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Evidence for two-photon exchange contributions in electron-proton and positron-proton elastic scattering

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The comparison of positron-proton and electron-proton elastic scattering cross sections is a sensitive test for the presence of two-photon exchange contributions. Thirty years ago, positron data were considered adequate to set tight limits on the size of two-photon corrections. More recently, these radiative corrections have again become a matter of great interest as a possible explanation for the discrepancy between Rosenbluth and polarization transfer measurements of the proton electromagnetic form factors. We have reexamined the electron and positron scattering data to see if they can accommodate two-photon effects of the size necessary to account for the Rosenbluth-polarization transfer discrepancy. The data are consistent with simple estimates of the two-photon contributions necessary to explain the discrepancy. In fact, they strongly favor a large ε-dependent correction to the positron to electron ratio, providing the first direct experimental evidence for a two-photon contribution to unpolarized lepton-proton scattering.

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Measurements of G_E/G_M, the ratio of the proton electric and magnetic form factors, from Rosenbluth separation and polarization transfer techniques yield significantly different results [1] at large values of Q^2, the four-momentum transfer squared. The systematic uncertainties of both the Rosenbluth [1] and polarization [2] measurements have been studied in detail, and no explanation for the discrepancy in terms of experimental problems has been found. If the discrepancy is not due to errors in the experiments or analyses, it may indicate a more fundamental problem with one of the techniques. Until this discrepancy is understood, there will be large uncertainties in our knowledge of the proton form factors. Since the polarization transfer measurements have only extracted the ratio G_E/G_M, cross section measurements are still needed to determine the magnitude of the individual form factors. So even if it is shown that the polarization transfer measurements are correct, and that the problem is due to unaccounted for corrections in the cross section measurements, there will still be uncertainties in the form factors until we fully understand these corrections [3].

Because the discrepancy grows rapidly with Q^2, it has typically been assumed that it is a problem with the cross section measurements, where a fixed error in the ε dependence of the cross sections would yield an error in $(G_E/G_M)^2$ that grows approximately linearly with Q^2. Assuming that the difference is due primarily to missing corrections in the cross section measurements, the discrepancy requires an error in the ε dependence of the cross section of $\approx 5-8\,\%$ for $1 < Q^2 < 6$ GeV^2 [1,3,4].

In order for a modification to the cross section to change the extracted form factor ratio, it must modify the ε dependence of the reduced cross section,

$$\sigma_R \equiv \frac{d\sigma}{d\Omega}\frac{\varepsilon(1+\tau)}{\sigma_{\mathrm{Mott}}} = \tau G_M^2(Q^2) + \varepsilon G_E^2(Q^2), \qquad (1)$$

where $\tau = Q^2/4M_p^2$ and ε is the longitudinal polarization of the virtual photon $[\varepsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta/2)]$. Several attempts have been made to find effects that might introduce an additional ε dependence to the measured cross section, thus modifying the extracted Rosenbluth form factors. Coulomb corrections, when implemented in a simple effective momentum approximation [5], do modify the ε dependence of the cross section, but yield a very small effect compared to the size needed to explain the discrepancy. For the most part, investigations have focussed on the effect of two-photon exchange corrections [4,6,7] beyond the limited contributions that are already included in the traditional calculations of radiative corrections [8–10].

While these works have shown that it is possible that a two-photon correction could explain the discrepancy, the only quantitative calculation [6] is limited to the elastic part of the two-photon contributions, i.e., the box and crossed-box diagrams considering only the case where the intermediate state is a proton, and yield only a 2% ε dependence, less than half the size necessary to explain the discrepancy. In this work, we reexamine positron measurements that were designed to test for two-photon contributions in elastic scattering in light of the possibility that they may be responsible for the discrepancy.

