Kerman-Klein-Dönau-Frauendorf model for odd-odd nuclei: Formal theory

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The Kerman-Klein-Dönau-Frauendorf (KKDF) model is a linearized version of the nonlinear Kerman-Klein (equations of motion) formulation of the nuclear many-body problem. In practice, it is a generalization of the standard core-particle coupling model that, like the latter, provides a description of the spectroscopy of odd nuclei in terms of the corresponding properties of neighboring even nuclei and of single-particle properties, which are the input parameters of the model. A divers sample of recent applications attests to the usefulness of the model. In this paper, we first present a concise general review of the fundamental equations and properties of the KKDF model. We then derive a corresponding formalism for odd-odd nuclei with proton-neutron number (Z,N) that relates their properties to those of the four neighboring even-even nuclei $(Z+1,N+1)$, $(Z-1,N+1)$, $(Z+1,N-1)$, and $(Z-1,N-1)$, all of which are required if one is to include both multipole and pairing forces. We treat these equations in two ways. In the first, we make essential use of the solutions of the neighboring odd nucleus problem, as obtained by the KKDF method. In the second, we relate the properties of the odd-odd nucleus directly to those of the even-even nuclei. For both choices, we derive equations of motion, normalization conditions, and an expression for transition amplitudes. We also resolve the problem of choosing the subspace of physical solutions that arises in an equation of motion approach that includes pairing interactions.

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I. INTRODUCTION

The Kerman-Klein-Dönau-Frauendorf (KKDF) model for odd nuclei was introduced and applied [1–6] as a semiphenomenological approximation to the Kerman-Klein (KK) self-consistent formulation of the equation of motion approach to nuclear collective motion [7–11]. As such it generalizes phenomenological core-particle coupling models, to which it can be shown to reduce in various limits [12]. The past decade has witnessed further development of the theory and additional applications [13–21] including, for example, a suggested solution of the Coriolis attenuation problem [17,18].

The main purpose of this paper is to show that a formalism of the KKDF type, at the same level of completeness as for odd nuclei, can be constructed for odd-odd nuclei. The first important step in this direction has already been made by Starosta *et al.* who have applied a restricted version of the formalism to the phenomenon of chirality in odd-odd triaxial nuclei [22]. The restriction is the omission of pairing interactions. When the latter are included, we face, among other difficulties, the problem that the manifold of solutions is four times the size of the manifold of physical solutions. More recently Koike *et al.* [23] have applied an approximate form of the formalism developed in Sec. III.

As a preliminary step, in Sec. II, we review the KKDF program for odd nuclei. We do this in a form which is both more general and more complete than can be found in our previously published work, and which sets the stage for the work on odd-odd nuclei that follows. It is more general in the sense that the equations are not restricted to deformed nuclei. It is more complete in the sense that in our published work, we have described up to three different methods for choosing the physical subspace of solutions, while here we discuss and compare them trying to indicate the most suitable one.

In Sec. III, we present the first of the two methods that can be used for odd-odd nuclei. We refer to this as the sequential method in that it solves the problem by two successive applications of the KKDF approach to odd nuclei, utilizing the solutions for neighboring odd nuclei to derive equations for an odd-odd nucleus relative to its neighboring odd nuclei, so that the method involves only single-particle coefficients of fractional parentage (CFP). In Sec. IV, in an approach that treats the pair of odd particles symmetrically, we derive a set of eigenvalue equations and attendant orthonormalization conditions for two-particle (proton-neutron) coefficients of fractional parentage. These amplitudes relate the given odd-odd nucleus to any of four neighboring even nuclei. For both approaches, we solve the problem of choosing the physical subspace of solutions. Finally we derive for each case formulas for single-particle transition matrix elements that clearly separate collective and single-particle contributions.

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II. REVIEW OF MODEL FOR ODD NUCLEI

A. Equations of motion

In this section we shall derive a version of the KK equations based on the Hamiltonian (1) given below. These equations, when taken literally, define a nonlinear problem for the self-consistent study of the properties of an odd nucleus and of its immediate even neighbors. However, the version of the theory developed here, referred to as the KKDF model, has a more modest goal. This goal is achieved by making such further approximations as to reduce the problem to a linear eigenvalue problem for the properties of odd nuclei, assuming the required properties of the neighboring even nuclei to be known. This can be done only if the Hamiltonian can be chosen of sufficiently simple form such that the matrix elements of its ingredient multipole and pairing operators can be related to observed properties of the even neighbors. Even with such simplification, the resulting theory generalizes previous core-particle coupling models.

We start with a shell-model Hamiltonian of the form

$$H = \sum_{\alpha} h_a a_\alpha^\dagger a_\alpha + \frac{1}{2} F_{\alpha\gamma\delta\beta} a_\alpha^\dagger a_\gamma a_\beta^\dagger a_\delta + \frac{1}{2} G_{\alpha\beta\gamma\delta} a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$
$$= \sum_{\alpha} h_a a_\alpha^\dagger a_\alpha + \frac{1}{2} \sum_{abcd} \sum_{LM_L} F_{acdb}(L) B^\dagger_{LM_L}(ac) B_{LM_L}(db)$$
$$+ \frac{1}{2} \sum_{abcd} \sum_{LM_L} G_{abcd}(L) A^\dagger_{LM_L}(ab) A_{LM_L}(cd). \quad (1)$$

Here h_a are the spherical single-particle energies referred to the nearest closed shell, α refers to the standard set of single-particle quantum numbers, including in particular the pair (j_a, m_a), and a refers to the same set with m_a omitted. The charge conservation requirement means that only the matrix elements of interactions F and G which fulfill the condition

$$q_a + q_b = q_c + q_d, \quad (2)$$

where q_a is the electric charge of a nucleon with the set of quantum numbers a, do not vanish and enter in the Hamiltonian of Eq. (1). In the KKDF model we assume additionally two more restrictive conditions for the interaction matrix elements, namely, (1) the charge exchange interactions are excluded, i.e., $q_a = q_c$ and $q_b = q_d$ for nonvanishing matrix elements $F_{\alpha\gamma\delta\beta}$; (2) only the pairs of like nucleons are correlated, i.e., $q_a = q_b$ and $q_c = q_d$ for nonvanishing matrix elements $G_{\alpha\beta\gamma\delta}$. $B^\dagger_{LM_L}$ is the particle-hole multipole operator,

$$B^\dagger_{LM_L}(ab) \equiv \sum_{m_a m_b} s_\beta (j_a m_a j_b - m_b | LM_L) a_\alpha^\dagger a_\beta$$
$$= (-1)^{j_a + j_b - M_L + 1} B_{L-M_L}(ba) \quad (3)$$

and $A^\dagger_{LM_L}$ is the particle-particle multipole operator,

$$A^\dagger_{LM_L}(ab) \equiv \sum_{m_a m_b} (j_a m_a j_b m_b | LM_L) a_\alpha^\dagger a_\beta^\dagger, \quad (4)$$

where $(j_1 m_1 j_2 m_2 | jm)$ is a Clebsch-Gordon coefficient, $s_\alpha = (-1)^{j_a - m_a}$. The coefficients F are the particle-hole matrix elements,

$$F_{acdb}(L) \equiv \sum_{m's} s_\gamma s_\beta (j_a m_a j_c - m_c | LM_L) \times (j_d m_d j_b - m_b | LM_L) F_{\alpha\gamma\delta\beta} \quad (5)$$

and G the particle-particle matrix elements,

$$G_{abcd}(L) \equiv \sum_{m's} (j_a m_a j_b m_b | LM_L)(j_c m_c j_d m_d | LM_L) G_{\alpha\beta\gamma\delta}. \quad (6)$$

Assuming the matrices F and G are real, we have

$$F_{acdb}(L) = F_{dbac}(L), \quad (7)$$

$$G_{acdb}(L) = G_{dbac}(L) = (-1)^{j_a + j_c - L + 1} G_{cadb} = (-1)^{j_b + j_d - L + 1} G_{acbd}. \quad (8)$$

The task is to obtain equations for the states and energies of an odd nucleus assuming that properties of immediately neighboring even nuclei are known. The states of the odd nucleus (particle number A) are designated below as $|J\mu\nu\rangle$, where ν denotes all quantum numbers besides the angular momentum J and its projection μ. The states of the neighboring even nuclei with particle numbers $(A \pm 1)$ are written, in a parallel notation, as $|IMn(A \pm 1)\rangle$. The corresponding eigenvalues are $E_{J\nu}$ and $E^{(A\pm1)}_{In}$, respectively. We first obtain the operator equations of motion (EOM), bar indicating reversal of the sign of the single-particle magnetic quantum number,

$$[a_{\bar\alpha}, H] = h'_a a_{\bar\alpha} + \sum_{bd\gamma} \sum_{LM} s_{\bar\gamma} (j_a - m_a j_c m_c | LM) \times \overline{F}_{acdb}(L) a_{\bar\gamma} B_{LM}(db) + \sum_{bd\gamma} \sum_{LM} (j_a - m_a j_c m_c | LM) \times G_{acbd}(L) a^\dagger_\gamma A_{LM}(bd), \quad (9)$$

$$[a^\dagger_\alpha, H] = -h''_a a^\dagger_\alpha - \sum_{bd\gamma} \sum_{LM} s_\gamma (j_a m_a j_c - m_c | LM) \times a^\dagger_\gamma \overline{F}_{acdb}(L) B^\dagger_{LM}(db) - \sum_{bd\gamma} \sum_{LM} (j_a m_a j_c - m_c | LM) \times a_{\bar\gamma} G_{acbd}(L) A^\dagger_{LM}(bd). \quad (10)$$

Here

$$\overline{F}_{acdb} = \frac{1}{2}[F_{acdb} + (-1)^{j_a + j_c + j_b + j_d} F_{bdca}] = F_{acdb}, \quad (11)$$

$$h'_a = h_a - \frac{1}{2} \sum_{Lj_c} F_{caca}(L) \frac{2L+1}{2j_a+1}, \quad (12)$$

$$h''_a = h_a + \sum_{Lj_c} \frac{2L+1}{2j_a+1} \left(2G_{acac} + \frac{1}{2} F_{acac} \right). \quad (13)$$

In consequence of Eq. (11), we may replace $\overline{F}$ by F.

The appearance of different single-particle energies in the two equations may be traced to the rearrangement of operators required to have the EOM in a form necessary to achieve

our aims. This requires, as we shall see below, that the multipole and pairing operators occur on the extreme right. The matrix elements of these equations provide expressions that determine the single-particle CFP,

$$V_{J\mu\nu}(\alpha IMn) = \langle J\mu\nu|a_{\overline{\alpha}}|IMn(A+1)\rangle, \tag{14}$$

$$U_{J\mu\nu}(\alpha IMn) = \langle J\mu\nu|a_{\alpha}^{\dagger}|IMn(A-1)\rangle. \tag{15}$$

To find equations for these quantities, we form the necessary matrix elements of the EOM and evaluate the interaction terms by inserting the completeness relation for the states of the appropriate even nuclei between the single-fermion operators and the multipole or pair operators.

In terms of a convenient and physically meaningful set of energy differences and sets of multipole fields and pairing fields defined below, we thereby obtain generalized matrix equations of the Hartree-Bogoliubov form:

$$\begin{aligned}(\mathcal{E}_{J\nu} - \epsilon'_a + \omega_{In}^{(A+1)})V_{J\mu\nu}(\alpha IMn) &= \Gamma^{(A+1)}(\overline{\alpha}IMn|\overline{\gamma}I'M'n')V_{J\mu\nu}(\gamma I'M'n') \\ &\quad + \Delta(\overline{\alpha}IMn|\gamma I'M'n')U_{J\mu\nu}(\gamma I'M'n'),\end{aligned} \tag{16}$$

$$\begin{aligned}(\mathcal{E}_{J\nu} + \epsilon''_a + \omega_{In}^{(A-1)})U_{J\mu\nu}(\alpha IMn) &= -\Gamma^{(A-1)\dagger}(\alpha IMn|\gamma I'M'n')U_{J\mu\nu}(\gamma I'M'n') \\ &\quad - \Delta^{\dagger}(\alpha IMn|\overline{\gamma}I'M'n')V_{J\mu\nu}(\gamma I'M'n').\end{aligned} \tag{17}$$

Here

$$\mathcal{E}_{J\nu} = -E_{J\nu} + \frac{1}{2}(E_0^{(A+1)} + E_0^{(A-1)}), \tag{18}$$

$$\epsilon'_a = h'_a - \lambda_A, \tag{19}$$

$$\lambda_A = \frac{1}{2}(E_0^{(A+1)} - E_0^{(A-1)}), \tag{20}$$

$$\omega_{In}^{(A\pm1)} = E_{In}^{(A\pm1)} - E_0^{(A\pm1)}, \tag{21}$$

and the matrices of the single-particle and pairing potentials read

$$\Gamma^{(A\pm1)}(\alpha IMn|\gamma I'M'n') = \sum_L \sum_{bd} s_\gamma (j_a m_a j_c - m_c|LM_L) F_{acdb}(L)\langle I'M'n'(A\pm1)|B_{LM_L}(db)|IMn(A\pm1)\rangle, \tag{22}$$

$$\Delta(\alpha IMn|\gamma I'M'n') = \sum_L \sum_{bd} (j_a m_a j_c m_c|LM_L) G_{acdb}(L)\langle I'M'n'(A-1)|A_{LM_L}(db)|IMn(A+1)\rangle. \tag{23}$$

Furthermore $E_0^{(A\pm1)}$ refer to the ground-state energies of the neighboring even nuclei, the matrix elements of $\Gamma^{\dagger}$ are derived from those of Eq. (22) simply by the replacement of the operator B by $B^{\dagger}$, and the matrix elements of $\Delta^{\dagger}$ are similarly derived from those of Δ by the replacement of A by $A^{\dagger}$ together with the interchange $A\pm1 \rightarrow A\mp1$. Finally ϵ''_a is obtained from ϵ'_a by the replacement of h'_a by h''_a.

