Staggering behavior of 0^+ state energies in the Sp(4) pairing model

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We explore, within the framework of an algebraic sp(4) shell model, discrete approximations to various derivatives of the energies of the lowest *isovector-paired* 0^+ states of atomic nuclei in the $40 \leqslant A \leqslant 100$ mass range. The results show that the symplectic model can be used to successfully interpret fine structure effects driven by the proton-neutron (*pn*) and like-particle isovector pairing interactions as well as interactions with higher J multipolarity. A finite energy difference technique is used to investigate two-proton and two-neutron separation energies, observed irregularities found around the $N=Z$ region, and the like-particle and *pn* isovector pairing gaps. A prominent staggering behavior is observed between groups of even-even and odd-odd nuclides. An oscillation, in addition to that associated with changes in isospin values, that tracks with alternating seniority quantum numbers related to the isovector pairing interaction is also found.

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I. INTRODUCTION

The observed staggering of energy levels in atomic nuclei requires a theory that goes beyond mean-field considerations [1]. Staggering data contain detailed information about the properties of the nucleonic interaction and suggest the existence of high-order correlations in the collective dynamics. Most studies of staggering focus on two aspects of the phenomena. There are discrete angular momentum dependent oscillations of physical observables; namely, of $M1$ transitions in nuclei [2] or of the energy levels themselves (e.g., in octupole [3–5], superdeformed [6–8], ground, and γ, [1,9,10] bands in atomic nuclei, as well as in molecular rotational bands [11]). And then there are sawtooth patterns of different physical quantities (most commonly binding energies) that track with changes in the number of particles in a system (both in nuclei [12] and in metallic clusters [13,14]).

In nuclear structure physics, staggering behavior of the second type is observed when one changes in a systematic way the usual nuclear characteristics such as proton (Z), neutron (N), mass (A), or isospin projection ($|Z-N|/2$) numbers. Examples of these nuclear phenomena include odd-even mass staggering (OEMS) [1,15–21], odd-even staggering in isotope/isotone shifts [22,23], and zigzag patterns of the first excited 2_1^+ state energies in even-even nuclei [24]. The staggering behavior of a nuclear observable is most easily seen when discrete derivatives of second or higher order in its variable(s) are considered. The aim of this approach is to filter out the strong mean-field (global) effects and in so doing reveal weaker specific features. In this way, for example, the OEMS, which is usually attributed to the nuclear pairing correlations, manifests itself in certain finite differences of the binding energies that can provide for a measure of the empirical pairing gap [1]. Likewise, various discrete approximation of derivatives (filters) of the binding energies can be considered to investigate detailed properties of the nuclear structure [25–32].

In this paper, we consider the binding energies of the 0^+ ground states of even-A nuclei in the mass range $40 \leqslant A \leqslant 100$, and for the rest odd-odd nuclei with a ($J^\pi \neq 0^+$) ground state, the energy of the lowest isobaric analog 0^+ excited state (which corresponds to the ground state of the even-even neighbor). We refer to these states as lowest *isovector-paired* 0^+ states [33]. Our aim is to investigate how various, comparatively small but not insignificant, parts of the interaction between nucleons influence these states when we consider higher-order discrete derivatives of their energies within the framework of a convenient systematics.

The algebraic pairing model [34,33] we exploit is based on a fermion realization of the symplectic sp(4) algebra which is isomorphic to so(5) [35–37]. It includes an isovector (isospin $T=1$) pairing interaction as well as a diagonal (in an isospin basis) proton-neutron (pn) isoscalar ($T=0$) part. The latter is proportional to a so-called $T(T+1)$ symmetry term[1] [33]. The operators of the reduction $\mathrm{sp}(4) \supset \mathrm{u}(2) \supset \mathrm{u}(1) \oplus \mathrm{su}(2)$ provide for a convenient and useful classification of nuclei and their corresponding ground and excited states. The systematics is in terms of the eigenvalues of these operators, namely, the total number n of valence nucleons and the third projection i of the isospin and their linear combinations.

We have already shown in Ref. [33] that the Sp(4) model[2] leads to a good reproduction of the experimental energies [38] of the lowest isovector-paired 0^+ state for even-A nuclei, $32 \leqslant A \leqslant 100$. As pointed out [33], although the $T=1$ like-particle pairing energy and the $T=1$ pn pairing energy yield $\Delta n=2$ staggering patterns that are of opposite phases, the total isovector pairing energy has a smooth behavior. It is the symmetry term that makes an accurate theoretical prediction of the regular zigzag pattern of the experi-

[1]It is common to address the symmetry energy in a slightly different way: the $T(T+1)$ term together with the isospin dependence of the isovector pairing term yield symmetry ($\sim T^2$) and Wigner ($\sim T$) energies.

[2]We distinguish between groups denoted with capital letters [e.g., Sp(4)] and the associated algebras of their generators denoted with lower-case letters [e.g., sp(4)].

mental energies in isobaric sequences possible. As a further and more detailed investigation, we now consider different types of discrete derivatives of the Coulomb corrected [39] energy function according to the Sp(4) classification and with no adjustable parameters.

The symplectic Sp(4) scheme not only allows for a systematic investigation of staggering patterns in the experimental energies of the even-A nuclei, it also offers a simple algebraic model for interpreting the results. Moreover, this detailed investigation serves as a test for the validity and reliability of the Sp(4) model and the interactions it includes.

II. Sp(4) CLASSIFICATION SCHEME

We start with a brief outline of the algebraic approach [34] used to interpret phenomena that have been observed experimentally and are related to the isovector ($T=1$ pairing correlations) and isoscalar interactions in nuclei. The sp(4) algebra is realized in terms of creation $c^{\dagger}_{jm\sigma}$ and annihilation $c_{jm\sigma}$ fermion operators with the standard anticommutation relations $\{c_{jm\sigma}, c^{\dagger}_{j'm'\sigma'}\}=\delta_{j,j'}\delta_{m,m'}\delta_{\sigma,\sigma'}$, where these operators create (annihilate) a particle of type $\sigma=\pm 1/2$ (proton/neutron) in a state of total angular momentum j (half integer) with projection m in a finite space $2\Omega=\Sigma_j(2j+1)$. In addition to the number operator $\hat{N}=\hat{N}_{+1}+\hat{N}_{-1}$ and the third isospin projection $T_0=(\hat{N}_{+1}-\hat{N}_{-1})/2$, the generators of Sp(4) are

$$T_{\pm}=\frac{1}{\sqrt{2\Omega}}\sum_{jm} c^{\dagger}_{jm,\pm 1/2}c_{jm,\mp 1/2}, \quad (1)$$

$$A^{\dagger}_{\mu=\sigma+\sigma'}=\frac{1}{\sqrt{2\Omega(1+\delta_{\sigma\sigma'})}}\sum_{jm}(-1)^{j-m}c^{\dagger}_{jm\sigma}c^{\dagger}_{j,-m,\sigma'},$$

$$A_{\mu}=(A^{\dagger}_{\mu})^{\dagger}, \quad (2)$$

where $\hat{N}_{\pm 1}$ are the valence proton (neutron) number operators, T_0 and $T_{\pm}$ are the valence isospin operators, and the generators $A^{\dagger}_{0,+1,-1}$ create a pn pair, a proton-proton (pp) pair, or a neutron-neutron (nn) pair of total angular momentum $J^{\pi}=0^{+}$ and isospin $T=1$. A totally symmetric finite space is spanned by the basis vectors constructed as ($T=1$)-paired fermions,

$$|n_{+1},n_0,n_{-1})=(A^{\dagger}_{+1})^{n_{+1}}(A^{\dagger}_{0})^{n_0}(A^{\dagger}_{-1})^{n_{-1}}|0\rangle, \quad (3)$$

where $n_{+1,0,-1}$ are the numbers of pairs of each kind, pp, pn, nn, respectively, and $|0\rangle$ denotes the vacuum state. In the like-particle pairing limit, n_0 gives the number of protons (neutrons) not coupled to $J=0$ pp (nn) pairs and hence defines the usual seniority quantum number [40,41], $\nu_1=n_0$. On the other hand, in the pn pairing limit another seniority number, $2\nu_0=2n_{+1}+2n_{-1}$, is recognized that counts the particles not coupled in $J=0$ pn pairs. The dependence of ν_0 on ν_1 within a given nucleus allows one to consider only ν_1 in the analysis; specifically, for a system of n valence particles with isospin projection $i=(Z-N)/2$, the fully paired states (3) differ in their coupling mode as the seniority quantum number $\nu_1(\nu_0=n/2-\nu_1)$ changes by $\pm 2(\mp 2)$.

