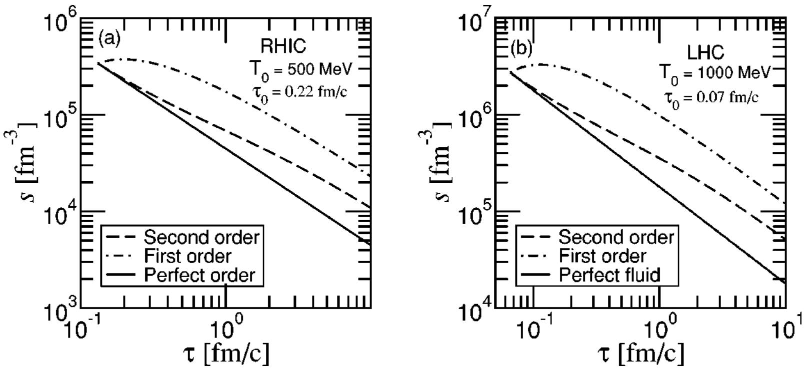

FIG. 5. The proper time evolution of entropy density for given primary initial conditions (T_0, τ_0) for (a) RHIC and (b) LHC scenarios.

will use the expected primary initial conditions with $\pi_0 = 0.3p_0$ and $\alpha_s = 0.4$.

As a benchmark both the fluid dynamical and cascade models have been solved numerically for same primary initial conditions and equation of state. This is done for consistency. It is apparent that hyperbolic models perform better than the parabolic ones, in agreement with VNI simulations. Also for energy density there is a peak in the parabolic model which is absent in the hyperbolic model. This spurious unphysical result highlights the difference between the parabolic and hyperbolic models in region of large gradients. We remark that the initial state under consideration presents very steep velocity gradients. Therefore this is an ideal benchmark for testing fluid dynamical models against transport models. Comparisons of Navier-Stokes-Fourier results with transport models were made in Ref. [48] with NSF failing terribly for smaller cross sections. In that particular study the NSF also brought in the problem of negative effective pressure. The transport results however gave a much better description. What is important however is that the second order theory seems to do a better job even in this case. The latest results on this latter point to be published elsewhere are still under investigation and comparison to previous work on the effective pressure of a saturated gluon plasma [49] is done.

The effect of dissipation is more pronounced at the very early stages of heavy ion collisions when gradients of temperature, velocity, etc., are large. This can be seen by comparing Figs. 5(a) and 5(b). One also sees that Euler hydrodynamics predicts the fastest cooling. The first order theory fails badly even for this case where we have a very high initial parton density. The first order theory significantly underpredicts the work done during the expansion relative to the Müller-Israel-Stewart and Euler predictions. Thus the entropy density decreases more slowly with the inclusion of dissipative effects. This would lead to greater yields of observables such as photons and dileptons. The system takes longer to cool down. This will delay freeze-out. More entropy is generated. This is important because entropy production can be related to the final multiplicity.

A legitimate question to ask is that do we really want to synchronize the initial conditions for both ideal fluid, first order, and second order theories. Given some initial conditions, we want to investigate the importance of second order theories as compared to first order theories and perfect fluids. That is, if one is given a set of well known initial conditions from experiment we want to see which of the theories best describes the dynamics of the system. Given an observable and a set of primary initial conditions we would like to see whether the microscopic cascade models, the ideal fluid, the first order theory, or second order theory best describes the evolution of the system.

Let us now analyze the differences between the second order and first order theories. The first thing we notice is that the Eckart-Landau-Lifshitz theory predicts that at early times the temperature will rise before falling off. This is more pronounced when we have small initial times. Naively one would expect that the system would cool monotonically as it expands, even in the case of dissipation where energy-momentum is conserved.