To specify fully solutions of the equations given above, we must develop orthonormalization conditions for the CFP that fix their scale. Orthogonality conditions can be derived from the equations of motion themselves. A normalization condition, on the other hand, is obtained by taking a suitable matrix element of the summed anticommutator,

$$\sum_{\alpha} \{a_\alpha, a_\alpha^{\dagger}\} = \Omega = \sum_{j_a} (2j_a + 1). \tag{24}$$

We thus find

$$\frac{1}{\Omega} \sum_{\alpha IMn} [|U_{J\mu\nu}(\alpha;IMn)|^2 + |V_{J\mu\nu}(\alpha;IMn)|^2] = 1. \tag{25}$$

B. Equations for reduced matrix elements

To apply the Wigner-Eckart theorem to obtain the EOM for the reduced matrix elements, we utilize the following definitions for the latter (which suppress nucleon number):

$$V_{J\mu\nu}(\alpha IMn) = (-1)^{j_a - m_a}(IMj_a m_a|J\mu)v_{J\nu}(aIn), \tag{26}$$

$$U_{J\mu\nu}(\alpha IMn) = (IMj_a m_a|J\mu)u_{J\nu}(aIn), \tag{27}$$

$$\langle I'M'n'|B_{LM_L}(bb')|IMn\rangle = (-1)^{L-M_L}(IML - M_L|I'M') \times [I'n'\|B_L(bb')\|In], \tag{28}$$

$$\langle I'M'n'|A_{LM_L}(bb')|IMn\rangle = (-1)^{L-M_L}(IML - M_L|I'M') \times [I'n'\|A_L(bb')\|In], \tag{29}$$

$$\langle I'M'n'|B^{\dagger}_{LM_L}(bb')|IMn\rangle = (IMLM_L|I'M')[I'n'\|B^{\dagger}_L(bb')\|In], \tag{30}$$

$$\langle I'M'n'|A^{\dagger}_{LM_L}(bb')|IMn\rangle = (IMLM_L|I'M')[I'n'\|A^{\dagger}_{L}(bb')\|In]. \tag{31}$$

Assuming the reality of the multipole and pairing matrix elements, we also have

$$\langle I'M'n'|B_{LM_L(bb')}|IMn\rangle = (I'M'LM_L|IM)[In\|B^{\dagger}_{L}(bb')\|I'n'], \tag{32}$$

$$\langle I'M'n'|A_{LM_L}(bb')|IMn\rangle = (I'M'LM_L|IM)[In\|A^{\dagger}_{L}(bb')\|I'n']. \tag{33}$$

With the help of these definitions, we can transform Eqs. (16) and (17) into the forms

$$\begin{aligned}\mathcal{E}_{J\nu}v_{J\nu}(aIn) = {} & (\epsilon'_a - \omega_n^{(A+1)})v_{J\nu}(aIn) \\ & + \sum_{a'I'n'} \Gamma_J^{(A+1)}(aIn|a'I'n')v_{j\nu}(a'I'n') \\ & + \sum_{a'I'n'} \Delta_J(aIn|a'I'n')u_{J\nu}(a'I'n'),\end{aligned} \tag{34}$$

$$\begin{aligned}\mathcal{E}_{J\nu}u_{J\nu}(aIn) = {} & -(\epsilon''_a + \omega_n^{(A-1)})u_{J\nu}(aIn) \\ & - \sum_{a'I'n'} \Gamma_J^{\dagger(A-1)}(aIn|a'I'n')u_{j\nu}(a'I'n') \\ & + \sum_{a'I'n'} \Delta_J^{\dagger}(aIn|a'I'n')v_{J\nu}(a'I'n'),\end{aligned} \tag{35}$$

where the corresponding reduced matrix elements of the single-particle and pairing potentials, $\Gamma_J^{(A\pm1)}(aIn|a'I'n')$ and $\Delta_J(aIn|a'I'n')$, respectively, are expressed by formulas of Eqs. (A1)–(A4) in Appendix A 1.

The normalization condition (25) becomes

$$\sum_{aIn}[|v_{J\nu}(aIn)|^2 + |u_{J\nu}(aIn)|^2] = \Omega. \tag{36}$$

The equations derived above define a linear eigenvalue problem, provided we supply from outside the single-particle energies h_a, the reduced matrix elements of the included multipole and pairing forces, and the excitation energies of the neighboring even nuclei. In the underlying (self-consistent) theory these quantities, other than the single-particle energies, can themselves be expressed in terms of the CFP v and u. In practice, characteristics of even nuclei expressed in terms of the reduced matrix elements of single-particle operators,

$$F_{LM_L} = \sum_{ac} f_{ac}(L)B_{LM_L}(ac) \tag{37}$$

and pair transfer operators,

$$G_{LM_L} = \sum_{ab} \gamma_{ab}(L)A^{\dagger}_{LM_L}(ab) \tag{38}$$

are available rather than the reduced matrix elements of two-body interactions. To make use of them in Eqs. (34) and (35) it is convenient to present the interactions appearing in Eqs. (A1)–(A4) as a sum of separable interactions of the form

$$F_{acdb}(L) = -\kappa_L(q_aq_b)f_{ac}(L)f_{db}(L), \tag{39}$$

$$G_{abcd}(L) = -g_L(q_a)\gamma_{ab}(L)\gamma_{cd}(L). \tag{40}$$

Then the interactions are parametrized by a few strengths κ_L and g_L which can be either fitted to the experimental data or estimated theoretically.

C. Physical solutions

The equations that we have derived have the form of generalized Hartree-Bogoliubov (HB) equations. We summarize the content of Eqs. (34) and (35) in the condensed form[1]

$$\mathcal{H}\psi_{J\nu} = \mathcal{E}_{J\nu}\psi_{J\nu}, \tag{41}$$

$$\mathcal{H} = \overline{\mathcal{H}} - \overline{\omega}, \tag{42}$$

where

$$\psi = \begin{pmatrix} v \\ u \end{pmatrix}, \tag{43}$$

$$\overline{\mathcal{H}} = \begin{pmatrix} \epsilon' + \Gamma^{(A+1)} & \Delta \\ \Delta^{\dagger} & -\epsilon'' - \Gamma^{\dagger(A-1)} \end{pmatrix}, \tag{44}$$

$$\overline{\omega} = \begin{pmatrix} \omega^{(A+1)} & 0 \\ 0 & \omega^{(A-1)} \end{pmatrix}. \tag{45}$$

The HB structure of these equations implies that only half of the solutions refer to physical states. In the standard ground-state problem, the solutions divide into two sets with reversed energies. These with the positive energies represent the physical solutions and a generalization of the notion of quasiparticles. The negative energy or the quasihole solutions are spurious. The solutions of Eq. (41) do not divide so neatly. The resolution of this dilemma starts by identifying a piece of the Hamiltonian $\mathcal{H}$ that has such a simple property and then initially to "turn off" the remainder of the operator. This is done with the aid of the orthogonal matrix C that interchanges particles and holes,

$$C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{46}$$

and its transpose $\tilde{C}$, and by defining the operator

$$\mathcal{H}_o = \frac{1}{2}(\mathcal{H} - C\mathcal{H}\tilde{C}) = \begin{pmatrix} (\mathcal{H}_o)_{11} & \frac{1}{2}(\Delta + \Delta^{\dagger}) \\ \frac{1}{2}(\Delta + \Delta^{\dagger}) & -(\mathcal{H}_o)_{11} \end{pmatrix}, \tag{47}$$

[1]The core excitation energy matrix $\overline{\omega}$ is usually not separated from the Hamiltonian $\mathcal{H}$ and we shall not do it in the following. Here $\mathcal{H}$ is decomposed into the odd-particle part $\overline{\mathcal{H}}$ and the core excitation energy matrix $\overline{\omega}$ in order to have an analogy qwith a similar decomposition for the odd-odd nucleus, Eqs. (109) and (133) below.

$$(\mathcal{H}_o)_{11} = \frac{1}{2}(\epsilon' + \epsilon'') + \frac{1}{2}(\Gamma^{(A+1)} + \Gamma^{\dagger(A-1)}) - \frac{1}{2}(\omega^{(A+1)} - \omega^{(A-1)}). \tag{48}$$

Because

$$C\mathcal{H}_o\tilde{C} = -\mathcal{H}_o, \tag{49}$$

if Ψ is an eigenstate of $\mathcal{H}_o$ with eigenvalue $\mathcal{E}$, then $C\Psi$ is an eigenstate with eigenvalue $-\mathcal{E}$. As in the simple case, the solutions with positive eigenvalues are the physical solutions for our limiting case. Again, these solutions represent quasiparticles which are now different for different states of the even-even cores.

Next we turn on the remainder of the Hamiltonian, namely, the even part

$$\mathcal{H}_e = \frac{1}{2}(\mathcal{H} + C\mathcal{H}\tilde{C}), \tag{50}$$

our aim being to keep track of the physical solutions. In the applications carried out to date several methods have been used for carrying out this program. The diagonalization of $\mathcal{H}_e$ within the subspace of physical (positive energy) states of $\mathcal{H}_o$ was performed originally when solving the model [1,3]. However, this can lead to a bad approximation of physical solutions of $\mathcal{H}$ as obtained by diagonalization within truncated basis, since matrix elements of $\mathcal{H}_e$ between physical and unphysical solutions need not be small. Therefore, a better procedure seemed to consider an auxiliary Hamiltonian

$$\mathcal{H}(\gamma) = \mathcal{H}_o + \gamma\mathcal{H}_e \tag{51}$$

and to turn on $\mathcal{H}_e$ adiabatically changing γ slowly from 0 to 1 [13]. Knowing the physical solutions for $\gamma=0$ one follows them for $0<\gamma\leqslant 1$ by using a wave function overlap argument; namely, the projection operator onto an eigenstate of $\mathcal{H}(\gamma)$, $\psi_{J\nu}(\gamma)$, for a given γ can in the condensed form be written as

$$\mathcal{R}_{J\nu}(\gamma) = \frac{1}{\Omega}\psi_{J\nu}(\gamma)\psi^{\dagger}_{J\nu}(\gamma). \tag{52}$$

Then, for a new value of the parameter, $\gamma+\delta\gamma$, close to γ, the wave function overlap $\psi^{\dagger}_{J\nu'}(\gamma+\delta\gamma)\mathcal{R}_{J\nu}(\gamma)\psi_{J\nu'}(\gamma+\delta\gamma)$ should be close to Ω, the norm of wave function squared for $\nu'=\nu$, the right continuation of state $J\nu$ at point $\gamma+\delta\gamma$ and close to zero for other states $\nu'\neq\nu$. This procedure is based on the assumption that the physical wave functions change slowly during such a procedure. The above assumption is valid when the levels are far apart. However, it fails when two levels approach and, possibly, cross each other.

