Bulk viscosity in neutron stars from hyperons

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The contribution from hyperons to the bulk viscosity of neutron star matter is calculated. Compared to previous works we use for the weak interaction the one-pion exchange model rather than a current-current interaction, and include the neutral current $nn \leftrightarrow n\Lambda$ process. Also the sensitivity to details of the equation of state is examined. Compared to previous works we find that the contribution from hyperons to the bulk viscosity is about two orders of magnitude smaller.

I. INTRODUCTION

The bulk viscosity of matter in neutron stars has recently received considerable attention in connection with damping of neutron star pulsations and gravitational radiation driven instabilities, especially in the damping of r modes [1]. If the r modes are unstable, i.e., if the damping time scales due to viscous processes in neutron star matter are longer than the gravitational radiation driving time scale, a rapidly rotating neutron star could emit a significant fraction of its rotational energy and angular momentum as gravitational waves, which could be detectable.

It has been shown [2–6] that the bulk viscosity in a neutron star is caused by energy dissipation associated with nonequilibrium weak interaction reactions in a pulsating dense matter. Strong interaction processes do not play a role, because the strong interaction equilibrium is reached so fast that these processes can be considered to be in thermal equilibrium compared to the typical pulsation time scales of 10^{-4}–10^{-3} s. The relevant nonequilibrium weak interaction reaction rates and the magnitude of the bulk viscosity depend on the density and the composition of neutron star matter.

In a relatively low density neutron star composed mainly of neutrons n with admixtures of protons p, electrons e, and muons μ, the bulk viscosity is mainly determined by the reactions of the nonequilibrium modified Urca process,

$$N + n \rightarrow N + p + l + \bar{\nu}_l, \quad N + p + l \rightarrow N + n + \nu_l, \quad (1)$$

with $N=n,p$ and $l=e,\mu$. The bulk viscosity was studied by Sawyer [7] for npe matter and by Haensel *et al.* [3] for $npe\mu$ matter. For the process in Eq. (1) the relaxation time, i.e., the time it takes to restore equilibrium in case of a perturbation, is strongly temperature dependent, namely, $\tau^{-1} \propto T^6$.

At densities n_B of a few times saturation density, n_0 ($n_0 \approx 0.15$ fm^{-3}), the direct Urca process,

$$n \rightarrow p + l + \bar{\nu}_l, \quad p + l \rightarrow n + \nu_l, \quad (2)$$

may also be allowed depending on whether or not the proton fraction exceeds the Urca limit of $x_p \approx 0.11$ [8]. The contribution of this process to the bulk viscosity was computed by Haensel and Schaeffer [9] for npe matter and by Haensel *et al.* [2] for $npe\mu$ matter. Compared to the modified Urca process a smaller number of particles are involved which lead to a weaker temperature dependence, $\tau^{-1} \propto T^4$. As a consequence at typical neutron star temperatures, $T \sim 10^9$–10^{10} K, its contribution to the bulk viscosity is typically four to six orders of magnitude larger than that from the modified Urca process. The largest difference compared with that from the modified Urca process is reached at the low temperatures.

At about the same densities hyperons may appear in the neutron star core, first the Σ^- and Λ hyperons, followed by Ξ^0, Ξ^-, and Σ^+ at higher densities. Here we will restrict ourselves to the Σ^- and Λ hyperons. In addition to the semileptonic hyperon processes, weak nonleptonic hyperon processes also occur, specifically the processes

$$n + n \leftrightarrow p + \Sigma^-, \quad (3)$$

$$p + n \leftrightarrow p + \Lambda, \quad (4)$$

and

$$n + n \leftrightarrow n + \Lambda. \quad (5)$$

At low temperatures these processes contribute more efficiently to the bulk viscosity than the direct Urca process and the semileptonic hyperon ones, because they contain no neutrino phase space factor; at typical neutron star temperatures, $T < 10^{10}$ K, the phase space of neutrinos is almost negligible compared to that of baryons. Hence, for the weak nonleptonic hyperon processes of Eqs. (3)–(5) the temperature dependence of the inverse relaxation time is $\tau^{-1} \propto T^2$.

Historically the first semiquantitative calculation of bulk viscosity in neutron matter was carried out by Jones [10]. In this and all later works the weak nonleptonic process was calculated using a baryon current-current interaction, i.e., a contact W exchange.

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More recently the contribution to the bulk viscosity from the various weak nonleptonic hyperon processes has been reconsidered by several authors using a modern equation of state (EoS). Haensel *et al.* [4] studied the bulk viscosity for the $nnp\Sigma^-$ process within the nonrelativistic limit, and they found the bulk viscosity to be several orders of magnitude larger than that of the direct and the modified Urca processes. Also, applying the contact interaction of W exchange Lindblom and Owen [6] calculated the contribution of the $pnp\Lambda$ process in addition to that of the $nnp\Sigma^-$ process.

One may question the validity of the W exchange process. First there is no W exchange contribution to the $nnn\Lambda$ process and therefore it has not been considered in the above approaches [4,6,10]. On the other hand it is well known that in nuclear physics experiments on weak Λ decay in large hypernuclei the $nnn\Lambda$ process is found equally important as the $pnp\Lambda$ process. Second, Jones noticed already several orders of magnitude difference between $\tau_{nnn\Lambda}$ and the theoretical W exchange based value of $\tau_{nnp\Sigma^-}$ [5]. This is interesting, because the rate for the $nnn\Lambda$ process must depend on a very wide class of possible weak hadronic processes. Because most of these processes also contribute to $nnp\Sigma^-$ process, the $\tau_{nnp\Sigma^-}$ in reality is probably of the same order of magnitude as $\tau_{nnn\Lambda}$.

