Determination of fission rate by mean last passage time

Jing-Dong Bao[1,2,*] and Ying Jia[1]
[1]*Department of Physics, Beijing Normal University, Beijing 100875, China*
[2]*Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing 100875, China*
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The mean last passage time is introduced instead of the mean first passage time for determining the decay rate of a nucleus after induced fission. The stationary fission rate calculated by the inverse of the mean last passage time at the saddle point is in agreement with the result of Langevin simulations and better than that of the mean first passing time at the scission point. In particular, we take into account the backstreaming effect where test particles pass over the potential barrier multiple times. It is shown that the oscillating time of a hot fissioning system around the saddle point is the longest one in time scales of the fission, thus more neutrons might be emitted during this period.

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The induced fission of compound nuclei has emerged as a topic of considerable experimental and theoretical interests in the past years [1–3]. A large number of numerical studies [4–7] have shown, by means of Langevin simulations, that the stationary fission rate at the saddle point of the potential was higher than that at the scission point [8]. Thus some authors [4,6,9] have already pointed out the fact that the saddle point is not a reasonable criterion in stochastic calculation of the fission rate. Nevertheless, an exit point (or an "absorbing bound") needs to be chosen to be away from the saddle point. As a critical point, the concept of saddle point plays an important role in nuclear fission as well as in many other problems, for instance, the angular distribution of fission fragments is determined at the saddle point.

A more general expression, viz., a mean first passage time (MFPT), taking the scission point explicitly into account, was derived by Gontchar and Fröbrich [4] and Hofmann *et al.* [10] in the overdamped case. Very recently, Hofmann *et al.* [10] used the MFPT to study the fission lifetime and the emission of light particles, where an absorbing boundary condition was chosen at the scission point and the stationary fission rate was approximately interpreted as the inverse of the mean first passage time. However, dynamical effect of the saddle point was neglected. We would like to emphasize that in the Kramers' theory the so called escape time actually is a mean last passage time (MLPT) at the saddle point.

In this Brief Report, we want to evaluate time-dependent fission rate at the saddle point by using the proposed method of test particles passing over the potential barrier multiple times, and use the inverse of MLPT across the saddle point to determine the stationary fission rate.

The Langevin equation for overdamped motion of elongation variable x of a fissioning nucleus reads

$$\gamma\dot{x}(t) = -\frac{\partial V(x)}{\partial x} + \sqrt{2\gamma T}\xi(t) \tag{1}$$

with $\langle\xi(t)\rangle=0$ and $\langle\xi(t)\xi(t')\rangle=\delta(t-t')$, where γ is the friction coefficient, $V(x)$ is the potential energy, and T the temperature of the compound nucleus.

*Electronic address: jdbao@bnu.edu.cn

In Fig. 1, we plot three kinds of schematic potentials: $V(x)=V_0$ for $x\leqslant 0.81$ and $V(x)=V_i$ $(i=1,2,3)$ for $x\geqslant 0.81$, where V_i $(i=0,1,2,3)$ are taken as the following forms:

$$V_0 = -80.11(0.2x^5 - 1.17x^4 + 2.41x^3 - 2.05x^2 + 0.63x) + 5,$$

$$V_1 = -20(x-0.81)^2 + 4,$$

$$V_2 = -10(x-0.81)^2 + 12(x-0.81)^3 - 6(x-0.81)^2 + 4,$$

$$V_3 = V_0. \tag{2}$$

We apply the stochastic Runge-Kutta algorithm to solve numerically Eq. (1) for 1.5×10^6 test particles. If a Langevin trajectory crosses finally the saddle point starting from the ground state, an escape event occurs, subsequently, time-dependent fission rate is determined as

$$r(t) = -\frac{1}{N(t)}\frac{\Delta N(t)}{\Delta t}, \tag{3}$$

where $N(t)$ denotes the number of test particles that have not undergone fission at time t, $\Delta N(t)$ is the number of test particles that have escaped within the time interval $t\to t+\Delta t$. We emphasize here the fact that $\Delta N(t)$ is the recorded number of test particles across the saddle point for the last time, which differs from the previous method of test particles passing over the saddle point first time. It is evident that the probability current over the saddle point takes into account the contribution of the positive velocity only in the method of test particles passing over the saddle point first time, because the trajectories cannot recross the boundary, if the saddle point is an absorbing boundary [11]. However, a quasistationary flow passing over the saddle point must contain both positive and negative currents in the Kramers' rate formula, which in fact coincides with the mechanism of test particles passing over the saddle point multiple times.