TABLE I. Realizations of the $\mathrm{u}^{\mu}(2)=\mathrm{u}^{\mu}(1)\oplus\mathrm{su}^{\mu}(2)$ subalgebras of sp(4).

Symmetry (μ)	$\mathrm{u}^{\mu}(1)$ (C_1^{μ})	Eigenvalues of C_1^{μ}	$\mathrm{su}^{\mu}(2)$
Isospin (T)	$\hat{N}$	n	T_+, T_0, T_-
pn pairs(0)	T_0	i	$A_0^{\dagger}, \frac{1}{2}\hat{N}-\Omega, A_0$
pp pairs(+)	$\hat{N}_{-1}$	N_{-1}	$A_{+1}^{\dagger}, \frac{1}{2}(\hat{N}_{+1}-\Omega), A_{+1}$
nn pairs(−)	$\hat{N}_{+1}$	N_{+1}	$A_{-1}^{\dagger}, \frac{1}{2}(\hat{N}_{-1}-\Omega), A_{-1}$

As a dynamical symmetry, the Sp(4) symplectic group describes isovector pairing correlations and isospin symmetry through the four different reduction chains $\mathrm{Sp}(4)\supset\mathrm{U}^{\mu}(2)\supset\mathrm{U}^{\mu}(1)\otimes\mathrm{SU}^{\mu}(2)$ with $\mu=T,0,\pm$ (Table I). The first-order invariant of $\mathrm{u}^{\mu}(2)$, $C_1^{\mu=\{T,0,\pm\}}=\{\hat{N},T_0,\hat{N}_{\mp 1}\}$, realizes the $\mathrm{u}^{\mu}(2)\supset\mathrm{su}^{\mu}(2)$ reduction and reduces the finite action space into a direct sum of unitary irreducible representations of $\mathrm{U}^{\mu}(2)$ ($\{n,i,N_{\mp 1}\}$ multiplets). Within a multiplet the third projection generator of $\mathrm{SU}^{\mu}(2)$ (middle operator in the fourth column in Table I) further reduces the $\mathrm{U}^{\mu}(2)$ representation to a vector with fixed quantum numbers (n,i), or alternatively (N_{+1},N_{-1}), to which corresponds a given nucleus (a cell in Table II). In this way the dynamical Sp(4) symmetry provides for a natural classification scheme of nuclei as belonging to a single-j level or a major shell (multi j), which are mapped to the algebraic multiplets. This classification also extends to the corresponding ground and excited states of the nuclei including their isovector-paired 0^{+} states.

The general model Hamiltonian with Sp(4) dynamical symmetry, which consists of one- and two-body terms, can be expressed through the Sp(4) group generators,

$$H=-GA_0^{\dagger}A_0-F(A_{+1}^{\dagger}A_{+1}+A_{-1}^{\dagger}A_{-1})-E\frac{\left(T^2-\frac{3\hat{N}}{4}\right)}{2\Omega}$$
$$-C\frac{\hat{N}(\hat{N}-1)}{2}-\left(D-\frac{E}{2\Omega}\right)\left(T_0^2-\frac{\hat{N}}{4}\right)-\epsilon\hat{N}, \quad (4)$$

where G,F,E,C, and D are strength parameters and $\epsilon>0$ is the Fermi level energy. This Hamiltonian conserves the number of particles and the third isospin projection, and changes the seniority quantum number ν_1 by zero or ± 2; the latter implies scattering of a pp pair and a nn pair into two pn pairs and vice versa. The isospin breaking Hamiltonian (4) includes an isovector ($T=1$) pairing interaction ($G\geqslant 0, F\geqslant 0$ for attraction) and a diagonal (in an isospin basis) isoscalar ($T=0$) force, which is related to a symmetry term E. Within a shell (a single-j level, $1f_{7/2}$, or a major shell, $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$), a reasonable estimate for the parameters in the Hamiltonian (4) is obtained in a fitting procedure of the maximum eigenvalues of $|H|$, E_0, to the Coulomb corrected [39] experimental energies [38] of the lowest isovector-paired 0^{+} states in even-A nuclei [33]. Although the fits yield quite good agreement with the relevant experi-

TABLE II. Classification scheme of even-A nuclei in the $1f_{7/2}$ shell. The shape of the table is symmetric with respect to the sign of i and $n-2\Omega$. $\Delta n=2$ in each i multiplet (columns), $\Delta i=1$ in each n multiplet (rows), $\Delta N_{\pm 1}=2$ in each $N_{\mp 1}$ multiplet (diagonals). The subsequent action of the $\mathrm{SU}^{\mu}(2)$ generators (shown in brackets) constructs the constituents in a given $\mathrm{SU}^{\mu}(2)$ multiplet ($\mu=T,0,\pm$).

$n \backslash i$	2	1	0	−1	−2	−3	−4
0			${}^{40}_{20}\mathrm{Ca}_{20}$				
2		${}^{42}_{22}\mathrm{Ti}_{20}$	${}^{42}_{21}\mathrm{Sc}_{21}$	${}^{42}_{20}\mathrm{Ca}_{22}$			
4	$\swarrow$	${}^{44}_{23}\mathrm{V}_{21}$	${}^{42}_{22}\mathrm{Ti}_{22}$	${}^{44}_{21}\mathrm{Sc}_{23}$	${}^{44}_{20}\mathrm{Ca}_{24}$		
6	$\cdots$	${}^{46}_{24}\mathrm{Cr}_{22}$	${}^{46}_{23}\mathrm{V}_{23}$	${}^{46}_{22}\mathrm{Ti}_{24}$	${}^{46}_{21}\mathrm{Sc}_{25}$	${}^{46}_{20}\mathrm{Ca}_{26}$	
8	$\leftarrow(T_+)$	${}^{48}_{25}\mathrm{Mn}_{23}$	${}^{48}_{24}\mathrm{Cr}_{24}$	${}^{48}_{23}\mathrm{V}_{25}$	${}^{48}_{22}\mathrm{Ti}_{26}$	${}^{48}_{21}\mathrm{Sc}_{27}$	${}^{48}_{20}\mathrm{Ca}_{28}$
10	$\cdots$	${}^{50}_{26}\mathrm{Fe}_{24}$	${}^{50}_{25}\mathrm{Mn}_{25}$	${}^{50}_{24}\mathrm{Cr}_{26}$	${}^{50}_{23}\mathrm{V}_{27}$	${}^{50}_{22}\mathrm{Ti}_{28}$	
12	$(A^{\dagger}_{-1})\searrow$	$\vdots$	$\downarrow(A^{\dagger}_{0})$	$\vdots$	$\swarrow(A^{\dagger}_{+1})$		

mental values, a reproduction of the fine properties of nuclear structure (typically of an order of magnitude or two less than the energies that were fit) is not guaranteed due to the strong mean-field contribution. For the present investigation these parameters in the energy operator (4) are not varied; their values are fixed as $G=0.53\Omega$, $F=0.45\Omega$, $C=0.47$, $D=-0.97$, $E=-1.12(2\Omega)$, $\epsilon=9.36$ MeV for the $1f_{7/2}$ level (with a ${}^{40}\mathrm{Ca}$ core) and $G=0.35\Omega$, $F=0.30\Omega$, $C=0.19$, $D=-0.80$, $E=-0.49(2\Omega)$, $\epsilon=9.57$ MeV for the $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ shell (with a ${}^{56}\mathrm{Ni}$ core) [33]. In the second case, the parameters of the effective interaction in the Sp(4) model with degenerate multi-j levels are likely to be influenced by the nondegeneracy of the orbits. Nevertheless, as the dynamical symmetry properties of the two-body interaction in nuclei from this region are not lost, the model remains a good multi-j approximation [33] and the extent to which it provides for a realistic description can be further tested with the use of various discrete derivatives of the energy function.