To settle the problem of crossing the no-crossing theorem [24–26] has been called [14,20]. According to the theorem the levels belonging to the equivalent irreducible representations of a symmetry group of a one-parameter Hamiltonian $\mathcal{H}(\gamma)$ most likely do not cross each other for any value of γ. The reasoning behind the theorem is outlined below. When the states in question belong to two nonequivalent irreducible representations, respectively, the corresponding nondiagonal matrix element of the Hamiltonian vanishes for an arbitrary γ. In such a case there is a single equation for the value of γ at the crossing point of the levels involved. The equation can, in general, be easily fulfilled, however, not necessary for γ within the range in question. The crossing is thus possible. However, when the representations are equivalent the matrix element is, in general, not equal to zero and then we have two equations for γ to be the crossing point. The equations are most likely inconsistent with each other and the crossing point does not exist. Besides, the existence of the crossing point would mean an additional symmetry of the Hamiltonian at some value of γ, which is not expected. Coming back to the Hamiltonian of Eq. (55), we expect that the levels of the same given angular momentum never cross each other and all crossings are avoided. Possible crossings of levels with different angular momenta are not of interest for current study, since we solve the problem separately for each angular momentum. In the case of no crossings it thus suffices to diagonalize the Hamiltonian $\mathcal{H}_e$ using the complete set of states (*physical and spurious*) generated by $\mathcal{H}_o$ and selecting the largest half of the eigenvalues as the physical solutions [16]. Unfortunately, this need not be always the case because when two levels approach each other close enough no matter if the crossing is real or avoided, the two wave functions in question interchange their character after the closest approach is passed. Therefore, instead of selecting the largest half of the eigenvalues we rather project all wave functions $\psi_{J\nu}(\gamma=1)$ onto the physical subspace of vectors $\psi_{J\nu}(\gamma=0)$ with $\mathcal{E}_{J\nu}(\gamma=0)>0$ and select the half of them with the largest overlaps. The corresponding projection operator is

$$\mathcal{R}_J(\gamma=0) = \sum_{\nu,\mathcal{E}_{J\nu}>0} \mathcal{R}_{J\nu}(\gamma=0). \tag{53}$$

In the ideal case the corresponding overlap of a physical wave function is Ω and that of a spurious one is 0. It may happen in practice that some states are half-and-half mixtures of the physical and spurious ones which means that those are reproduced badly within the present model. Figure 1 shows an exemplary level scheme of Hamiltonian $\mathcal{H}(\gamma)$. From the above discussion the physical and spurious levels can be identified.

This brings us to another issue that is both technical and physical. The simplest application of the KKDF method is to cases where there is well established band structure, either rotational or vibrational, of the same type for both neighboring even nuclei. The problem is then to classify the states of the odd nucleus into bands. For this case, the study initially of $\mathcal{H}_o$ can be useful. This is because for the states belonging to the same band, states of different J are practically degenerate, because of the smallness of $\omega^{(A+1)}-\omega^{(A-1)}$. This was the method used in our early work [13–16]. For more complicated situations, we can identify different band members by the structure of the states, in the sense that the expansion coefficients in terms of a given basis of states vary slowly with angular momentum [20]. Consistent with the identification by state vector, we should equally be able to associate states into bands by calculating transition rates of a suitable collective operator, usually the electric quadrupole operator.

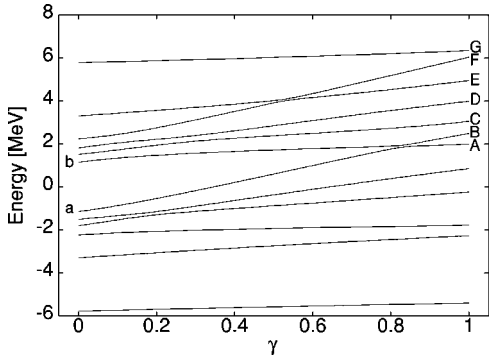

FIG. 1. An exemplary scheme of the energy levels of Hamiltonian $\mathcal{H}(\gamma)$ as functions of γ. At $\gamma=0$ the positive energy level b and all higher levels are physical (quasiparticle) whereas the all negative energy ones starting from a down are spurious (quasihole). At $\gamma=1$ there is no doubt that the physical levels are *C*, *D*, *E*, *F*, *G*. An estimation of overlaps should determine which of levels *A* and *B* is the physical level. The diagonalization of $\mathcal{H}(\gamma=1)$ within the basis of only the quasiparticle states would certainly give the lowest physical state close to *A* rather than to *B*.

D. Matrix elements of transition operators

We complete the exposition of the general formalism for present purposes by deriving formulas for transition amplitudes of a general (charge-conserving) one-body operator. We choose this operator to be a tensor of rank L, T_{LM_L}, that we write in the form

$$T_{LM_L} = \sum_{\beta\gamma} t_{\beta\gamma} a^\dagger_\beta a_\gamma. \tag{54}$$

The notation is such that the quantities $t_{\alpha\beta}$ include a product of matrix elements of single-particle operators and of associated coupling strengths (charges, gyromagnetic ratios, etc.) We wish to calculate the matrix element $\langle J'\mu'\nu'|T_{LM_L}|J\mu\nu\rangle$. To carry through the calculation, we substitute for the ket a formally exact expression in terms of the action of single-particle operators on the states of the core[2]

$$|J\mu\nu\rangle = \frac{1}{\Omega}\sum_{\alpha IMn} [U_{J\mu\nu}(\alpha IMn) a^\dagger_\alpha |\underline{IMn}\rangle + V_{J\mu\nu}(\alpha IMn) a_{\bar\alpha} |\overline{IMn}\rangle], \tag{55}$$

where an underline identifies the lighter of the two cores and an overline the heavier one [5,6]. That Eq. (59) represents an orthonormal set can be proved by first showing that the orthogonality of different states follows from the equations of motion (16) and (17) and then showing that the normalization follows from the CFP normalization condition (25). By using the commutation relations and completeness, this leads to the following expression for the transition element:

$$\begin{aligned}\langle J'\mu'\nu'|T_{LM_L}|J\mu\nu\rangle &= \frac{1}{\Omega}\sum_{\alpha IMnI'M'n'} [U_{J'\mu'\nu'}(\alpha I'M'n') U_{J\mu\nu}(\alpha IMn) \\ &\times \langle \underline{I'M'n'}|T_{LM_L}|\underline{IMn}\rangle + V_{J'\mu'\nu'}(\alpha I'M'n') \\ &\times V_{J\mu\nu}(\alpha IMn)\langle \overline{I'M'n'}|T_{LM_L}|\overline{IMn}\rangle] \\ &+ \frac{1}{\Omega}\sum_{\alpha,\alpha',IMn} t_{\alpha\alpha'}[U_{J'\mu'\nu'}(\alpha IMn) U_{J\mu\nu}(\alpha' IMn) \\ &- V_{J\mu\nu}(\bar\alpha IMn) V_{J'\mu'\nu'}(\bar{\alpha'} IMn)].\end{aligned} \tag{56}$$

We thus have a clear separation into collective and single-particle contributions. The corresponding formula for reduced matrix elements of T_L is presented in Appendix C 1, Eq. (C4).

III. ODD-ODD NUCLEI

A. Equations of motion

We turn to the problem of deriving a general core-particle coupling model for odd-odd nuclei analogous to the model derived for odd nuclei in Sec. II. Given an odd-odd nucleus with Z protons and N neutrons, we shall relate its properties to those of four neighboring even nuclei with proton-neutron numbers $(Z+1,N+1)$, $(Z+1,N-1)$, $(Z-1,N+1)$, and $(Z-1,N-1)$, respectively. In the following development, we shall continue to use greek letters for a general single-particle level, but shall use p, p', etc., to indicate proton levels and n, n', etc., to indicate neutron levels. To relate the properties of the odd-odd nucleus to its four even-even neighbors, we need the equations of motion for four pairs of operators that we present for initial convenience in an uncoupled form:

$$\begin{aligned}[a_{\bar p} a_{\bar n}, H] &= (h'_p + h'_n) a_{\bar p} a_{\bar n} + F_{\overline{pp'}\beta\beta'} a_{\bar{p'}} a_{\bar n} (a^\dagger_{\beta'} a_\beta) \\ &+ F_{\overline{nn'}\beta\beta'} a_{\bar p} a_{\bar{n'}} (a^\dagger_{\beta'} a_\beta) + G_{\overline{pp'}p''p'''} a^\dagger_{p'} a_{\bar n} (a_{p'''} a_{p''}) \\ &+ G_{\overline{nn'}n''n'''} a_{\bar p} a^\dagger_{n'} (a_{n'''} a_{n''}) - F_{\overline{pp'}\bar{n'}\bar n} a_{\bar{p'}} a_{\bar{n'}},\end{aligned} \tag{57}$$

$$\begin{aligned}[a_{\bar p} a^\dagger_{n}, H] &= (h'_p - h''_n) a_{\bar p} a^\dagger_n + F_{\overline{pp'}\beta\beta'} a_{\bar{p'}} a^\dagger_n (a^\dagger_{\beta'} a_\beta) \\ &- F_{nn'\beta\beta'} a_{\bar p} a^\dagger_{n'} (a^\dagger_\beta a_{\beta'}) + G_{\overline{pp'}p''p'''} a^\dagger_{p'} a^\dagger_n (a_{p'''} a_{p''}) \\ &- G_{n\bar{n'}n''n'''} a_{\bar p} a_{\bar{n'}} (a^\dagger_{n''} a^\dagger_{n'''}) + F_{\overline{pp'}nn'} a_{\bar{p'}} a^\dagger_{n'},\end{aligned} \tag{58}$$

$$\begin{aligned}[a^\dagger_{p} a_{\bar n}, H] &= (-h''_p + h'_n) a^\dagger_p a_{\bar n} - F_{pp'\beta\beta'} a^\dagger_{p'} a_n (a^\dagger_\beta a_{\beta'}) \\ &+ F_{\overline{nn'}\beta\beta'} a^\dagger_p a_{\bar{n'}} (a^\dagger_{\beta'} a_\beta) - G_{p\overline{p'}p''p'''} a_{\bar{p'}} a_{\bar n} (a^\dagger_{p''} a^\dagger_{p'''}) \\ &+ G_{\bar{n}n'n''n'''} a^\dagger_p a^\dagger_{n'} (a_{n'''} a_{n''}) + F_{pp'\overline{nn'}} a^\dagger_{p'} a_{\bar{n'}},\end{aligned} \tag{59}$$

[2]This expression was first called to the attention of one of the authors (A.K.) in 1965 by Do Dang and was later used by him in his unpublished lectures on nuclear theory. It reappears in the work of Dönau and Frauendorf [3,5].

$$[a_p^\dagger a_n^\dagger, H] = (-h_p'' - h_n'')a_p^\dagger a_n^\dagger F_{pp'\beta\beta'} a_{p'}^\dagger a_n^\dagger (a_\beta^\dagger a_{\beta'}) - F_{nn'\beta\beta'} a_p^\dagger a_{n'}^\dagger (a_\beta^\dagger a_{\beta'}) - G_{p\bar{p}^\tau p''p'''} a_{\bar{p}^\tau}^\dagger a_n^\dagger (a_{p''}^\dagger a_{p'''}^\dagger) - G_{n\bar{n}^\tau n''n'''} a_p^\dagger a_{\bar{n}^\tau}^\dagger (a_{n''}^\dagger a_{n'''}^\dagger) - F_{pp'n'n} a_{p'}^\dagger a_{n'}^\dagger. \quad (60)$$

Notice that we have not included neutron-proton pairing interactions. We shall study matrix elements of these equations between the states $\langle JM_J s|$ on the left, an included state of the odd-odd nucleus (Z,N), and the appropriate one of the states $|\sigma\tau RM_R r\rangle$ of the even nucleus $(Z\sigma 1, N\tau 1)$, $\sigma=\pm$, $\tau=\pm$. We use the following notation for the matrix elements in question which are the two-particle CFP:

$$\langle JM_J s|a_{\bar{p}} a_{\bar{n}}|+\ +\ IM_I r\rangle = \Psi^{(++)}_{JM_J s}(pnIM_I r), \quad (61)$$

$$\langle JM_J s|a_{\bar{p}} a_n^\dagger|+\ -\ IM_I r\rangle = \Psi^{(+-)}_{JM_J s}(pnIM_I r), \quad (62)$$

$$\langle JM_J s|a_p^\dagger a_{\bar{n}}|-\ +\ IM_I r\rangle = \Psi^{(-+)}_{JM_J s}(pnIM_I r), \quad (63)$$

$$\langle JM_J s|a_p^\dagger a_n^\dagger|-\ -\ IM_I r\rangle = \Psi^{(--)}_{JM_J s}(pnIM_I r). \quad (64)$$

To shorten the notation we introduce a two-component operator a_p^σ for protons,

$$a_p^\sigma = \begin{cases} a_{\bar{p}}(-1)^{j_p - m_p} & \text{for } \sigma = +, \\ a_p^\dagger & \text{for } \sigma = -, \end{cases} \quad (65)$$

and a corresponding pair, a_n^τ, $\tau=\pm$, for neutrons. Then Eqs. (61)–(64) resolve themselves into formula

$$\Psi^{(\sigma\tau)}_{JM_J s}(pnRM_R r) = \langle JM_J s|a_p^\sigma a_n^\tau|\sigma\tau RM_R r\rangle. \quad (66)$$

The next step is to write out equations for the amplitudes $\Psi^{(\sigma\tau)}_{JM_J s}(pnIM_I r)$. To fix additional notation, which will be understood immediately to be a modified form of the notation of Sec. II, we exhibit just one of these equations (using the summation convention),

$$\begin{aligned}(-E_{Js} + E_{++Ir})\Psi^{(++)}_{JM_J s}(pnIM_I r) &= (h_p' + h_n')\Psi^{(++)}_{JM_J s}(pnIM_I r) \\ &+ \Gamma^{(++)}(\bar{p}IM_I r|\bar{p}'I'M_{I'}r')\Psi^{(++)}_{JM_J s}(p'nI'M_{I'}r') \\ &+ \Delta^{(+)}(\bar{p}IM_I r|p'I'M_{I'}r')\Psi^{(-+)}_{JM_J s}(p'nI'M_{I'}r') \\ &+ \Gamma^{(++)}(\bar{n}IM_I r|\bar{n}'I'M_{I'}r')\Psi^{(++)}_{JM_J s}(pn'I'M_{I'}r') \\ &+ \Delta^{(+)}(\bar{n}IM_I r|n'I'M_{I'}r')\Psi^{(+-)}_{JM_J s}(pn'I'M_{I'}r') \\ &- F_{p\bar{p}'\bar{n}'n}\Psi^{(++)}_{JM_J s}(p'n'IM_I r), \end{aligned} \quad (67)$$

where the single-particle and pairing potentials

$$\Gamma^{(\sigma\tau)}(aIM_I r|a'I'M_{I'}r') = F_{aa'\beta\beta'}\langle\sigma\tau IM_I r|a_{\beta'}^\dagger a_\beta|\sigma\tau I'M_{I'}r'\rangle \quad (68)$$

and

$$\Delta^{(\tau)}(pIM_I r|p'I'M_{I'}r') = G_{pp'p''p'''}\langle+\ \tau IM_I r|a_{p'''}a_{p''}|-\ \tau I'M_{I'}r'\rangle \quad (69)$$

are essentially the same as those of Eqs. (22) and (23) but now for different even cores.