However, in nuclear physics it is customary to model the weak nonleptonic $NY \rightarrow NN$ process in terms of meson exchange (pion and kaon) with one phenomenological weak πNY or KNN vertex. In this approach one can describe both the observed weak decay rate of the Λ and branching ratios in hypernuclei quite well [11]. Therefore, in the following we will calculate the bulk viscosity using the meson exchange picture to describe the hyperon processes in Eqs. (3)–(5).

In practice the rates also depend on the details of the equation of state, e.g., the hyperon fractions as a function of the density. A variety of model equations of state has been constructed with widely varying properties during the last decades, some of these based upon a microscopic free space NN and NY interactions while others are phenomenological parametrizations of the energy density to higher densities. In particular the relativistic mean field approaches, as used in Refs. [4,6], although microscopic in nature, do not start from a realistic nucleon-nucleon interaction. Here we employ the effective EoS based on the work of Balberg and Gal [12]. To get some idea about the sensitivity of the bulk viscosity to the details of the EoS we consider two different parameter sets for the density dependence of the multibody potential energy.

Superfluidity can also influence the bulk viscosity. Below the critical temperature, superfluidity [4,6] suppresses the reaction rates by roughly a factor $\exp(-\Delta/T)$ with gap energy Δ. It generally leads to a smaller bulk viscosity. Above the critical temperature, it has no influence on the reaction rates. Many properties of superfluid matter such as the gaps and the critical temperatures are still known with large uncertainties, e.g., $T_c \sim 10^8 - 10^{10}$ K. Hence, the bulk viscosity is only considered here for nonsuperfluid matter. For recent papers about the effects of superfluidity we refer to Haensel *et al.* [4] and Lindblom and Owen [6].

The main goal in this study is threefold: (i) to compute the viscosity using the more realistic one-pion exchange (OPE) instead of the contact W exchange description in the nonleptonic weak hyperon processes, (ii) to include the weak neutral current $nnn\Lambda$ process, and (iii) to examine the sensitivity of the bulk viscosity to the EoS. The starting point of our derivation of the bulk viscosity for the $nnp\Sigma^-$, $pnp\Lambda$, and $nnn\Lambda$ processes is the finite temperature Green's function formalism in the quasiparticle approximation (QPA). The coupling constants for these processes using OPE can be verified from pionic hyperon decay experiments. The application of these coupling constants gives the correct order of magnitude for the rate of nonmesonic hyperon decay in large hypernuclei. Covariant expressions for the matrix elements are derived.

The plan of this paper is as follows. The EoS is discussed in Sec. II. The bulk viscosity for neutron star matter is discussed in Sec. III. Section IV is devoted to the collision rate of the weak nonleptonic hyperon process as in Eqs. (3)–(5). The results are presented and discussed in Sec. V. Finally the conclusion is given in Sec. VI.

II. EQUATION OF STATE

In this paper we use the equation of state constructed by Balberg and Gal [12]. It is a generalization of the (npe) EoS of Lattimer and Swesty [13]. The energy density is parametrized in terms of the densities of the constituents. The parameters for the nucleon-nucleon interactions are fitted to the binding energy, symmetry energy, and incompressibility of saturated symmetric nuclear matter. The values of the parameters for hyperon-nucleon and hyperon-hyperon interactions mainly rely on experimental data from hypernuclei. The equilibrium fractions are obtained in the standard manner, by assuming a given nucleon density n and solving the imposed conditions of baryon conservation, charge neutrality, and weak equilibrium:

$$n_B = n_n + n_p + n_{\Sigma^-} + n_\Lambda, \tag{6}$$

$$n_p = n_e + n_{\Sigma^-}, \tag{7}$$

$$\mu_p = \mu_n - \mu_e, \tag{8}$$

$$\mu_e = \mu_\mu, \tag{9}$$

$$\mu_{\Sigma^-} = \mu_n + \mu_e, \tag{10}$$

$$\mu_\Lambda = \mu_n. \tag{11}$$

To obtain some idea about the sensitivity of the resulting bulk viscosity to details of the EoS we use two parametrizations which differ in the density dependence of the contributions of the nucleons (δ) and hyperons (γ) to the energy density, specifically given by the parameter sets $\gamma = \delta = 2$ and $\gamma = \delta = 5/3$, respectively (Figs. 1 and 2). The difference between the two parametrizations of the equation of state has no influence on the particle fractions in the nuclear sector. For densities below the appearance density of the hyperons the neutrons are abundantly present ($x_n > 0.9$). The leptons
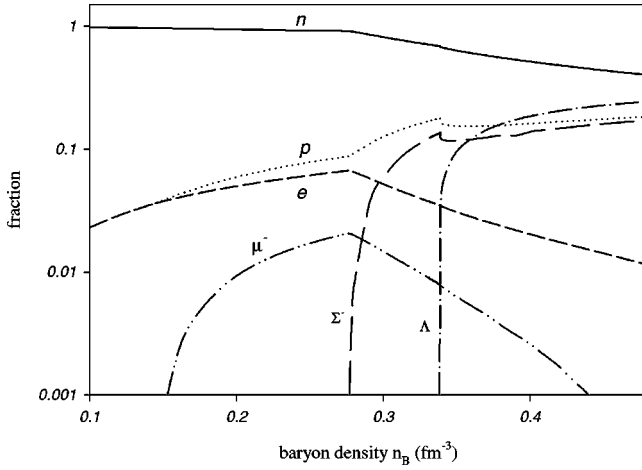
FIG. 1. The particle fraction as a function of the baryon number density (Balberg and Gal [12] case 1 with $\gamma = \delta = 2$).