III. DISCRETE DERIVATIVES AND FINE STRUCTURE EFFECTS

The symplectic Sp(4) model [namely, the E_0 maximal eigenvalues of $|H|$ Eq. (4)] reproduces the Coulomb corrected energies of the isovector-paired 0^+ states quite well [33]. A more detailed investigation and a significant test for the theory is achieved through the discrete approximation of the $\partial^m E_0/\partial x^m$ derivatives of the E_0 energy function,

$$Stg^{(m)}_{\delta}(x)=\frac{Stg^{(m-1)}_{\delta}\left(x+\frac{\delta}{2}\right)-Stg^{(m-1)}_{\delta}\left(x-\frac{\delta}{2}\right)}{\delta},\quad m\geqslant 2,$$

$$Stg^{(1)}_{\delta}(x)=\begin{cases}\dfrac{E_0\left(x+\frac{\delta}{2}\right)-E_0\left(x-\frac{\delta}{2}\right)}{\delta}, & m\text{ even}\\[2ex] \dfrac{E_0(x+\delta)-E_0(x)}{\delta}, & m\text{ odd},\end{cases}\tag{5}$$

expressed recursively with two terminating conditions depending on the order m (even or odd) of the derivative, where the variable is $x=\{n,i,N_{+1},N_{-1}\}$ according to the Sp(4) classification and $\delta\geqslant 1$ is a discrete integer step. The present investigation is focused predominantly on the $\delta=1$ or 2 cases [the way the different variables, n,i,N_{+1}, or N_{-1} vary in the Sp(4) systematics can be recognized in Table II].

The first ($m=1$) discrete derivative defined in Eq. (5), $[E_0(x+\delta)-E_0(x)]/\delta$, is related to the δ-particle separation energy, when x counts the (total, proton, or neutron) number of particles. The general $Stg^{(m)}_{\delta}(x)$ quantity represents a finite difference between the E_0 energies of neighboring nuclei, for example,

$$Stg^{(2)}_{\delta}(x)=\frac{E_0(x+\delta)-2E_0(x)+E_0(x-\delta)}{\delta^2},\tag{6}$$

when $m=2$, and

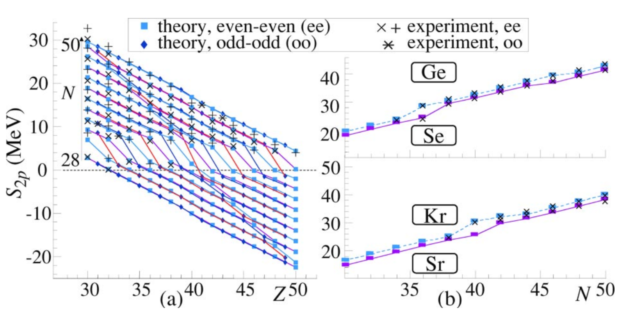

FIG. 1. (Color online) The S_{2p} two-proton separation energy in MeV for the even-even (ee) and odd-odd (oo) nuclei in the $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ major shell (a) vs number of protons for different isotones (N=28–50) (the Coulomb repulsion energy is taken into account); (b) vs number of neutrons for Ge, Se, Kr, Sr isotopes.

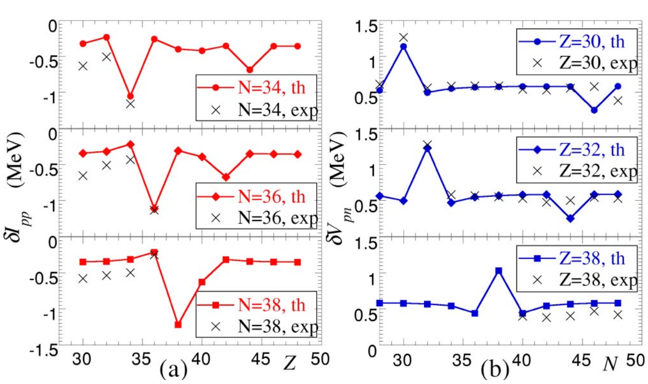

FIG. 2. (Color online) Second discrete derivatives of the E_0 energy ($1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ shell) (a) with respect to N_{+1}, $\delta I_{pp}(N_{+1})$, as an estimation of the nonpairing like-particle nuclear interaction in MeV for the $N=34,36,38$ multiplets; (b) with respect to N_{+1} and N_{-1}, $\delta V_{pn}(N_{+1},N_{-1})$, for Zn, Ge, Sr isotopes.

$$Stg_{\delta}^{(3)}(x) = \frac{E_0(x+2\delta) - 3E_0(x+\delta) + 3E_0(x) - E_0(x-\delta)}{\delta^3}, \tag{7}$$

when $m=3$, and filters out contributions to E_0 proportional to x^{m-1}.

The filters (5) are $(m+1)$-point expressions that account for deviations from the common behavior of neighboring nuclei. When $m \geqslant 3$ the $Stg_{\delta}^{(m)}(x)$ discrete derivative is independent of strong mean-field effects, strictly speaking it cancels out all regularly varying linear and quadratic in x contributions to the energy, which are typically large, and only can provide for a description of higher-order terms in the variable x, as well as for discontinuities in the energy function. In this way, the finite energy difference isolates specific parts of the interaction that are comparatively smaller and may vary substantially from one nucleus to its neighbors. While these interactions do not contribute much to the overall trend of the E_0 energies, they play a very significant role in determining nuclear structure properties.

The mixed derivatives also provide useful information about the nuclear fine structure effects and are defined as

$$Stg_{\delta_1,\delta_2}^{(2)}(x,y) = \frac{E_0(x+\delta_1,y+\delta_2) - E_0(x+\delta_1,y) - E_0(x,y+\delta_2) + E_0(x,y)}{\delta_1\delta_2}, \tag{8}$$

where the variables represent quantities among the set $(x,y)=\{n,i,N_{+1},N_{-1}\}$ and $\delta_{1,2} \geqslant 1$ is a discrete increment in accordance with the Sp(4) classification scheme (Table II).

Different types of discrete derivatives are considered and various staggering patterns are investigated in the following sections. The corresponding components of the interaction isolated through the energy difference filters can be explained in analogous ways as in Ref. [27,28], in addition to the advantage that because they are free of Coulomb effect they reflect phenomena related only to nuclear forces.