B. Matrix elements of transition operators

It still remains the problem of deriving a general formula for the transition matrix element, $\langle J'M_{J'}s'|T_{LM_L}|JM_J s\rangle$ of a one-body operator, T_{LM_L} in a manner analogous to the calculation carried out in Sec. II D. For this purpose, we utilize a formula for the state $|JM_J s\rangle$ [compare Eq. (55)]

$$|JM_J s\rangle = \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\sigma\tau pnRM_R r} \Psi^{(\sigma\tau)}_{JM_J s}(pnRM_R r)a_p^\sigma a_n^\tau|\sigma\tau RM_R r\rangle. \quad (70)$$

Equation (70) describes a set of orthonormal states, as follows from the equations of motion and the normalization condition (108). We thus derive the formula

$$\begin{aligned}\langle J'M_{J'}s'|T_{LM_L}|JM_J s\rangle &= \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\sigma\tau pnRM_R rR'M_{R'}r'} \Psi^{(\sigma\tau)}_{JM_J s}(pnRM_R r)\Psi^{(\sigma\tau)}_{J'M_{J'}s'}(pnR'M_{R'}r')\langle\sigma\tau R'M_{R'}r'|T_{LM_L}|\sigma\tau RM_R r\rangle \\ &+ \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\tau pp'nRM_R r} \Psi^{(-\tau)}_{JM_J s}(pnRM_R r)\Psi^{(-\tau)}_{J'M_{J'}s'}(p'nRM_R r)t_{p'p} + \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\sigma pnn'RM_R r} \Psi^{(\sigma-)}_{JM_J s}(pnRM_R r) \\ &\times \Psi^{(\sigma-)}_{J'M_{J'}s'}(pn'RM_R r)t_{n'n} - \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\tau pp'nRM_R r} \Psi^{(+\tau)}_{JM_J s}(pnRM_R r)\Psi^{(+\tau)}_{J'M_{J'}s'}(p'nRM_R r) \\ &\times t_{\bar{p}\bar{p}'}(-1)^{j_p - m_p + j_{p'} - m_{p'}} - \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\sigma pnn'RM_R r} \Psi^{(\sigma+)}_{JM_J s}(pnRM_R r)\Psi^{(\sigma+)}_{J'M_{J'}s'}(pn'RM_R r)t_{\bar{n}\bar{n}'}(-1)^{j_n - m_n + j_{n'} - m_{n'}}, \end{aligned} \quad (71)$$

in which we have again a clear separation into collective part of the four even cores, and the single-proton and single-neutron contributions coming from the odd particles.

For the further development of the formalism, in particular the reduction to equations for reduced matrix elements, there are, however, several choices which are to be discussed in the two following sections. First, we shall develop a method that makes maximal use of the formalism for odd nuclei, and, as a consequence involves only single-particle CFP. This method, which treats the neutron and proton asymmetrically, we shall refer to as sequential coupling. Then, we shall develop an alternative, referred to as symmetrical coupling, that bypasses any use of the results for odd nuclei.

IV. SEQUENTIAL COUPLING OF ODD NUCLEONS

A. Equations of motion

By introducing a complete set of states of the appropriate odd nucleus between the neutron and proton single-particle operators in matrix elements of Eqs. (61)–(64), we write

$$\Psi^{(++)}_{JM_Js}(pnIM_Ir) = \sum_{J_nM_nr_n} X_{JM_Js}(pJ_nM_nr_n)V^{(+)}_{J_NM_nr_n}(nIM_Ir), \tag{72}$$

$$\Psi^{(+-)}_{JM_Js}(pnIM_Ir) = \sum_{J_nM_nr_n} X_{JM_Js}(pJ_nM_nr_n)U^{(+)}_{J_NM_nr_n}(nIM_Ir), \tag{73}$$

$$\Psi^{(-+)}_{JM_Js}(pnIM_Ir) = \sum_{J_nM_nr_n} Y_{JM_Js}(pJ_nM_nr_n)V^{(-)}_{J_NM_nr_n}(nIM_Ir), \tag{74}$$

$$\Psi^{(--)}_{JM_Js}(pnIM_Ir) = \sum_{J_nM_nr_n} Y_{JM_Js}(pJ_nM_nr_n)U^{(-)}_{J_NM_nr_n}(NIM_Ir). \tag{75}$$

Here

$$V^{(\sigma)}_{J_nM_nr_n}(nIM_Ir) = \langle \sigma J_nM_nr_n|a^{\dagger}_{\overline{n}}|\sigma + IM_Ir\rangle, \tag{76}$$

$$U^{(\sigma)}_{J_nM_nr_n}(nIM_Ir) = \langle \sigma J_nM_nr_n|a^{\dagger}_{\overline{n}}|\sigma - IM_Ir\rangle \tag{77}$$

are two sets of CFP amplitudes for odd neutron nuclei, which can be calculated using the formalism for odd nuclei developed in Sec. II. On the other hand the amplitudes

$$X_{JM_Js}(pJ_nM_nr_n) = \langle JM_Js|a^{\dagger}_{\overline{p}}|+ J_nM_nr_n\rangle, \tag{78}$$

$$Y_{JM_Js}(pJ_nM_nr_n) = \langle JM_Js|a^{\dagger}_{p}|- J_nM_nr_n\rangle \tag{79}$$

are single-particle CFP relating odd and odd-odd nuclei. The aim of the present coupling scheme is to obtain equations to determine the amplitudes X and Y. Before proceeding along these lines, we remark that there is a related sequential scheme obtained by starting with two-particle amplitudes in which the order of the single-particle operators is interchanged.

Into Eq. (67) and its three partners, we substitute Eqs. (72)–(75) and recognize that the result can be simplified by the use of equations such as

$$\begin{aligned}(E_{J_nr_n} - E_{++Ir} - h'_n)V^{(+)}_{J_nM_nr_N}(nIM_Ir) &= \Gamma^{(++)}(\overline{n}IM_Ir|\overline{n'}I'M_{I'}r')V^{(+)}_{J_nM_nr_n}(n'I'M_{I'}r') \\ &\quad + \Delta^{(+)}(\overline{n}IM_Ir|n'I'M_{I'}r')U^{(+)}_{J_nM-nr_n}(n'I'M_{I'}r')\end{aligned} \tag{80}$$

and its partners. The resulting equation for $\Psi^{(++)}$ is then combined with the corresponding equation for $\Psi^{(+-)}$, contracting the first with a $V^{(+)}$ factor and the second with a $U^{(+)}$ factor so as to permit use of the normalization condition (25). We carry through a corresponding procedure for the pair of amplitudes $\Psi^{(-+)}$ and $\Psi^{(--)}$.

We thus obtain the pair of equations

$$\begin{aligned}(-E_{J_S} + E_{+J_nr_n})X_{JM_Js}(pJ_nM_nr_n) &= h'_pX_{JM_Js}(pJ_nM_nr_n) \\ &\quad + \Gamma^{(+)}(\overline{p}J_nM_nr_n|\overline{p'}J_{n'}M_{n'}r_{n'})X_{JM_Js}(p'J_{n'}M_{n'}r_{n'}) \\ &\quad + \Delta(\overline{p}J_nM_nr_n|p'J_{n'}M_{n'}r_{n'})Y_{JM_Js}(p'J_{n'}M_{n'}r_{n'}) \\ &\quad + \mathcal{V}^{(+)}(\overline{p}J_nM_nr_n|\overline{p'}J_{n'}M_{n'}r_{n'})X_{JM_Js}(p'J_{n'}M_{n'}r_{n'}),\end{aligned} \tag{81}$$

$$\begin{aligned}(-E_{J_S} + E_{-J_nr_n})Y_{JM_Js}(pJ_nM_nr_n) &= -h''_pY_{JM_Js}(pJ_nM_nr_n) \\ &\quad - \Gamma^{\dagger(-)}(pJ_nM_nr_n|p'J_{n'}M_{n'}r_{n'})Y_{JM_Js}(p'J_{n'}M_{n'}r_{n'}) \\ &\quad - \Delta^{\dagger}(pJ_nM_nr_n|\overline{p'}J_{n'}M_{n'}r_{n'})X_{JM_Js}(p'J_{n'}M_{n'}r_{n'}) \\ &\quad + \mathcal{V}^{(-)}(pJ_nM_nr_n|p'J_{n'}M_{n'}r_{n'})Y_{JM_Js}(p'J_{n'}M_{n'}r_{n'}),\end{aligned} \tag{82}$$

where the effective single-particle and pairing potentials for the proton on top of odd nuclei, $\Gamma^{(\pm)}$ and Δ, and the effective proton-neutron interaction $V^{(\pm)}$ depend on the single-neutron CFP, $V^{(\pm)}$ and $U^{(\pm)}$. The corresponding formulas are presented in Appendix A, Eqs. (A5)–(A10).

The final goal of this section is to obtain equations of motion for the reduced matrix elements. For this purpose the only definitions needed to supplement Eqs. (26)–(31) are those for the reduced CFP relating the odd to the odd-odd nuclei,

$$X_{JM_Js}(pJ_nM_nr_n) = (-1)^{j_p-m_p}(J_nM_nj_pm_p|JM_J)\chi_{Js}(j_pJ_nr_n), \tag{83}$$

$$Y_{JM_Js}(pJ_nM_nr_n) = (J_nM_nj_pm_p|JM_J)\,\eta_{Js}(j_pJ_nr_n). \tag{84}$$

It is then a straightforward exercise in angular momentum algebra to derive the equations

$$(-E_{Js}+E_{+J_nr_n})\chi_{Js}(j_pJ_nr_n)$$
$$=h'_p\chi_{Js}(j_pJ_nr_n)+\Gamma^{(+)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\chi_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\Delta_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\eta_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\mathcal{V}^{(+)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\chi_{Js}(j_{p'}J_{n'}r_{n'}), \quad (85)$$

$$(-E_{Js}+E_{-J_nr_n})\eta_{Js}(j_pJ_nr_n)$$
$$=-h''_p\eta_{Js}(j_pJ_nr_n)-\Gamma^{\dagger(-)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\eta_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\Delta^{\dagger}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\chi_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\mathcal{V}^{(-)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\eta_{Js}(j_{p'}J_{n'}r_{n'}), \quad (86)$$

where the reduced matrix elements of $\Gamma^{(\pm)}$, Δ, and $\mathcal{V}^{(\pm)}$ are given by Eqs. (A11)–(A16) in Appendix A 2.

The normalization condition associated with this formalism is

$$\sum_{j_pJ-nr_n}[|\chi_{Js}(j_pJ_nr_n)|^2+|\eta_{Js}(j_pJ_nr_n)|^2]=\Omega_{(p)}. \quad (87)$$

B. Physical solutions

The problem of choosing the physical solutions of Eqs. (85) and (86) can solved by simply repeating the arguments given in Sec. II C. This is seen immediately if we rearrange the energies in these equations so that they resemble exactly the corresponding equations (34) and (35). We thus write

$$\mathcal{E}_{Js}\chi_{Js}(j_pJ_nr_n)=(\epsilon'_p-\omega_{+J_nr_n})\chi_{Js}(j_pJ_nr_n)$$
$$+\Gamma^{(+)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\chi_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\Delta_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\eta_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\mathcal{V}^{(+)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\chi_{Js}(j_{p'}J_{n'}r_{n'}), \quad (88)$$

$$\mathcal{E}_{Js}\eta_{Js}(j_pJ_nr_n)=(-\epsilon''_p-\omega_{-J_nr_n})\eta_{Js}(j_pJ_nr_n)$$
$$-\Gamma^{\dagger(-)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\eta_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\Delta^{\dagger}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\chi_{Js}(j_{p'}J_{n'}r_{n'})$$
$$+\mathcal{V}^{(-)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'})\eta_{Js}(j_{p'}J_{n'}r_{n'}), \quad (89)$$

with

$$\mathcal{E}_{Js}=-E_{Js}+\tfrac{1}{2}(E_++E_-), \quad (90)$$

$$\epsilon=h-\tfrac{1}{2}(E_+-E_-), \quad (91)$$

$$\omega_{\pm J_nr_n}=E_{\pm J_nr_n}-E_\pm. \quad (92)$$

Here $E_\pm$ are the ground states of the heavier and lighter odd neutron nuclei, respectively. With these forms one has an exact parallel to Eqs. (34) and (35), and thus the arguments for choosing physical solutions can be repeated without modification.

