$T=3/2$ levels in $A=11$ nuclei

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Potential-model calculations for the energies and widths of $T=3/2$ levels in $A=11$ nuclei have previously been used to argue that the $1/2^+, T=3/2$ levels in ^{11}B and ^{11}C have been misidentified. Here we repeat these calculations using different, more reasonable, definitions for the energy and width of an unbound level, and also using a many-channel R-matrix model. We find reasonable agreement with the experimental energies of the $1/2^+, T=3/2$ levels in ^{11}B and ^{11}C but possible disagreement in their widths.

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I. INTRODUCTION

The ground state of ^{11}Be has $J^\pi=1/2^+$, and the first two excited states at 0.32 and 1.78 MeV are $1/2^-$ and (presumably) $5/2^+$ [1], respectively. Corresponding $T=3/2$ levels should occur in ^{11}B, ^{11}C, and ^{11}N, although isospin mixing with $T=1/2$ levels is possible in ^{11}B and ^{11}C. Such $T=3/2$ levels have been identified in ^{11}B at 12.56 MeV, 12.92 MeV, and 14.34 MeV, in ^{11}C at 12.16 MeV (presumably $1/2^+$), 12.51 MeV, and 13.90 MeV (presumably $5/2^+$) [1], and at somewhat uncertain energies in ^{11}N (see below). Widths of these levels that are unbound have also been measured [1].

Sherr and Fortune (hereafter referred to as SF) [2] used a Woods-Saxon (W-S) potential model, with conventional values of the radius parameter and diffuseness, and depths adjusted to fit the energies of the ^{11}Be levels, to predict the energies of the $T=3/2$ levels in the other three nuclides; likewise, they used this model to calculate single-particle widths and, by fitting experimental widths, they obtained values of the spectroscopic factors. For the $1/2^-$ and $5/2^+$ levels, SF find satisfactory agreement between predicted and experimental energies and between the spectroscopic factors in the four nuclides. For the $1/2^+$ levels, however, there is disagreement in the values of both the energies and the spectroscopic factors for ^{11}B and ^{11}C, though possible agreement for ^{11}N. SF suggest that the $1/2^+, T=3/2$ levels in ^{11}B and ^{11}C have been misidentified, and that the correct levels have not yet been observed.

The 12.56 MeV level of ^{11}B and the 12.16 MeV level of ^{11}C have each been seen in various reactions, with consistent values for the energy and width (see Tables 11.15 and 11.18 of Ref. [1]). The $T=3/2$ assignments for these levels were made partly because their energy separations from the other $T=3/2$ levels agreed with those in ^{11}Be, and partly because most of the reactions in which these levels have been seen are isospin allowed for $T=3/2$ contributions. A level in ^{11}B at 12.55 MeV was, however, also seen in ^{7}Li$(\alpha,\alpha)^7$Li [3], which would indicate at least some $T=1/2$ component. SF pointed out that, relative to the $1/2^-, T=3/2$ level, the 12.56 MeV level in ^{11}B was much more strongly produced in the ^{9}Be$(^3$He$,p)^{11}$B reaction [4] than was the ground state of ^{11}Be in the ^{9}Be$(t,p)^{11}$Be reaction [5], though equal ratios are expected under equivalent kinematic conditions (also see Sec. IV). In ^{10}B$(p,p')^{10}$B*, where production of a $T=3/2$ level is isospin forbidden in the entrance channel, the 12.16 MeV level of ^{11}C was seen in the ^{10}B(1.74 MeV, $T=1$) exit channel, which is isospin allowed, but not seen in isospin-forbidden channels involving ^{10}B in $T=0$ states (ground state, first and third excited states) [6]. In the reaction ^{13}C$(p,t)^{11}$C, where contributions from a $T=3/2$ level would be isospin allowed, the 12.16 MeV state was not seen, but this can be explained because production of a positive-parity level is forbidden if the reaction is a direct pickup [7]. Thus the experimental evidence in favor of a $1/2^+, T=3/2$ assignment to each of these two levels in ^{11}B and ^{11}C is reasonable though not compelling. Also, no nearby level is known in either nucleus that could have this assignment [1].

The potential-model procedure used by SF is a modification of the earlier procedure of Sherr and Bertsch [8], which had been shown to give reasonable agreement with experiment in many other cases. The modification consists of the inclusion of contributions from channels with $2^+, T=1$ parent states of the $A=10$ nuclei, in addition to $0^+, T=1$ states. Fortune *et al.* [9] had previously included only $0^+, T=1$ channels in their use of the Sherr and Bertsch procedure to calculate the energies and widths of ^{11}N levels by using potentials that fitted energies of ^{11}Be levels. In these earlier calculations [8,9] there were problems when unbound levels were involved.