A. Discrete derivatives with respect to N_{+1} and N_{-1}: the $N=Z$ region

For even-even nuclei, the discrete approximation of the $\partial E_0^C/\partial N_{\pm 1}$ first derivative of the binding energies (including the Coulomb repulsion energy) is related to the well-known two-proton (two-neutron) separation energy, which is usually defined as $S_{2p(2n)}(N_{\pm 1}) = E_0^C(N_{\pm 1}) - E_0^C(N_{\pm 1}-2)$ [see Fig. 1(a) for a relation to proton number and Fig. 1(b) for the difference of the Coulomb corrected energies E_0 versus neutron number]. The Sp(4) theory reproduces very well the available experimental data [38] (shown as "×" or "+" symbols for even-even nuclei and as "*" for odd-odd nuclei in Fig. 1(a)), especially the irregularity at $N_{+1}=N_{-1}$. The zero point of S_{2p} along an isotone sequence determines the two-proton-drip line (dashed black line in Fig. 1), which according to the Sp(4) model for the $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ major shell lies near the following even-even nuclei:

$$^{60}\mathrm{Ge}_{28}, {}^{64}\mathrm{Se}_{30}, {}^{68}\mathrm{Kr}_{32}, {}^{72}\mathrm{Sr}_{34}, {}^{76}\mathrm{Zr}_{36},$$
$$^{78}\mathrm{Zr}_{38}, {}^{82}\mathrm{Mo}_{40}, {}^{86}\mathrm{Ru}_{42}, {}^{90}\mathrm{Pd}_{44}, {}^{94}\mathrm{Cd}_{46}, \tag{9}$$

beyond which the higher-Z isotones are unstable with respect to diproton emissions. These nuclei are not yet explored as seen in Fig. 1 and an experimental comparison for the two-proton-drip line is expected to be soon possible due to radioactive beam experiments near the limits of stability. Yet, the findings of our model are in close agreement with the results of other theoretical predictions [42–45]. Particularly, the estimate for the two-proton separation energies in Refs. [43–45] confirms the division in nuclides such that the isotones with lower/higher Z values than the nuclei in (9) have positive/negative S_{2p} energies (compare to Fig. 1). In addition, the two-proton separation energies for those of the nuclei in (9) considered also in the other studies are close in their estimates: the quadratic mean of the difference in S_{2p} between our model and Ref. [43] is 0.32 MeV [in a comparison of the first three nuclei in (9)], is 0.78 MeV when all the nuclei in (9) are compared to Ref. [44], and is 0.43 MeV in a comparison to Ref. [45] of the first four nuclei in (9). For odd-odd nuclei the zero point of S_{2p} can be also determined ($^{60}\mathrm{Ga}_{29}$, $^{64}\mathrm{As}_{31}$, $^{68}\mathrm{Br}_{33}$, $^{72}\mathrm{Rb}_{35}$, $^{76}\mathrm{Y}_{37}$, $^{78}\mathrm{Y}_{39}$, $^{82}\mathrm{Nb}_{41}$, $^{86}\mathrm{Tc}_{43}$, $^{90}\mathrm{Rh}_{45}$, $^{94}\mathrm{Ag}_{47}$) although it does not define the drip line, as S_{2p} is a relation of the lowest 0^+ state energies E_0 rather than of the binding energies for most odd-odd nuclei.

As a whole, the higher-order derivatives with respect to proton (neutron) number have a smooth behavior. This is because these derivatives reflect changes only within a sequence of either even-even or odd-odd nuclei. The discretization of the $\partial^2 E_0/\partial N_{\pm 1}^2$ second-order derivative, $4\delta I_{pp(nn)}(N_{\pm 1}) = E_0(N_{\pm 1}+2) - 2E_0(N_{\pm 1}) + E_0(N_{\pm 1}-2)$ [$=4Stg_2^{(2)}(N_{\pm 1})$, Eq. (5)], accounts for the interaction between the last two pp (nn) pairs in the $(N_{\pm 1}+2)$ nucleus [Fig. 2(a)]. The average interaction $\delta I_{pp(nn)}$ may be used as an alternative way to Ref. [28] to estimate the nonpairing like-particle interaction[3] [of the last two protons (neutrons)]. It shows no outlined staggering pattern but a repulsive peak around the $N=Z$ nuclei in very good agreement with the experiment

[3]The meaning of "nonpairing" relates to $J \neq 0$ and $T \neq 1$ interaction or any interaction that is different from the isovector pairing. Also, here the approximation is of $O(1/\Omega)$.

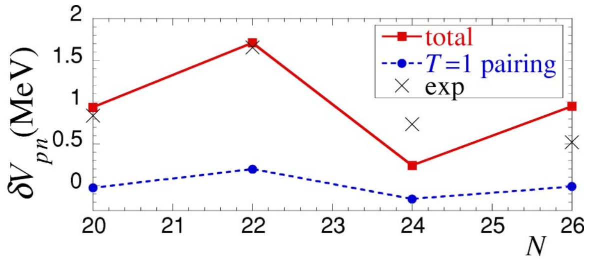

FIG. 3. (Color online) δV_{pn} in MeV of the total binding energy (■) and of the $T=1$ pairing energy (•) in comparison to experiment (×) for Ti isotopes in the $1f_{7/2}$ shell. The isovector pairing interaction is not enough to reproduce the experimental peak at $N=Z$.

[38] and with the results and discussions of Ref. [28]. Another smaller peak is observed around midshell [Fig. 2(a)], which is due to the particle-hole discontinuity introduced in the pairing theory. The analysis yields that as a whole the Sp(4) model reproduces the fine structure effects in interactions isolated via the $Stg_2^{(2)}(N_{\pm 1})$ filters.

Another aspect of the nuclear interaction is revealed by the second-order discrete mixed derivative of the energy [46], $\delta V_{pn}(N_{+1},N_{-1})=[E_0(N_{+1}+2,N_{-1}+2)-E_0(N_{+1}+2,N_{-1})-E_0(N_{+1},N_{-1}+2)+E_0(N_{+1},N_{-1})]/4$, Eq. (8). For even-even nuclei it was found to represent the residual interaction between the last proton and the last neutron [27,47] and it was empirically approximated by $40/A$ [29]. The theoretical discrete derivative [Fig. 2(b)] agrees remarkably well with the experiment [38], especially in reproducing the typical behavior at $N_{+1}=N_{-1}$, and is consistent with the empirical trend (on average, ~ 0.71 for $1f_{7/2}$ and ~ 0.52 for the major shell above the ^{56}Ni core). It is well known that the attractive peak in the self-conjugate nuclei cannot be described by a model with an isovector interaction only [47] and in this respect our model achieves this result due to the additional terms included in the Hamiltonian, mainly the symmetry term (Fig. 3). The δV_{pn} energy difference provides for a powerful test for the symplectic model: the theory not only gives a thorough description of the isovector *pn* and like-particle pairing but additionally accounts for $J>0$ components of the *pn* interaction in a consistent way with the experiment. As a result the model can be used to provide for a reasonable prediction of δV_{pn} of proton-rich exotic nuclei as well as odd-odd nuclei.

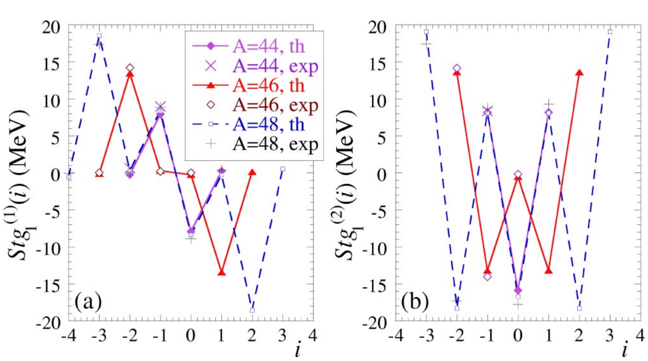

FIG. 4. (Color online) The $Stg_1^{(1,2)}(i)$ discrete derivatives for different isobaric multiplets for even-A nuclei with valence nucleons in the $1f_{7/2}$ shell with a core ^{40}Ca.

