Transverse momentum fluctuations and percolation of strings

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The behavior of the transverse momentum fluctuations with the centrality of the collision shown by the Relativistic Heavy Ion Collider data is naturally explained by the clustering of color sources. In this framework, elementary color sources—*strings*—overlap forming clusters, so the number of effective sources is modified. These clusters decay into particles with mean transverse momentum that depends on the number of elementary sources that conform each cluster, and the area occupied by the cluster. The transverse momentum fluctuations in this approach correspond to the fluctuations of the transverse momentum of the particles produced by the different clusters, and they behave essentially as the number of effective sources.

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Event-by-event fluctuations of the mean transverse momentum are considered to be one of the most important tools to identify a phase transition in the evolution of the system created in relativistic heavy ion collisions, since a second-order phase transition may lead to a divergence of the specific heat which could be observed as fluctuations in mean transverse momentum [1–3]. These fluctuations have been extensively studied both theoretically [4–10] and experimentally [11–17]. Recently, the PHENIX Collaboration [15,16] of the Relativistic Heavy Ion Collider (RHIC) has reported a peculiar behavior of the variable F_{p_T} that measures the transverse momentum fluctuations as a function of the number of participants in Au-Au collisions. F_{p_T} quantifies the deviation of the observed fluctuations from statistically independent particle emission,

$$F_{p_T} = \frac{\omega_{data} - \omega_{random}}{\omega_{random}} \tag{1}$$

where

$$\omega = \frac{\sqrt{\langle p_T^2\rangle - \langle p_T\rangle^2}}{\langle p_T\rangle}, \tag{2}$$

and $\langle p_T\rangle$ is the mean transverse momentum averaged over all particles and all events. The data show that F_{p_T} increases with the number of participants, reaching a maximum around $N_{part}=150\div 200$ and decreasing at higher centrality. In this paper we show that this behavior is naturally explained by the clustering of elementary color sources that may take place in heavy ion collisions.

Multiparticle production is currently described in terms of color strings stretched between the partons of the projectile and the target. These strings decay into new ones by sea $q-\overline{q}$ production, and subsequently hadronize to produce the observed hadrons. In our approach, the strings are equivalent to effective color sources with a fixed transverse size πr_0^2, with $r_0 \simeq 0.2$ fm, filled with the color field created by the colliding partons. With increasing energy and/or atomic number of the colliding nuclei, the number of exchanged strings grows, so they start to interact forming clusters. In the transverse space that means that the transverse areas of the strings overlap, as it happens for disks in the two-dimensional percolation theory.

In the case of a nuclear collision, the density of disks—elementary strings—corresponds to

$$\eta = \frac{N_s S_1}{S_A}, \tag{3}$$

where N_s is the total number of strings created in the collision, each one of an area $S_1=\pi r_0^2$, and S_A corresponds to the nuclear overlap area $S_A=\pi R_A^2$ for central collisions.

Moreover, at a certain critical density of disks, $\eta \simeq 1.12-1.18$, a macroscopical cluster appears which marks the percolation phase transition [18,19], which is a second order, nonthermal, phase transition.

The percolation theory governs the geometrical pattern of the string clustering. Its observable implications, however, required the introduction of some dynamics in order to describe the behavior of the cluster formed by several overlapping strings [20,21]. We assume that a cluster of n strings that occupies an area S_n behaves as a single color source with a higher color field, generated by a higher color charge Q_n. This charge corresponds to the vectorial sum of the color charges of each individual string $\mathbf{Q}_1$. The resulting color field covers the area S_n of the cluster. As $Q_n^2=(\Sigma_1^n \mathbf{Q}_1)^2$, and the individual string colors may be oriented in an arbitrary manner respective to one another, the average $\mathbf{Q}_{1i}\mathbf{Q}_{1j}$ is zero, so $Q_n^2=nQ_1^2$. Q_n depends also on the area S_1 of each individual string that comes into the cluster, as well as on the total area of the cluster $S_n, Q_n=\sqrt{(nS_n/S_1)}Q_1$.[1] We take S_1 constant and equal to a disk of radius $r_0 \simeq 0.2$ fm. S_n corresponds to the total area occupied by n disks [20]. One could do reasonable alternative assumptions about the interaction among the strings, but they have incompatibilities with correlation data [22,23].