The effect of two-photon exchange terms can be observed in several processes. The imaginary part of the two-photon amplitude can, in principle, be measured in polarization observables. Measurements of the normal polarization P_N, which is zero in the Born approximation, have been made [11–13], but no statistically significant indication of two-photon contributions has been seen. Similarly, the asymmetry A_N has been measured for both elastic and inelastic scatterings [14,15], again with only null results. Thus far, the only observations of possible two-photon effects are in the asymmetry of scattering of transversely polarized electrons from protons. These asymmetries are extremely small, of order 10^{-5}, and so extremely difficult to measure. However, they have been observed by the SAMPLE experiment at MIT-Bates [16] and the PVA4 Collaboration at Mainz [17].

However, these polarization observables are related to the imaginary part of the two-photon amplitude, while the cross section measurements are related to the real part. Therefore, while these data can be used to test models of the two-photon exchange, they do not directly constrain the two-photon contributions to the unpolarized cross sections, which might explain the discrepancy.

There are two ways to look for the effects of two-photon exchange corrections in the unpolarized elastic electron-proton cross section. First, one can look for deviations from the linear ε dependence in Eq. (1). There are a few Rosenbluth separation measurements [18–20] that cover both large and small ε values and have small uncertainties (1–2%). These measurements do not show any significant deviations from linearity, but they have limited sensitivity because they have little data below $\varepsilon=0.4$ and no data below $\varepsilon=0.2$. Data from different experiments can be combined to expand the ε range, but normalization uncertainties between different experiments reduce the significance of such tests, while attempts to normalize across the data sets [1,21] rely on the linearity when determining normalization factors. So while these data set limits on nonlinearities at large ε, the limits are less significant at low ε. In addition, these measurements are insensitive to two-photon correction terms that are constant or vary linearly with ε. Such corrections would modify the extracted values of G_E and G_M without spoiling the linearity of the Rosenbluth plot.

The second approach is to compare positron-proton scattering to electron-proton scattering. For positron-proton scattering, the interference term between the one-photon and two-photon amplitudes changes sign, yielding a ratio $R \equiv \sigma(e^+p)/\sigma(e^-p) \approx 1+4\mathrm{Re}(B)/A$, where B is the two-photon amplitude and A is the one-photon amplitude [22]. The modification to the electron cross section is $\approx 1-2\ \mathrm{Re}(B)/A$, and so any change in the electron cross section will yield roughly twice the change in R, but with the opposite sign. In the simplest approximation, one expects an additional factor of α in the two-photon amplitude relative to one-photon amplitude, yielding corrections to the electron cross section of roughly $2\alpha \approx 1.5\%$, and to the ratio R of $\approx 3\%$. Additional differences come from Bremsstrahlung corrections where proton recoil is taken into account, but these are included in the usual radiative corrections. An analysis by Mar and collaborators [22] found these additional differences to be relatively small, typically less than 1–2% for their kinematics, and to be identical to better than 0.3% in different prescriptions [10,23] of the radiative corrections.

Figure 1 shows the existing data [22,24–30] for the ratio of positron-proton to electron-proton elastic cross sections as a function of Q^2. While there is some hint of a Q^2 dependence, the large Q^2 data have large uncertainties, and a fit to the data of the form $R=a+bQ^2$ yields $b=0.0085\pm0.0063$, less than 1.5 standard deviations from zero. A fit of the ratios to a constant value yields $\langle R\rangle=1.003\pm0.005$, with $\chi^2_\nu=0.87$, which corresponds to a two-photon correction to the electron cross sections of $(-0.15\pm0.25)\%$. This result has been interpreted to mean that the two-photon corrections must be even smaller than the naive estimate, limiting the effect on the electron-proton cross section to less than 1%.