In the calculation of SF, the levels in ^{11}B, ^{11}C, and ^{11}N are unbound in some (mostly proton) channels. For these, SF "assume the resonances are those of $d\theta/dE$, where θ is the scattering phase at proton energy E." This presumably means that the level energy is taken as the energy at which $d\theta/dE$ is a maximum, and the single-particle width as the full width at half maximum (FWHM) of the function $d\theta/dE$ [8]. This procedure has been criticized, and alternative definitions proposed [10,11] that are based on R-matrix theory [12]. In cases where the resonance energy approaches the top of the Coulomb plus centrifugal barrier, SF "obtain resonance energies by extrapolation and single-particle widths by matching smoothly to asymptotic penetrabilities." It is not clear what this means. Here we repeat in Sec. II the potential-model calculation of SF, except that we use R-matrix definitions of the energy and width of an unbound level. This is similar to the potential-model calculation in Ref. [11], but

here we include contributions from $2^+, T=1$ parent states using the SF procedure, and also make calculations for $T=3/2$ levels in ^{11}B and ^{11}C.

An alternative method in Ref. [11] used many-channel R-matrix formulas to calculate the Coulomb displacement energies relating ^{11}Be and ^{11}N levels; in Sec. III we extend this calculation to obtain properties of the analog ^{11}B and ^{11}C levels. Section IV gives a discussion of the SF procedure and of the results obtained here.

II. POTENTIAL-MODEL CALCULATION

This potential-model calculation uses the procedure of SF, differing from it mainly by the use of R-matrix definitions for the energy (and width) of an unbound level. Initially we concentrate on this difference, in particular for the $1/2^+, T=3/2, A=11$ levels and for the $0^+, T=1, A=10$ proton channels (these levels are bound for all neutron channels and for all $2^+, T=1$ channels). As in SF, we use a Woods-Saxon potential with parameter values $r_0=1.25$ fm and $a_0=0.65$ fm, except that we cut off the potential at the channel radius, for which we use the conventional value $a=1.45(A_1^{1/3}+A_2^{1/3})$ fm $=4.57$ fm. The potential depth is obtained by fitting the energy of the bound $1/2^+$ ^{11}Be ground state ($V_0=59.1$ MeV). With this depth, the s-wave proton nuclear phase shift δ (called θ in SF) is calculated for each of the ^{11}B, ^{11}C, and ^{11}N cases. We introduce various definitions of the energy and width of these $1/2^+$ unbound levels. The energy $E_{\rm SF}$ and width $\Gamma_{\rm SF}$ are defined as in SF (using $d\theta/dE$). In R-matrix theory [12], the resonant phase shift β is defined by

$$\beta = \delta + \phi \tag{1}$$

and the density-of-states function in the one-level approximation by

$$\rho = c \sin^2 \beta / P, \tag{2}$$

where $-\phi$ is the hard-sphere phase shift and P is the penetration factor, both being functions of E and of a [12]. Then the resonance energy E_r is defined by $\beta(E_r)=90°$ and the corresponding width by $\Gamma_r = 2/(d\beta/dE)_{E_r}$ (this is approximately the "observed" width Γ^0 of R-matrix theory [12]). Also one defines E_m as the energy at which $\rho(E)$ is a maximum and Γ_m as the FWHM of $\rho(E)$ [10].

A. Level energies

Table I gives values of $E_{\rm SF}$, E_r, and E_m for the $1/2^+, T=3/2, A=11$ levels in the $0^+, T=1, A=10$ proton channels. The first row gives the values of $E_{\rm SF}$ used by SF (deduced from Table IV of SF). The second row gives our values of $E_{\rm SF}$, which are somewhat different possibly because of the cutoff W-S potential used here. The third and fourth rows give values of E_m and E_r. It is seen that all the present values are greater than those from SF; it may also be noted that Fortune *et al.* [9] give the ^{11}N energy as 1.60 ± 0.22 MeV, which is much higher than the SF value, presumably because of different definitions used for the energy. The higher values of E_m and E_r lead to reduced discrepancies between the calculated and experimental energies for the $1/2^+$ levels in ^{11}B and ^{11}C, and a higher predicted energy for the $1/2^+$ level in ^{11}N, as is seen in Table II.

TABLE I. Potential-model energies for $1/2^+, T=3/2, A=11$ levels and for $0^+, T=1, A=10$ proton channels. Energies are channel energies in MeV.