Note that if the strings are just touching each other, $S_n = nS_1$ and $Q_n = nQ_1$, so the strings behave independently. On

[1] Q_n would be equal to $\sqrt{n}Q_1$ if the strings overlap completely. Since the strings may overlap only partially we introduce a dependence on the area of the cluster. See Ref. [20(a)] for more details.

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the contrary, if they fully overlap, $S_n = S_1$ and $Q_n = \sqrt{n} Q_1$. Knowing the color charge Q_n, one can compute the multiplicity μ_n and the mean transverse momentum $\langle p_T \rangle_n$ of the particles produced by a cluster of n strings. According to the Schwinger mechanism for the fragmentation of the cluster, one finds

$$\mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1 \quad \text{and} \quad \langle p_T \rangle_n = \left(\frac{nS_1}{S_n}\right)^{1/4} \langle p_T \rangle_1 \tag{4}$$

for the multiplicity μ_n and the average transverse momentum $\langle p_T \rangle_n$ of the particles produced by a cluster formed by n strings, where μ_1 and $\langle p_T \rangle_1$ correspond to the mean multiplicity and the mean transverse momentum of the particles produced by one individual string. These equations constitute the main tool of our evaluations.

The behavior of the transverse momentum fluctuations can be understood as follows: At low density, most of the particles are produced by individual strings with the same $\langle p_T \rangle_1$, so the fluctuations are small. Similarly, at large density above the percolation critical point, there is essentially only one cluster formed by most of the strings created in the collision and therefore fluctuations are not expected either. Instead, the fluctuations are expected to be maximal below the percolation critical density, where the number of clusters is larger. Moreover, there are clusters formed by very different numbers of strings, with different size, and therefore with different $\langle p_T \rangle_n$.

In order to develop quantitatively this idea, we introduce the function [4] ϕ defined by

$$\phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle \mu \rangle}} - \sqrt{\langle z^2 \rangle}. \tag{5}$$

F_{p_T} is related to ϕ [14], approximately

$$F_{p_T} = \frac{\phi}{\sqrt{\langle z^2 \rangle}} = \frac{1}{\sqrt{\langle z^2 \rangle}} \sqrt{\frac{\langle Z^2 \rangle}{\langle \mu \rangle}} - 1. \tag{6}$$

For each particle we define $z_i = p_{Ti} - \langle p_T \rangle$, where p_{Ti} is the transverse momentum of the particle i and $\langle p_T \rangle$ is the mean transverse momentum of all particles averaged over all events. $\sqrt{\langle z^2 \rangle}$ is the second moment of the single particle inclusive z distribution, and it is averaged over all events. Z is defined for each event,

$$Z_i = \sum_{j=1}^{N_i} z_j, \tag{7}$$

where N_i is the number of particles produced in an event i.

In this way, introducing our formulas for the multiplicity and the mean p_T we get

$$\langle p_T \rangle = \frac{\sum_{i=1}^{N_{events}} \sum_j \mu_{n_j} \langle p_T \rangle_{n_j}}{\sum_{i=1}^{N_{events}} \sum_j \mu_{n_j}}. \tag{8}$$

The sum over j goes over all individual clusters j, each one formed by n_j strings and occupying an area S_{n_j}. The quantities n_j and S_{n_j} are obtained for each event, using a Monte Carlo code [24,25], based on the quark gluon string model. Each string is generated at an identified impact parameter in the transverse space. Knowing the tranverse area of each string, we identified all the clusters formed in each event, the number of strings n_j that conforms each cluster j, and the area occupied by each cluster S_{n_j}. Note that for two different clusters, j and k, formed by the same number of strings $n_j = n_k$, the areas S_{n_j} and S_{n_k} can vary. Because of this we do the sum over all individual clusters.