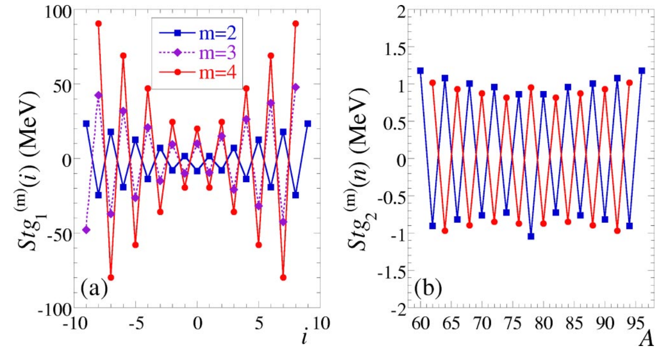

FIG. 5. (Color online) Discrete derivatives $Stg_\delta^{(m)}(i)$ ($1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$, a ^{56}Ni core): (a) $\delta=1$, $m=2,3,4$ for $A=76$ isobars; (b) $\delta=2$, $m=2,4$ for $(i=-1)$ multiplet $[N=Z+2]$.

B. Discrete derivatives with respect to n and i: staggering behavior

The Sp(4) classification scheme can also be used to investigate energy differences with respect to the total number of particles n and their isospin projection i. Indeed, in contrast with the typical smooth behavior observed for discrete derivatives with respect to N_{+1} and N_{-1} that was highlighted in the preceding section, the derivatives with respect to n and i are the ones that reveal distinct staggering effects. They give a relation between even-even (*ee*) and odd-odd (*oo*) nuclei and the patterns can be referred as an "*ee*−*oo*" staggering.

1. Second- and higher-order derivatives in one variable

The discrete derivatives, $Stg_1^{(m)}(i)$, $m=1,2,\ldots$, show a prominent $\Delta i=1$ staggering of the experimental energies [38] of the lowest 0^+ isovector-paired states for different isobaric multiplets [see Fig. 4 for the $1f_{7/2}$ shell and Fig. 5(a) for nuclei above the ^{56}Ni core]. The theory reproduces this staggering very well.

For each of the i multiplets (i fixed), a $\Delta n=2$ staggering effect is also observed for the experimental values [38] via the energy filters $Stg_2^{(m)}(n)$, $m=1,2,\ldots$, and successfully predicted by the symplectic model [Fig. 6 ($1f_{7/2}$) and Fig. 5(b) ($1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$)].

The staggering amplitudes of both $Stg_1^{(m)}(i)$ and $Stg_2^{(m)}(n)$, while almost independent of the total number of

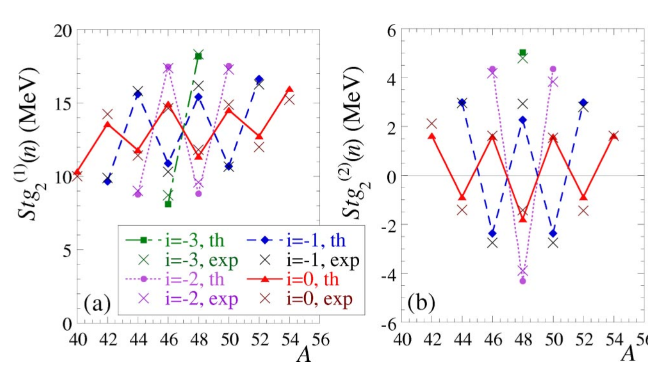

FIG. 6. (Color online) Discrete derivatives $Stg_2^{(1,2)}(n)$ for different i multiplets for even-A nuclei ($1f_{7/2}$, a ^{40}Ca core).

particles n, increase with increasing difference in proton and neutron numbers, i, and hence the $ee-oo$ staggering effect is greater for the proton- (neutron-) rich nuclei than around $N \approx Z$. Also, the amplitude of $Stg_1^{(m)}(i)$ increases in higher-order derivatives. This analysis shows a more complicated dependence of the energy function on the isospin projection i than on the mass number A.

The first, $m=1$, discrete derivative, $S_{pn}=2Stg_2^{(1)}(n)=E_0(n+2)-E_0(n)$, where i is fixed, corresponds to the energy gained when a $T=1$ pn pair is added [Fig. 6(a) ($1f_{7/2}$) and Fig. 7 (a ^{56}Ni core)]. S_{pn} is the true pn separation energy only when E_0 is the binding energy of the odd-odd nucleus involved in its calculation. The experimental data, where available [38], are also shown in Fig. 7 and the Sp(4) model follows the distinctive zigzag pattern very well. A $\Delta n=4$ bifurcation separates the nuclei into two groups: one of even-even nuclei [$(n/2+i)$ even] and another of odd-odd nuclei [$(n/2+i)$ odd]. The S_{pn} energy difference has a smooth behavior within each group. The magnitude of S_{pn} is proportional to the total number of particles and increases (decreases) with i for odd-odd (even-even) nuclei (Fig. 7).[4] Furthermore, the $Stg_4^{(1)}(n)=[Stg_2^{(1)}(n+2)+Stg_2^{(1)}(n)]/2$ energy difference shows no $\Delta n=4$ staggering (average values of two consecutive data points in Fig. 7). This indicates that the addition of an α-like cluster has almost the same effect for both even-even and odd-odd nuclei. This statement does not contradict the stronger binding of even-pairs nuclei as compared to odd-pairs ones, which is detected via S_{pn} and the binding energy (BE) filter, $\mathrm{BE}(Z+2,N+2)-[\mathrm{BE}(Z+2,N)+\mathrm{BE}(Z,N+2)]/2$ [26].

2. Pairing gaps

The $Stg_1^{(m)}(i)$ and $Stg_2^{(m)}(n)$ energy differences, $m=1,2,\ldots$, described above, isolate effects related to the various types of pairing in addition to nonmonopole interactions resulting in changes in energy due to the different isospin values (symmetry term). As noted in [27,28], the significance of the various energy filters can be understood using phenomenological arguments that can be given by a simple and useful graphical representation. Specifically, each nucleus can be represented by an inactive core, schematically illustrated by a box $\square$, in which the interaction between the constituent particles does not change. Active particles beyond this core can be represented by solid or empty dots, for protons or neutrons, above the box.

The second-order filter

$$Stg_1^{(2)}(i)=E_0(i+1)-2E_0(i)+E_0(i-1)=E_0(N_{+1}+1,N_{-1}-1)-2E_0(N_{+1},N_{-1})+E_0(N_{+1}-1,N_{-1}+1),\quad n=\text{const}, \tag{10}$$

when centered at an odd-odd [$(n/2+i)$ odd] self-conjugate ($i=0$) nucleus, represents the pairing gap relation $2\tilde{\Delta}$,

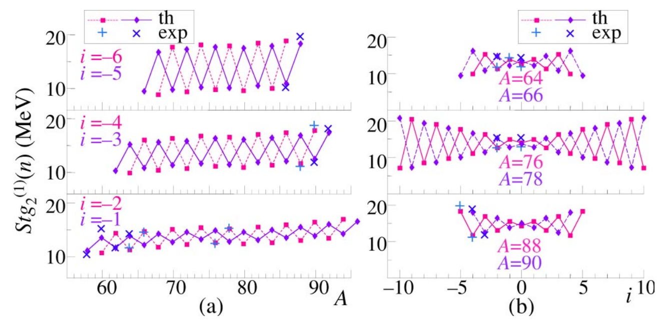

FIG. 7. (Color online) The $Stg_2^{(1)}(n)$ discrete approximation of the first derivative $\partial E_0/\partial n$ (^{56}Ni core) with respect to (a) A for several i multiplets and (b) i for different isobars.

$$Stg_1^{(2)}(i=0) \overset{(n/2-odd)}{=} \overset{\bullet\bullet}{\square}+\overset{\circ\circ}{\square}-2\overset{\bullet\circ}{\square}\approx 2\tilde{\Delta}\equiv 2\Delta_{pp}+2\Delta_{nn}-4\Delta_{pn}. \tag{11}$$

