Global and consistent analysis of the heavy-ion elastic scattering and fusion processes

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We have developed a model for the nuclear interaction which is based on the effects of the Pauli nonlocality. In earlier works, we have successfully used this interaction to describe the elastic scattering for several systems in a very wide energy range. In the present work, we have checked the validity of the same interaction in the description of about 2500 fusion cross section data for 165 different systems. By introducing only one energy- and system-independent effective parameter, the nonlocal model describes the global behavior of the fusion process with good precision.

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The heavy-ion fusion process has been extensively studied over the last decades [1]. It is well known that fusion cross sections for heavy-ion systems have shown large enhancements at sub-barrier energies in comparison with theoretical predictions from the barrier penetration model [2]. These enhancements can be described by introducing effective barrier parameters, which have been studied in a global way for a large number of systems [2–4]. On the other hand, the enhancements have been explained for several particular systems by considering the internal structure of the participating nuclei through couped-channel calculations (e.g., Refs. [5,6]). A few models have also been presented to describe the elastic scattering process and the energy and system dependences of the corresponding optical potential [7]. However, the consistency between the models for the fusion and elastic scattering processes has only been verified in certain particular cases (e.g., Ref. [8]). We have developed a model for the nuclear interaction which has been successful in describing the elastic scattering for several systems in a wide energy range [9–15]. The model is based on the effects of the Pauli nonlocality and is totally parameter free. In this work, we use this interaction in the description of about 2500 fusion cross section data [16–53] for 165 different systems from sub-barrier to high energies.

Within the nonlocal model, the bare interaction V_N is connected with the folding potential V_F through [54]

$$V_N(R,E) = V_F(R)e^{-4v^2/c^2}, \qquad (1)$$

where c is the speed of light and v is the local relative velocity between the two nuclei,

$$v^2(R,E) = \frac{2}{\mu}[E - V_C(R) - V_N(R,E)]. \qquad (2)$$

The folding potential [Eq. (3)] can be obtained in two different ways [54]: (i) using the nucleon distributions of the nuclei and an appropriate form for the nucleon-nucleon interaction, and (ii) using the matter distributions of the nuclei with a zero-range approach for $v(\vec{r})$. We distinguish the matter density from the nucleon one by taking into account the finite size of the nucleon. Both alternatives are equivalent in describing the heavy-ion nuclear potential [54], and we have adopted the zero-range approach to describe the fusion process:

$$V_F(R) = \int \rho_1(r_1)\rho_2(r_2)v(\vec{R} - \vec{r}_1 + \vec{r}_2)d\vec{r}_1 d\vec{r}_2. \qquad (3)$$

With the aim of providing a global description of the nuclear interaction, we proposed [54] an extensive systematization of nuclear densities, based on experimental charge distributions and theoretical densities calculated through the Dirac-Hartree-Bogoliubov model. In that work, we adopted the two-parameter Fermi (2pF) distribution to describe the nuclear densities. The radii of the 2pF distributions are well described by

$$R_0 = 1.31A^{1/3} - 0.84 \text{ fm}, \qquad (4)$$

where A is the number of nucleons of the nucleus. The matter densities present an average diffuseness value $a = 0.56$ fm. Owing to specific nuclear structure effects (single particle and/or collective), the parameters R_0 and a show small variations around the corresponding average values throughout the periodic table. This systematization of the nuclear distributions is essential to obtain a parameter-free interaction, since the folding potential depends on the densities of the partners in the collision. In the present work, we use the nonlocal model in the context of this systematics, i.e., assuming the average diffuseness value and Eq. (4) for the radii of the distributions. Therefore, the interaction does not contain any free parameter and is quite appropriate to connect experimental results for different systems in a realistic manner. In the present work, we have also calculated the Coulomb potential through a folding procedure using the realistic charge densities of Ref. [54].