For the quantities $\langle p_T \rangle_{n_j}$ (mean p_T of the particles produced by a cluster j of n_j strings and area S_{n_j}) and μ_{n_j} (mean multiplicity of a cluster j formed by n_j strings and of area S_{n_j}), we apply the analytical expressions given by Eqs. (4). Finally we do the average over all events.

By introducing Eqs. (4) for $\langle p_T \rangle_{n_j}$ and μ_{n_j} we get

$$\langle p_T \rangle = \frac{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1 \left(\frac{n_j S_1}{S_{n_j}}\right)^{1/4} \langle p_T \rangle_1}{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1} = \frac{\left\langle \sum_j n_j^{3/4} \left(\frac{S_{n_j}}{S_1}\right)^{1/4} \right\rangle}{\left\langle \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \right\rangle} \langle p_T \rangle_1 = f_2 \langle p_T \rangle_1, \tag{9}$$

where the mean value in the right-hand side corresponds to an average over all events.

For the quantities $\langle z^2 \rangle$ and $\langle Z^2 \rangle$ we obtain

$$\langle z^2 \rangle = \frac{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1 \left[\left(\frac{n_j S_1}{S_{n_j}}\right)^{1/4} \langle p_T \rangle_1 - \langle p_T \rangle\right]^2}{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1} = \left[\frac{\left\langle \sum_j n_j \right\rangle}{\left\langle \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \right\rangle} - f_2^2\right] \langle p_T \rangle_1^2 = (f_1 - f_2^2) \langle p_T \rangle_1^2 \tag{10}$$

and

$$\frac{\langle Z^2\rangle}{\langle\mu\rangle}=\frac{\sum_{i=1}^{N_{events}}\left\{\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\mu_1\left[\left(\frac{n_jS_1}{S_{n_j}}\right)^{1/4}\langle p_T\rangle_1-\langle p_T\rangle\right]\right\}^2}{\sum_{i=1}^{N_{events}}\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\mu_1}$$

$$=\left[\frac{\left\langle\left[\sum_j n_j^{3/4}\left(\frac{S_{n_j}}{S_1}\right)^{1/4}\right]^2\right\rangle}{\left\langle\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\right\rangle}+\frac{\left\langle\left[\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\right]^2\right\rangle}{\left\langle\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\right\rangle}f_2^2-\frac{\left\langle\sum_j n_j^{3/4}\left(\frac{S_{n_j}}{S_1}\right)^{1/4}\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\right\rangle}{\left\langle\sum_j\left(\frac{n_jS_{n_j}}{S_1}\right)^{1/2}\right\rangle}2f_2\right]\mu_1\langle p_T\rangle_1^2$$

$$=[f_3+f_4f_2^2-2f_2f_5]\mu_1\langle p_T\rangle_1^2. \tag{11}$$

Finally we arrive to

$$F_{p_T}=\sqrt{\mu_1}\sqrt{\frac{f_3+f_4f_2^2-2f_2f_5}{f_1-f_2^2}}-1, \tag{12}$$

where f_1, f_2, f_3, f_4, and f_5 are defined in the expressions (9)–(11).

In order to compute Eq. (12), several ingredients are necessary. On one hand we need a Monte Carlo code [24,25] for the cluster formation, in order to compute the number of strings that come into each cluster and the area of the cluster. On the other hand, we do not use a Monte Carlo code for the decay of the cluster, since we apply analytical expressions [Eqs. (4)] for the transverse momentum and the multiplicities of the clusters.

We also need the value of μ_1—multiplicity produced by one individual string. It was previously fixed from a comparison of the model to SPS and RHIC data [20,21] on multiplicities. In the first reference of [20(a)], the total multiplicity per unit rapidity produced by one string has been taken as $\mu_{0tot}\simeq 1$. If we assume that 2/3 of the created particles are charged, which would lead to a charged particle multiplicity per unit rapidity for each individual string of $\mu_{0ch}=0.65$. In order to compare with experimental data we define $\mu_1=\mu_{0ch}y$, where y is the rapidity interval of the produced particles. We don't introduce any dependence of μ_0 with the energy or the centrality of the collision. Note that in Eq. (12) the value of $\langle p_T\rangle_1$ cancels.

