$\Delta(1232)$ in $\pi N \rightarrow \pi\pi N$ reaction

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The contribution of $\Delta(1232)$ to the $\pi N \rightarrow \pi\pi N$ reaction is examined by making use of the chiral reduction formula developed by Yamagishi and Zahed. The influence of the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions on this reaction, which has not been regarded as important so far, is considered for all channels with the initial $\pi^{\pm}p$ states. The total cross sections are calculated to tree level for the energy up to $T_{\pi}=400$ MeV. Although the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions give a small effect on the $\pi^{+}p \rightarrow \pi^{+}\pi^{+}n$, $\pi^{-}p \rightarrow \pi^{+}\pi^{-}n$, and $\pi^{-}p \rightarrow \pi^{0}\pi^{0}n$ channels, the $\pi^{\pm}p \rightarrow \pi^{\pm}\pi^{0}p$ channels are found to be sensitive to these interactions.

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I. INTRODUCTION

The $\pi N \rightarrow \pi\pi N$ reaction is important in the analysis of the low energy πN scattering because of its role as a major inelastic process. Since the pion mass is small owing to the spontaneous breaking of chiral symmetry, the contribution of the $\pi\pi N$ channel is observed even around the $\Delta(1232)$ resonance.

Many theoretical approaches have been taken to study this reaction: for example, the phenomenological model using effective Lagrangian [1–3] and a series of systematic analyses based on the chiral perturbation theory [4–6]. Also, the several kinds of global fits of the experimental data have been performed [7–9].

These studies show that the $\pi N \rightarrow \pi\pi N$ reaction is useful to extract some important aspects of hadron physics. The $\pi\pi$ scattering length is obtained by using the $\pi N \rightarrow \pi\pi N$ data. Furthermore valuable information taken from the $\pi N \rightarrow \pi\pi N$ reaction benefits the study of baryon resonances which have a considerable influence on this reaction. In particular, the contribution of $\Delta(1232)$ is remarkable around the threshold region.

Because $\Delta(1232)$ contributes to the $\pi N \rightarrow \pi\pi N$ reaction through three typical interactions, i.e., the $\pi N\Delta$, $\pi\Delta\Delta$, and $\rho N\Delta$ interactions, the information of these interactions can be accessible through the analysis of this reaction. However the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions have not been taken seriously in the theoretical analyses so far. These interactions are not included in Ref. [1], and their contributions to the $\pi^{-}p \rightarrow \pi^{+}\pi^{-}n$ channel are shown to be negligible in Refs. [2,3]. On the other hand, many types of interactions including the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions are considered in the global fits [8,9], but their individual roles in each channel of the $\pi N \rightarrow \pi\pi N$ reaction are not discussed. The $\pi\Delta\Delta$ and $\rho N\Delta$ interactions still remain controversial in contrast to the well known $\pi N\Delta$ interaction.

Chiral symmetry is a key to systematic understanding of the pion related reactions including the $\pi N \rightarrow \pi\pi N$ reaction. This symmetry has been accepted as a fundamental concept in the hadron physics due to the successes of the various low energy theorems [10] and the chiral perturbation theory in the threshold region [11,12]. These studies are based on spontaneous broken chiral symmetry, where the pions are considered as the Nambu-Goldstone bosons realized by this symmetry breaking.

Recently, a general framework analyzing hadron reactions has been developed by Yamagishi and Zahed [13] on the basis of chiral symmetry. This framework introduces a new type of reduction formula, i.e., the chiral reduction formula, which manifests a requirement of chiral symmetry satisfied by the invariant amplitudes not only for the threshold region but for the resonance region. It is worth noting that the chiral reduction formula offers the Ward identity satisfied by the quantum amplitudes without relying on any model or expansion scheme at the beginning. This identity allows us to take a flexible and consistent view, which is free from restrictions given by specific model, of the theoretical approach to resonances [14,15].

In this paper, applying the chiral reduction formula to the invariant amplitude of the $\pi N \rightarrow \pi\pi N$ reaction, we try to clarify the contribution of $\Delta(1232)$ in this reaction to tree level. We particularly discuss the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions in all channels with the initial $\pi^{\pm}p$ state. We assume the Rarita-Schwinger field for $\Delta(1232)$ without taking account of its internal structure. We do not aim to fix their coupling constants but to clarify their importance in the $\pi N \rightarrow \pi\pi N$ reaction. Our tree level calculation is enough to make a qualitative discussion about the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions because of the smallness of loop collections in this reaction as shown in Ref. [6].

In Sec. II, we show the total cross section of the $\pi N \rightarrow \pi\pi N$ reaction. We take into account the isospin symmetry breaking when we compare our numerical results with the experimental values. We explain the chiral reduction of the $\pi N \rightarrow \pi\pi N$ reaction in Sec. III, and we show the form factors of the current matrix elements appeared due to the reduction of the invariant amplitude in Sec. IV. Our numerical results are presented in Sec. V and we discuss the contributions of $\Delta(1232)$ in this reaction. The summaries are given in Sec. VI. In the Appendix, we show the phenomenological Lagrangians which are necessary to evaluate the form factors appeared in the current matrix elements, and give a brief explanation of the Rarita-Schwinger field.

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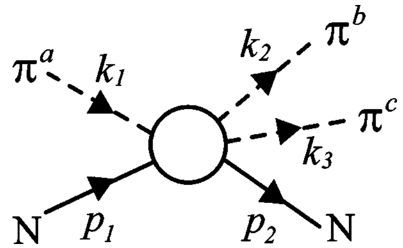

FIG. 1. The $\pi N \to \pi\pi N$ reaction. Each pion has the isospin index (a,b,c) and the four-momentum k_i $(i=1,2,3)$, and each nucleon has the four-momentum p_j $(j=1,2)$.