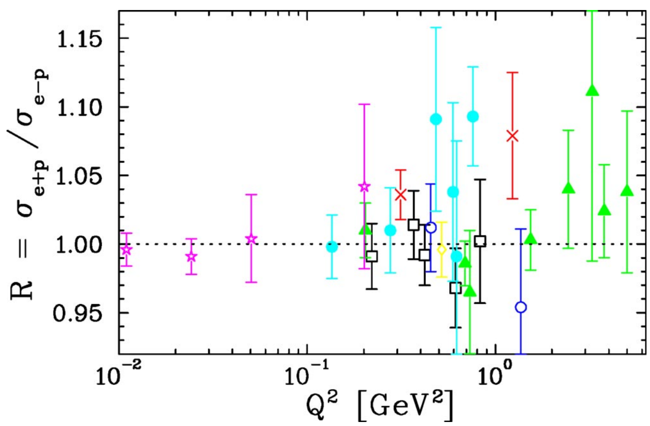

FIG. 1. (Color online) The cross section ratio, $R=\sigma_{e+}/\sigma_{e-}$, as a function of Q^2. The experiments are differentiated by color and symbol: black squares [24], red crosses [25], green solid triangles [22], blue hollow circles [26], yellow diamonds [27], cyan filled circles [28], and magenta stars [29].

However, the low intensity of the secondary positron beams used in these experiments makes it difficult to perform precise measurements where the cross section is small. Because of this, the data in Fig. 1 are limited to low Q^2 values ($\lesssim 1\ \mathrm{GeV}^2$) or small scattering angles ($\varepsilon > 0.7$), where the cross section is large. While the existing data do place tight limits on the size of two-photon corrections in some regions, they do not place any limits on two-photon contributions at low ε except at relatively low Q^2 values ($\lesssim 1\ \mathrm{GeV}^2$). So it is still possible that the discrepancy in the extracted form factors is due to two-photon corrections to the cross sections, if the correction is only large for small ε values.

If we assume that the two-photon corrections are responsible for the discrepancy between polarization transfer and Rosenbluth measurements, we can make specific predictions about how these corrections would impact the positron measurements, and use this to examine the existing data more carefully. In order to explain the discrepancy, the effect must increase the slope of the Rosenbluth plot, and so must increase the cross section at large ε relative to the low ε. Based on the size and Q^2 dependence of the discrepancy, the ε dependence in the electron cross section must be 5–8%, depending only weakly on Q^2, for $Q^2 \gtrsim 2\ \mathrm{GeV}^2$. It must also be reasonably close to linear in ε, or else it would introduce visible nonlinearities in the Rosenbluth plot. This implies that the ratio R should have a 10–15% ε dependence, approximately linear in ε and of the opposite sign as in the electron cross section, i.e., the positron to electron ratio must either increase at small ε or decrease at large ε.

Unfortunately, there is very little positron data above $Q^2=2$, and it covers a very limited ε range. The data by Mar *et al.* [22] has four points above $Q^2=2\ \mathrm{GeV}^2$, yielding $\langle R\rangle=1.034\pm0.024$. These data are all at large ε values ($\langle\varepsilon\rangle=0.88$), and so do not exclude significant two-photon corrections at large Q^2. If the two-photon correction is small at $\varepsilon=0$, then a 10–15 % decrease in R at large ε would be nec-

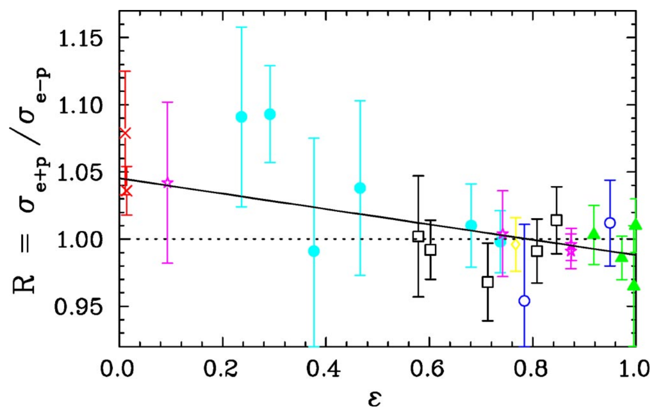

FIG. 2. (Color online) σ_{e+}/σ_{e-} cross section ratio as a function of ε for the measurements below $Q^2=2$ GeV^2. The solid line is a fit assuming a linear ε dependence and no Q^2 dependence to the ratio, and yields a slope of $-(5.7\pm1.8)$%. The symbols are identical to Fig. 1.