In the context of the barrier penetration model (BPM), the effective potential is a sum of the Coulomb, nuclear, and centrifugal parts:

$$V_{eff}(R,E) = V_C(R) + V_N(R,E) + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2}. \qquad (5)$$

The fusion cross section is associated with the transmitted flux through

$$\sigma_{BPM}(E) = \frac{\pi}{k^2} \sum (2\ell + 1) T_\ell. \tag{6}$$

In our calculations, the sum in Eq. (6) is performed up to a maximum ℓ wave, which is the greatest ℓ value that results a pocket (and a barrier) in the corresponding effective potential. For ℓ waves with effective barrier heights below the center of mass energy, we have approximated the effective potential by a parabola with curvature $\hbar\omega_\ell$. In such cases, the transmission coefficients can be obtained through the Hill-Wheeler formula [55]:

$$T_\ell = \left\{ 1 + \exp\left[\frac{2\pi(V_{B\ell} - E)}{\hbar\omega_\ell} \right] \right\}^{-1}, \tag{7}$$

$$\hbar\omega_\ell = \left| \frac{\hbar^2}{\mu} \frac{d^2 V_{eff}}{dR^2} \right|_{R_{B\ell}}^{1/2}, \tag{8}$$

where $V_{B\ell}$ and $R_{B\ell}$ are the barrier height and the corresponding radius, respectively. On the other hand, for ℓ waves with effective barriers above the center of mass energy, instead of the Hill-Wheeler formula we have used the more appropriate WKB method:

$$T_\ell = [1 + \exp(S_\ell)]^{-1}, \tag{9}$$

$$S_\ell = \int_{R_1}^{R_2} \sqrt{\frac{8\mu}{\hbar^2}[V_{eff}(R,E) - E]} dR, \tag{10}$$

where R_1 and R_2 are the classical turning points. At low energies, the WKB method results in values for the transmission coefficients quite different from those of the Hill-Wheeler formula. In this case, we have defined the barrier curvature connecting expressions (7) and (9) as

$$\hbar\omega_\ell = \frac{2\pi(V_{B\ell} - E)}{S_\ell}. \tag{11}$$

Within the context of parabolic transmission coefficients, and considering $R_{B\ell} \approx R_{B0}$ and $\hbar\omega_\ell \approx \hbar\omega_0$, Wong has demonstrated [56] that

$$\sigma^{Wong}(E) = \frac{R_{B0}^2 \hbar\omega_0}{2E} \ln\left\{ 1 + \exp\left[\frac{2\pi(E - V_{B\ell})}{\hbar\omega_0} \right] \right\}. \tag{12}$$

With the aim of obtaining system-independent quantities, we have defined the following reduced cross section and energy:

$$\sigma_{red} = \frac{2E}{R_{B0}^2 \hbar\omega_0} \sigma_{fus}, \tag{13}$$

$$E_{red} = \frac{E - V_{B0}}{\hbar\omega_0}. \tag{14}$$

By using these adimensional quantities, Eq. (12) can be recast in the following system-independent form:

$$\sigma_{red}^{Wong} = \ln[1 + \exp(2\pi E_{red})]. \tag{15}$$

However, we have verified that, in some cases, the results obtained from the Wong expression [Eq. (12)] present sig-
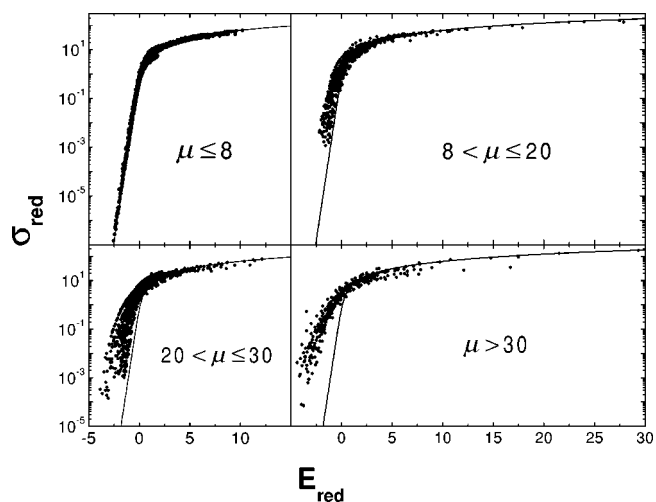

FIG. 1. The reduced fusion cross section as a function of the reduced energy, using barrier parameters obtained with the nonlocal model. The figure shows the results for different ranges of the reduced mass of the system. The solid lines represent the reduced Wong equation.