II. TOTAL CROSS SECTION

We consider the total cross section of the $\pi N \to \pi\pi N$ reaction in the isospin symmetric limit with the averaged values $m_N = 939$ MeV and $m_\pi = 138$ MeV for the nucleon and the pion masses, respectively (see Fig. 1).

The isospin symmetric total cross section is given by

$$\sigma_{\rm sym} = \frac{\mathcal{B}}{2\sqrt{(s-m_+^2)(s-m_-^2)}} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3k_2}{(2\pi)^3 2\omega_2} \frac{d^3k_3}{(2\pi)^3 2\omega_3} \times (2\pi)^4 \delta^{(4)}(p_1 + k_1 - p_2 - k_2 - k_3) \overline{|\mathcal{T}|^2}, \tag{1}$$

where $s = (p_1 + k_1)^2$ and $m_\pm = m_N \pm m_\pi$. The energy of outgoing nucleon and pions are denoted as E_2 and $\omega_i (i=2,3)$, respectively. The Bose factor $\mathcal{B}$ is equal to 1/2 if the outgoing pions are identical, otherwise this factor is 1. The invariant amplitude is denoted as $\mathcal{T}$, for which the incoming (outgoing) nucleon spin average (sum) is taken, $\overline{|\mathcal{T}|^2} = (1/2)\Sigma_{spins}|\mathcal{T}|^2$.

We will show the numerical result as a function of the incoming pion kinetic energy in the laboratory system, T_π, which incorporates the isospin symmetry breaking observed in the mass difference in each reaction channel. The invariant amplitude is sensitive to this difference through the pion kinetic energy of each channel near the threshold. We consider this symmetry breaking by shifting the value of the kinetic energy from the isospin symmetric threshold to the physical threshold [5]. The total cross section $\sigma_{\rm expt}$ measured by the experiments is related to Eq. (1) by

$$\sigma_{\rm expt}(T_\pi) = \sigma_{\rm sym}(\overline{T_\pi}), \tag{2}$$

where $\overline{T_\pi}$ is the isospin symmetric pion kinetic energy and $\delta T_\pi = T_\pi - \overline{T_\pi}$. In Table I, we summarize the value of δT_π in each reaction channel.

TABLE I. The values of δT_π in each reaction channel.

Channel	δT_π(MeV)
$\pi^+ p \to \pi^+\pi^+ n$	+3.97
$\pi^+ p \to \pi^+\pi^0 p$	−3.66
$\pi^- p \to \pi^+\pi^- n$	+3.97
$\pi^- p \to \pi^0\pi^0 n$	−7.92
$\pi^- p \to \pi^-\pi^0 p$	−3.66

III. CHIRAL REDUCTION FOR $\pi N \to \pi\pi N$ REACTION

In this section, we explain the chiral reduction of the invariant amplitude for the $\pi N \to \pi\pi N$ reaction,

$$(2\pi)^4 \delta^{(4)}(p_1 + k_1 - p_2 - k_2 - k_3) i\mathcal{T} = \langle N(p_2)|a^c(k_3) a^b(k_2) \hat{S} a^{a\dagger}(k_1)|N(p_1)\rangle|_{\phi=0}, \tag{3}$$

where $a^a(k)[a^{a\dagger}(k)]$ is an annihilation (creation) operator of the pion with the isospin component a and the four-momentum k. $\hat{\mathcal{S}} = \hat{\mathcal{S}}[\phi]$ is the extended S matrix operator which is a functional of $\phi = (a,v,s,J)$; the axial vector, vector, scalar, and pseudoscalar c-number external fields [13]. At $\phi=0$, $\hat{S}$ is reduced to the ordinary S matrix operator. These external fields play an important role in the formulation of the chiral reduction formula.

Using the chiral reduction formula, we decompose the invariant amplitude $i\mathcal{T}$ as [16]

$$i\mathcal{T} = (i\mathcal{T}_\pi + i\mathcal{T}_A + i\mathcal{T}_{SA} + i\mathcal{T}_{VA}) + (k_1, a \leftrightarrow -k_3, c) + (k_2, b \leftrightarrow k_3, c) + i\mathcal{T}_{AAA}, \tag{4}$$

where $(\leftrightarrow)$ represents a permutation of the momentum and isospin indices of the pion in the first four terms. Each term on the right-hand side of Eq. (4) is explicitly written as

$$i\mathcal{T}_\pi = \frac{i}{f_\pi^2}[(k_1 - k_2)^2 - m_\pi^2]\delta^{ab}\langle N(p_2)|\hat{\pi}^c(0)|N(p_1)\rangle, \tag{5}$$

$$i\mathcal{T}_A = \frac{1}{2f_\pi^3}(k_2 - k_1)^\mu \delta^{ab}\langle N(p_2)|j^c_{A\mu}(0)|N(p_1)\rangle, \tag{6}$$

$$i\mathcal{T}_{SA} = -i\frac{m_\pi^2}{f_\pi^2} k_3^\mu \delta^{ab} \int d^4x e^{-i(k_1-k_2)x} \times \langle N(p_2)|T^*(\hat{\sigma}(x) j^c_{A\mu}(0))|N(p_1)\rangle, \tag{7}$$

$$i\mathcal{T}_{VA} = \frac{1}{2f_\pi^3}(k_1 + k_2)^\mu k_3^\nu \varepsilon^{abe} \times \int d^4x e^{-i(k_1-k_2)x}\langle N(p_2)|T^*(j^e_{V\mu}(x) j^c_{A\nu}(0))|N(p_1)\rangle, \tag{8}$$

$$i\mathcal{T}_{AAA} = -\frac{1}{f_\pi^3} k_1^\mu k_2^\nu k_3^\lambda \int d^4x_1 d^4x_2 e^{-ik_1x_1 + ik_2x_2} \times \langle N(p_2)|T^*(j^a_{A\mu}(x_1) j^b_{A\nu}(x_2) j^c_{A\lambda}(0))|N(p_1)\rangle. \tag{9}$$

The pseudoscalar density $\hat{\pi}^a(x)$ is identified with the asymptotic pion field $\pi_{as}(x)$ as $x_0 \to \pm\infty$. The one-pion reduced axial current $j^a_{A\mu}(x)$ is defined by $j^a_{A\mu}(x) = \mathbf{A}^a_\mu(x) + f_\pi \partial_\mu \hat{\pi}^a(x)$, where $\mathbf{A}^a_\mu(x)$ is the ordinary axial current with the asymptotic form $\mathbf{A}^a_\mu(x) \to -f_\pi \partial_\mu \pi_{as}(x) + \cdots$ as $x_0 \to \pm\infty$.