The result (11) follows from the well-known definition of the empirical like-particle pairing gap [1],

$$\Delta_{pp(nn)}\equiv\tfrac{1}{2}\{\mathrm{BE}(N_{+1}\pm 1,N_{-1}\mp 1)-\mathrm{BE}(N_{+1}-1,N_{-1}-1)-2[\mathrm{BE}(N_{\pm 1},N_{\mp 1}-1)-\mathrm{BE}(N_{+1}-1,N_{-1}-1)]\}=\tfrac{1}{2}(\overset{\bullet\bullet}{\square}-\square-2[\overset{\bullet}{\square}-\square]), \tag{12}$$

which isolates the isovector pairing interaction of the $(N_{\pm 1})$th and $(N_{\pm 1}+1)$th protons (neutrons) for an even-even $(N_{+1}-1,N_{-1}-1)$ core (marked by a square) [28]. We also define the pn isovector pairing gap

$$\Delta_{pn}\equiv\tfrac{1}{2}\{E_0(N_{+1},N_{-1})-\mathrm{BE}(N_{+1},N_{-1}-1)-[\mathrm{BE}(N_{+1}-1,N_{-1})-\mathrm{BE}(N_{+1}-1,N_{-1}-1)]\}=\tfrac{1}{2}(\overset{\bullet\circ}{\square}-\overset{\bullet}{\square}-[\overset{\circ}{\square}-\square]) \tag{13}$$

as the pairing interaction of the (N_{+1})th proton and the (N_{-1})th neutron. In order to account correctly for the $T=1$ mode of the pn pairing one should consider in Eq. (13) the E_0 energy of the odd-odd (N_{+1},N_{-1}) nucleus (that is, the energy of the isobaric analog state rather than its ground state energy, BE). For the remaining even-even nuclei in Eq. (10), replacing the symbol E_0 with BE is justified. In the computation of $\tilde{\Delta}$, all odd-A binding energies in Eqs. (12) and (13) cancel so their theoretical calculation is not required.

The $\tilde{\Delta}$ relation of the gaps is a measure of the difference in the isovector pairing energy between even-even and odd-odd nuclei. For odd-odd $N=Z$ nuclei information about $\tilde{\Delta}$ is extracted via the $Stg_1^{(2)}(i)$ energy filter (10). Both experimental and model estimations yield $\tilde{\Delta}\cong 0$ for all the odd-odd $i=0$ nuclei in the $1f_{7/2}$ shell [for example, see solid (purple) line with open squares in Fig. 8 for $A=46$, $i=0$]. The result reflects the fact that in this case all three isovector pairing gaps Δ_{pp}, Δ_{nn}, and Δ_{pn} are equal [31,32].

[4] When $(n/2+i)$ corresponds to an odd-odd nucleus S_{pn} is related to the properties of the even-even $(n+2)$ nucleus.

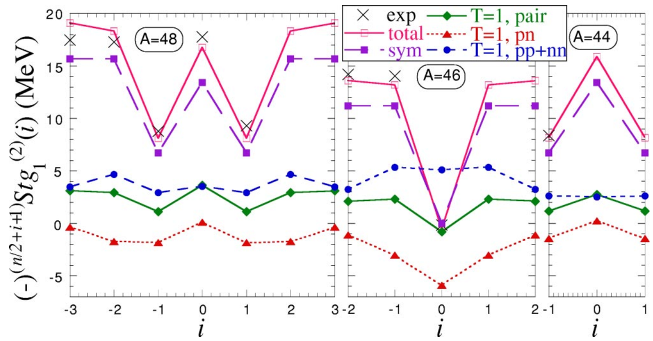

FIG. 8. (Color online) Theoretical staggering amplitudes for the total energy in comparison to experiment [38] for the isovector pairing energy, the *pn* and the like-particle pairing energies, and for the symmetry energy for $A=48$, $A=46$ and $A=44$ nuclei in the $1f_{7/2}$ shell (a ^{40}Ca core).

A different scenario regarding two aspects is encountered when one considers the $Stg_1^{(2)}(i)$ discrete derivative centered at an even-$(n/2+i)$ $N=Z$ nucleus [relative to a $(N_{+1}-2,N_{-1}-2)$-core]:

$$Stg_1^{(2)}(i=0) = \square + \square - 2\square, \quad \left(\frac{n}{2}+i\right) \text{ even}$$

$$\approx -\tfrac{2}{3}\tilde{\Delta} + I_2^{J\neq 0,T\neq 1}, \qquad (14)$$

where an additional nonpairing two-body interaction $I_2^{J\neq 0,T\neq 1}$ is not filtered out in this case. Here, for example, $I_2^{J\neq 0,T\neq 1}$ is related to the nonpairing interaction of the three protons and of the three neutrons in the odd-odd nuclei (14). Another new feature of Eq. (14) is that $Stg_1^{(2)}(i=0)$ does not simply account for the energy gained when two *pn* pairs are created (in the first two odd-odd nuclei) and the energy lost to destroy a *pp* pair and a *nn* pair in the even-even $N=Z$ nucleus. The straightforward reason is that *pp*, *nn*, and *pn* $T=1$ pairs coexist. A good approximation that serves well in estimating the pairing gaps is to assume that a $2p$-$2n$ formation above the inactive core ($\square$) consists of $n_0=2/3$ *pn* pairs, $n_1=2/3$ *pp* pairs, and $n_{-1}=2/3$ *nn* pairs [rather than a proton pair ($n_1=1$) and a neutron pair ($n_{-1}=1$)]. This is in analogy to an even-even $n=4$ nucleus where the *pp*, *nn*, and *pn* "numbers of pairs" are the same and equal to one-third the total number of pairs, $n/2$ [37,33]. Additionally, the relations, such as, Eqs. (11)–(14), are based on the assumptions that the interaction of a particle with the core is independent of the type of added/removed particles and is the same for all protons (neutrons) above the core. Finally, all the approximations are of an order $O(1/\Omega)$.

The additional nonmonopole two-body residual interaction $I_2^{J\neq 0,T\neq 1}$ should be also taken into account for the rest $i\neq 0$ of the (*ee*, and *oo*) nuclei:

$$Stg_1^{(2)}(i\neq 0) \approx \begin{cases} -\tfrac{4}{3}\tilde{\Delta} + I_2^{J\neq 0,T\neq 1}, & ee \\ \tfrac{4}{3}\tilde{\Delta} + I_2^{J\neq 0,T\neq 1}, & oo. \end{cases} \qquad (15)$$

The main contribution to the $I_2^{J\neq 0,T\neq 1}$ interaction is due to the symmetry energy as is apparent from the Sp(4) model.

The very close theoretical reproduction of the experimental staggering allows us to use the symplectic model as a microscopic explanation of the observed effects through the investigation of the different terms in the Hamiltonian (4) (Fig. 8). According to the Sp(4) model, the *ee*-*oo* staggering patterns appear due to the discontinuous change of the seniority numbers driven by the $T=1$ pairing interaction [33]. Even values of the seniority quantum number ν_1 in even-even nuclei and odd values for odd-odd nuclei lead to a change in *pn* and like-particle pairing energies in opposite directions. After the contribution from the isovector pairing energy is taken away, the theoretical staggering amplitude, $(-)^{n/2+i+1}Stg_1^{(2)}(i)$, has still a (typically large) component from the remaining $(J\neq 0,T\neq 1)$ interactions in the Hamiltonian (4), mainly the symmetry (T^2) term [Fig. 8, long-dashed (purple) line with squares]. This is the same nonmonopole nuclear interaction, $I_2^{J\neq 0,T\neq 1}$, that was suggested in Eqs. (14) and (15) using phenomenological arguments. Indeed, the symmetry energy contribution is significant and nonzero in all nuclei but the odd-odd $N=Z$ (4) (Fig. 8), which is consistent with the discussion above [Eqs. (11), (14), and (15)]. Also, an estimation of the pairing gaps is possible based on the examination of the model Hamiltonian but the theoretical staggering amplitudes of the $T=1$ pairing energies (shown in Fig. 8) need to be rescaled in accordance with Eqs. (11), (14), and (15).