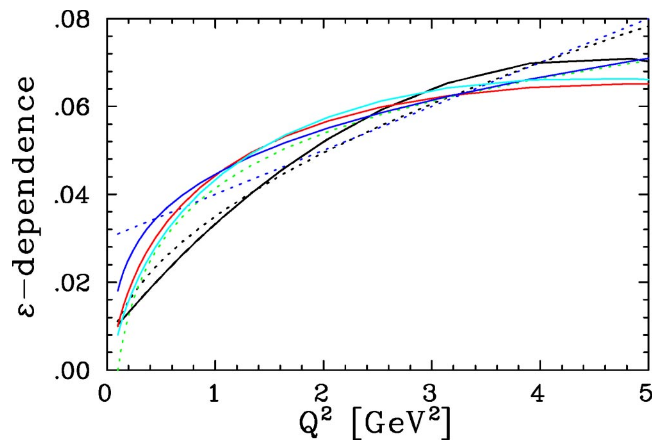

FIG. 3. (Color online) The ε dependence of the electron-proton cross section as a function of Q^2 estimated from the discrepancy between cross section and polarization transfer measurements. The four curves correspond to four different parametrizations for the Q^2 dependence. The ε dependence of the positron-to-electron ratio should be of opposite sign and approximately twice the size of the ε dependence in the electron cross section.

essary to explain the discrepancy, and this is clearly ruled out by the high ε data. So any two-photon corrections would have to increase R (decrease the electron cross section) at low ε in order to explain the discrepancy and still be consistent with the positron data.

The positron data with significant ε range is limited to $Q^2<2$ GeV^2. Figure 2 shows these data as a function of ε, and a significant ε dependence can be seen. A linear fit, neglecting any Q^2 dependence, yields slope of $-(5.7\pm1.8)$%, with $\chi^2=11.1$ for 22 degrees of freedom. The extremely low χ^2 indicates that the uncertainties in the data have most likely been overestimated, and that the effect may be more significant than indicated by the fit uncertainty.

The observed increase in the positron to electron ratio at low ε corresponds to an increase of 2.8% in the observed slope in the Rosenbluth extraction. This implies that the value of G_E extracted from the Rosenbluth separation will be larger than the true value, while the extracted G_M value will be smaller. While the 2.8% ε dependence is only half the size necessary to explain the discrepancy at large Q^2 value, data covering a wide range of ε values are only available at low Q^2. The average Q^2 value of the data in Fig. 2 is only 0.5 GeV^2, and the lower Q^2 data are generally more precise, making the weighted average Q^2 value less than 0.4 GeV^2.

We can estimate the ε dependence necessary to explain the discrepancy in the form factors at large Q^2, but at these low Q^2 values the polarization transfer and Rosenbluth form factors are not precise enough to determine if there is an inconsistency, and so cannot be used to estimate the size of the two-photon corrections. A decrease in the size of the ε dependence at low Q^2 could easily yield the slope observed in Fig. 2, yet still be large enough to fully explain the discrepancy between polarization and Rosenbluth extractions at larger Q^2 values. At larger Q^2 values, where the size of the corrections can be estimated from the discrepancy, the effect decreases somewhat as Q^2 decreases, and is approximately 5% for $Q^2=1-2$ GeV^2. In addition, the correction must become smaller for very small Q^2 values (0.01–0.1 GeV^2), or the decrease in the low-ε cross sections would lead to significant reductions in the extracted values of G_M. The extractions of G_M are not precise enough to conclude that the corrections must go to zero, but they must be significantly smaller than the 5% corrections observed at larger Q^2 values. Thus, we expect this slope in the positron to electron comparison to be less than 10%.