As we already mentioned, the procedure is not symmetric in treatment of the odd proton and the odd neutron. We have the two separate Hamiltonians for the two odd neutron nuclei with even proton numbers $Z\mp1$, respectively. The odd parts of these Hamiltonians represent the two sets of the corresponding neutron quasiparticles. Then, the single set of proton quasiparticles is introduced on top of the two odd neutron nuclei. The roles of odd neutron and odd proton can, of course, be exchanged.

C. Matrix elements of transition operators

The reduced matrix elements for a one-body transition operator can be calculated from Eq. (71) by substituting decompositions of Eqs. (72)–(75) of the two-particle CFP's, $\Psi^{(\sigma\tau)}$, into the single-particle CFP's, X, Y, $V^{(\pm)}$, and $U^{(\pm)}$, and applying the Wigner-Eckart theorem to these latter ones. However, the result we want can be read off directly from Eq. (C4) if we replace, appropriately, the single-particle CFP v and u by χ and η and the reduced matrix elements of the transition operator T_L between states of even nuclei by the corresponding matrix elements between states of the appropriate odd nuclei. The final result is presented in Appendix C 2, Eq. (C5).

V. FORMALISM FOR SYMMETRICAL TREATMENT OF ODD NUCLEONS

A. Equations for reduced matrix elements

With the aid of the Wigner-Eckart theorem and suitable definitions, we proceed to the transformation of these raw equations for $\Psi^{(\sigma\tau)}$ [Eq. (67) and the three similar ones] to equations for reduced matrix elements. We couple the products of operators a^σ_p and a^τ_n to a given multipolarity:

$$a^\sigma_pa^\tau_n=\sum_l(j_pm_pj_nm_n|lm)\mathcal{B}^{\sigma\tau}_{lm}(pn), \quad (93)$$

and define reduced matrix elements $b^{\sigma\tau}_{Js}(pnlRr)$,

$$\langle JM_Js|\mathcal{B}^{\sigma\tau}_{lm}(pn)|\sigma\tau RM_Rr\rangle=(RM_rlm|JM_j)b^{\sigma\tau}_{Js}(pnlRr). \quad (94)$$

Here $|JM_Js\rangle$ is a state of the odd-odd nucleus (Z,N) and $|\sigma\tau RM_Rr\rangle$ is a state of the even nucleus $(Z\sigma1,N\tau1)$.

In the following, we also require reduced matrix elements of the multipole and pairing operators, defined as follows:

$$\langle\sigma\tau R'M_{R'}r'|B_{LM_L}(bb')|\sigma\tau RM_Rr\rangle$$
$$=(-1)^{L-M_L}(RM_RL-M_L|R'M_{R'})[R'r'\|B^{(\sigma\tau)}_L(bb')\|Rr], \quad (95)$$

$$\langle\sigma\tau R'M_{R'}r'|B^\dagger_{LM_L}(bb')|\sigma\tau RM_Rr\rangle$$
$$=(RM_RLM_L|R'M_{R'})[R'r'\|B^{\dagger(\sigma\tau)}_L(bb')\|Rr], \quad (96)$$

$$\langle\sigma-R'M_{R'}r'|A_{LM}(nn')|\sigma+RM_Rr\rangle$$
$$=(-1)^{L-M_L}(RM_RL-M_L|R'M_{R'})[R'r'\|A^{(\sigma)}_L(nn')\|Rr], \quad (97)$$

$$\langle \sigma + R'M_{R'}r'|A^{\dagger}_{LM}(nn')|\sigma - RM_Rr\rangle = (RM_RLM_L|R'M_{R'})[R'r'\|A^{\dagger(\sigma)}_L(nn')\|Rr], \quad (98)$$

$$\langle -\tau R'M_{R'}r'|A_{LM}(pp')|+\tau RM_Rr\rangle = (-1)^{L-M_L}(RM_RL-M_L|R'M_{R'})[R'r'\|A^{(\tau)}_L(pp')\|Rr], \quad (99)$$

$$\langle +\tau R'M_{R'}r'|A^{\dagger}_{LM}(pp')|-\tau RM_Rr\rangle = (RM_RLM_L|R'M_{R'})[R'r'\|A^{\dagger(\tau)}_L(pp')\|Rr]. \quad (100)$$

In the final equations of motion given below, we also introduce, in as close analogy as possible with our procedure for the odd-nucleus case, various combinations of energies. The energies of the odd states will naturally be specified by E_{Js}, those of the four neighboring even nuclei by $E_{\sigma\tau Rr}$, and the ground states of the latter by $E_{\sigma\tau}$. We then introduce the following differences:

$$\mathcal{E}_{Js} = -E_{Js} + \frac{1}{4}(E_{++}+E_{+-}+E_{-+}+E_{--}),$$

$$\omega_{\sigma\tau Rr} = E_{\sigma\tau Rr} - E_{\sigma\tau}, \quad (101)$$

$$\epsilon'_n = h'_n - \frac{1}{4}(E_{++}-E_{+-}) - \frac{1}{8}(E_{++}-E_{--}),$$

$$\epsilon'_p = h'_p - \frac{1}{4}(E_{++}-E_{-+}) - \frac{1}{8}(E_{++}-E_{--}), \quad (102)$$

$$\epsilon''_n = h''_n - \frac{3}{4}(E_{++}-E_{+-}) + \frac{1}{8}(E_{++}-E_{--}),$$

$$\epsilon''_p = h''_p - \frac{3}{4}(E_{++}-E_{-+}) + \frac{1}{8}(E_{++}-E_{--}), \quad (103)$$

$$\epsilon'''_n = h''_n + \frac{1}{4}(E_{--}-E_{+-}) + \frac{1}{8}(E_{--}-E_{++}),$$

$$\epsilon'''_p = h''_P + \frac{1}{4}(E_{--}-E_{-+}) + \frac{1}{8}(E_{--}-E_{++}). \quad (104)$$

Starting from Eq. (67) and its three partners we thereby obtain the following equations of motion for the reduced matrix elements:

$$\mathcal{E}_{Js}b^{\sigma\tau}_{Js}(pnlRr) = \sum_{\sigma'\tau'p'n'l'R'r'} \mathcal{H}(\sigma\tau pnlRr|\sigma'\tau'p'n'l'R'r') \times b^{\sigma'\tau'}_{Js}(p'n'l'R'r'). \quad (105)$$

Expressions for the matrix elements of the effective Hamiltonian matrix $\mathcal{H}$ are given in Appendix B.

To the equations of motion, we add a normalization condition that can be derived from the anticommutation relation

$$\sum_{pn}\{a_p,a^{\dagger}_p\}\{a_n,a^{\dagger}_n\} = \Omega_{(p)}\Omega_{(n)}, \quad (106)$$

$$\Omega_{(p)} = \sum_p (2j_p+1). \quad (107)$$

Rearranging the order of the operators, taking a diagonal matrix element in the state $|JM_Js\rangle$, and utilizing completeness and the definitions (93) and (94), we obtain the expected result

$$\sum_{\sigma\tau j_p j_n lRr} |b^{(\sigma\tau)}_{Js}(pnlRr)|^2 = \Omega_{(p)}\Omega_{(n)}. \quad (108)$$

B. Physical solutions

We expect the space of physical solutions to be only a quarter of the total space of solutions. With a little care, we can generalize the method used to identify physical solutions for the case of odd nuclei. If we examine the Hamiltonian matrix $\mathcal{H}$ given by Eqs. (B1)–(B12), we see that it can be decomposed into a sum

$$\mathcal{H} = \mathcal{H}_p + \mathcal{H}_n - \overline{\omega} + \mathcal{V}, \quad (109)$$

describing in turn an odd-neutron nucleus, an odd-proton nucleus, a core excitation energy matrix, and a neutron-proton effective interaction. We initially turn off the last two terms. Next we define two 4×4 matrices, C_n and C_p,

$$C_n = \begin{pmatrix} \underline{C} & \underline{0} \\ \underline{0} & \underline{C} \end{pmatrix}, \quad (110)$$

$$C_p = \begin{pmatrix} \underline{0} & -\underline{1} \\ \underline{1} & \underline{0} \end{pmatrix}, \quad (111)$$

where the underlined entries are each 2×2 matrices and the matrix $\underline{C}$ is the particle-hole conjugation matrix defined in Eq. (46). The matrices C_p, C_n commute with each other.

We then observe that the averages

$$\overline{\mathcal{H}}_p = \frac{1}{2}(\mathcal{H}_p + C_n\mathcal{H}_p\tilde{C}_n), \quad (112)$$

$$\overline{\mathcal{H}}_n = \frac{1}{2}(H_n + C_p\mathcal{H}_n\tilde{C}_p) \quad (113)$$

each have a structure more symmetrical than their individual terms. Thus, the nonvanishing elements of $\overline{\mathcal{H}}_p$ are (in a condensed notation)

$$\overline{\mathcal{H}}_p(++|++) = \overline{\mathcal{H}}_p(+-|+-) = \epsilon'_p + \frac{1}{2}(\Gamma^{(++)}_p + \Gamma^{(+-)}_p), \quad (114)$$

$$\overline{\mathcal{H}}_p(-\,+|-\,+)=\overline{\mathcal{H}}_p(--|--)$$
$$=-\frac{1}{2}(\epsilon_p''+\epsilon_p'''+\Gamma_p^{\dagger(-+)}+\Gamma_p^{\dagger(--)}), \quad (115)$$

$$\overline{\mathcal{H}}_p(+\,+|-\,+)=\overline{\mathcal{H}}_p(+-|--)=\frac{1}{2}(\Delta_p^{(+)}+\Delta_p^{(-)}), \quad (116)$$

$$\overline{\mathcal{H}}_p(-\,+|+\,+)=\overline{\mathcal{H}}_p(--|+-)=\frac{1}{2}(\Delta_p^{\dagger(+)}+\Delta_p^{\dagger(-)}), \quad (117)$$

whereas $\overline{\mathcal{H}}_n$ has the block-diagonal structure,

$$\overline{\mathcal{H}}_n=\begin{pmatrix}\underline{\mathcal{H}}_n & 0\\ 0 & \underline{\mathcal{H}}_n\end{pmatrix}, \quad (118)$$

in terms of 2×2 matrices, and

$$\underline{H}_n=\begin{pmatrix}\epsilon_n'+\frac{1}{2}(\Gamma_n^{(++)}+\Gamma_n^{(+-)}) & \frac{1}{2}(\Delta_n^{(+)}+\Delta_n^{(-)})\\ \frac{1}{2}(\Delta_n^{\dagger(+)}+\Delta_n^{\dagger(-)}) & -\frac{1}{2}(\epsilon_n''+\epsilon_n'''+\Gamma_n^{\dagger(-+)}+\Gamma_n^{\dagger(--)})\end{pmatrix}. \quad (119)$$

We infer from their structure that it is the barred matrices that form suitable starting points for the antisymmetrization that was the essential step for identifying physical solutions for the theory of odd nuclei. We thus define the matrices $\overline{\mathcal{H}}_{po}$ and $\overline{\mathcal{H}}_{no}$,

$$\overline{\mathcal{H}}_{po}=\frac{1}{2}(\overline{\mathcal{H}}_p-C_p\overline{H}_p\widetilde{C}_p), \quad (120)$$

$$\overline{\mathcal{H}}_{no}=\frac{1}{2}(\overline{\mathcal{H}}_n-C_n\overline{\mathcal{H}}_n\widetilde{C}_n), \quad (121)$$

which are antisymmetric with respect to the particle-hole conjugation matrices C_p and C_n for protons and neutrons, respectively,

$$C_p\overline{\mathcal{H}}_{po}\widetilde{C}_p=-\overline{\mathcal{H}}_{po}, \quad (122)$$

$$C_n\overline{\mathcal{H}}_{no}\widetilde{C}_n=-\overline{\mathcal{H}}_{no}. \quad (123)$$