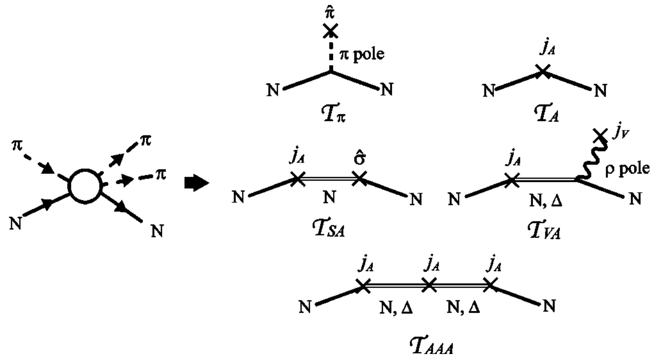

FIG. 2. Diagrammatical interpretation for the chiral reduction of $\pi N \to \pi\pi N$ reaction to tree level.

The vector current and the scalar density are represented by $j^a_{V\mu}(x)$ and $\hat{\sigma}(x)$, respectively.[1]

Equation (4) is an exact relation between the invariant amplitude and the current correlations constrained by chiral symmetry with the partially conserved axial-vector current (PCAC) condition, $\partial^\mu \mathbf{A}^a_\mu(x) \to f_\pi m_\pi^2 \pi^a_{as}(x)$ as $x_0 \to \pm\infty$. Thus the chiral reduction formula can naturally deal with the explicit breaking of chiral symmetry. For example, $\mathcal{T}_{SA}$ is due to this breaking and vanishes in chiral limit $m_\pi \to 0$.

In Fig. 2, we show the diagrammatical interpretation of Eq. (4) to tree level. The solid line corresponds to the external nucleon, and the double line to the propagation of the nucleon or $\Delta(1232)$. The cross denotes that the hadron couples to the current. The pion pole appears in the $\hat{\pi}$ current, while the j_A current is free of this pole by definition. We assume the vector meson dominance (VMD) for the j_V current, and take into account the ρ meson pole in $\mathcal{T}_{VA}$. The $\Delta(1232)$ resonance does not propagate in $\mathcal{T}_{SA}$ because the N-Δ transition is not brought about by the $\hat{\sigma}$ current.

IV. CURRENT MATRIX ELEMENTS

Taking account of the Lorentz, isospin, parity, and time reversal invariances and the vector current conservation $\partial^\mu j^a_{V\mu}(x)=0$, we can write the general forms of the current matrix elements necessary to evaluate Eqs. (5)–(9). We summarize those matrix elements in Table II.

A. Vector-isovector part

The vector current matrix elements are generally written as

$$\langle N(p')|j^a_{V\mu}(0)|N(p)\rangle = \overline{u}(p')\left[F^N_{V,1}(t)\gamma_\mu + F^N_{V,2}(t)\frac{i}{2m_N}\sigma_{\mu\nu}q^\nu\right]\frac{\tau^a}{2}u(p) \tag{10}$$

for the nucleon, and

[1]We call $\hat{\pi}^a(x)$ and $\hat{\sigma}(x)$ the "current" instead of the "density."

TABLE II. The current matrix elements necessary to evaluate each diagram in Fig. 2.

$\mathcal{T}_\pi$	$\mathcal{T}_A$	$\mathcal{T}_{SA}$	$\mathcal{T}_{VA}$	$\mathcal{T}_{AAA}$
$\langle N\|\hat{\pi}\|N\rangle$	$\langle N\|j_A\|N\rangle$	$\langle N\|j_A\|N\rangle$	$\langle N\|j_V\|N\rangle$	$\langle N\|j_A\|N\rangle$
		$\langle N\|\hat{\sigma}\|N\rangle$	$\langle \Delta\|j_V\|N\rangle$	$\langle \Delta\|j_A\|N\rangle$
			$\langle N\|j_A\|N\rangle$	$\langle \Delta\|j_A\|\Delta\rangle$
			$\langle \Delta\|j_A\|N\rangle$	

$$\langle \Delta(p')|j^a_{V\mu}(0)|N(p)\rangle = \overline{U}^\nu(p')[F^{N\Delta}_{V,1}(t)g_{\nu\mu} + F^{N\Delta}_{V,2}(t)Q_\nu\gamma_\mu + F^{N\Delta}_{V,3}(t)Q_\nu Q_\mu + iF^{N\Delta}_{V,4}(t)Q_\nu\sigma_{\mu\lambda}Q^\lambda]\gamma_5 I^a\left(\tfrac{3}{2},\tfrac{1}{2}\right)u(p) \tag{11}$$

for the N-Δ transition, where $q^\mu=(p'-p)^\mu$, $Q^\mu=-q^\mu$, and $t=(p'-p)^2$, τ^a is the isospin Pauli matrix and $I^a(i,j)$ is the $j\to i$ isospin transition $(2i+1)\times(2j+1)$ matrix. The isoquadruplet Rarita-Schwinger vector-spinor and the isodoublet Dirac spinor are denoted as $U^\mu(p)$ and $u(p)$, respectively. We note that γ_5 appears in Eq. (11) because the Rarita-Schwinger field with spin-parity $(3/2)^+$ is employed for $\Delta(1232)$. The form factors $[F^N_{V,i}(t)$ with $i=1,2$ and $F^{N\Delta}_{V,j}(t)$ with $j=1-4]$ are the functions of t.