Time-dependent fission rates calculated by different approaches are shown in Figs. 2(a), 2(b), and 2(c), where the presaddle potentials are the same, however, the postsaddle potentials are considered to be V_1 in (a), V_2 in (b), and V_3 in

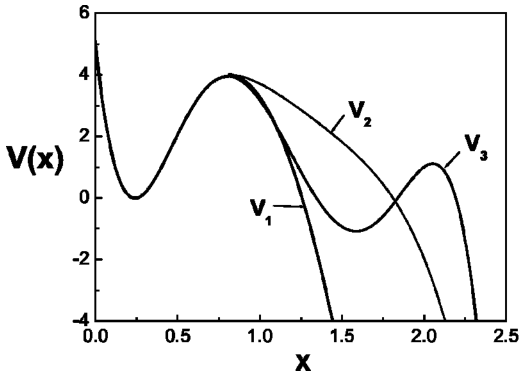

FIG. 1. The three kinds of schematic potentials.

(c), respectively. For comparison, the inverses of the MFPT calculated numerically at the saddle point $[\tau_{\rm MFPT}(x_0 \to x_b)]$ and at the scission point $[\tau_{\rm MFPT}(x_0 \to x_{sc})]$, as well as the MLPT calculated numerically at the saddle point $[\tau_{\rm MLPT}(x_0 \to x_b)]$ are also plotted in this figure, where x_0, x_b, and x_{sc} are the coordinates of the ground state, the saddle point, and the scission point, respectively. In each part, the upper line is the result simulated at the saddle point with test particles first passing over the saddle point; while the lower line is the result calculated with test particles reaching at the scission point; and the middle line is calculated also at the saddle point but with test particles passing over the saddle point multiple times. It is seen that the stationary fission rates calculated by using test particles passing over the saddle point first time are definitely higher than those determined by test particles passing over the saddle point multiple times. Indeed, the latter approaches the fission rate defined at the scission point. It is found that the *backstreaming* as the difference between the rates calculated by test particles passing over the saddle point first time and multiple times, is quite large if the postsaddle potential is gentle or the potential have structure. This is due to the fact that in the description of the MFPT at the saddle point, the particles cannot recross back over the boundary if the saddle point is chosen to be an absorbing boundary.

In Figs. 3(a) and 3(b), we show the case of the potential having a second minimum and maximum. The numerical result of the stationary fission rate is shown as a function of the temperature and friction strength, respectively. This is because we know that the fission rate increases with increasing temperature and decreasing friction strength. It is seen that the inverse of the MLPT across the saddle point produces the best data which are in agreement with the present Langevin simulation.

In the approach of the MFPT, trajectories recrossing the boundary of the specified domain have not taken into account the mean escape time of the particle from a metastable well. Here we propose a relation between the MLPT at the saddle point and the MFPT at the exit position (scission point). Restricting to the overdamped case, such an analysis can be performed in an analytic fashion. We have

$$\tau_{\rm MLPT}(x_0 \longrightarrow x_b) = \tau_{\rm MFPT}(x_0 \longrightarrow x_{ex}) - \tau_{b\to ex} \qquad (4)$$