In a way analogous to that used in Eq. (15), the second-order discrete derivative with respect to n (can be compared to the filter used in Ref. [31]),

$$Stg_2^{(2)}(n) = \frac{E_0(n+2) - 2E_0(n) + E_0(n-2)}{4}, \quad i = \text{const}, \qquad (16)$$

is related to the pairing gap relation

$$Stg_2^{(2)}(n) \approx \begin{cases} -\dfrac{\tilde{\Delta}}{3} + I_2^{J\neq 0,T\neq 1}, & ee \\ \dfrac{\tilde{\Delta}}{3} + I_2^{J\neq 0,T\neq 1}, & oo, \end{cases} \qquad (17)$$

where in the odd-odd case, for example, $I_2^{J\neq 0,T\neq 1}$ is the nonpair interaction of the last two protons with the last two neutrons in the $(n+2)$ nucleus. The effects due to $\tilde{\Delta}$ cannot be isolated via Eq. (17) because of the additional nonzero contribution due to the symmetry energy. However, the staggering amplitude of the discrete derivative (16), $-3(-)^{n/2+i}Stg_2^{(2)}(n)$, of the theoretical total, *pp* (*nn*), and *pn* pairing energies can provide for estimation of the pairing gaps $\tilde{\Delta}$, $\Delta_{pp(nn)}$, and $-2\Delta_{pn}$, respectively [Fig. 9(a)]. The like-particle pairing gap can be compared to the empirical value of $\Delta_{pp}+\Delta_{nn}=24/A^{1/2}$ [1] [solid (purple) line]. The gap is smaller in odd-odd nuclei as compared to their even-even neighbors. This is a consequence of a decrease in the like-particle pairing energy in the odd-odd nuclei due to the blocking effect while there is an increase in energy due to the *pn* pairing. The *pn* isovector pairing gap increases

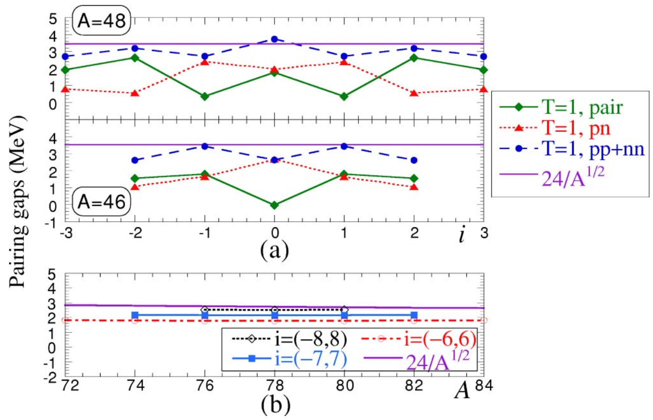

FIG. 9. (Color online) Estimation of the pairing gaps: (a) total isovector pairing gap $\widetilde{\Delta}$, $2\Delta_{pn}$, and $\Delta_{pp}+\Delta_{nn}$, as well as the empirical like-particle pairing gap $\Delta_{pp}+\Delta_{nn}=24/A^{1/2}$ shown for comparison, for $A=48$ and $A=46$ nuclei vs i ($1f_{7/2}$ shell); (b) like-particle pairing gap [according to Eq. (18)] vs A for $i=\pm 6,\pm 7,\pm 8$ multiplets in the $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ shell.

toward $i=0$ and eventually gets almost equal to $\Delta_{pp(nn)}$ for odd-odd nuclei around the $N=Z$ region, which is in agreement with the discussion of Refs. [31,32].

Furthermore, an average of the additional nonpair interaction is achieved by the fourth-order derivatives both in $n[Stg_2^{(4)}(n)]$ and $i[Stg_1^{(4)}(i)]$:

$$\widetilde{\Delta}_{|i|\neq 0,1} \approx \frac{3}{16}(-)^{n/2+i}[Stg_1^{(4)}(i) - I_2^{J\neq 0,T\neq 1}] \qquad (18)$$

$$\approx 3(-)^{n/2+i}[Stg_2^{(4)}(n) - I_2^{J\neq 0,T\neq 1}]. \qquad (19)$$

Assuming that the pn pairing gap is negligible for high-i nuclei in large shells, such as the $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ major shell, the gap relation (18) or (19) provides for a rough estimation of the like-particle pairing gaps. With the use of the model Hamiltonian (4) we can estimate the additional $I_2^{J\neq 0,T\neq 1}$ interaction with the major input being the symmetry

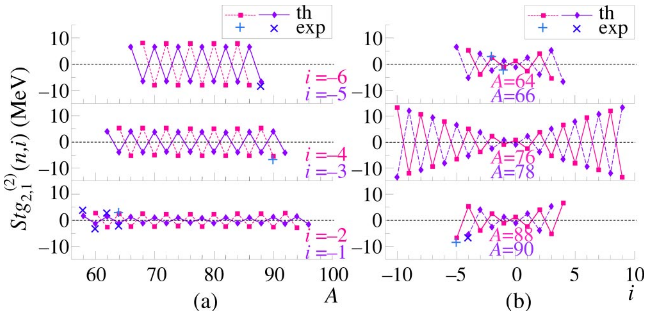

FIG. 10. (Color online) Second-order energy filter $Stg_{2,1}^{(2)}(n,i)$ for nuclei above the ^{56}Ni core with respect to A (a) and i (b).

energy. Although the existence of a very small mixing of isospin values complicates the computation of the symmetry energy for nuclear systems with very large interaction matrices, as a very good approximation one may use $E_{sym,T} = (E/2\Omega)T(T+1)$ with isospin values $T=|i|$ for even-even nuclei and $T=|i|+1$ for odd-odd nuclei. Once the fourth-order discrete derivative (5) of the approximated symmetry energy is removed from $Stg_1^{(4)}(i)$, Eq. (18), the like-particle pairing gaps $\Delta_{pp}+\Delta_{nn}$ are found to be in a very good agreement with the experimental approximation of $24/\sqrt{A}$ for the ($i=\pm 6,\pm 7,\pm 8$) multiplets in the $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ major shell [Fig. 9(b)]. For lower $|i|$ values the difference increases due to an increase in the pn pairing gap as mentioned above. As a whole, the agreement would not be possible if the significant energy contribution due to the symmetry energy was not taken into account.

3. Second-order mixed derivatives

Next we consider the second-order discrete mixed derivative of the relevant energies with respect to the total number n and the third projection i:

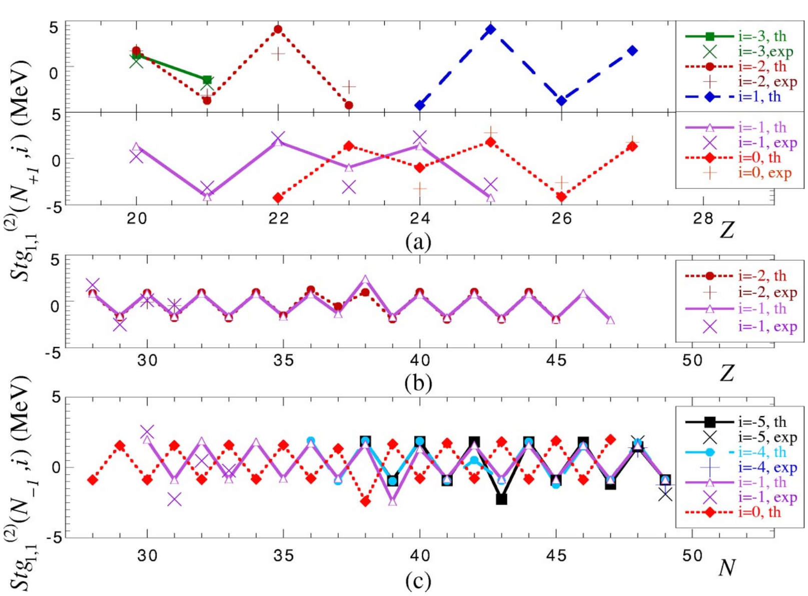

FIG. 11. (Color online) Discrete derivative, $Stg_{1,1}^{(2)}(x,i)$, for various i multiplets for even-A nuclei: (a) $x=N_{+1}$, $1f_{7/2}$ level; (b) $x=N_{+1}$, $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ shell; (c) $x=N_{-1}$, $1f_{(5/2)}2p_{(1/2,3/2)}1g_{(9/2)}$ shell.