Notice that C_n commutes with $\overline{\mathcal{H}}_{po}$ and C_p commutes with $\overline{\mathcal{H}}_{no}$, but the two Hamiltonians do not commute with each other. Because of that these Hamiltonians cannot generate the separate "average" quasiparticle (quasihole) proton and neutron states. Instead, the eigenvectors of matrix,

$$\overline{\mathcal{H}}_o=\overline{\mathcal{H}}_{no}+\overline{\mathcal{H}}_{po} \quad (124)$$

represent the neutron-proton pair states. To identify the physical basis we introduce an auxiliary Hamiltonian

$$\overline{\mathcal{H}}_o(\varepsilon)=\overline{\mathcal{H}}_{no}+\varepsilon\overline{\mathcal{H}}_{po}, \quad (125)$$

which has the following properties:

$$C_pC_n\overline{\mathcal{H}}_o(\varepsilon)\widetilde{C}_n\widetilde{C}_p=-\overline{\mathcal{H}}_o(\varepsilon), \quad (126)$$

$$C_p\overline{\mathcal{H}}_o(\varepsilon)\widetilde{C}_p=\overline{\mathcal{H}}_o(-\varepsilon). \quad (127)$$

A quarter of the set of eigenvectors of $\overline{\mathcal{H}}_o(1)\equiv\overline{\mathcal{H}}_o$ form the physical basis. From Eq. (126) we see that for each positive eigenvalue, $\overline{\mathcal{E}}_{Js}(\varepsilon)>0$ of $\overline{\mathcal{H}}_o(\varepsilon)$ which enter in equation

$$\overline{\mathcal{H}}_o(\varepsilon)\psi_{Js}(\varepsilon)=\overline{\mathcal{E}}_{Js}(\varepsilon)\psi_{Js}(\varepsilon), \quad (128)$$

there is a corresponding negative eigenvalue $-\overline{\mathcal{E}}_{Js}(\varepsilon)$ with the associated eigenvector $C_pC_n\psi_{Js}(\varepsilon)$. Following the standard reasoning of superconductivity theory we reject half of the eigenvectors, those belonging to negative eigenvalues as nonphysical. These spurious states are interpreted as the quasihole-quasihole and quasihole-quasiparticle pairs. However that still leaves too many states.

From Eq. (127) it follows that the Hamiltonians $\overline{\mathcal{H}}_o(\varepsilon)$ and $\overline{\mathcal{H}}_o(-\varepsilon)$ have the same set of eigenvalues, i.e., $\overline{\mathcal{E}}_{Js}(\varepsilon)=\overline{\mathcal{E}}_{Js}(-\varepsilon)$, with the corresponding eigenvectors $\psi_{Js}(-\varepsilon)=C_p\psi_{Js}(\varepsilon)$. This has as a further consequence that each eigenvalue of $\overline{\mathcal{H}}_o(0)$, $\overline{\mathcal{E}}_{Js}(0)$ is twofold degenerate with the two eigenvectors $\psi_{Js}(0)$ and $C_p\psi_{Js}(0)$. The degeneracy of this pair of levels at $\varepsilon=0$ means the real crossing, which is not surprising since there is additionally the particle-hole symmetry C_p for proton. Next we solve Eq. (128) for $0\leqslant\varepsilon\leqslant 1$ and a given J. For $\varepsilon>0$, level $\overline{\mathcal{E}}_{Js}(0)$ splits into a pair of levels, labeled $\overline{\mathcal{E}}_{Js>}(\varepsilon)$ and $\overline{\mathcal{E}}_{Js<}(\varepsilon)$, distinguishing the larger from the smaller value. The corresponding eigenvectors are labeled $\psi_{Js>}(\varepsilon)$ and $\psi_{Js<}(\varepsilon)$, respectively. Let us suppose that the expectation value of matrix $\overline{\mathcal{H}}_{po}$ in state $\psi_{Js}(0)$ is positive:

$$\delta\overline{\mathcal{E}}_{Js} = \psi^{\dagger}_{Js}(0)\overline{\mathcal{H}}_{po}\psi_{Js}(0) > 0. \tag{129}$$

If it is not the case we exchange the vectors $\psi_{Js}(0)$ and $C_p\psi_{Js}(0)$ since $\psi^{\dagger}_{Js}(0)\widetilde{C}_p\overline{\mathcal{H}}_{po}C_p\psi_{Js}(0) = -\delta\mathcal{E}_{Js}$. For small values of ε the perturbed energies of unperturbed states $\psi_{Js>}(0) = \psi_{Js}(0)$ and $\psi_{Js<}(0) = C_p\psi_{Js}(0)$ are equal to

$$\overline{\mathcal{E}}_{Js>}(\varepsilon) = \overline{\mathcal{E}}_{Js}(0) + \varepsilon\delta\overline{\mathcal{E}}_{Js} \tag{130}$$

and

$$\overline{\mathcal{E}}_{Js<}(\varepsilon) = \overline{\mathcal{E}}_{Js}(0) - \varepsilon\delta\overline{\mathcal{E}}_{Js}, \tag{131}$$

respectively. For larger ε's we solve Eq. (128) in the same way as the eigenvalue problem of matrix of Eq. (51) in Sec. II C. We choose the larger of the two eigenvalues as the physical one, recognizing its role as the analog of the quasiparticle-quasiparticle pair energy. The lower levels correspond to the quasiparticle-quasihole pair states and are still spurious. The corresponding projection operator onto physical subspace is

$$\overline{\mathcal{R}}_J(\varepsilon) = \frac{1}{\Omega_{(p)}\Omega_{(n)}} \sum_{\substack{s> \\ \overline{\varepsilon}_{Js>}>0}} \psi_{Js>}(\varepsilon)\psi^{\dagger}_{Js>}(\varepsilon). \tag{132}$$

Increasing the value of ε to unity we define the physical eigenstates of the Hamiltonian $\overline{\mathcal{H}}_o(\varepsilon=1) \equiv \overline{\mathcal{H}}_o$ as the quarter of all 1's having large overlap with physical subspace determined through the projection operator of Eq. (132). Since the number of positive eigenvalues is even, in the case of no crossings for a given J, the physical solutions are the odd-numbered positive eigenvalues, counting from the largest value.

The separation of $\overline{\mathcal{H}}_o$ from the original Hamiltonian of Eq. (109) is achieved by the following decomposition of $\mathcal{H}$:

$$\mathcal{H} = \overline{\mathcal{H}}_o + \overline{\mathcal{H}}_e - \overline{\omega} + \mathcal{V}, \tag{133}$$

where

$$\overline{\mathcal{H}}_e = \frac{3}{4}(\mathcal{H}_p + \mathcal{H}_n) + \frac{1}{4}C_pC_n(\mathcal{H}_p + \mathcal{H}_n)\widetilde{C}_n\widetilde{C}_p + \frac{1}{4}C_p(\mathcal{H}_p - \mathcal{H}_n)\widetilde{C}_p + \frac{1}{4}C_n(\mathcal{H}_n - \mathcal{H}_p)\widetilde{C}_n. \tag{134}$$

The physical eigenvectors of $\mathcal{H}$ can further be found using again the methods similar to those discussed in Sec. II C for odd nuclei.

C. Reduced matrix elements of transition operators

To calculate the reduced matrix elements of transition operators we apply the Wigner-Eckart theorem to Eq. (71) and use the formula

$$\Psi^{(\sigma\tau)}_{JM_Js}(pnRM_Rr) = \sum_{lm}(j_pm_pj_nm_n|lm) \times (RM_Rlm|JM_J)b^{(\sigma\tau)}_{Js}(pnlRr), \tag{135}$$

which combines the contents of Eqs. (93), (94), and (66). The final result is written out in Appendix C 2, Eq. (C6).

VI. SUMMARY AND CONCLUSIONS

A linearized version of the equation of motion approach to the nuclear many-body problem, considered as a generalization of traditional core-particle coupling models, has proved its worth in a number of recent applications to deformed odd nuclei. In this method, the basic object studied is a single-particle coefficient of fractional parentage (CFP) relating the states of the even nuclei to those of a neighboring odd nucleus.

In this paper we showed how the same general method can be applied to odd-odd nuclei. We started with a review of the formalism for odd nuclei, since it plays an essential role in some of the considerations that follow. We then showed that there are three possible formulations for the odd-odd case, two of which we label as sequential and a third as symmetrical, terms that characterize the way in which we couple an extra neutron (or neutron hole) and an extra proton (or proton hole) to nearby even nuclei, treated as cores. First we study in detail the case where we initially couple the odd neutron to the even cores, an example of our method for odd nuclei. We then couple the odd proton to the odd neutron nuclei, introducing new CFP for this relationship, and making essential use of the odd neutron calculations for energies and CFP. The second sequential method, not discussed in detail, reverses the order of the odd-particle couplings. In the symmetrical coupling, we first couple the two odd particles together and study directly the relationship of the odd-odd nucleus to the core even nuclei by means of two-particle CFP. In principle all three methods are equivalent, but in practice results will differ owing to the need to approximate. In this regard, the existence of alternatives that may be compared may be of some practical advantage.

Because of the presence of pairing interactions the equations for the odd-odd case yield four times as many solutions as are physical. In the sequential method, the problem of choosing physical solutions can be solved by sequential use of essentially the same method as for the odd case. For the symmetrical coupling case, a more elaborate method has been devised.

Concerning applications, approximate versions of the sequential method have already been carried out [22,23]. The symmetrical approach remains to be tried.

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APPENDIX A: MATRIX ELEMENTS OF EFFECTIVE INTERACTIONS

1. Odd nuclei

The single-particle and pairing potential matrices occurring in Eqs. (16) and (17) for the single-particle CFP are given by Eqs. (22) and (23), respectively. The corresponding reduced matrix elements appearing in Eqs. (34) and (35) are equal to

$$\Gamma_J^{(A+1)}(aIn|a'I'n') = \sum_{Lbb'} (-1)^{j_a+J+I}\sqrt{(2I'+1)(2L+1)} \times \begin{Bmatrix} I & I' & L \\ j_{a'} & j_a & J \end{Bmatrix} F_{aa'bb'}(L) \times [I'n'\|B_L(bb')\|In], \tag{A1}$$

$$\Delta_J(aIn|a'I'n') = \sum_{Lbb'} (-1)^{j_a+J+I+1}\sqrt{(2I'+1)(2L+1)} \times \begin{Bmatrix} I & I' & L \\ j_{a'} & j_a & J \end{Bmatrix} G_{aa'bb'}(L) \times [In\|A_L^\dagger(bb')\|I'n'], \tag{A2}$$

$$\Gamma_J^{\dagger(A-1)}(aIn|a'I'n') = \sum_{Lbb'} (-1)^{j_a+J+I}\sqrt{(2I'+1)(2L+1)} \times \begin{Bmatrix} I & I' & L \\ j_{a'} & j_a & J \end{Bmatrix} F_{aa'bb'}(L) \times [I'n'\|B_L^\dagger(bb')\|In], \tag{A3}$$

$$\Delta_J^\dagger(aIn|a'I'n') = \sum_{Lbb'} (-1)^{j_a+J+I+1}\sqrt{(2I'+1)(2L+1)} \times \begin{Bmatrix} I & I' & L \\ j_{a'} & j_a & J \end{Bmatrix} G_{aa'bb'}(L) \times [I'n'\|A_L^\dagger(bb')\|In]. \tag{A4}$$