So far our focus has been on the quark-gluon plasma where the composition of the parton fluid enters the description through the form of the conservation laws and the equation of state. Now we study the dynamics of a pion fluid. Pions are the lightest hadrons. They are produced in abundance in ultrarelativistic collisions compared to heavier hadrons, particularly in the central region. It is therefore important to study their influence on the expansion. If pions are produced by hadronization of quark-gluon plasma, then dissipation encountered during their subsequent expansion may change the observables. The expansion in the central region conserves pion energy and momentum. Since pions carry baryon number zero, their total number is not conserved. Therefore, we expect the equilibrium number density of pions in a given volume to vary with temperature.

The equation of state is approximated by that of a massless pion gas. Thus the pressure is given by $p = aT^4$ with $a = g_h \pi^2/90$, where $g_h = 3$ is the number of degrees of freedom. The energy density and entropy density are given by $\varepsilon = 3aT^4$ and $s = 4aT^3$, respectively. The bulk viscous pressure equation does not contribute for massless particles, since $\zeta \to 0$ [37]. For the $(1+1)$-dimensional Bjorken-type hydrodynamics the heat term in the energy equation will not contribute. Thus we need only the shear viscous pressure for this presentation. The energy density evolution equation is determined as before since the equation of state is the same except for the degeneracy factor. However this time the shear viscosity coefficient is approximated by $\eta = \tau_\pi/(2\beta_2)$. For massless particles $\beta_2 = 3/(4p)$, and this is used in the expression for τ_π. The primary transport coefficients of a massless pion gas are not that well known. For chiral pions the expressions for shear viscosity and thermal conductivity are given in Ref. [26]. We will estimate the shear viscosity from the mean

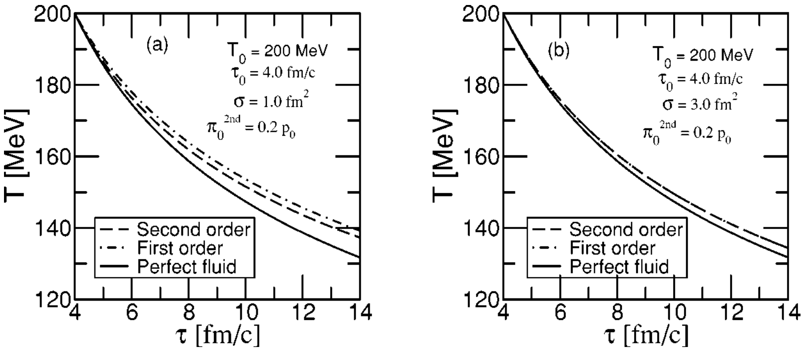

FIG. 6. The proper time evolution of temperature (a) for or $\sigma = 1.0\ \mathrm{fm}^2$ and (b) for $\sigma = 3.0\ \mathrm{fm}^2$. The initial conditions (T_0, τ_0) are arbitrarily chosen. The first order and second order theories overlap.

collision time of the pions. The mean time between collisions of pions moving at $\langle v \rangle \approx 1$ is given by

$$\tau_\pi = \frac{1}{\sigma n}, \tag{84}$$

where $n(T) = bT^3$ with $b = 3\zeta(3)/\pi^2$ is the pion density and $\sigma \approx 1\ \mathrm{fm}^2$ is an effective cross section. The quantity $\tau_\pi T^3 = 1/(\sigma b)\ \mathrm{fm}^2$ is roughly constant for temperatures $T > 100$ MeV. The shear viscosity can therefore be represented by

$$\eta = f_\eta T \quad \text{with} \quad f_\eta = \frac{2}{3}\frac{a}{b}\frac{1}{\sigma}. \tag{85}$$

Using the transport and thermodynamic properties outlined here the energy and transport equations can be written as before with the equation for π given by

$$\frac{d\pi}{d\tau} = -\sigma b T^3 \pi - \frac{1}{2}\left(\frac{1}{\tau} - 5\frac{1}{T}\frac{dT}{d\tau}\right) + \frac{8}{9}\frac{aT^4}{\tau}. \tag{86}$$

The energy equation can be solved analytically for the perfect fluid and the first order (provided η is constant) cases. But since we want η to depend on temperature or time one must then solve the equations numerically, or first find the temperature evolution as done in the preceding section.