nificant differences in comparison with those from the full BPM calculations [Eq. (6)]. Thus, in order to compare experimental results for very different systems, we have defined the experimental reduced fusion cross section through the following trivial equation:

$$\sigma_{red}^{exp} = \frac{\sigma_{fus}}{\sigma_{BPM}} \sigma_{red}^{Wong}. \tag{16}$$

The reduced fusion cross section data, according to Eq. (16), are presented in Fig. 1, as a function of the reduced energy [Eq. (14)]. The data set includes 165 quite different systems ranging in reduced mass from about 4 to 40 amu. A very good agreement between data and theoretical predictions is obtained for energies above the s-wave barrier ($E_{red} \geq 0$) for the whole set of systems, while sub-barrier data present large enhancements in comparison with the BPM calculations. The sub-barrier deviation is negligible for $\mu \leq 8$ and increases as a function of the reduced mass of the system in a relatively smooth manner. A similar behavior of the sub-barrier fusion data has already been very well established [2]. An inspection of the slopes of the data in Fig. 1 indicates that the enhancements are more closely connected with the barrier curvature than with the barrier height itself. Taking into account all these considerations, we propose a simple model to describe the enhancements by introducing effective barrier curvatures. This effective parameter is assumed to be a simple linear function of the reduced mass of the system, as given in Eq. (17). The fit to the data results in $\lambda = 0.1/$amu:

$$\hbar\omega_{\ell eff} = \begin{cases} \hbar\omega_\ell & \text{for } \mu \leq 8 \text{ amu} \\ \hbar\omega_\ell[1 + \lambda(\mu - 8)] & \text{for } \mu \geq 8 \text{ amu}. \end{cases} \tag{17}$$

Using Eq. (17) with $\lambda = 0.1$, the barrier penetration model describes with good precision the global behavior of the 2500 experimental fusion cross section data in the whole energy range (see Fig. 2). In fact, it is possible to describe
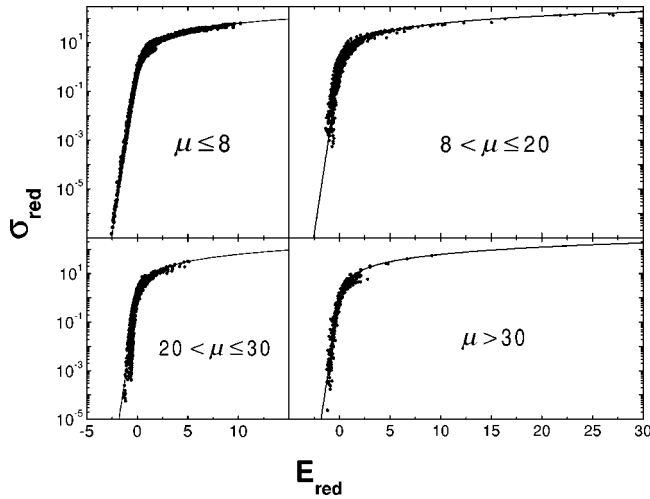
FIG. 2. The same as Fig. 1, considering the correction [Eq. (17) with $\lambda = 0.1/amu$] for the barrier curvatures.

the above-barrier data within 20% precision (standard deviation) and the sub-barrier data within an average factor of about 3. We consider this dispersion rather small because our analysis has been performed for a large number of different systems and over a very wide energy range (from about 18 MeV below the barrier to 120 MeV above it) with only one system- and energy-independent free parameter λ. With the aim of including the whole data set, Figs. 1 and 2 have been made with reduced scales. Figures 3–5 present the results in usual scales for a few particular systems, in order to provide further examples of the quality of our predictions.