A global analysis of the cross section and polarization transfer data was used to try and estimate the low Q^2 behavior. In Ref. [3], a global analysis of the cross section and polarization transfer data, assuming a fixed 6% ε-dependent correction to the cross section, was used to extract the "polarization form factors." A modified version of this global analysis was performed, but rather than extracting G_E and G_M with a fixed two-photon correction, we extract G_E, G_M, and the Q^2 dependence of the slope of the linear ε corrections. Several different functional forms were tried, and a range of curves, which all gave good fits, are shown in Fig. 3. While the fits were not constrained to go to zero, they all yield a much smaller value as $Q^2\rightarrow0$. When these curves are used to estimate the ε dependence for the correction to the electron cross section at $Q^2=0.4$ GeV^2, they yield slopes of (1.8–3.3)%, implying a slope in the positron to electron ratio of $-(3.7-6.8)$%, in agreement with the observed -5.7% slope.

The ε dependence extracted above assumed no Q^2 dependence to the size of the correction, and a simple linear ε dependence. While it is in agreement with the estimated ε dependence from the form factor discrepancy, the estimate relied on an extrapolation to lower Q^2 values. However, while the above analysis had to make some assumptions about the Q^2 dependence of the two-photon effects, we can also make some significant model-independent statements from this data.

For the low ε data ($\varepsilon<0.5$)$\langle Q^2\rangle=0.5$ GeV^2 (weighted average) and $\langle R\rangle = 1.049 \pm 0.014$, clearly demonstrating that two-photon effects decrease the electron cross section at low ε and low Q^2. Note that only one point below $\varepsilon = 0.5$ is above $Q^2 = 1$ GeV^2, and it has a positron to electron ratio of 1.079 ± 0.046. This is consistent with the 10–12% increase that would explain the discrepancy in form factor measurements for $Q^2 = 1-2$ GeV^2, but also only two standard deviations from $R = 1$.

In addition to the observation of a significant ε dependence at low Q^2 values, the data also set significant limits on possible two-photon exchange corrections at large Q^2 for $\varepsilon \gtrsim 0.8$. For $Q^2 > 1$ GeV^2, $\langle R\rangle = 1.020 \pm 0.015$. So the 95% confidence region for R at large ε is 0.99–1.05, yields limits on the electron cross section modification of −2.5% to +0.5%. To increase the slope of the Rosenbluth plot, they would have to increase the high-ε electron cross section, and such an enhancement is limited to <0.5%.

The ε dependence of the two-photon effects seen here is consistent with the calculations of Refs. [6,31], which have small corrections at large ε and a significant decrease of the electron cross section at low ε. However, it rules out the form of Ref. [7] as an explanation for the discrepancy between polarization and Rosenbluth extractions of G_E/G_M. In Ref. [7], the authors predict a specific ε dependence, which is zero at $\varepsilon = 0$, and grows rapidly at large ε. If the size of the correction is made small enough to be consistent with the constraints from the positron measurements at large ε, then the two-photon effects at smaller ε values will be negligible.

The positron measurements are also inconsistent with the corrections obtained in Ref. [4], at least given the specific approximations that the authors use to obtain the two-photon effects from the discrepancy in form factor measurements. They do not assume single photon exchange, but instead write a more general expression for the cross section in terms of two generalized form factors $\tilde{G}_E$ and $\tilde{G}_M$ along with a third term $\tilde{F}_3$ which is zero in the Born approximation. By assuming that the two-photon contributions are negligible in $\tilde{G}_E$ and $\tilde{G}_M$, they extract values of $Y_{2\gamma}$, a dimensionless parameter related to the size of $\tilde{F}_3$, such that the effect of $Y_{2\gamma}$ on the cross section and polarization transfer data resolves the discrepancy. Under this assumption, the two-photon effects on the cross section measurements are approximately proportional to ε, and are $\gtrsim 5\%$ at $\varepsilon = 1$ for $Q^2 > 1$ GeV^2. This would yield $R \approx 0.9$ for the large Q^2 positron measurements, which is clearly ruled out by the data (Fig. 1). It is possible that the two-photon effects in their $Y_{2\gamma}$ terms could be canceled by two-photon effects that modify $\tilde{G}_E$ and $\tilde{G}_M$. If this is the case, the formalism may still allow a connection between the two-photon effects in polarization transfer and cross section measurements, but it is no longer possible to determine the two-photon terms directly from the extraction of $Y_{2\gamma}$, because of the sizable two-photon contributions to $\tilde{G}_E$ and $\tilde{G}_M$.