with

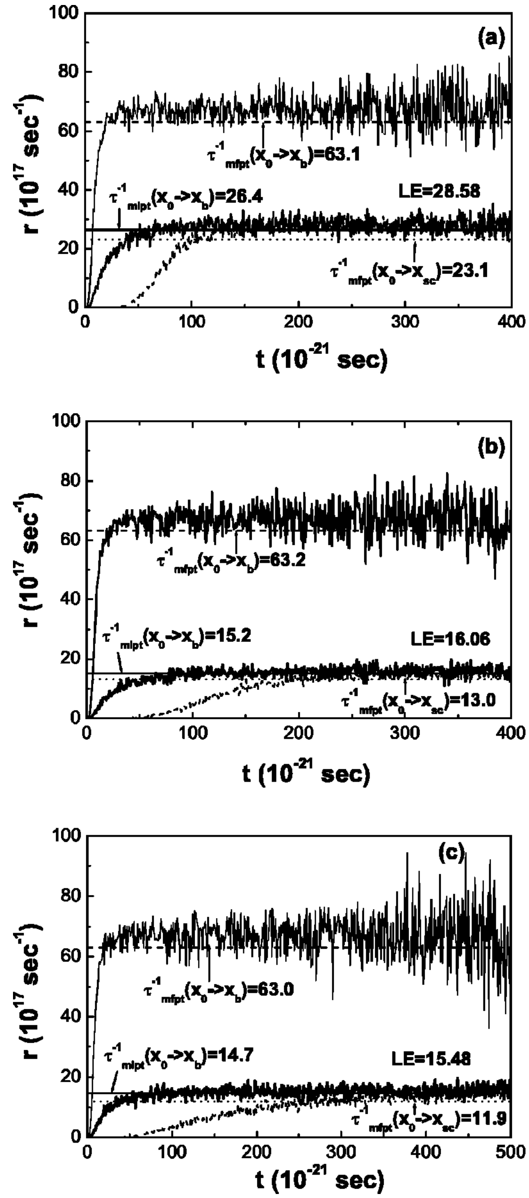

FIG. 2. Time-dependent fission rate calculated by Langevin simulation using different methods and various potentials. The parameters used are $T=4$ MeV, $\gamma=3.4$ MeV/$\hbar$, and the scission position $x_{sc}=2.7$.

$$\tau_{\rm MFPT}(x_0 \longrightarrow x_{ex}) = \frac{\gamma}{T}\int_{x_0}^{x_{ex}} e^{V(y)/T}dy \int_{-\infty}^{y} e^{-V(z)/T}dz, \qquad (5)$$

$$\tau_{b\to ex} = \frac{\gamma}{T}\int_{x_b}^{x_{ex}} e^{-V(y)/T}dy \int_{y}^{\infty} e^{V(z)/T}dz, \qquad (6)$$

where $\tau_{b\to ex}$ is the mean descent time from the saddle to scission points [12–15], and for the double-saddle case we have

$$\tau_{b\to ex} = \tau_{b\to x_{min}^{(2)}} + \tau_{\rm MFPT}(x_{min}^{(2)} \longrightarrow ex), \qquad (7)$$

where $x_{min}^{(2)}$ is the position of the right minimum of the potential. Thus the stationary fission rate is approximately equal to

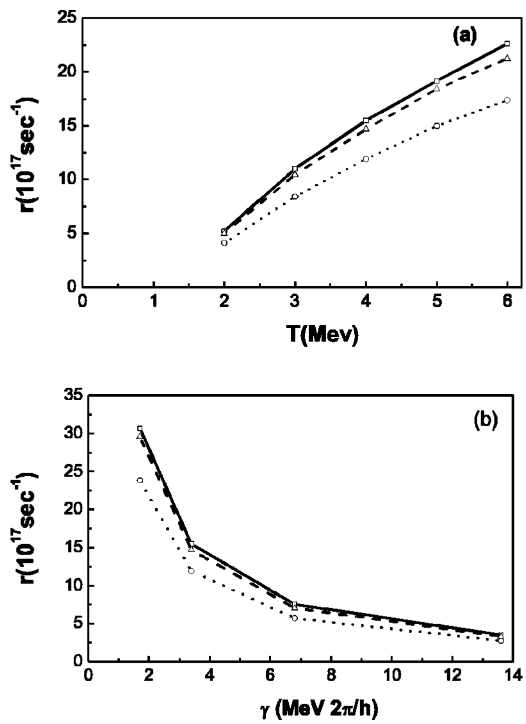

FIG. 3. Dependence of the stationary fission rate on the temperature T at fixed $\gamma = 3.4$ MeV/$\hbar$ in (a) and on the friction strength γ at fixed $T = 4$ MeV in (b). Here the potential V_3 with double saddle potential is used. The solid lines are the results of Langevin simulation, the dashed lines are inverse of the mean last passage time across the saddle point, and the dotted lines are the mean first passage time at the scission point.