We have neglected the subsequent rescattering of hadrons and resonances that takes place after the decay of the clusters. It gives rise to correlations which would be similar in the clustering approach and in an independent string picture, unless additional dynamics were taken into account. Therefore its contribution to F_{p_T} cancels in our approach.

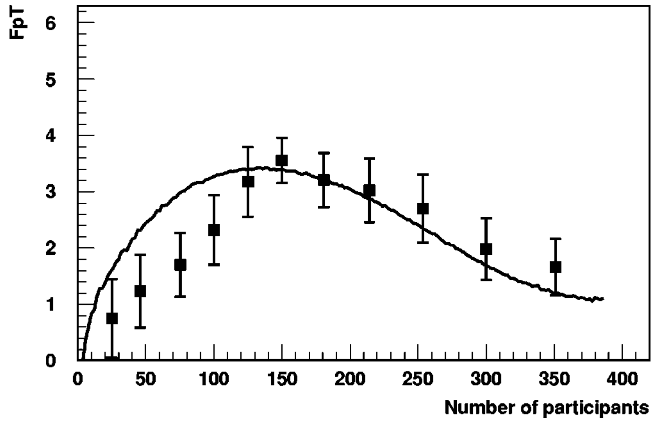

FIG. 1. F_{p_T}(%) vs the number of participants. Experimental data from PHENIX at $\sqrt{s}=200$ GeV are compared with our results (solid line).

The comparison of our results for the dependence of F_{p_T} on the number of participants N_p with the PHENIX data [16] is shown in Fig. 1. The calculation is done for charged particles in the rapidity range $|\eta|<0.35$, $\mu_1=0.7\mu_{0ch}$. An acceptable overall agreement is obtained.

In order to compute our value for F_{p_T}, we take into account all possible transverse momenta, whereas in the experiment there is a limited acceptance, $0.2\ \text{GeV}/c<p_T<p_T^{max}$. PHENIX [15,16] has studied the variation of F_{p_T} with the maximal value of the acceptance for p_T,p_T^{max}. The maximum of F_{p_T} is reached for the largest acceptance, $p_T^{max}=4\ \text{GeV}/c$ [15]. So we can expect that our value for F_{p_T} is going to be higher than the experimental one, specially for a moderate number of participants N_p since the truncated average p_T [26],

$$\langle p_T^{trunc}\rangle=\frac{\int_{p_T^{min}}^{\infty}p_TdN/dp_T}{\int_{p_T^{min}}^{\infty}dN/dp_T}-p_T^{min},$$

decreases with the number of participants for $p_T^{min}>2\ \text{GeV}/c$. This means that, for momenta higher than 2 GeV/c, the high p_T contribution would be due to collisions with a moderate number of participants. These considerations may explain the difference between our results and PHENIX data—with a limited acceptance of $0.2\ \text{GeV}/c<p_T<2.0\ \text{GeV}/c$—at low N_p.

Moreover, our values for the mean transverse momentum $\langle p_T\rangle$, for the dispersion $\sigma_{p_T}=\sqrt{\langle p_T^2\rangle-\langle p_T\rangle^2}$ and for the mean multiplicity per unit rapidity dN^{ch}/dy, for Au-Au central (0–5 %) collisions at $\sqrt{s}=200$ GeV are $\langle p_T\rangle=455$ MeV/c, $\sigma_{p_T}=379$ MeV/c, and $dN^{ch}/dy=700$. These results are compatible with the PHENIX data of Ref. [27], where the mean p_T is calculated by integrating over the measured p_T range

TABLE I. Centrality dependence of mean transverse momentum and dispersion for Pb-Pb collisions at 158*A* GeV. The experimental data are taken from NA49 Collaboration.

N_{part}	$\langle p_T \rangle$ (MeV/c)	σ_{p_T} (MeV/c)	$\langle p_T \rangle$ (MeV/c)	σ_{p_T} (MeV/c)
	Experimental data		Model results	
42	299	220	290	237
88	305	226	296	242
134	309	230	302	248
204	312	233	313	256
281	315	234	324	265
352	317	236	336	274

from the data, and adding the fitted contribution of the integral from zero to the first data point and from the last data point to infinity. On the contrary, our mean p_T value is smaller than the experimental one of Ref. [16], where the mean value is obtained in a limited range, $0.2\ \mathrm{GeV}/c < p_T < 2.0\ \mathrm{GeV}/c$.