We believe that the effective curvature simulates, in some way, the average effect of a large number of coupled channels that contribute to the fusion process. The effect of the couplings depends on the reduced mass of the system because heavier systems present a greater number of reaction
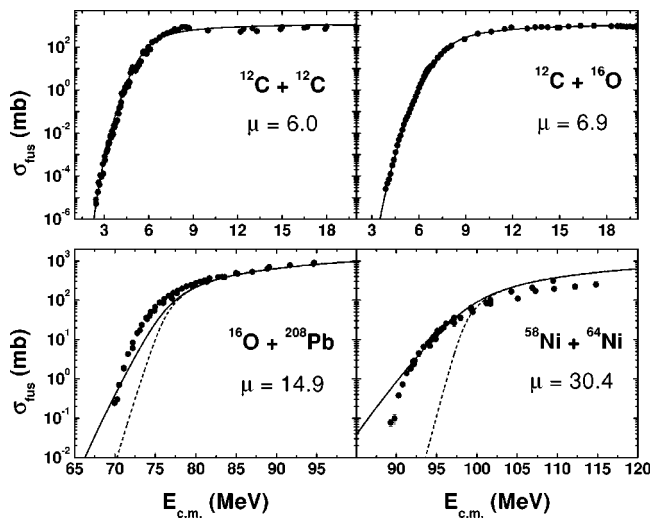
channels and also larger coupling amplitudes (which are connected with the size of the nuclei). The dispersion observed in Fig. 2 is probably due [57] to particular strong couplings that were not included in our model, and is also connected with variations of the densities, arising from nuclear structure effects, which have an influence on the nuclear potential [54]. For example, the great isotopic dependence (see Fig. 6—top) of the sub-barrier fusion cross section for the $^{16}O + {}^{144,148,150,152,154}Sm$ systems is decreased by using the effective curvature (Fig. 6—bottom). Even so, the difference is not totally eliminated probably due to structure effects not included in our model.

The effect of the nonlocality is not very significant at near-barrier energies (low velocities), where Eq. (1) indicates that $V_N(R,E) \approx V_F(R)$. The effect becomes greater as the energy increases, such that at energies of about 200 MeV/nucleon $V_N(R,E)$ is about one order of magnitude less intense than the corresponding folding potential [9,10].

FIG. 4. The same of Fig. 3 for the $^{16}O + {}^{144,148,150,154}Sm$ systems.

FIG. 3. The fusion cross section for the $^{12}C + {}^{12}C$, $^{12}C + {}^{16}O$, $^{16}O + {}^{208}Pb$, and $^{58}Ni + {}^{64}Ni$ systems. The lines represent full barrier penetration model calculations with (solid lines) or without (dashed lines) including the effect of the effective curvature.
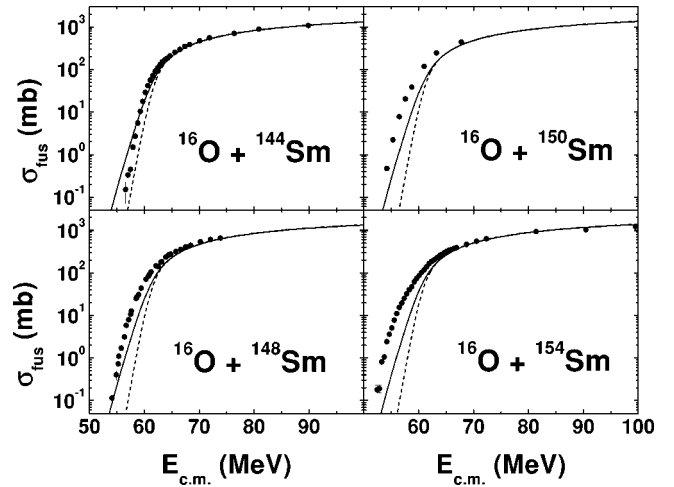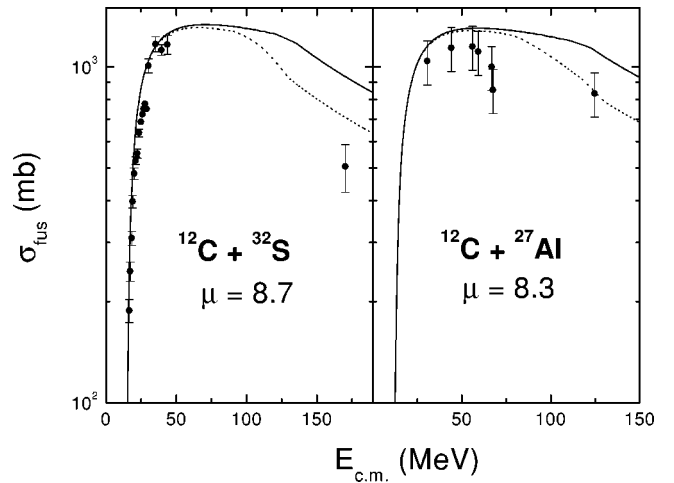