In order to determine the t dependence of the form factors to tree level, we make use of the current-field identity [17]

$$j^a_{V\mu}(x) = \frac{m_\rho^2}{f_\rho}\rho^a_\mu(x), \tag{12}$$

where m_ρ is the ρ meson mass and f_ρ corresponds to the gauge coupling constant of the hidden local symmetry model for the vector mesons [18,19]. Based on this identity which expresses the phenomenology of the VMD, we write the nucleon form factors as

$$F^N_{V,1}(t) = \frac{m_\rho^2}{m_\rho^2 - t - im_\rho\Gamma_\rho(t)}, \tag{13}$$

$$F^N_{V,2}(t) = \kappa_V F^N_{V,1}(t), \tag{14}$$

where we use the standard ρNN interaction (see the Appendix) and the universality relation $g_{\rho NN}=f_\rho$. The isovector magnetic moment is denoted as κ_V. The phenomenological width of the ρ meson is parametrized as

$$\Gamma_\rho(t) = \Gamma_\rho\frac{m_\rho}{\sqrt{t}}\left(\frac{t-4m_\pi^2}{m_\rho^2-4m_\pi^2}\right)^{3/2}\theta(t-4m_\pi^2), \tag{15}$$

where Γ_ρ is the total width at $t=m_\rho^2$ [14]. As for the N-Δ transition form factors, we obtain

$$F^{N\Delta}_{V,1}(t) = \frac{f_{\rho N\Delta}}{f_\rho}\left(\frac{m_N+m_\Delta}{m_\rho}\right)\frac{m_\rho^2}{m_\rho^2 - t - im_\rho\Gamma_\rho(t)}, \tag{16}$$

$$F_{V,2}^{N\Delta}(t)=\frac{1}{m_N+m_\Delta}F_{V,1}^{N\Delta}(t), \tag{17}$$

where we use the $\rho N\Delta$ interaction (A5). The $\rho N\Delta$ coupling constant is denoted as $f_{\rho N\Delta}$.

We note that the other form factors $F_{V,3}^{N\Delta}(t)$ and $F_{V,4}^{N\Delta}(t)$ in Eq. (11) are fixed to zero as long as we consider the Lagrangian (A5) for the $\rho N\Delta$ interaction.

B. Axial-isovector part

The matrix elements of the one-pion reduced axial current are generally written as [20,21]

$$\langle N(p')|j_{A\mu}^a(0)|N(p)\rangle = \overline{u}(p')[F_{A,1}^N(t)\gamma_\mu + F_{A,2}^N(t)q_\mu]\gamma_5\frac{\tau^a}{2}u(p), \tag{18}$$

$$\begin{aligned}\Delta(p')|j_{A\mu}^a(0)|N(p)\rangle &= \overline{U}^\nu(p')[F_{A,1}^{N\Delta}(t)g_{\nu\mu}+F_{A,2}^{N\Delta}(t)Q_\nu\gamma_\mu \\ &\quad + F_{A,3}^{N\Delta}(t)Q_\nu Q_\mu + iF_{A,4}^{N\Delta}(t)Q_\nu\sigma_{\mu\lambda}Q^\lambda]I^a\left(\tfrac{3}{2},\tfrac{1}{2}\right)u(p),\end{aligned} \tag{19}$$

$$\begin{aligned}\langle\Delta(p')|j_{A\mu}^a(0)|\Delta(p)\rangle &= \overline{U}^\nu(p')[F_{A,1}^\Delta(t)g_{\nu\lambda}\gamma_\mu + F_{A,2}^\Delta(t)g_{\nu\lambda}q^\mu + F_{A,3}^\Delta(t) \\ &\quad \times(q_\nu g_{\mu\lambda}+g_{\nu\mu}q_\lambda)+F_{A,4}^\Delta(t)q_\nu\gamma_\mu q_\lambda \\ &\quad + F_{A,5}^\Delta(t)q_\nu q_\mu q_\lambda]\gamma_5 I^a\left(\tfrac{3}{2},\tfrac{3}{2}\right)U^\lambda(p).\end{aligned} \tag{20}$$

Owing to the nature of $\Delta(1232)$ in our treatment, γ_5 does not appear in Eq. (19) in contrast to the familiar form of the axial currents given by Eqs. (18) and (20). Note that the pion pole does not contribute to the above form factors by definition.

Some of the form factors in Eqs. (18)–(20) are exactly related to the renormalized coupling constants for the pion-baryon interaction,

$$f_{\pi NN}(t)=\frac{m_\pi}{f_\pi}\left[\frac{1}{2}F_{A,1}^N(t)+\frac{t}{4m_N}F_{A,2}^N(t)\right], \tag{21}$$

$$f_{\pi N\Delta}(t)=\frac{m_\pi}{f_\pi}[F_{A,1}^{N\Delta}(t)+(m_N-m_\Delta)F_{A,2}^{N\Delta}(t)+tF_{A,3}^{N\Delta}(t)], \tag{22}$$

$$f_{\pi\Delta\Delta}(t)=\frac{m_\pi}{f_\pi}\left[F_{A,1}^\Delta(t)+\frac{t}{2m_\Delta}F_{A,2}^\Delta(t)\right], \tag{23}$$

where we consider the effective πNN, $\pi N\Delta$, and $\pi\Delta\Delta$ interactions (A1)–(A3) given in the Appendix. The other form factors [$F_{A,4}^{N\Delta}(t)$, $F_{A,3}^\Delta(t)$, $F_{A,4}^\Delta(t)$, and $F_{A,5}^\Delta(t)$] do not appear in our calculation.