In Figs. 6(a) and 6(b) we show the τ dependence of temperature for the three different cases: a perfect fluid, a first order theory of dissipative fluids, and a second order theory of dissipative fluids. Here we assume that the pion gas is produced at hadronization of quark-gluon plasma at $\tau = 4\ \mathrm{fm}/c$. As expected, in this regime, with the given initial conditions, the first order and second order theories converge. This convergence is faster with increasing cross sections. The effects of viscosity are small but non-negligible. In this regime the first order theory describes the dynamics well. However it is clearly unable to deal with the evolution towards this regime, or with the overall dynamics of the fluid, in a satisfactory way.

In Figs. 7(a) and 7(b) we assume that the pion gas is formed at $\tau_0 \sim 1/(3T_0)$. As we know by now, the difference between the three theories is noticeable and first order theories are not suitable. We see here also that the convergence of first order theory results and second order theory results will occur for large cross sections.

IV. CONCLUSIONS

In this work I have given a comprehensive exposition of the nonequilibrium properties of a new state of matter produced in heavy ion reactions. In doing so I presented some basic features of nonequilibrium fluid dynamics. I studied the space-time description of high energy nuclear collisions. The main aim is to bridge the phenomenological theory with the kinetic theory of the matter produced in heavy ion collisions. In doing so I made use of the dissipative fluid dynamics. The connection between the macroscopic theory and microscopic theory enters through the transport coefficients of the matter. The equation of state provided closure to the system of conservation equations.

I demonstrated that extended irreversible thermodynamics provides a consistent framework to simulate and study the space-time evolution of ultrarelativistic nuclear collisions from some initial time to the final particle yield. Although this approach relies on a number of fundamental assumptions and is far from providing an accurate quantitative description, it has the advantage of wide applicability.

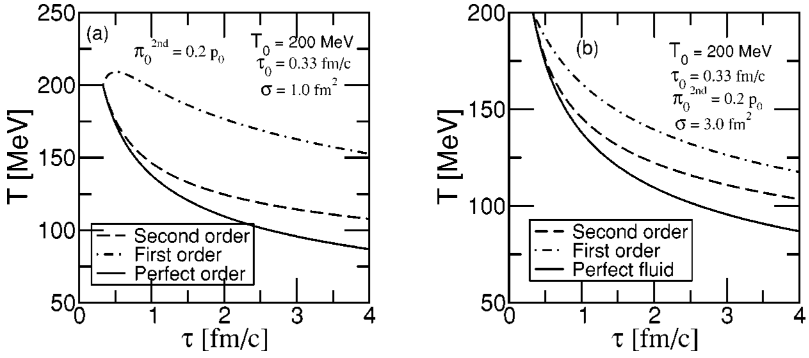

FIG. 7. The proper time evolution of temperature (a) for $\sigma = 1.0\ \mathrm{fm}^2$ and (b) $\sigma = 3.0\ \mathrm{fm}^2$. The primary initial conditions are those from uncertainty principle.

The advent of accelerators such as RHIC and LHC provides an opportunity for studying the dynamics and properties of the matter at very high energy density. In the description of the evolution of such a system, it is mandatory to evaluate, as accurately as possible, the order of magnitude of different characteristic time scales, since their relationship with the time scale of observation will determine, along with the relevant equations, the evolution pattern. This is rather general when dealing with dissipative systems. It has been my purpose here, by means of simple model with simple equation of state and arguments related to a wide range of time scales, to emphasize the convenience of resorting to hyperbolic theories when dissipative processes, either outside the steady-state regime or when the observation time is of the order of or shorter than some characteristic time of the system, are under consideration. Furthermore, dissipative processes may affect the way in which the system tends to equilibrium, thereby affecting the future of the system even for time scales much larger than the relaxation time.