This observation of the form of the two-photon effects can also be used to assist in the extraction of form factors from Rosenbluth and polarization transfer data. To have a consistent extraction of the form factors from the cross section and polarization data, we have to assume something about the nature of the discrepancy. An ε dependence of the cross section, of the form observed in the positron data, is consistent with the assumption used in Ref. [3]. In this case, it was assumed that the cross sections were modified by two-photon exchange terms that were zero at $\varepsilon = 1$, linear in ε, and large enough to explain the discrepancy. This assumed modification to the cross sections was removed to correct for the two-photon effects, with no correction at $\varepsilon = 1$ and a 6% increase in the cross sections at small ε (σ_{c2} of Ref. [3]). The size of this correction is such that the Rosenbluth data approximately reproduce the polarization transfer values of G_E/G_M, and the 6% increase in the $\varepsilon = 0$ cross section yields a value of G_M that is ≈3% higher than a direct Rosenbluth extraction from the unmodified cross sections (e.g., the parametrization of Ref. [32] or the "Rosenbluth form factors," of Ref. [3]).

A similar combined extraction of form factors in Ref. [33] used the polarization transfer values for G_E/G_M to fix the slope of the reduced cross section, and used the uncorrected cross sections to extract the magnitude of G_E and G_M. For a data point at $\varepsilon = 1$, the change in the assumed slope of Eq. (1) yields an increase in the extrapolation to $\varepsilon = 1$ of 5–8 %, the size implied by the discrepancy, and so gives similar results to the extraction of Ref. [3]. For a measurement at very low ε, the change in slope leaves the $\varepsilon = 0$ extrapolation unchanged. So depending on the mean ε value of the data in a given Q^2 range, the extracted value of G_M will be 0–4 % lower in this analysis, compared to Ref. [3], with a typical difference of 1–2 %, as there are more data at large ε values. Note that both of these combined extractions rely on the assumption that the two-photon exchange terms have little or no effect on the polarization transfer results.

This is the first direct experimental evidence for large two-photon corrections in the unpolarized elastic electron-proton cross section. The effect is only observed for low Q^2 values, and so cannot be directly compared to the two-photon contributions necessary to explain the discrepancy between Rosenbluth and polarization transfer measurements of the proton form factors. However, the size and ε dependence of these effects are consistent with simple estimates based on the observed discrepancy, and so this observation supports the idea that two-photon contributions may significantly modify the Rosenbluth extraction of nucleon form factors.

Additional comparisons of positron to electron scattering over a range in Q^2 and ε would provide the most direct extraction of these two-photon corrections. With precise measurements over an adequate range in ε and Q^2, we could determine if the two-photon effects can fully explain the difference between polarization transfer and Rosenbluth measurements of the form factors, and could also provide significant data with which to constrain models of the real part of the two-photon amplitude. However, at the present time it is unclear where such a program could be carried out over the necessary kinematic range, and with the precision needed to map out these corrections. In the meantime, the existing data can be used to test calculations of the two-photon effects, and already are sufficient to rule out approaches with large electron cross section enhancements at large ε values. These data also provide information about the size and ε dependence of the two-photon effects, important when attempting to extract the proton form factors from a combined analysis of Rosenbluth and polarization transfer data.

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