$$r_k = \tau^{-1}_{\mathrm{MLPT}}(x_0 \rightarrow x_b). \tag{8}$$

Finally, we show in Fig. 4 the ratio of the fission rate calculated theoretically by the inverses of $\tau_{\mathrm{MLPT}}(x_0 \rightarrow x_b)$ and $\tau_{\mathrm{MFPT}}(x_0 \rightarrow x_{ex})$ to the resulting rate of Langevin simulation as a function of the exit position; here the potential V_2 is used. It is seen that the fission rate determined by the inverse of the MLPT at the saddle point is better than that of the MFPT at the scission point, because there is still room for dynamical saddle-to-scission time in the MFPT defined at the scission point.

The fission lifetime can be written as $\tau_f = \tau_{\mathrm{MFPT}}(x_0 \rightarrow x_b) + \Delta\tau_b + \tau_{b \rightarrow sc}$ in the present dynamical model. This would be of considerable interest since still more neutrons are emitted during the period the system oscillates around the saddle point. The time for which the compound nucleus after induced fission oscillates around the saddle point is evaluated theoretically as $\Delta\tau_b = \tau_{\mathrm{MLPT}}(x_0 \rightarrow x_b) - \tau_{\mathrm{MFPT}}(x_0 \rightarrow x_b)$. It can be found from Figs. 2(a), 2(b), and 2(c) that the oscillating time around the saddle point might be the longest one in the above three time scales. Therefore, the number of neutrons emitted might be more during a large elongation oscillation for a hot heavy compound nucleus. Moreover, the value of $\Delta\tau_b$ increases when the nucleus becomes heavier, because the distance between the saddle point and scission point increases, which increases with increasing the friction.

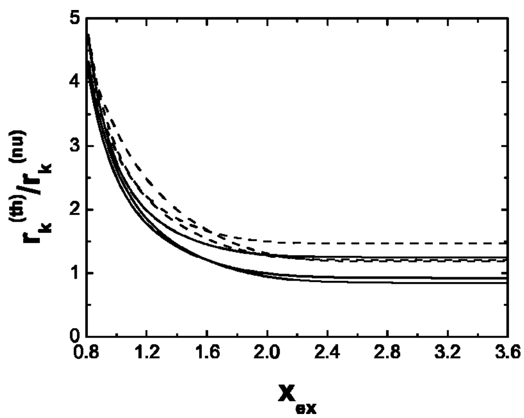

FIG. 4. The ratio between the resulting rates calculated by theoretical expression and numerical simulation as a function of the exit position. The solid and dashed lines are the results of MLPT across the saddle point and MFPT arriving at the exit point, respectively. The temperatures are equal to 2, 3, and 4 MeV from top to bottom and $\gamma = 3.4$ MeV/$\hbar$.

In summary, we propose a mean last passage time for the fission rate defined at the saddle point rather than the one of the mean first passage time at the scission point as suggested recently by some authors. It is concluded that the saddle point is still a reasonable criterion for the exit point in stochastic calculations of time-dependent fission rate as soon as the backstreaming across the saddle point is taken into account, i.e., test particles pass over the saddle point multiple times. This method can also be applied to calculate the fusion probability of massive nuclei [16]. In particular, the position of scission point only plays a weak role in the proper determination of fission rate in the calculation of the mean last passage time across the saddle point, and a dynamical effect of descent from the saddle point to scission point has been induced in the mean last passage time. Therefore we think that it is a concept better than the mean first passage time.

A hot heavy nucleus has a long oscillating time around the saddle point, thus more neutrons might be emitted in this period of time. The number of prescission neutrons with different energies emitted at three deformation regions of the fissioning nuclei is expected to be tested by experiments. As one knows that the fission is most of the time accompanied by light particle and γ-ray emission. Particle emission may modify the collective potential which is no longer static but changes with time. This might require one to study the effect of particle emission on the characteristic times of fission process.

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