are present because of charge neutrality to compensate the positively charged protons, although they are expensive in terms of energy density. The appearance of the hyperons creates the possibility to lower the neutron excess without lepton formation, whereas the negatively charged hyperons allow charge neutrality to be maintained within the baryon sector. Therefore, the appearance of the hyperons is accompanied by a strong deleptonization.

The first hyperon to appear is the Σ^-, followed by the lower mass Λ. This can be explained by the higher chemical potential of the Σ^- due to its negative charge, which compensates the mass difference of about 80 MeV. However, the growth of the Σ^- number density fraction is soon hindered by charge-dependent forces, which disfavor an excess of Σ^-'s over Σ^+'s and a joint excess of Σ^-'s and neutrons. The Λ is not affected by charge-dependent forces and its fraction continues to increase. For the EoS with the smaller values for $\delta = \gamma = 5/3$ the onset of the various hyperons occur at higher densities and the corresponding hyperon fractions are smaller.
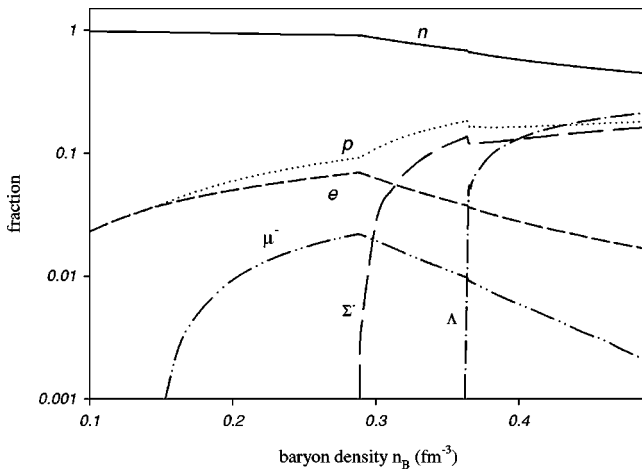

FIG. 2. The particle fraction as a function of the baryon number density (Balberg and Gal [12] case 2 with $\gamma = \delta = 5/3$).

III. THE BULK VISCOSITY

In general bulk viscosity is a dissipative process in which the compression is converted into heat. It is due to the instantaneous difference between the local physical pressure p and the thermodynamic pressure $\tilde{p}$

$$p - \tilde{p} = -\zeta \vec{\nabla} \cdot \vec{v},$$

where $\vec{v}$ is the local velocity. In neutron star matter one is interested in describing small deviations (oscillations) $\delta x_i = x_i - \tilde{x}_i$ around the thermodynamic equilibrium, characterized by the variables (densities) $\tilde{x}_i$.

We will now turn to the calculation of ζ for the specific case of a neutron star with hyperons. Since the three reactions (3)–(5) involve neutrons the fluctuation of the neutron fraction x_n can be used as the primary parameter. For this situation a general expression for the bulk viscosity has been derived in Ref. [6],

$$\zeta = \frac{-n_B \tau}{1 - i\omega\tau} \left(\frac{\partial p}{\partial x_n} \right)_{n_B} \frac{d\tilde{x}_n}{dn_B}, \tag{12}$$

with τ the relaxation time, ω the pulsation frequency of the neutron star, p the pressure, and n_B the baryon density.

We restrict ourselves to the densities, at which nucleons, Σ^-'s, and Λ's are present. We define the chemical imbalance as

$$\chi = 2\mu_n - \mu_p - \mu_{\Sigma^-} = \mu_n - \mu_\Lambda, \tag{13}$$

and assume that in first order the difference $\Delta\Gamma_a$ between the rates for the various direct reactions Γ_a and inverse reactions $\bar{\Gamma}_a$ in Eqs. (3)–(5) is proportional to χ,

$$\Delta\Gamma = \sum_a \Delta\Gamma_a = \lambda\chi, \tag{14}$$

with λ determining the viscosity. Hence, the relaxation time τ can be expressed in terms of the variation of the imbalance with the neutron density fluctuation,

$$\tau = \frac{n_B \chi}{\Delta\Gamma(d\chi/dx_n)} \approx \frac{n_B}{\lambda(d\chi/dx_n)}. \tag{15}$$