At tree level, all the form factors are reduced to constants. We write $F_{A,1}^N=g_A$ and $F_{A,2}^N=-2\overline{\Delta}_{\pi N}/m_\pi^2$, where g_A is the axial coupling constant and $\overline{\Delta}_{\pi N}$ characterizes the discrepancy between $f_{\pi NN}(t=0)$ and $f_{\pi NN}(t=m_\pi^2)$ [16]. It is difficult to determine the differences in their coupling constants between $t=0$ and $t=m_\pi^2$ in the present situation of experimental data. Then we eliminate $F_{A,3}^{N\Delta}$ and $F_{A,2}^\Delta$ naturally by using the PCAC hypothesis $f_{\pi N\Delta,\pi\Delta\Delta}(m_\pi^2)\simeq f_{\pi N\Delta,\pi\Delta\Delta}(0)$. In order to determine $F_{A,1}^\Delta$, we use the ratio $R_{N\Delta}=f_{\pi\Delta\Delta}(0)/f_{\pi NN}(0)$ and obtain

$$F_{A,1}^\Delta=\frac{R_{N\Delta}}{2}g_A. \tag{24}$$

In the quark model, $R_{N\Delta}$ becomes $4/5$ [22].

TABLE III. The value of constants in this paper. The mass and width of each particle or resonance are shown in MeV. Note that $F_{A,2}^{N\Delta}$ has the opposite sign with G in Ref. [20] due to the difference of definition for the N-Δ transition form factors of j_A.

Masses and widths	(MeV)	Parameters	
m_N	939	g_A	1.265
m_π	138	f_ρ	5.80[a]
m_Δ	1232	κ_V	3.71
m_ρ	770	$F_{A,1}^{N\Delta}$	1.382[b]
Γ_Δ	120	$\overline{\Delta}_{\pi N}$	−54 MeV[c]
Γ_ρ	149	$\sigma_{\pi N}$	45 MeV[d]
		f_π	93 MeV
		$F_{A,2}^{N\Delta}$	-4.235×10^{-4} MeV^{-1}[b]

[a]See p. 33 in Reference [19].
[b]Reference [20].
[c]Reference [16].
[d]Reference [23].

C. Scalar-isoscalar part

Since $\Delta(1232)$ dose not contribute to $\mathcal{T}_{SA}$, we only need the nucleon matrix element for the $\hat{\sigma}$ current,

$$\langle N(p')|\hat{\sigma}(0)|N(p)\rangle = S(t)\overline{u}(p')u(p). \tag{25}$$

According to the definition in Ref. [13], the form factor $S(t)$ is equal to $-\sigma_{\pi N}(t)/f_\pi m_\pi^2$, where $\sigma_{\pi N}(t)$ is the pion-nucleon sigma term which becomes independent of t at tree level.

D. Pseudoscalar-isovector part

The nucleon matrix element of the $\hat{\pi}$ current appearing in $\mathcal{T}_\pi$ is written as

$$\langle N(p')|\hat{\pi}^a(0)|N(p)\rangle = P(t)\overline{u}(p')i\gamma_5\tau^a u(p). \tag{26}$$

This current satisfies

$$(\Box+m_\pi^2)\hat{\pi}^a(x)=\frac{1}{f_\pi}\partial^\mu j_{A\mu}^a(x), \tag{27}$$

where the external fields are set to zero [13]. From Eqs. (18), (27), and (A1), we obtain the general relation between the form factor $P(t)$ and the form factors $F_{A,1}^N$, $F_{A,2}^N$ or the renormalized πNN coupling constant,

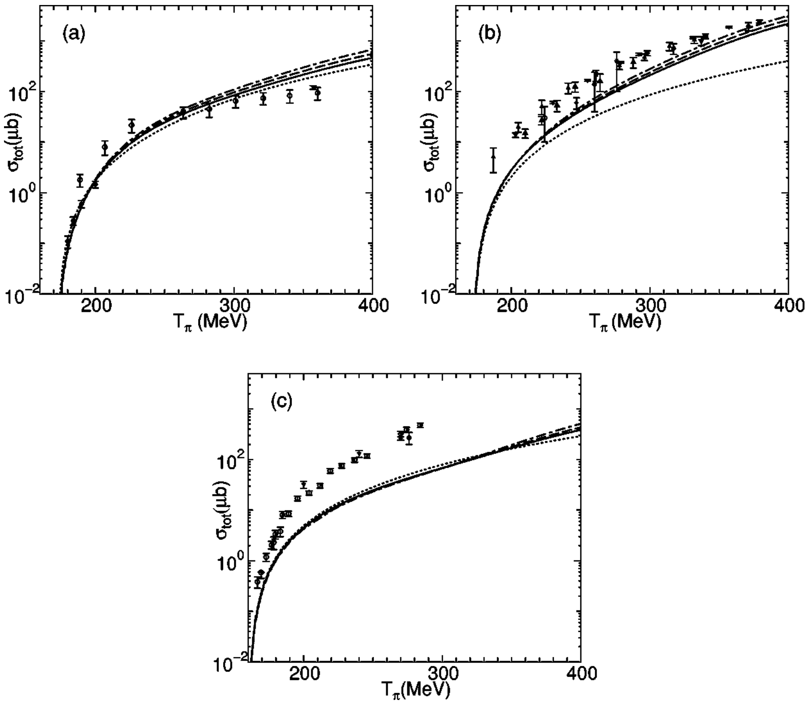

FIG. 3. The dependence of the cross section on the $\pi\Delta\Delta$ interaction; (a) $\pi^+p \to \pi^+\pi^+n$, (b) $\pi^-p \to \pi^+\pi^-n$, and (c) $\pi^-p \to \pi^0\pi^0n$. It is taken $R_{N\Delta}=0$ for the solid line, $R_{N\Delta}=0.4$ for the dashed line, and $R_{N\Delta}=0.8$ for the dashed-dotted line. In addition, the dotted line is the result with the π and N only. The data from Refs. [24–26] for $\pi^+p \to \pi^+\pi^+n$, [27–36] for $\pi^-p \to \pi^+\pi^-n$, and [37–41] for $\pi^-p \to \pi^0\pi^0n$.