2. Sequential coupling in odd-odd nuclei

The effective single-proton, proton-pairing, and proton-neutron potentials in Eqs. (81) and (82) for single-proton CFP, X and Y, relating the odd-neutron and odd-odd nuclei, are dependent on the single-neutron CFP, $V^{(\pm)}$ and $U^{(\pm)}$, and read

$$\Gamma^{(+)}(pJ_nM_nr_n|p'J_{n'}M_{n'}r_{n'}) = \sum_{nIM_Ir'M_{I'}r'} \frac{1}{\Omega_{(n)}}[V^{(+)}_{J_nM_nr_n}(nIM_Ir)\Gamma^{(++)}(pIM_Ir|p'I'M_{I'}r')V^{(+)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r') + U^{(+)}_{J_nM_nr_n}(nIM_Ir)\Gamma^{(+-)}(pIM_Ir|p'I'M_{I'}r')U^{(+)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r')], \tag{A5}$$

$$\Delta(pJ_nM_nr_n|p'J_{n'}M_{n'}r_{n'}) = \sum_{nIM_Ir'M_{I'}r'} \frac{1}{\Omega_{(n)}}[V^{(+)}_{J_nM_nr_n}(nIM_Ir)\Delta^{(+)}(pIM_Ir|p'I'M_{I'}r')V^{(-)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r') + U^{(+)}_{J_nM_nr_n}(nIM_Ir)\Delta^{(-)}(pIM_Ir|p'I'M_{I'}r')U^{(-)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r')], \tag{A6}$$

$$\mathcal{V}^{(+)}(\overline{pI}_nM_nr_n|\overline{p'}J_{n'}M_{n'}r_{n'}) = \sum_{nIM_Ir'M_{I'}r'} \frac{1}{\Omega_{(n)}}[-V^{(+)}_{J_nM_nr_n}(nIM_Ir)V^{(+)}_{J_{n'}M_{n'}r_{n'}}(n'IM_Ir)F_{\overline{pp'}\,\overline{n'}\,\overline{n}} + U^{(+)}_{J_nM_nr_n}(nIM_Ir)U^{(+)}_{J_{n'}M_{n'}r_{n'}}(n'IM_Ir)F_{\overline{pp'}nn'}], \tag{A7}$$

$$\Gamma^{\dagger(-)}(pJ_nM_nr_n|p'J_{n'}M_{n'}r_{n'}) = \sum_{nIM_Ir'M_{I'}r'} \frac{1}{\Omega_{(n)}}[V^{(-)}_{J_nM_nr_n}(nIM_Ir)\Gamma^{\dagger(-+)}(pIM_Ir|p'I'M_{I'}r')V^{(-)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r') + U^{(-)}_{J_nM_nr_n}(nIM_Ir)\Gamma^{\dagger(--)}(pIM_Ir|p'I'M_{I'}r')U^{(-)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r')], \tag{A8}$$

$$\Delta^\dagger(pJ_nM_nr_n|p'J_{n'}M_{n'}r_{n'}) = \sum_{nIM_Ir'M_{I'}r'} \frac{1}{\Omega_{(n)}}[V^{(-)}_{J_nM_nr_n}(nIM_Ir)\Delta^{\dagger(+)}(pIM_Ir|p'I'M_{I'}r')V^{(+)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r') + U^{(-)}_{J_nM_nr_n}(nIM_Ir)\Delta^{\dagger(-)}(pIM_Ir|p'I'M_{I'}r')U^{(+)}_{J_{n'}M_{n'}r_{n'}}(nI'M_{I'}r')], \tag{A9}$$

$$\mathcal{V}^{(-)}(pI_nM_nr_n|p'J_{n'}M_{n'}r_{n'}) = \sum_{nIM_IrI'M_{I'}r'} \frac{1}{\Omega_{(n)}}[V^{(-)}_{J_nM_nr_n}(nIM_Ir)V^{(-)}_{J_{n'}M_{n'}r_{n'}}(n'IM_Ir)F_{pp'\overline{nn'}} - U^{(-)}_{J_nM_nr_n}(nIM_Ir)U^{(-)}_{J_{n'}M_{n'}r_{n'}}(n'IM_Ir)F_{pp'nn'}]. \tag{A10}$$

The correctness of the above equations can be verified independently. By starting with Eqs. (9) and (10), we can derive equations of the form (16) and (17) with potentials Γ and Δ that refer appropriately to the odd systems, rather than the even system and with no overt sign of the neutron-proton interaction terms. These equations are readily transformed into the results given above by the application of Eq. (55), just as the latter was applied in Sec. II D to express transition matrix elements between odd states in terms of matrix elements between even states and single-particle CFP.

Passing to Eqs. (85) and (86) for the reduced CFP, χ and η, we encounter the following reduced matrix elements of the effective interactions given above:

$$\Gamma^{(+)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'}) = \frac{1}{\Omega_{(n)}}\sum_{Laa'j_nIrI'r'} \begin{Bmatrix} J_n & j_p & J \\ j_{p'} & J_{n'} & L \end{Bmatrix}\begin{Bmatrix} I & J_n & j_n \\ J_{n'} & I' & L \end{Bmatrix}(-1)^{j_p+j_n+J+I'+1-L}\sqrt{(2L+1)(2J_n+1)(2J_{n'}+1)(2I'+1)} \times F_{pp'aa'}(L)\{v^{(+)}_{J_nr_n}(j_nIr)v^{(+)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|B^{(++)}_L(aa')\|Ir] + u^{(+)}_{J_nr_n}(j_nIr)u^{(+)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|B^{(+-)}_L(aa')\|Ir]\}, \tag{A11}$$

$$\Delta_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'}) = \frac{1}{\Omega_{(n)}}\sum_{Laa'j_nIrI'r'} \begin{Bmatrix} J_n & j_p & J \\ j_{p'} & J_{n'} & L \end{Bmatrix}\begin{Bmatrix} I & J_n & j_n \\ J_{n'} & I' & L \end{Bmatrix}(-1)^{j_p+j_n+J+I'-L}\sqrt{(2L+1)(2J_n+1)(2J_{n'}+1)(2I'+1)} \times G_{pp'aa'}(L)\{v^{(+)}_{J_nr_n}(j_nIr)v^{(-)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|A^{(+)}_L(aa')\|Ir] + u^{(+)}_{J_nr_n}(j_nIr)u^{(-)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|A^{(-)}_L(aa')\|Ir]\}, \tag{A12}$$

$$\mathcal{V}^{(+)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'}) = \frac{1}{\Omega_{(n)}}\sum_{Lj_nj_{n'}Ir} \begin{Bmatrix} J_n & j_p & J \\ j_{p'} & J_{n'} & L \end{Bmatrix}\begin{Bmatrix} j_{n'} & j_n & L \\ J_n & J_{n'} & I \end{Bmatrix}(-1)^{j_p+j_n+J_n+J+L}(2L+1)\sqrt{(2J_n+1)(2J_{n'}+1)} \times [v^{(+)}_{J_nr_n}(j_nIr)v^{(+)}_{J_{n'}r_{n'}}(j_{n'}Ir)F_{pp'n'n}(L) + (-1)^{j_n+j_{n'}-L}u^{(+)}_{J_nr_n}(j_nIr)u^{(+)}_{J_{n'}r_{n'}}(j_{n'}Ir)F_{pp'nn'}(L)], \tag{A13}$$

$$\Gamma^{\dagger(-)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'}) = \frac{1}{\Omega_{(n)}}\sum_{Laa'j_nIrI'r'} \begin{Bmatrix} J_n & j_p & J \\ j_{p'} & J_{n'} & L \end{Bmatrix}\begin{Bmatrix} I & J_n & j_n \\ J_{n'} & I' & L \end{Bmatrix}(-1)^{j_p+j_n+J+I'+1-L}\sqrt{(2L+1)(2J_n+1)(2J_{n'}+1)(2I'+1)} \times F_{pp'aa'}(L)\{v^{(-)}_{J_nr_n}(j_nIr)v^{(-)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|B^{(++)}_L(aa')\|Ir] + u^{(+)}_{J_nr_n}(j_nIr)u^{(+)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|B^{(+-)}_L(aa')\|Ir]\}, \tag{A14}$$

$$\Delta^{\dagger}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'}) = \frac{1}{\Omega_{(n)}}\sum_{Laa'j_nIrI'r'} \begin{Bmatrix} J_n & j_p & J \\ j_{p'} & J_{n'} & L \end{Bmatrix}\begin{Bmatrix} I & J_n & j_n \\ J_{n'} & I' & L \end{Bmatrix}(-1)^{j_p+j_n+J+I'-L}\sqrt{(2L+1)(2J_n+1)(2J_{n'}+1)(2I'+1)} \times G_{pp'aa'}(L)\{v^{(-)}_{J_nr_n}(j_nIr)v^{(+)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|A^{\dagger(+)}_L(aa')\|Ir] + u^{(-)}_{J_nr_n}(j_nIr)u^{(+)}_{J_{n'}r_{n'}}(j_nI'r')[I'r'\|A^{\dagger(-)}_L(aa')\|Ir]\}, \tag{A15}$$

$$\mathcal{V}^{(-)}_J(j_pJ_nr_n|j_{p'}J_{n'}r_{n'}) = \frac{1}{\Omega_{(n)}}\sum_{Lj_nj_{n'}Ir} \begin{Bmatrix} J_n & j_p & J \\ j_{p'} & J_{n'} & L \end{Bmatrix}\begin{Bmatrix} j_{n'} & j_n & L \\ J_n & J_{n'} & I \end{Bmatrix}(-1)^{j_p+j_{n'}+J_n+J+1}(2L+1)\sqrt{(2J_n+1)(2J_{n'}+1)} \times [v^{(-)}_{J_nr_n}(j_nIr)v^{(-)}_{J_{n'}r_{n'}}(j_{n'}Ir)F_{pp'nn'}(L) + (-1)^{j_n+j_{n'}-L}u^{(-)}_{J_nr_n}(j_nIr)u^{(-)}_{J_{n'}r_{n'}}(j_{n'}Ir)F_{pp'n'n}(L)]. \tag{A16}$$

APPENDIX B: THE EFFECTIVE HAMILTONIAN FOR TWO-PARTICLE CFP

The matrix of effective Hamiltonian occurring in Eq. (105) for the reduced matrix elements of two-particle CFP are equal to

$$\mathcal{H}(++pnlRr|++p'n'l'R'r') = (\epsilon'_p + \epsilon'_n - \omega_{++Rr})\delta_{p,p'}\delta_{n,n'}\delta_{l,l'}\delta_{R,R'}\delta_{r,r'} + \Gamma_J^{(++)}(plRr|p'l'R'r')\delta_{n,n'} + \Gamma_J^{(++)}(nlRr|n'l'R'r')\delta_{p,p'} + \mathcal{V}^{(++)}(pn|p'n')\delta_{l,l'}\delta_{R,R'}\delta_{r,r'}, \tag{B1}$$

$$\mathcal{H}(+-pnlRr||+-p'n'l'R'r') = (\epsilon'_p - \epsilon''_n - \omega_{+-Rr})\delta_{p,p'}\delta_{n,n'}\delta_{l,l'}\delta_{R,R'}\delta_{r,r'} + \Gamma_J^{(+-)}(plRr|p'l'R'r')\delta_{n,n'} - \Gamma_J^{\dagger(+-)}(nlRr|n'l'R'r')\delta_{p,p'} + \mathcal{V}^{(+-)}(pn|p'n')\delta_{l,l'}\delta_{R,R'}\delta_{r,r'}, \tag{B2}$$

$$\mathcal{H}(-+pnlRr|-+p'n'l'R'r') = (-\epsilon''_p + \epsilon'_n - \omega_{-+Rr})\delta_{p,p'}\delta_{n,n'}\delta_{l,l'}\delta_{R,R'}\delta_{r,r'} - \Gamma_J^{\dagger(-+)}(plRr|p'l'R'r')\delta_{n,n'} + \Gamma_J^{(-+)}(nlRr|n'l'R'r')\delta_{p,p'} + \mathcal{V}^{(-+)}(pn|p'n')\delta_{l,l'}\delta_{R,R'}\delta_{r,r'}, \tag{B3}$$

$$\mathcal{H}(--pnlRr|--p'n'l'R'r') = (-\epsilon'''_p - \epsilon'''_n - \omega_{--Rr})\delta_{p,p'}\delta_{n,n'}\delta_{l,l'}\delta_{R,R'}\delta_{r,r'} - \Gamma_J^{\dagger(--)}(plRr|p'l'R'r')\delta_{n,n'} - \Gamma_J^{\dagger(--)}(nlRr|n'l'R'r')\delta_{p,p'} + \mathcal{V}^{(--)}(pn|p'n')\delta_{l,l'}\delta_{R,R'}\delta_{r,r'}, \tag{B4}$$

$$\mathcal{H}(++pnlRr|-+p'n'l'R'r') = \Delta_J^{(+)}(plRr|p'l'R'r')\delta_{n,n'}, \tag{B5}$$

$$\mathcal{H}(++pnlRr|+-p'n'l'R'r') = \Delta_J^{(+)}(nlRr|n'l'R'r')\delta_{p,p'}, \tag{B6}$$

$$\mathcal{H}(+-pnlRr|--p'n'l'R'r') = \Delta_J^{(-)}(plRr|p'l'R'r')\delta_{n,n'}, \tag{B7}$$

$$\mathcal{H}(+-pnlRr|++p'n'l'R'r') = \Delta_J^{\dagger(+)}(nlRr|n'l'R'r')\delta_{p,p'}, \tag{B8}$$

$$\mathcal{H}(-+pnlRr|++p'n'l'R'r') = \Delta_J^{\dagger(-)}(plRr|p'l'R'r')\delta_{n,n'}, \tag{B9}$$

$$\mathcal{H}(-+pnlRr|--p'n'l'R'r') = \Delta_J^{(-)}(nlRr|n'l'R'r')\delta_{p,p'}, \tag{B10}$$

$$\mathcal{H}(--pnlRr|+-p'n'l'R'r') = \Delta_J^{\dagger(-)}(plRr|p'l'R'r')\delta_{n,n'}, \tag{B11}$$

$$\mathcal{H}(--pnlRr|-+p'n'l'R'r') = \Delta_J^{\dagger(-)}(nlRr|n'l'R'r')\delta_{p,p'}. \tag{B12}$$

The remaining matrix elements $\mathcal{H}(\sigma\tau|-\sigma-\tau)$ vanish.