$$Stg^{(2)}_{2,1}(n,i) = \frac{E_0(n+2,i+1) - E_0(n+2,i) - E_0(n,i+1) + E_0(n,i)}{2} \quad (20)$$

$$\approx \begin{cases} \frac{2}{3}\tilde{\Delta} + I_2^{J\neq 0,T\neq 1}, & ee \\ -\frac{2}{3}\tilde{\Delta} + I_2^{J\neq 0,T\neq 1}, & oo, \end{cases} \quad (21)$$

where in addition to the pairing gaps relation, $\tilde{\Delta}$, there is the contribution due to the nonpairing interaction, $I_2^{J\neq 0,T\neq 1}$. For example, for the odd-odd (even-even) case it is the positive (negative) nonpairing average interaction between the last three protons (neutrons) in the $[n+2(n),i+1]$ nucleus with a $[n-2(n-4),i]$ core. Within the Sp(4) framework the additional nonpairing contribution corresponds to the staggering of the symmetry energy approximation, $E_{sym,T}$, of $(-)^{n/2+i+1}(E/2\Omega)(2|i|+3)$.

The filter (20) isolates fine structure effects between two i multiplets [Fig. 10(a)] and two consecutive isobaric sequences [Fig. 10(b)]. Clearly, it reveals a $\{\Delta n,\Delta i\}=\{2,1\}$ symmetric oscillating pattern as it is observed in the experiment [38]. Its positive (negative) value is centered at even-even (odd-odd) nuclei and its amplitude increases (decreases) with $|i|$. This mixed discrete derivative (20) serves as another test for the Sp(4) model and allows for a detailed investigation of the nonpairing, like-particle interactions involved.

To isolate the effect of nonpairing interactions (again, it is understood to order $1/\Omega$), an energy difference with respect to both $N_{\pm 1}$ and i can be considered. The second discrete derivative of the energy,

$$Stg^{(2)}_{1,1}(N_{\pm 1},i) = \frac{E_0(N_{\pm 1}+1,i+1) - E_0(N_{\pm 1}+1,i) - E_0(N_{\pm 1},i+1) + E_0(N_{\pm 1},i)}{2}, \quad (22)$$

represents the negative (positive) nonpairing two-body interaction of the last two neutrons (protons) with a proton and a neutron in the $[N_{\pm 1}+1,i(+1)]$ nucleus. It shows prominent $\Delta i=1$ staggering patterns for different i multiplets (Fig. 11). While in the framework of the Sp(4) model its amplitude does not depend on $N_{\pm 1}$ and i except for irregularities around the midshell, the magnitude of the few experimental values [38] (where data exist) tends to be slightly lower away from the closed shell. As a whole, the results show that the staggering behavior of this interaction is due to the fine structure features in the relationship between the like-particle and *pn* nonpairing interactions and differs between proton-rich and neutron-rich nuclei.

Regarding Eq. (22) and the other discrete approximations of the derivatives in Sec III B, it is clear that the oscillating patterns that exist and their regular appearance throughout the nuclear chart cannot be a simple artifact due to errors in the experimental or theoretical energies. Even more, the staggering amplitudes are usually (very) large compared to the energy uncertainties.

For all the discrete derivatives that we have investigated above and that show *ee-oo* staggering behavior, the discontinuity of the symmetry term (due to discrete changes in the isospin value) plays an important role. In contrast, when these discrete derivatives include states of odd-odd nuclei with a dominant $T=0$ *pn* coupling there is a constant or no contribution due to the symmetry energy, and hence yield patterns of different shapes and interpretations. Our investigation does not aim to account for such effects. It is focused on the *ee-oo* staggering behavior of the E_0 energies of the lowest isovector-paired states as observed from the experimental data and reproduced remarkably well by the Sp(4) model.

IV. CONCLUSIONS

A dynamical Sp(4) symmetry was used to provide for a natural classification scheme of nuclei and to describe isovector pairing correlations and high-J interactions. In a previous study [33], it was found that the Sp(4) model reproduced reasonably well the experimental energies of the lowest isovector-paired 0^+ states and provided for an estimation of the interaction strength parameters.

Here the sp(4) algebraic approach has been further tested through second- and higher-order discrete derivatives of the energies of the lowest isovector-paired 0^+ states in the Sp(4) systematics, without any parameter variation. If reality were only a mean-field theory, none of the finite energy differences would reveal regular or irregular staggering effects. The reason is that any effect due to a smoothly varying mean-field part of the nuclear interaction is either entirely canceled out in a finite energy difference filter or contributes regularly to the isolated part of the interaction. Indeed, the results obtained show that this is not the case and staggering behavior is observed. The theoretical discrete derivatives investigated not only followed the experimental patterns but their magnitude was also found to be in a remarkable agreement with the data. The proposed model successfully interpreted the following: the two-proton (two-neutron) separation energy $S_{2p(2n)}$ (hence determined the two-proton drip line) for even-even nuclei, the S_{pn} energy difference when a *pn* $T=1$ pair is added, the observed irregularities around $N=Z$, the prominent *ee-oo* staggering when even-even and odd-odd nuclides are considered simultaneously, the like-particle and *pn* isovector pairing gaps, and the large contribution to the finite energy differences due to the symmetry term. The oscillating effects, where observed, were found to develop due to the discontinuity of the seniority numbers for the *pn* and like-particle isovector pairing, which is in addition to the larger staggering due to the discontinuous change in isospin values (symmetry term) between even-even and odd-odd nuclei.

We found a finite energy difference that, for a specific case, can be interpreted as an isovector pairing gap $\tilde{\Delta}=\Delta_{pp}+\Delta_{nn}-2\Delta_{pn}$, which is related to the like-particle and *pn* isovector pairing gaps. They correspond to the $T=1$ pairing mode because we do not consider the binding energies for all the nuclei but the respective isobaric analog 0^+ states for the odd-odd nuclei with a $J\neq 0^+$ ground state. This investigation is the first of its kind. Moreover, the relevant energies are corrected for the Coulomb interaction and therefore the isolated effects reflect solely the nature of the nuclear interaction.

The outcome of this investigation shows that, in comparison to the experiment, the simple Sp(4) model reproduces not only global trends of the relevant energies but as well the smaller fine structure effects driven by isovector pairing correlations and higher-*J* *pn* and like-particle nuclear interactions. In particular, the sp(4) algebraic model was used to interpret specific phenomena revealed in finite energy differences and to investigate the contribution of the underlying interactions. In this way, it provides for an estimation of the isovector pairing gaps. For $N=Z$ odd-odd nuclei all three pairing gaps were found equal while the *pn* pairing was found to weaken relative to the like-particle pairing strengths with increasing proton (neutron) excess. The like-particle pairing gaps were found to be in a good agreement with the empirical value of $12/\sqrt{A}$. Additionally, the discrete derivatives give insight into particular small parts of the various non-$(J=0,T=1)$ interactions, mainly into the detailed contribution of the interaction related to the $T(T+1)$ term (symmetry energy). Small deviations from the experimental data are attributed to other two-body interactions or higher-order correlations that are not included in the theoretical model.

We explored independent finite energy differences based on a simple sp(4) algebraic classification scheme. The results suggest that this theoretical framework can be used to reproduce various experimental results including observed staggering behavior in fine structure effects of nuclear collective motion.

ACKNOWLEDGMENTS

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