Thus the main task is to evaluate the $d\chi/dx_n$. We wish to include several weak nonleptonic hyperon reactions. To do this in a proper way one has to take into account that the four relevant baryon number densities x_n, x_p, x_Λ, and x_Σ are related to each other by the following three constraints: baryon conservation, charge neutrality, and chemical equilibrium for strong processes and in particular for the reaction $n + \Lambda \leftrightarrow p + \Sigma^-$. By using these conditions one can express χ in terms of the fractions x_i and assuming that all leptonic reaction rates are much smaller than those which produce the hyperon bulk viscosity, one obtains in the density region with $np\Sigma^-$ matter [6]

$$\frac{d\chi}{dx_n} = 2\left[\alpha_{nn} - (\alpha_{pn} + \alpha_{\Sigma^-n} + \alpha_{np} + \alpha_{n\Sigma^-}) + (1/2)(\alpha_{pp} + \alpha_{\Sigma^-p}\right.$$

$$\left. + \alpha_{p\Sigma^-} + \alpha_{\Sigma^-\Sigma^-})\right], \qquad (16)$$

and in the region with $np\Sigma^-\Lambda$ matter

$$\frac{d\chi}{dx_n} = \alpha_{nn} + \frac{(\beta_n - \beta_\Lambda)(\alpha_{np} - \alpha_{\Lambda p} + \alpha_{n\Sigma^-} - \alpha_{\Lambda\Sigma^-})}{2\beta_\Lambda - \beta_p - \beta_{\Sigma^-}} - \alpha_{\Lambda n}$$

$$- \frac{(2\beta_n - \beta_p - \beta_{\Sigma^-})(\alpha_{n\Lambda} - \alpha_{\Lambda\Lambda})}{2\beta_\Lambda - \beta_p - \beta_{\Sigma^-}}, \qquad (17)$$

where the α's correspond to the partial derivatives of the chemical potentials with respect to the various fractions

$$\alpha_{ij} = \left(\frac{\partial \mu_i}{\partial x_j}\right)_{n_k, k\neq j}, \qquad (18)$$

and

$$\beta_i = \alpha_{ni} + \alpha_{\Lambda i} - \alpha_{pi} - \alpha_{\Sigma^-i}, \qquad (19)$$

where i and j stand for n, p, Σ^-, and Λ. These quantities are obtained from the EoS.

The real part of the hyperon bulk viscosity is

$$\mathrm{Re}\ \zeta = \frac{-n_B \tau}{1 + \omega^2 \tau^2}\left(\frac{\partial P}{\partial x_n}\right)\frac{dx_n}{dn_B}, \qquad (20)$$

where τ is given in Eq. (15). The core of the neutron star is assumed to pulsate with a frequency of about $\omega \sim 10^3 - 10^4\ \mathrm{s}^{-1}$. In the high frequency limit $1 \ll \omega\tau$ the bulk viscosity ζ is proportional to the inverse of the relaxation time, $\zeta \propto \tau^{-1} \propto \lambda$, whereas in the low frequency limit $1 \gg \omega\tau$ the bulk viscosity is proportional to the relaxation time, $\zeta \propto \tau \propto \lambda^{-1}$. In the following section the difference in the rates given by $\Delta\Gamma$ in Eq. (14) in first order in χ is derived, from which $\lambda = \Delta\Gamma/\chi$ can be obtained.

IV. COLLISION RATE

In this section the various rates $\Delta\Gamma_{N1N2N'Y'}$ for the processes $N_1 + N_2 \leftrightarrow N' + Y'$ are considered, where the N is a nucleon and the Y is a hyperon. First we consider the one-pion exchange in Born approximation. For completeness the rates using the contact W exchange interaction [6] are given in Sec. IV B.

A. One-pion exchange

In order to obtain $\Delta\Gamma_{N1N2N'Y'}$, first the rate $\Gamma_{N1N2\to N'Y'}$ is considered. The relevant free space diagrams are shown in Fig. 3. The strangeness changing weak vertex is given by

$$F_{NY'}^w = G_F m_\pi^2 (\bar{A}_{NY'} + \bar{B}_{NY'}\gamma_5), \qquad (21)$$

for which the constants $\bar{A}$ and $\bar{B}$ determine the strengths of the parity violating and parity conserving $Y \to N + \pi$ amplitudes, respectively, and are specified in Sec. V. The strong vertex is
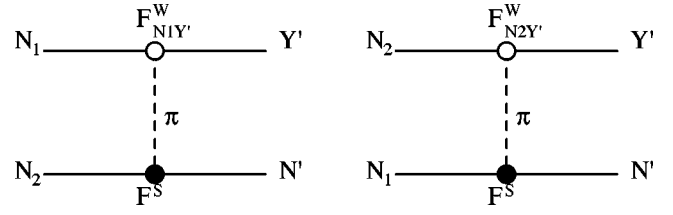

FIG. 3. The pion exchange diagrams in Born approximation for the weak nonleptonic hyperon processes with only one hyperon involved.

$$F_{NN'}^s = g_{NN'}\gamma_5. \qquad (22)$$

To compute the rates in the medium one needs to account for Pauli blocking; it is convenient to use the optical theorem to convert the free space diagrams of Fig. 3 to the closed diagrams of Fig. 4. Using finite temperature Green's functions in the QPA, one can express the collision rate as

$$\Gamma_{N1N2\to N'Y'} = \frac{1}{S}\int \frac{d^4p_{N1}}{(2\pi)^4}\frac{d^4p_{N2}}{(2\pi)^4}\frac{d^4p_{N'}}{(2\pi)^4}\frac{d^4p_{Y'}}{(2\pi)^4}Z(2\pi)^4$$

$$\times \delta^4(p_{N1} + p_{N2} - p_{Y'} - p_{N'}). \qquad (23)$$

In Eq. (23) one has

$$Z = Z^a + Z^b, \qquad (24)$$

with

$$Z^a = \mathrm{Tr}\left[G_{N1}^{-+}(p_{N1})F_{N1N'}^s G_{N'}^{+-}(p_{N'})F_{N1N'}^{s\dagger}\right]$$