FIG. 5. The fusion cross section for the $^{12}C + {}^{27}Al$, ^{32}S systems. The lines represent full barrier penetration model calculations with (dotted lines) or without (solid lines) the effect of the Pauli nonlocality.
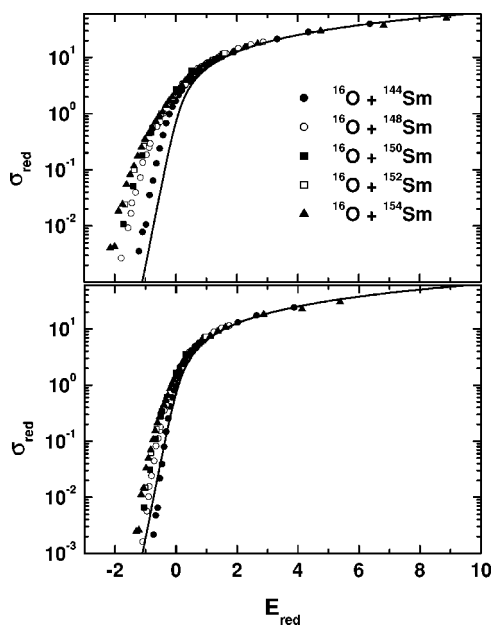
FIG. 6. The reduced fusion cross section for the $^{16}O + ^{144,148,150,152,154}Sm$ systems as a function of the reduced energy, with (bottom) or without (top) including the effect of the effective curvature. The same data set is presented in usual scales in Fig. 4.

However, there are few available fusion data at high energies. Thus, for most of the cases considered in this work, the theoretical results for the BPM cross sections obtained using Eq. (1) differ by less than 5% in comparison from those obtained considering $V_N(R,E) = V_F(R)$. Even so, we have found two cases in the present data set in which the energy is high enough to emphasize such a difference (see Fig. 5). Here the decrease of the cross section for higher energies is connected with the competition between centrifugal repulsion and nuclear attraction. The energy dependence of the nuclear interaction that arises from nonlocality plays an important role by providing further reduction of the cross section. On the other hand, the nonlocality has been fundamental in our description of the heavy-ion elastic scattering process, in which the energy dependence of the potential has

been successfully taken into account by Eq. (1). Therefore, we consider the major reason for using the nonlocal interaction to be the goal of obtaining a unified description of both the elastic scattering and fusion processes, within a consistent model from the sub-barrier region to intermediate energies.

In summary, our parameter-free nonlocal model for the nuclear interaction has been successful in describing the heavy-ion elastic scattering process from sub-barrier energies up to 200 MeV/nucleon. This interaction has also been successfully tested in some cases for inelastic scattering and transfer at sub-Coulomb and intermediate energies [11,12,15]. In the present work, we have also obtained good predictions for fusion cross sections, using the nonlocal interaction in the context of the barrier penetration model, for a very large number of different systems and over a wide energy range (from the sub-barrier region to energies about 15 MeV/nucleon, where there are still a few existing experimental data). We emphasize that the theoretical calculations have no free parameters, except for the system- and energy-independent one connected with the effective barrier curvature. We also emphasize that our model provides remarkable predictions for the light heavy-ion systems over nine orders of magnitude (see Fig. 1 for $\mu \leq 8$). In this range of reduced mass there is no need to include any correction in the barrier curvatures. Therefore, the parameter-free nonlocal model seems to be a good basis for studying the fusion process in important cases for astrophysics, which involve mainly light systems. The model also provides a good description, again without using any free parameter, for the whole set of systems in energies above the barrier (see Fig. 1 for $E_{red} \geq 0$). Clearly, there is room for improvement of the model [57] for heavier systems in lower energies, by considering two points: (i) the development of a more realistic model for the effective barrier parameters, based on results obtained from coupled-channel calculations; and (ii) inclusion of the experimental results obtained for fusion barrier distributions in the analysis.

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