We have also compared our results for $\langle p_T \rangle$ and σ_{p_T} with the data from NA49 Collaboration [28]. Here again the data are obtained in a determined p_T range, $0.005\ \mathrm{GeV}/c < p_T < 1.5\ \mathrm{GeV}/c$, while our results are obtained taking into account all possible transverse momenta. Nevertheless, the experimental acceptance in this case covers the small p_T region, which gives the biggest contribution at these energies. Because of this, we obtain a good agreement for $\langle p_T \rangle$. Concerning σ_{p_T}, our value is bigger than the experimental one, since we don't have a limited acceptance.[2] These results together with experimental data are shown in Table I. The fact that we obtain a bigger $\sigma_{p_T} = \sqrt{\langle z^2 \rangle}$ could have some influence on the value of ϕ_{p_T}. Nevertheless this is compensated, since we obtain a similar deviation in the value of $\sqrt{\langle Z^2 \rangle / \langle u \rangle}$. This also explains the acceptable agreement that we obtain when we compare our results on F_{p_T} with the PHENIX data. Note that we calculate F_{p_T} through Eq. (6), where the effects on $\sqrt{\langle z^2 \rangle}$ and $\sqrt{\langle Z^2 \rangle / \langle u \rangle}$ are compensated.

In Fig. 2 our results for ϕ_{p_T} of charged particles in Pb-Pb central collisions at 158*A* GeV are compared with the experimental data of NA49 Collaboration [28,29]. In this case the data correspond to the forward rapidity range $4.0 < y < 5.5$. For this reason we use $\mu_1 = 1.5\ \mu_{0ch}$, which in principal implicates larger correlations. However we see that this effect is compensated, since we have a lower value for the mean number of strings at fixed N_p, due to:

(a) lower energy at SPS than at RHIC so less strings are produced,

(b) the mean number of strings in this rapidity region is proportional to the number of participants N_p, while in the central region it is proportional to $N_p^{4/3}$ due to the contribution of $q-\bar{q}$ strings from the sea.

For the computation of ϕ_{p_T} we use $\langle p_T \rangle_1 = 0.3\ \mathrm{GeV}/c$.

The CERES Collaboration has also measured ϕ_{p_T} [29] at four different centralities: 0–5%, 5–10%, 10–15%, and 15–20% for Pb-Au collisions at 40, 80, and 158 *A* GeV/c in the central pseudorapidity range $2.2 < \eta < 2.7$ and restricted to tracks with transverse momenta $0.1 < p_T < 1.5\ \mathrm{GeV}/c$. Due to the narrower rapidity and transverse momentum range one would expect a lower ϕ_{p_T} value. However this is compensated with the increase of the number of strings at central rapidities. The data at 158*A* GeV, after short range removal, are 3.3, 3.6, 4.4, and 4.1 for the above mentioned centralities, with errors of the order of 1.5—for smaller energies the data are lower as expected. These values and their dependence with centrality are compatible with our results of Fig. 2.

In order to have a better understanding of the behavior of F_{p_T} and ϕ_{p_T} on the number of participants, we plot in Figs. 3 and 4 the mean number of clusters M and the dispersion on the number of clusters multiplied by the number of clusters $\sigma_M M$ at RHIC and SPS energies. The ratio σ_M^2/M would be one in the case of a Poisson distribution. The p_T fluctuations are due in our approach to the different mean transverse momenta of the clusters. These momenta depend on the number of strings that comes into the cluster and the area occupied by the cluster through our Eq. (4), therefore M and σ_M should be the key quantities. However, σ_M^2/M ranges between 1/2 and 2 in the whole N_p range, what indicates a lower variation than the one for M, as can be seen from Figs.