$$P(t) = \frac{1}{m_\pi^2 - t}\frac{1}{f_\pi}\left(m_N F^N_{A,1}(t) + \frac{t}{2}F^N_{A,2}(t)\right)$$
$$= \frac{1}{m_\pi^2 - t}\left(\frac{2m_N}{m_\pi}\right) f_{\pi NN}(t), \qquad (28)$$

where the pion pole contribution is taken into account.

V. RESULTS AND DISCUSSIONS

In this section we show the numerical results for the $\pi N \to \pi\pi N$ total cross section below $T_\pi=400$ MeV. We discuss the influence of the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions on this reaction. The free parameters are $f_{\rho N\Delta}$ and $R_{N\Delta}$. In Table III, we summarize the value of constants in this paper.

First we consider the $\pi\Delta\Delta$ interaction. This interaction probes a double-Δ part of the contribution in $\mathcal{T}_{AAA}$ when only Δ(1232) appears as the intermediate baryon. Figures 3 and 4 show the dependence of each cross section on the values of $R_{N\Delta}$. The $\rho N\Delta$ interaction is not included there. Two values for $R_{N\Delta}$ are chosen for the calculation, i.e., 0.8 and 0.4. The former is taken from the quark model result [22] and the latter is selected based on the argument in Ref. [47].

In the $\pi^+p \to \pi^+\pi^+n$ and $\pi^-p \to \pi^+\pi^-n$ channels, the cross sections are increased by about 10–30% around $T_\pi=300$ MeV due to the double-Δ propagation in $\mathcal{T}_{AAA}$ compared to the results with $R_{N\Delta}=0$. In $\pi^-p \to \pi^0\pi^0n$ channel, the effect of the double-Δ propagation is less than a few percent. In the same figure, we also display the experimental data taken from Refs. [24–46]. As for the $\pi^-p \to \pi^+\pi^-n$ and $\pi^-p \to \pi^0\pi^0n$ channel, the disagreement between the theory and data is far beyond the variation in the cross section due to the $\pi\Delta\Delta$ interaction. This result is consistent with that of Ref. [2] for the $\pi^-p \to \pi^+\pi^-n$ channel.

In contrast with the above results, the $\pi^\pm p \to \pi^\pm\pi^0p$ channels are sensitive to the $\pi\Delta\Delta$ interaction (see Fig. 4). The total cross section for the $\pi^+p \to \pi^+\pi^0p$ channel below 300 MeV increases by about 50% with $R_{N\Delta}=0.4$, and the cross section with $R_{N\Delta}=0.8$ becomes twice the result with $R_{N\Delta}=0$. As for the $\pi^-p \to \pi^-\pi^0p$ channel, the total cross section indicates the increase by about 25% with $R_{N\Delta}=0.4$ and 65% with $R_{N\Delta}=0.8$ around $T_\pi=300$ MeV. These two channels exhibit large influence of the $\pi\Delta\Delta$ interaction on the total cross section. However, if the fit with data is considered, it is still difficult for the $\pi^+p \to \pi^+\pi^0p$ channel to improve the result by including the $\pi\Delta\Delta$ interaction alone.

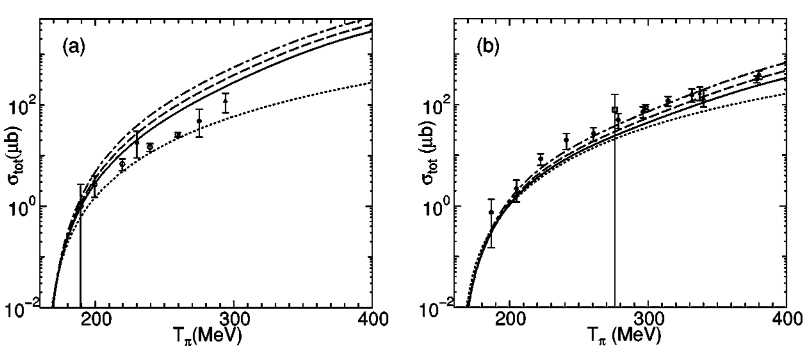

FIG. 4. The dependence of the cross section on the $\pi\Delta\Delta$ interaction; (a) $\pi^+p \to \pi^+\pi^0p$ and (b) $\pi^-p \to \pi^-\pi^0p$. Each line is the same as that of Fig. 3. The data from Refs. [42–44] for $\pi^+p \to \pi^+\pi^0p$ and [29,31,33,36,45,46] for $\pi^-p \to \pi^-\pi^0p$.