The effective interactions that occur in the above equations are

$$\Gamma_J^{(\sigma\tau)}(plRr|p'l'R'r') = \sum_{Lbb'}(-1)^{j_p+j_n+l+l'+L+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L\end{Bmatrix}\begin{Bmatrix} j_n & j_p & l \\ L & l' & j_{p'}\end{Bmatrix} \times \overline{F}_{pp'bb'}(L)[R'r'\|B_L^{(\sigma\tau)}(bb')\|Rr], \tag{B13}$$

$$\Gamma_J^{(\sigma\tau)}(nlRr|n'l'R'r') = \sum_{Lbb'}(-1)^{j_p+j_{n'}+L+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L\end{Bmatrix}\begin{Bmatrix} j_p & j_n & l \\ L & l' & j_{n'}\end{Bmatrix} \times \overline{F}_{nn'bb'}(L)[R'r'\|B_L^{(\sigma\tau)}(bb')\|Rr], \tag{B14}$$

$$\Gamma_J^{\dagger(\sigma\tau)}(plRr|p'l'R'r') = \sum_{Lbb'}(-1)^{j_p+j_n+L+l+l'+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L\end{Bmatrix}\begin{Bmatrix} j_n & j_p & l \\ L & l' & j_{p'}\end{Bmatrix} \times \overline{F}_{pp'bb'}(L)[R'r'\|B^{\dagger(\sigma\tau)}L^{\sigma}(bb')\|Rr], \tag{B15}$$

$$\Gamma_J^{\dagger(\sigma\tau)}(nlRr|n'l'R'r') = \sum_{Lbb'} (-1)^{j_p+j_{n'}+L+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L \end{Bmatrix}\begin{Bmatrix} j_p & j_n & l \\ L & l' & j_{n'} \end{Bmatrix} \times \overline{F}_{nn'bb'}(L)[R'r'\|B_L^{\dagger(\sigma\tau)}(bb')\|Rr], \tag{B16}$$

$$\Delta_J^{(\tau)}(plRr|p'l'R'r') = -\sum_{Lp''p'''} (-1)^{j_p+j_n+l+l'+L+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L \end{Bmatrix}\begin{Bmatrix} j_n & j_p & l \\ L & l' & j_{p'} \end{Bmatrix} \times G_{pp'p''p'''}(L)[R'r'\|A_L^{(\tau)}(p''p''')\|Rr], \tag{B17}$$

$$\Delta_J^{(\sigma)}(nlRr|n'l'R'r') = -\sum_{Ln''n'''} (-1)^{j_p+j_{n'}+L+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L \end{Bmatrix}\begin{Bmatrix} j_p & j_n & l \\ L & l' & j_{n'} \end{Bmatrix} \times G_{nn'n''n'''}(L)[R'r'\|A_L^{(\sigma)}(n''n''')\|Rr], \tag{B18}$$

$$\Delta_J^{\dagger(\tau)}(plRr|p'l'R'r') = -\sum_{Lp''p'''} (-1)^{j_p+j_n+L+l+l'+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L \end{Bmatrix}\begin{Bmatrix} j_n & j_p & l \\ L & l' & j_{p'} \end{Bmatrix} \times G_{pp'p''p'''}(L)[R'r'\|A_L^{\dagger(\tau)}(p''p''')\|Rr], \tag{B19}$$

$$\Delta_J^{\dagger(\sigma)}(nlRr|n'l'R'r') = -\sum_{Ln''n'''} (-1)^{j_p+j_{n'}+L+R+J}\sqrt{(2R'+1)(2L+1)(2l+1)(2l'+1)}\begin{Bmatrix} l & R & J \\ R' & l' & L \end{Bmatrix}\begin{Bmatrix} j_p & j_n & l \\ L & l' & j_{n'} \end{Bmatrix} \times G_{nn'n''n'''}(L)[R'r'\|A_L^{\dagger(\sigma)}(n''n''')\|Rr], \tag{B20}$$

$$\mathcal{V}^{(++)}(pn|p'n') = -\sum_L (-1)^{j_p+j_{n'}+l}(2L+1)\begin{Bmatrix} j_p & j_n & l \\ j_{n'} & j_{p'} & L \end{Bmatrix}\overline{F}_{pp'n'n}(L), \tag{B21}$$

$$\mathcal{V}^{(+-)}(pn|p'n') = \sum_L (-1)^{j_p+j_n+l+L}(2L+1)\begin{Bmatrix} j_p & j_n & l \\ j_{n'} & j_{p'} & L \end{Bmatrix}\overline{F}_{pp'nn'}(L), \tag{B22}$$

$$\mathcal{V}^{(-+)}(pn|p'n') = \sum_L (-1)^{j_{p'}+j_{n'}+l+L}(2L+1)\begin{Bmatrix} j_p & j_n & l \\ j_{n'} & j_{p'} & L \end{Bmatrix}\overline{F}_{pp'nn'}(L), \tag{B23}$$

$$\mathcal{V}^{(--)}(pn|p'n') = -\sum_L (-1)^{j_{p'}+j_n+l}(2L+1)\begin{Bmatrix} j_p & j_n & l \\ j_{n'} & j_{p'} & L \end{Bmatrix}\overline{F}_{pp'n'n}(L). \tag{B24}$$

APPENDIX C: REDUCED MATRIX ELEMENTS OF TRANSITION OPERATORS

1. Odd nuclei

The matrix elements of one-body transition operators within states of an odd nucleus are given by Eq. (56). We use the following definitions of the reduced matrix elements:

$$\langle J'\mu'\nu'|T_{LM_L}|J\mu\nu\rangle = \frac{(-1)^{J-\mu}}{\sqrt{2L+1}}(J'\mu' J-\mu|LM_L)\langle J'\nu'||T_L||J\nu\rangle, \tag{C1}$$

$$\langle I'M'n'|T_{LM_L}|IMn\rangle = \frac{(-1)^{I-M}}{\sqrt{2L+1}}(I'M'I-M|LM_L) \times\langle I'n'||T_L||In\rangle, \tag{C2}$$

$$t_{\alpha\gamma} = \frac{(-1)^{j_c-m_c}}{\sqrt{2L+1}}(j_a m_a j_c - m_c|LM_L)t_{ac}. \tag{C3}$$

Applying the Wigner-Eckart theorem with the help of Eqs. (26) and (27) to Eq. (56) we find the following expression for the reduced matrix element of T_L:

$$\langle J'\nu'||T_L||J\nu\rangle = \frac{1}{\Omega}\sum_{aInI'n'}(-1)^{j_a+J'+I+L}\begin{Bmatrix} I & I' & L \\ J' & J & j_a \end{Bmatrix}\sqrt{(2J+1)(2J'+1)}[u_{J\nu}(aIn)u_{J'\nu'}(aI'n')\langle \underline{I'n'}||T_L||\underline{In}\rangle$$
$$+ v_{J\nu}(aIn)v_{J'\nu'}(aI'n')\langle \overline{I'n'}||T_L||\overline{In}\rangle] + \frac{1}{\Omega}\sum_{aa'In} t_{aa'}\sqrt{(2J+1)(2J'+1)}\Bigg[(-1)^{j_{a'}+I+J'+L}$$
$$\times\begin{Bmatrix} j_a & j_{a'} & L \\ J & J' & I \end{Bmatrix}u_{J'\nu'}(aIn)u_{J\nu}(a'In) + (-1)^{j_{a'}+I+J'+1}\begin{Bmatrix} j_a & j_{a'} & L \\ J' & J & I \end{Bmatrix}v_{J\nu}(aIn)v_{J'\nu'}(a'In)\Bigg]. \quad \text{(C4)}$$

2. Odd-odd nuclei

By means of methods discussed in Sec. IV C we obtain the following formula for the reduced matrix element of transition operator T_L within states of an odd-odd nucleus as a function of the reduced matrix elements of only the single-particle CFP, v and u, and χ and η:

$$\langle J's'||T_L||Js\rangle = \frac{1}{\Omega_{(n)}}\sum_{j_pJ_nr_nJ_{n'}r_{n'}}(-1)^{j_p+J'+J_n+L}\begin{Bmatrix} J_n & J_{n'} & L \\ J' & J & j_p \end{Bmatrix}\sqrt{(2J+1)(2J'+1)}[\eta_{Js}(j_pJ_nr_n)\eta_{J's'}(j_pJ_{n'}r_{n'})\langle \underline{J_{n'}r_{n'}}||T_L||\underline{J_nr_n}\rangle$$
$$+ \chi_{Js}(j_pJ_nr_n)\chi_{J's'}(j_pJ_{n'}r_{n'})\langle \overline{J_{n'}r_{n'}}||T_L||\overline{J_nr_n}\rangle] + \frac{1}{\Omega_n}\sum_{j_pj_{p'}J_nr_n} t_{pp'}\sqrt{(2J+1)(2J'+1)}\Bigg[(-1)^{j_{p'}+J_n+J'+L}\begin{Bmatrix} j_p & j_{p'} & L \\ J & J' & J_n \end{Bmatrix}$$
$$\times\ \eta_{J's'}(j_pJ_nr_n)\eta_{Js}(j_{p'}J_nr_n) + (-1)^{j_{p'}+J_n+J'+1}\begin{Bmatrix} j_n & j_{n'} & L \\ J' & J & J_n \end{Bmatrix}\chi_{Js}(j_pJ_nr_n)\chi_{J's'}(j_{p'}J_nr_n)\Bigg], \quad \text{(C5)}$$

where the reduced matrix elements of T_L within states of odd nuclei occurring on the right-hand side of Eq. (C5) are given by Eq. (C4).

The same reduced matrix elements as these of Eq. (C5) but expressed by the reduced matrix elements of the two-particle CFP, b, take the following form coming directly from Eqs. (71) and (135), and the Wigner-Eckart theorem:

$$\langle J's'||T_L||Js\rangle = \frac{1}{\Omega_{(p)}\Omega_{(n)}}\sum_{\sigma\tau lLj_pj_nRrR'r'}(-1)^{J+R'+l+L}\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} J & J' & L \\ R' & R & l \end{Bmatrix}\langle\sigma\tau R'r'||T_L||\sigma\tau Rr\rangle b_{Js}^{(\sigma\tau)}(pnlRr)b_{J's'}^{(\sigma\tau)}(pnlR'r')$$
$$+ \frac{1}{\Omega_{(p)}\Omega_{(n)}}\sum_{\tau ll'Lj_pj_{p'}j_nRr}(-1)^{R+J'+j_{p'}+j_n}\frac{2j_{p'}+1}{2L+1}\sqrt{(2l+1)(2l'+1)(2J+1)(2J'+1)}\begin{Bmatrix} L & J & J' \\ R & l' & l \end{Bmatrix}$$
$$\times\begin{Bmatrix} j_p & j_n & l \\ l' & L & j_{p'} \end{Bmatrix}t_{p'p}b_{Js}^{(-\tau)}(pnlRr)b_{J's'}^{(-\tau)}(pnlR'r') + \frac{1}{\Omega_{(p)}\Omega_{(n)}}\sum_{\tau ll'Lj_pj_{n'}j_nRr}(-1)^{R+J'+j_p+j_n+l+l'}\frac{2j_{n'}+1}{2L+1}$$
$$\times\sqrt{(2l+1)(2l'+1)(2J+1)(2J'+1)}\begin{Bmatrix} L & J & J' \\ R & l' & l \end{Bmatrix}\begin{Bmatrix} j_n & j_p & l \\ l' & L & j_{n'} \end{Bmatrix}t_{n'n}b_{Js}^{(\sigma-)}(pnlRr)b_{J's'}^{(\sigma-)}(pn'lR'r')$$
$$+ \frac{1}{\Omega_{(p)}\Omega_{(n)}}\sum_{\tau ll'Lj_pj_{p'}j_nRr}(-1)^{R+J'+j_{p'}+j_n}\frac{2j_{p'}+1}{2L+1}\sqrt{(2l+1)(2l'+1)(2J+1)(2J'+1)}\begin{Bmatrix} L & J & J' \\ R & l' & l \end{Bmatrix}\begin{Bmatrix} j_p & j_n & l \\ l' & L & j_{p'} \end{Bmatrix}$$
$$\times(-1)^{j_p+j_{p'}-L}t_{pp'}b_{Js}^{(+\tau)}(pnlRr)b_{J's'}^{(+\tau)}(pnlR'r') + \frac{1}{\Omega_{(p)}\Omega_{(n)}}\sum_{\tau ll'Lj_pj_{n'}j_nRr}(-1)^{R+J'+j_p+j_n+l+l'}\frac{2j_{n'}+1}{2L+1}$$
$$\times\sqrt{(2l+1)(2l'+1)(2J+1)(2J'+1)}\begin{Bmatrix} L & J & J' \\ R & l' & l \end{Bmatrix}\begin{Bmatrix} j_n & j_p & l \\ l' & L & j_{n'} \end{Bmatrix}(-1)^{j_n+j_{n'}-L}t_{nn'}b_{Js}^{(\sigma-)}(pnlRr)b_{J's'}^{(\sigma-)}(pn'lR'r'). \quad \text{(C6)}$$

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