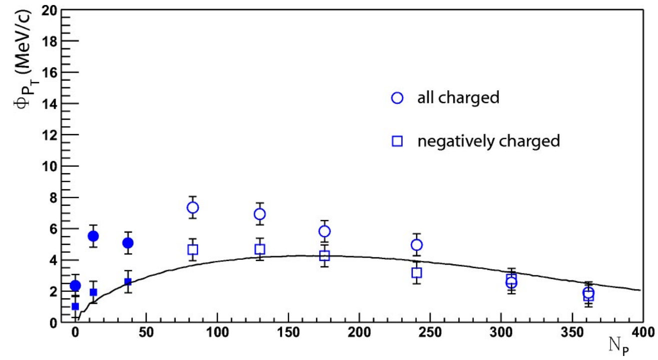

FIG. 2. (Color online) ϕ_{p_T} vs the number of participants. Experimental data from NA49 Collaboration at SPS energies are compared with our results (solid line).

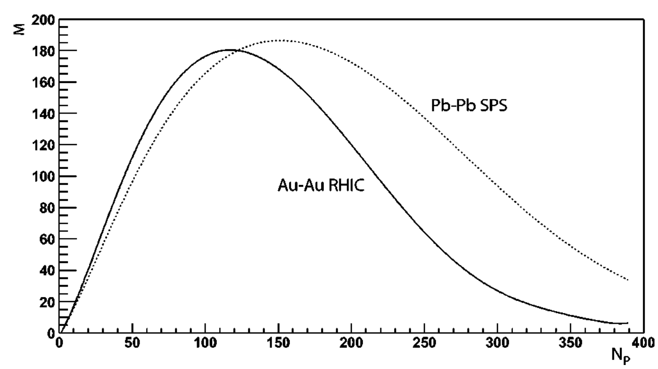

FIG. 3. (Color online) Mean number of clusters M vs the number of participants for Pb-Pb collisions at SPS energies (dotted line) and Au-Au collisions at RHIC energies (solid line).

[2]In fact, our values for $b \leq 3.5$ fm lie between the experimental data of Ref. [28] and the ones of Ref. [12].

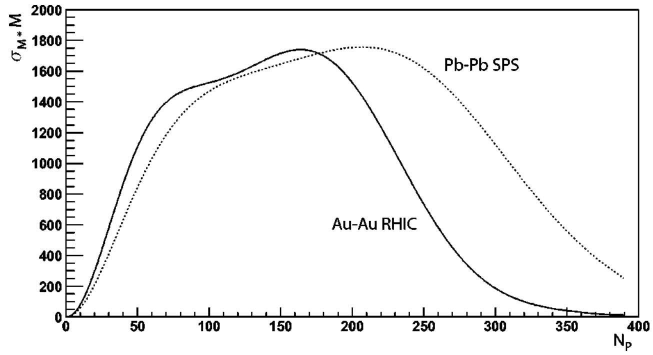

FIG. 4. (Color online) Dispersion on the number of clusters multiplied by the number of clusters $\sigma_M M$ vs the number of participants for Pb-Pb collisions at SPS energies (dotted line) and Au-Au collisions at RHIC energies (solid line).

3 and 4 where M and $\sigma_M M$ are plotted. The only effect of σ_M is to shift the maximum of M. Because of this we expect the dependence of F_{p_T} and ϕ_{p_T} on N_p to be more similar to the M behavior, as it is actually. In other words, a decrease in the number of effective sources leads to a decrease of the tranverse momentum fluctuations.

Similar conclusions have been reached in Ref. [31], where a formula for F_{p_T} as a function of the cluster dispersion over the mean number of strings per cluster has been obtained.

Note that we only need to know the number of strings formed for each centrality and their location in the impact parameter space in order to form clusters. This information, together with Eq. (4), is enough for us to calculate F_{p_T} and ϕ_{p_T}. The same variables have been able to describe the behavior of the strength of two [22] and three [23] body Bose-Einstein correlations with centrality and the dependence of the multiplicities and transverse momentum distributions [21,30] on the centrality. All that points out that the percolation approach may be appropriate to describe the relativistic heavy ion collisions.

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