In the early stages of heavy ion collisions, nonequilibrium effects play a dominant role. A complete description of the dynamics of heavy ion reactions needs to include the effects of dissipation through dissipative or nonequilibrium fluid dynamics. As is well known, hyperbolic theories of fluid dissipation were formulated to get rid of some of the undesirable features of parabolic theories, such as acausality. It seems appropriate therefore to resort to hyperbolic theories instead of parabolic theories in describing the dynamics of heavy ion collisions. Thus in ultrarelativistic heavy ion collisions, where the fluid evolution occurs very rapidly, the second order theories, due to their universality, should be used to analyze collision dynamics.

Unlike in first order theories, where the transport equations are just the algebraic relations between the dissipative fluxes and the thermodynamic forces, second order theories describe the evolution of the dissipative fluxes from an arbitrary initial state to a final steady state using the transport equations. The presence of relaxation terms in second order theories makes the structure of the resulting transport equations hyperbolic and they thus represent a well-posed initial value problem.

The consequences of nonideal fluid dynamics, both first order (if applicable) and second order, were demonstrated here in a simple situation, that of scaling solution assumption and simple equation of state. A more careful study of the effects of the nonideal fluid dynamics on the observables is therefore important. Conversely, measurements of the observables related to thermodynamic quantities would allow us to determine the importance and strength of dissipative processes in heavy ion collisions.

In summary, although parabolic theories have proved very useful for many practical purposes, they appear to fail hopelessly in describing the dynamics of heavy ion collisions. In contrast, hyperbolic theories successfully give a better description in agreement with transport models and hopefully they will be able to predict the experimental results. Thus hyperbolic theories are more reliable. In the steady state, under the conditions mentioned before and for times exceeding τ both theories may converge. Based on the results presented here one can only stress the convenience of using hyperbolic transport equations when parabolic theories either fail or the problem under consideration happens to lie outside the range of applicability of parabolic theories.

V. OUTLOOK

Here we have used only a simple equation of state in a simplified model of high energy nuclear collisions. A more realistic situation (including transverse expansion) will require careful analysis of both the transport coefficients and the equation of state which are employed in the full set of the equations. It is then that one may have a better understanding of when to use either of these theories in the context of relativistic heavy ion collisions.

There are important questions that need to be investigated in order to tackle the challenges faced by hyperbolic theories. An important question is the measurability of the dissipative fluxes. The heat flux through a system may be simply evaluated by measuring the amount of energy transported per unit area and time. The viscous pressure can be measured from the tangential shear force exerted per unit area. In practice, it may be difficult to evaluate these quantities at each instant and at every point. From kinetic theory these fluxes can be simulated from microscopic transport models such as HIJING, VNI, and UrQMD.

A thorough study of transport coefficients is needed. The results from lattice QCD should give clear predictions for these coefficients. So far there are no reliable phenomenological expressions for the coefficients. The shear viscosity results presented in Ref. [52] using the microscopic model UrQMD serve as a starting point for future calculations of transport coefficients.

A thorough study of the equation of state is required in order to be consistent with the nonequilibrium description of matter. In addition to the initial/boundary conditions for the primary/equilibrium variables a thorough study of initial/boundary conditions for the dissipative fluxes is necessary. This is required for the evolution equations of the fluxes. For example, one would like to know how much of a particular dissipative flux is generated as a response to an associated thermodynamics force in the early stages of the heavy ion collisions. This in turn gives us information about the initial entropy generated as a result of dissipation.

For the complete description of the dynamics of viscous, heat conducting matter we need to consider more realistic situations: a system that expands in both the longitudinal and transverse directions [53] and we need a full (3+1)-dimensional solutions to the conservation and evolution equations. This will require extensive numerical computation. This is a challenging but interesting problem. In order to understand the observables we need a full formulation of hyperbolic theory that should be tested against other models/theories.

Solving imperfect hydrodynamics amounts to knowledge of (i) a realistic equation of state, (ii) reliable transport coefficients (iii) realistic initial/boundary conditions, and (iv) numerical computational algorithm. In order to understand the observables from RHIC and LHC knowledge of these requirements is needed and that is what I have started to do. I hope that this will gather momentum in due course.

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