$$\times \mathrm{Tr}\left[G_{N2}^{-+}(p_{N2})F_{N2Y'}^w G_{Y'}^{+-}(p_{Y'})F_{N2Y'}^{w\dagger}\right]D_\pi(k_1^2)^2$$

$$+ \{N_1 \leftrightarrow N_2\} \qquad (25)$$

with $D_\pi(k^2) = 1/(\vec{k}^2 + m_\pi^2)$, and $\vec{k}_i = \vec{p}_{Ni} - \vec{p}_{N'}$ (for $i = 1, 2$), corresponding to the diagram on the left in Fig. 4, and

$$Z^b = \mathrm{Tr}\left[G_{N1}^{-+}(p_{N1})F_{N1N'}^s G_{N'}^{+-}(p_{N'})F_{N2N'}^{s\dagger}\right.$$

$$\times G_{N2}^{-+}(p_{N2})F_{N2Y'}^w G_{Y'}^{+-}(p_{Y'})F_{N1Y'}^{w\dagger}\right]D_\pi(k_1^2)D_\pi(k_2^2)$$

$$+ \{N_1 \leftrightarrow N_2\} \qquad (26)$$

corresponding to that of the right in Fig. 4. The symmetry factor S for the $pnp\Lambda$ process is $S=1$ and for the $nnp\Sigma$ and $nnn\Lambda$ processes $S=2$. After evaluation of the p^0 integrals in the QPA, one obtains
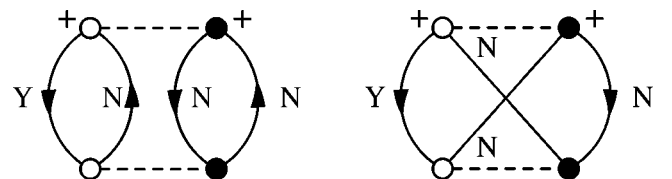

FIG. 4. The two types of closed diagrams at the two-loop level.

$$\Gamma_{N1N2 \to N'Y'} = \int \frac{d^3 p_{N1}}{2\,m_{N1}(2\pi)^3} \frac{d^3 p_{N2}}{2\,m_{N2}(2\pi)^3}$$

$$\times \frac{d^3 p_{N'}}{2\,m_{N'}(2\pi)^3} \frac{d^3 p_{Y'}}{2\,m_{Y'}(2\pi)^3}$$

$$\times \frac{1}{S} |M_{N1N2N'Y}|^2 f_{N1} f_{N2}(1 - f_{N'})(1 - f_{Y'})$$

$$\times (2\pi)^4 \delta(E_{N1} + E_{N2} - E_{N'} - E_{Y'}) \delta^3(p_{N1} + p_{N2}$$

$$- p_{N'} - p_{Y'}), \tag{27}$$

where p_i and E_i are the particle momentum and energy, respectively, and $f_i = \{1 + \exp[(\epsilon_i - \mu_i)/T]\}^{-1}$ is the Fermi-Dirac distribution function. The matrix element is

$$M_{N1N2N'Y'} = [\bar{u}_{N'} F^s_{N1N'} u_{N1} \bar{u}_{Y'} F^w_{N2Y'} \bar{u}_{N2} D_\pi(k_1^2)$$

$$- \bar{u}_{N'} F^s_{N2N'} u_{N2} \bar{u}_{Y'} F^w_{N1Y'} \bar{u}_{N1} D_\pi(k_2^2)]. \tag{28}$$

The evaluation of $\Gamma_{N1N2 \to N'Y'}$ with the standard technique [14] takes advantage of the strong degeneracy of the participating particles in neutron star matter. The multidimensional integral is decomposed into an energy and an angular integration. All momenta p_i are placed on the appropriate Fermi spheres wherever possible. Furthermore, we introduce the dimensionless quantities

$$y_i = \frac{\epsilon_i - \mu_i}{T}, \quad \xi = \frac{\chi}{T}. \tag{29}$$

The rate in Eq. (27) can be factorized in the form

$$\Gamma_{N1N2 \to N'Y'} = \Gamma^{(0)}_{N1N2N'Y'} I(\xi), \tag{30}$$

with

$$I(\xi) = \prod_{i=1}^{4} \int_{-\infty}^{\infty} dy_i f(y_i) \delta\left(\sum_{i=1}^{4} y_i + \xi\right) = \frac{e^\xi}{e^\xi - 1} \frac{4\pi^2 \xi + \xi^3}{6}, \tag{31}$$

and

$$\Gamma^{(0)}_{N1N2N'Y'} = \frac{p_{FN1} T^3}{8(2\pi)^6} \int d\theta_{N'} d\theta_{Y'} \frac{\Theta(1 - |C(\theta_{N'}, \theta_{Y'})|)}{\sqrt{1 - C^2(\theta_{N'}, \theta_{Y'})}}$$

$$\times \frac{1}{S} \sum_{spin} |M_{N1N2N'Y'}|^2, \tag{32}$$

where p_{FNi} is the Fermi momentum of baryon N_i, $\theta_{N'}$ is the angle between $\vec{p}_{N1}$ and $\vec{p}_{N'}$, $\theta_{Y'}$ is the angle between $\vec{p}_{N1}$ and $\vec{p}_{Y'}$, and

$$C(\theta_{N'}, \theta_{Y'}) = [-p_{FN1}^2 - p_{FN'}^2 - p_{FY'}^2 + 2\,p_{FN1}\,p_{FN'}\cos(\theta_{N'})$$