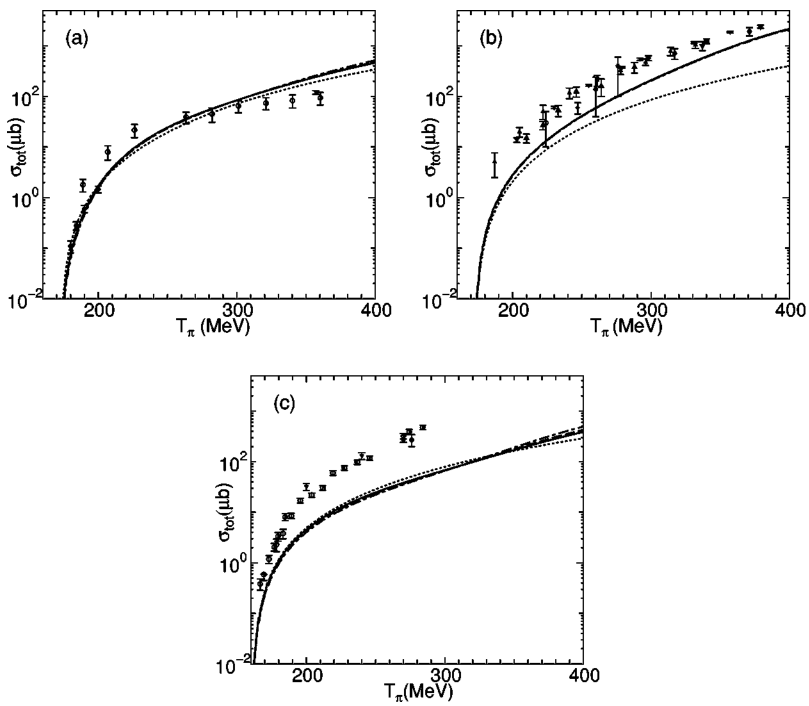

FIG. 5. The dependence of the total cross section on the $\rho N\Delta$ interaction; (a) $\pi^+p\rightarrow\pi^+\pi^+n$, (b) $\pi^-p\rightarrow\pi^+\pi^-n$, and (c) $\pi^-p\rightarrow\pi^0\pi^0n$. It is taken $f_{\rho N\Delta}=0$ for the solid line, $f_{\rho N\Delta}=3.5$ for the dashed line, and $f_{\rho N\Delta}=7.8$ for the dashed-dotted line. The data and the dotted line are the same as in Fig. 3.

Next we show the dependence of each cross section on the value of $f_{\rho N\Delta}$ in Figs. 5 and 6. We do not include the $\pi\Delta\Delta$ interaction instead. We take two values for $f_{\rho N\Delta}$, 3.5 and 7.8, referring to the self-consistent calculation of the $\rho N\Delta$ vertex function [48] and the *NN* phase shift analysis using the meson exchange diagram [49], respectively.

As for the three channels in Fig. 5, the effect of the $\rho N\Delta$ interaction is negligible. This situation is similar to the case of Fig. 3, in which the dependence on the $\pi\Delta\Delta$ interaction is examined. From the results given in Figs. 3 and 5, we can say that the $\pi^+p\rightarrow\pi^+\pi^+n$ and $\pi^-p\rightarrow\pi^0\pi^0n$ channels are insensitive to $\Delta(1232)$ and ρ. Since the experimental data for the $\pi^+p\rightarrow\pi^+\pi^+n$ channel are well reproduced by the theoretical calculation, this channel is saturated by the reaction mechanisms and contains only the pion and the nucleon. This result is in agreement with other calculations [1,5].

On the other hand, the effect of the $\rho N\Delta$ interaction is seen in the two channels in Fig. 6. In the $\pi^+p\rightarrow\pi^+\pi^0p$ channel, the cross section with $f_{\rho N\Delta}=3.5$ and 7.8 around $T_\pi=200$ MeV decreases about 30% in comparison with the cross section with $f_{\rho N\Delta}=0$. In the $\pi^-p\rightarrow\pi^-\pi^0p$ channel, the cross section shows large increase over all values of T_π. Many reports on the value of $f_{\rho N\Delta}$ are settled in the range of $3.5\lesssim f_{\rho N\Delta}\lesssim 7.8$, and the $\pi^-p\rightarrow\pi^-\pi^0p$ channel is sensitive to the variation of $f_{\rho N\Delta}$ in this range.

Here we comment on the Roper resonance. The experimental data for the $\pi^-p\rightarrow\pi^+\pi^-n$ and $\pi^-p\rightarrow\pi^0\pi^0n$ channels cannot be reproduced in this paper. It is known that this failure can be cured by the Roper resonance $N^*(1440)$ with $N^*(1440)\rightarrow N(\pi\pi)_{I=0}$ decay [1,2]. However, this resonance was not included in this paper because we concentrate our attention on the $\Delta(1232)$ resonance. We note that $N^*(1440)\rightarrow N(\pi\pi)_{I=0}$ contribution appears, at tree level, only in $\mathcal{T}_{SA}$ which results from explicit breaking of chiral symmetry. Although several improvements are suggested for the treatment of the Roper resonance [50], the role of this famous resonance is still in question.

VI. SUMMARY

We have calculated the total cross section of the $\pi N\rightarrow\pi\pi N$ reaction up to $T_\pi=400$ MeV and discussed the contribution of $\Delta(1232)$ to this reaction. Applying the chiral reduction formula to the invariant amplitude, we have considered the $\pi\Delta\Delta$ and $\rho N\Delta$ interactions which have not been taken seriously so far. Using the numerical values of the

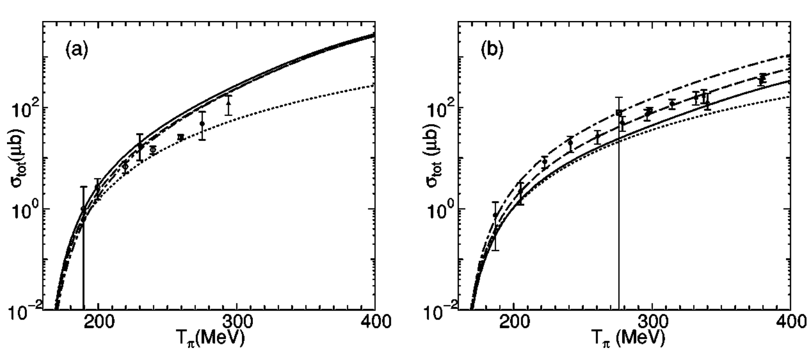

FIG. 6. The dependence of the total cross section on the $\rho N\Delta$ interaction: (a) $\pi^+p\rightarrow\pi^+\pi^0p$ and (b) $\pi^-p\rightarrow\pi^-\pi^0p$. Each line is the same as that of Fig. 5. The data is same as in Fig. 4.

coupling constants given in the past studies, we have shown that the $\pi^{\pm}p \to \pi^{\pm}\pi^0 p$ channels are sensitive to these two interactions. If we hope to determine concrete values of these coupling constants, we need to extend our treatment by, for example, the systematic inclusion of the Roper resonance.