$$+ 2\,p_{FN1}\,p_{FY'}\cos(\theta_{Y'})$$

$$- 2\,p_{FN'}^2\,p_{FY'}^2\cos(\theta_{N'})\cos(\theta_{Y'})$$

$$+ p_{FN2}^2]/[2\,p_{FN'}^2\sin(\theta_{N'})\sin(\theta_{Y'})]. \tag{33}$$

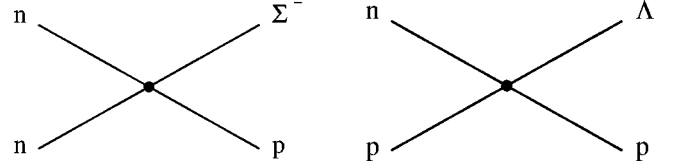The conditions in Sec. II, Eqs. (6)–(11), apply in thermody-

FIG. 5. The diagram of the contact W exchange interaction for the $nnp\Sigma^-$ process and the $pnp\Lambda$ process.

namic equilibrium. However, small deviations from thermodynamic equilibrium occur just after a neutron star is born. For the bulk viscosity the small deviations from the weak chemical equilibrium are important. For this purpose we need to consider $\Delta\Gamma_{N1N2N'Y'} = \Gamma_{N1N2 \to N'Y'} - \Gamma_{N'Y' \to N1N2}$, where the inverse rate is $\Gamma_{N'Y' \to N1N2} = \Gamma^{(0)}_{N1N2N'Y'} I(-\xi)$. Therefore, in the linear approximation, valid for small deviations $|\xi| = |\chi|/T \ll 1$ one has

$$\Delta\Gamma_{N1N2N'Y'} = \Gamma^{(0)}_{N1N2N'Y'} \Delta I, \tag{34}$$

with

$$\Delta I = I(\xi) - I(-\xi) = \frac{4\pi^2}{3} \xi. \tag{35}$$

The ΔI is the same for the weak nonleptonic hyperon processes of Eqs. (3)–(5). The total rate considered for the hyperon bulk viscosity is

$$\Delta\Gamma = \Delta\Gamma_{pnp\Lambda} + \Delta\Gamma_{nnn\Lambda} + 2\Delta\Gamma_{nnp\Sigma^-}. \tag{36}$$

This result is inserted into Eq. (15) to obtain the relaxation time.

B. Contact W exchange interaction

In order to illustrate the difference between the pion exchange approach and the contact W exchange picture (Fig. 5), we also consider the latter. The rates for W exchange have been calculated before by Haensel *et al.* [4] and by Lindblom and Owen [6], however, in the angle-averaged approximation. Performing an analogous derivation as in the preceding section, one obtains the expression in Eq. (15). For convenience of the reader here we give the simpler matrix elements in the nonrelativistic limit (for the general expression for the matrix elements we refer to Ref. [6]):

$$|M_{nnp\Sigma^-}|^2 = 8G_F^2 \sin^2(2\,\theta_C) m_n^2 m_p\,m_{\Sigma^-}(1 + 3\,c_A^{np}\,c_A^{n\Sigma^-})^2 \tag{37}$$

and

$$|M_{pnp\Lambda}|^2 = 8G_F^2 \sin^2(2\,\theta_C) m_n\,m_p^2\,m_\Lambda(1 + 3|c_A^{np}|^2|c_A^{p\Lambda}|^2), \tag{38}$$

respectively. Note that no meson propagators $D(k^2)$ are present in Eqs. (37) and (38), i.e., in the medium these matrix elements have a different density dependence than the OPE ones. The $nnn\Lambda$ process has no simple W exchange contribution. The results are shown in the following section.
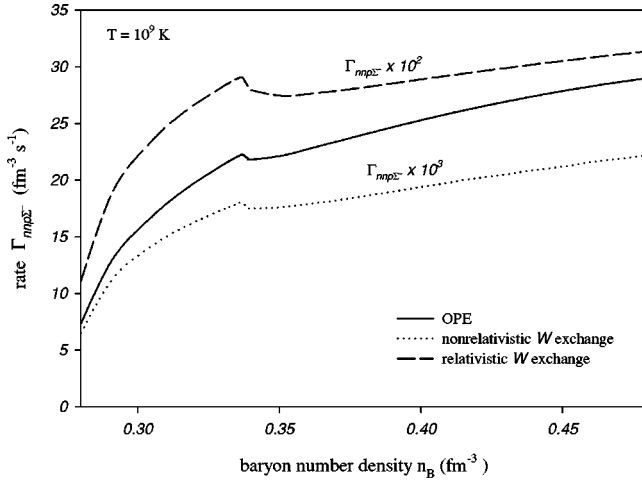
FIG. 6. The equilibrium rate Γ for $n+n \rightarrow p+\Sigma^-$ as a function of the baryon number density at $T=10^9$ K.

V. RESULTS AND DISCUSSION

Some remarks about the differences between the meson exchange and the W exchange picture are appropriate. First in the OPE (in which phenomenological input is used for both the weak and strong πNN couplings) the pion exchange leads to a finite range interaction and the pseudoscalar πNN coupling to the presence of momentum dependent terms, p_i/m. These effects are absent in the simple W point coupling. Hence, in addition to a different overall strength one also expects to find a different density dependence of the matrix elements for the two processes in the medium. Second, the neutral current process, such as $nnn\Lambda$ process, cannot be described in the simple W exchange approach.

The EoS applied for the following figures is given by Balberg and Gal [12] with parametrization $\gamma=\delta=2$ (case 1) and also $\gamma=\delta=5/3$ (case 2). In the calculation for the case of pion exchange we use the following values for the strong and weak vertices: $g_{nn}^2/(4\pi)=14.2$ and $g_{np}=\sqrt{2}g_{nn}$. The values $\bar{A}_{n\Sigma^-}=1.93$, $\bar{A}_{n\Lambda}=-1.07$, $\bar{A}_{p\Lambda}=1.46$, $\bar{B}_{n\Sigma^-}=-0.63$, $\bar{B}_{n\Lambda}=-7.19$, and $\bar{B}_{p\Lambda}=9.95$ are based upon the experimental data on pionic decay of hyperons [15,16]. In the W exchange picture the standard values of the weak coupling constants are $c_A^{np}=-g_A=-1.26$, $c_A^{p\Lambda}=-0.72$, and $c_A^{n\Sigma^-}=0.34$ and G_F is the Fermi weak coupling constant. These values are also used by Lindblom and Owen [6], while Haensel *et al.* [4] treat the interaction matrix as a free parameter. Compared to the work of Lindblom and Owen [6] and Haensel *et al.* [4] effects of superfluidity are not taken into account and there is no angle averaging in this work.

In Fig. 6 we compare the rates for the $n+n \rightarrow p+\Sigma^-$ process for the W exchange and the pion exchange as a function of density for case 1 of the EoS. Concerning the W exchange, we note that the nonrelativistic result is a poor approximation to the relativistic result since there is an almost complete cancellation between the leading order vector and axial-vector contributions, $(1+3c_A^{np}c_A^{n\Sigma^-})^2 \approx 1/16$. The overall rate calculated with OPE is almost two orders of magnitude larger than the one corresponding to relativistic W exchange.
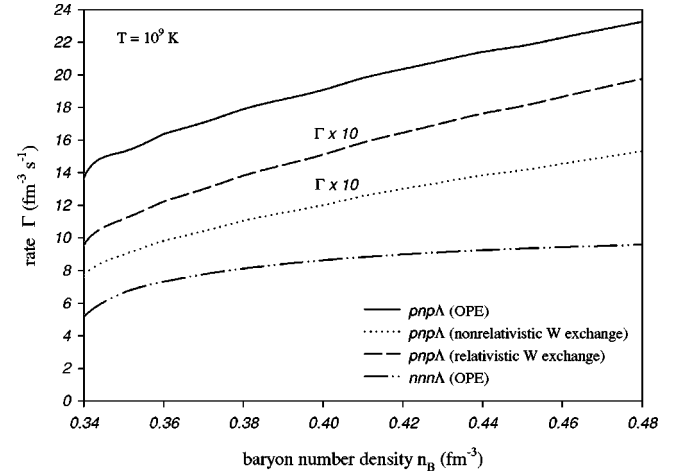

FIG. 7. The equilibrium rate Γ for $p+n \rightarrow p+\Lambda$ and $n+n \rightarrow n+\Lambda$ as a function of the baryon number density at $T=10^9$ K.

The difference can be attributed to larger values of the couplings, $\bar{A}_{NY'}, \bar{B}_{NY'}$ and $g_{NN'}>1$, in the OPE case. At low momentum transfer (low density), this effect would be compensated for by the p-wave character of the πNN coupling, $\vec{\sigma}\cdot\vec{k}$. However, for momenta relevant in neutron stars this does not happen. In addition, a stronger density dependence is observed, which is related to finite range pion exchange in Eq. (28).

Because of the rapid increase of the Λ fraction after its appearance, the proton and Σ^- fraction drop abruptly and also their Fermi momenta. Therefore, the kink in the rate can be attributed to the appearance of the Λ at $n_B \approx 0.33$ fm^{-3}.

The rates for $pnp\Lambda$ process are shown in Fig. 7. As mentioned above the OPE shows a stronger density dependence because of the presence of the meson propagator. Although at baryon densities where the hyperons are present a partial cancellation between the direct and exchange OPE matrix elements occurs, the overall OPE rate is about one order of magnitude larger than the one obtained with W exchange, which again can be attributed to the larger values of the meson couplings.
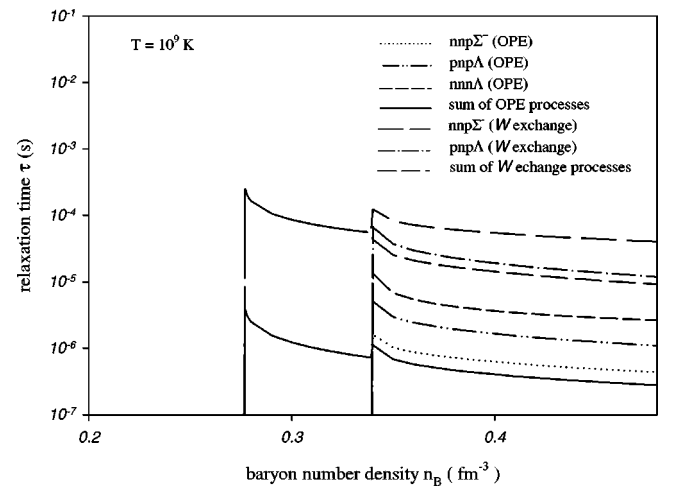