The chiral reduction formula is found to be effective to use to analyze the $\pi N \to \pi\pi N$ reaction. Owing to this formula we can consider the detail of each reaction mechanism separately from the general framework of pion induced reactions constrained by chiral symmetry. This approach has an advantage to gain deeper insight for the nonperturbative features of the hadron physics.

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APPENDIX

In this appendix, we show the phenomenological Lagrangians and the Δ propagator. Some useful relations for the isospin matrices are listed, too.

The Lagrangians are written in the following forms,

$$\mathcal{L}_{\pi NN} = \frac{f_{\pi NN}}{m_\pi}\bar{N}\gamma_\mu\gamma_5\tau^a N\partial^\mu\pi^a, \tag{A1}$$

$$\mathcal{L}_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi}\bar{\Delta}^\nu\Theta_{\nu\mu}(Z)I^a\left(\tfrac{3}{2},\tfrac{1}{2}\right)N\partial^\mu\pi^a + \text{H.c.}, \tag{A2}$$

$$\mathcal{L}_{\pi\Delta\Delta} = \frac{f_{\pi\Delta\Delta}}{m_\pi}\bar{\Delta}^\alpha\Theta_{\alpha\beta}(Z')\gamma_\mu\gamma_5 I^a\left(\tfrac{3}{2},\tfrac{3}{2}\right)\Theta^\beta{}_\delta(Z')\Delta^\delta\partial^\mu\pi^a, \tag{A3}$$

$$\mathcal{L}_{\rho NN} = g_{\rho NN}\bar{N}\left[\gamma_\mu\rho^{\mu a} - \frac{\kappa_V}{2m_N}\sigma_{\mu\nu}\partial^\nu\rho^{\mu a}\right]\frac{\tau^a}{2}N, \tag{A4}$$

$$\mathcal{L}_{\rho N\Delta} = i\frac{f_{\rho N\Delta}}{m_\rho}\bar{\Delta}^\sigma\Theta_{\sigma\mu}(Z'')\gamma_\nu\gamma_5 I^a\left(\tfrac{3}{2},\tfrac{1}{2}\right)N(\partial^\nu\rho^{\mu a} - \partial^\mu\rho^{\nu a}) + \text{H.c.}, \tag{A5}$$

where $(\Delta_\mu)^{\mathrm{T}} = (\Delta_\mu^{++}, \Delta_\mu^{+}, \Delta_\mu^{0}, \Delta_\mu^{-})$ and $N^{\mathrm{T}} = (p, n)$. The second-rank Lorentz tensor $\Theta_{\mu\nu}$ is defined by $\Theta_{\mu\nu}(Y) \equiv g_{\mu\nu} - \frac{1}{2}(1+2Y)\gamma_\mu\gamma_\nu$ (where we take $A=-1$ [51]). The second term of this tensor vanishes if Δ is on the mass shell [because of $\gamma_\mu U^\mu(p)=0$], so that Y is called the off-shell parameter. In this paper, we assume $Z=Z'=Z''=-\frac{1}{2}$ for simplicity, that is, $\Theta_{\mu\nu} \to g_{\mu\nu}$.

The $\Delta(1232)$ propagator is

$$S_{\mu\nu}(p) = \frac{(\not{p} + m_\Delta)}{3(p^2 - m_\Delta^2)}\left[-2g_{\mu\nu} + \frac{2p_\mu p_\nu}{m_\Delta^2} - i\sigma_{\mu\nu} + \frac{\gamma_\mu p_\nu - \gamma_\nu p_\mu}{m_\Delta}\right]. \tag{A6}$$

In order to include the $\Delta(1232)$ width phenomenologically, we modify the denominator of the Δ propagator as $p^2 - m_\Delta^2 \to p^2 - m_\Delta^2 + i m_\Delta \Gamma_\Delta(p^2)$. The width $\Gamma_\Delta(s)$ is [1]

$$\Gamma_\Delta(s) = \Gamma_\Delta\frac{m_\Delta}{\sqrt{s}}\frac{|\mathbf{q}(\sqrt{s})|^3}{|\mathbf{q}(m_\Delta)|^3}\theta(s - (m_N + m_\pi)^2), \tag{A7}$$

where $\mathbf{q} = \mathbf{q}(\sqrt{s})$ is the pion spatial momentum in the center of mass πN system with the total energy $\sqrt{s}$.

Finally we list the following relations for the isospin matrices (see Appendix A in Ref. [2]),

$$I^{a\dagger}\left(\tfrac{3}{2},\tfrac{1}{2}\right)I^b\left(\tfrac{3}{2},\tfrac{1}{2}\right) = \delta^{ab} - \tfrac{1}{3}\tau^a\tau^b, \tag{A8}$$

$$I^{a\dagger}\left(\tfrac{3}{2},\tfrac{1}{2}\right)I^b\left(\tfrac{3}{2},\tfrac{3}{2}\right)I^c\left(\tfrac{3}{2},\tfrac{1}{2}\right) = \tfrac{5}{6}i\varepsilon^{abc} - \tfrac{1}{6}\delta^{ab}\tau^c + \tfrac{2}{3}\delta^{ac}\tau^b - \tfrac{1}{6}\delta^{bc}\tau^a. \tag{A9}$$

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