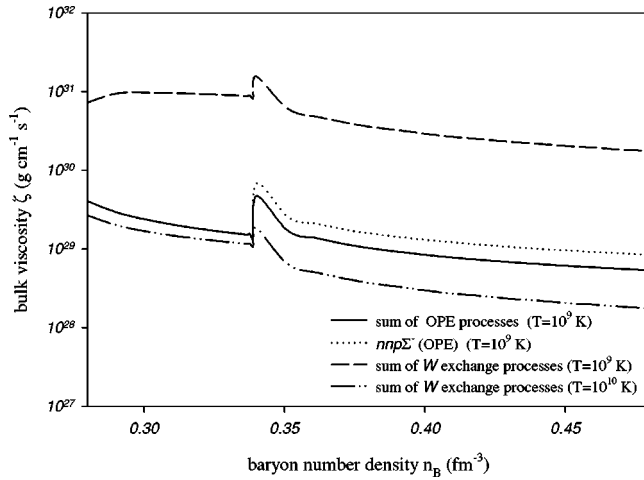FIG. 8. The relaxation time τ as a function of the baryon number density at $T=10^9$ K.

FIG. 9. The bulk viscosity ζ as a function of the baryon number density.

FIG. 10. The bulk viscosity ζ as a function of the baryon number density for different equations of state at $T=10^{10}$ K.

As mentioned above the neutral current $nnn\Lambda$ process does not receive a contribution from W exchange. In the OPE approach the magnitude of this rate is a factor 2–3 smaller than that of the $pnp\Lambda$ process. We note that there exists experimental information on the ratio of the $nnn\Lambda$ and $pnp\Lambda$ processes in the weak decay of hypernuclei which suggests that the simple OPE mechanism is not sufficiently accurate [11]. The inclusion of other mesons, such as kaons, could improve the situation.

The relaxation time is shown as a function of the baryon density in Fig. 8. One sees that the relaxation time in the OPE picture is about two orders of magnitude smaller than in the W picture. In the latter case after the appearance of Σ^-, the relaxation time is determined by the $nnp\Sigma^-$ process; after the occurrence of the Λ, the $pnp\Lambda$ process takes over and it dominates the relaxation time. Therefore, a drop in the relaxation time occurs at the appearance density of the Λ. In the OPE picture after the appearance of the Λ, the $nnp\Sigma^-$ process remains the most important process for the relaxation time. Therefore at the appearance density of the Λ a small abrupt increase in the relaxation time occurs.

To compute the viscosity one needs to assume a value for the frequency; typical values of the frequency of the pulsations are $\omega=10^3-10^4$ s^{-1}. We have used $\omega=10^4$ s^{-1} in Fig. 9. For the OPE picture, the high frequency limit is applicable. Whereas for W exchange at $T=10^9$ K and at low density we are even in the low frequency limit. The bulk viscosity in the OPE picture is about one to two orders of magnitude smaller than that in the W exchange picture.

To investigate the sensitivity of the bulk viscosity to details of the EoS we also considered different values of the parameters. As can be seen in Fig. 10 the bulk viscosity is rather insensitive to the details of the EoS except with respect to the appearance of the Λ.
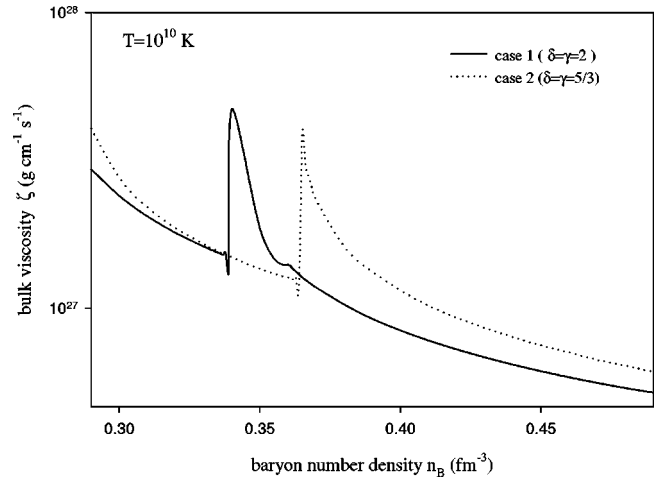
VI. CONCLUSION

In this paper the bulk viscosity due to weak nonleptonic hyperon processes has been studied. This viscosity is relevant in connection with damping of neutron star pulsations, especially in the damping of r modes. In particular we considered pion exchange, which is based upon empirical input, in Born approximation to describe the weak nonleptonic hyperon processes instead of the W exchange [4–6]. The conclusions are the following:

(i) The bulk viscosity in the OPE picture is about one to two orders of magnitude smaller than that in the W exchange picture.

(ii) The rates of the $nnp\Sigma$ and $pnp\Lambda$ process are of the same order of magnitude using OPE. This result is in contrast with the case of W exchange, because the $pnp\Lambda$ process is one order of magnitude larger than $nnp\Sigma$ process.

(iii) The $nnn\Lambda$ process can be included in the calculation for the bulk viscosity using OPE.

(iv) The bulk viscosity is rather insensitive for the EoS used except with respect to the appearance of the Λ.

Therefore, the contact W exchange interaction is probably too naive for quantitative calculations. In a more realistic OPE approach the contributions of the hyperons to the bulk viscosity are less pronounced.

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