	^{11}B	^{11}C	^{11}N
$E_{\rm SF}$ (from Ref. [2])	0.503	0.780	1.081
$E_{\rm SF}$ (present)	0.589	0.881	1.192
E_m	0.679	1.036	1.417
E_r	0.778	1.202	1.652

Table II is similar to Table IV of SF, which gives the deviations between the calculated and experimental energies of the $T=3/2, A=11$ levels; it should be noted that the errors on the deviations given by SF are experimental errors only, and contain no contributions from any uncertainty in the calculated values. Table II uses values of E_m rather than the $E_{\rm SF}$ used by SF; also it gives separately values for neutron and proton channels. It is based on the SF procedure of adding weighted contributions from the (0^+) and (2^+) parent channels, the weights being given by appropriate spectroscopic factors, for which we use the SF values. Some comments on the justification for this procedure are given in Sec. IV. The deviations between the calculated and experimental energies are now somewhat greater for the $1/2^-$ and $5/2^+$ levels than in SF, but the deviations for the $1/2^+$ levels are considerably less. For ^{11}B, the deviation is reduced from 0.113 MeV in SF to 0.063 MeV here, and for ^{11}C the reduction is from 0.32 MeV to 0.17 MeV. If values of E_r were used rather than E_m, the deviations would be further reduced to 0.036 MeV and 0.08 MeV, respectively. These deviations are of the same order as those for the $1/2^-$ and $5/2^+$ levels. The predicted energies of the levels in ^{11}N are correspondingly increased; for the $1/2^+$ level to 1.64 MeV (E_m) or 1.82 MeV (E_r), for $1/2^-$ to 2.36 or 2.42 MeV, and for $5/2^+$ to 3.77 or 3.79 MeV. These values for the $1/2^-$ and $5/2^+$ levels are higher than most experimental values, but agree well with the values 2.31 and 3.78 MeV (for E_m) or 2.36 and 3.79 MeV (E_r) that were found recently [13]. The predicted energies for the $1/2^+$ level are higher than all published experimental values, including 1.31 MeV in Ref. [13]—the high values follow because all the calculated energies approximately satisfy the isobaric multiplet mass equation (IMME) (with $d=0$), so that agreement with the experimental energies for ^{11}Be, ^{11}B, and ^{11}C ensures that the calculated values for ^{11}N will be close to the IMME values given by SF in their Table II.

The smaller values of the differences between calculated and experimental energies for the $1/2^+, T=3/2$ levels of ^{11}B and ^{11}C as compared with SF weakens their argument that these levels have been misidentified. The differences are now of the order of the changes in the differences caused by changes in the definition of the energy of an unbound level. They are also of an order that might be caused by isospin mixing with $T=1/2$ levels; for non-normal-parity levels of somewhat similar structure in other nuclei, the calculated

TABLE II. Energies of $T=3/2, A=11$ levels using SF procedure, except that E_m is used for the energy of an unbound level. Energies are excitation energies except for ^{11}N, where they are measured from the $^{10}C(0^+)+p$ threshold. All energies are in MeV.

J^π	Configuration	^{11}Be	^{11}B			^{11}C			^{11}N
		n	n	p	Total	n	p	Total	p
$1/2^+$	(0^+)	0	12.691	11.907	12.430	12.617	11.466	11.850	1.417
	(2^+)	0	12.747	12.766	12.753	12.603	12.520	12.548	2.511
	$0.80(0^+)+0.20(2^+)$	0			12.494			11.989	1.636
	Expt.	0			12.557			12.16	1.27–1.63
	Calc-Expt.	0			−0.063			−0.17	0.37–0.01
$1/2^-$	(0^+)	0.320	13.011	12.601	12.874	12.937	12.220	12.459	2.212
	(2^+)	0.320	13.067	13.049	13.061	12.923	12.793	12.836	2.770
	$0.74(0^+)+0.26(2^+)$	0.320			12.923			12.557	2.357
	Expt.	0.320			12.916			12.51	2.01–2.24
	Calc-Expt.	0			0.007			0.05	0.35–0.12
$5/2^+$	(0^+)	1.778	14.464	14.248	14.392	14.390	13.900	14.063	3.889
	(2^+)	1.778	14.525	14.133	14.394	14.381	13.770	13.974	3.619
	$0.57(0^+)+0.43(2^+)$	1.778			14.393			14.025	3.773
	Expt.	1.778			14.34			13.90	3.61–3.75
	Calc-Expt.	0			0.05			0.12	0.16–0.02

isospin-mixing matrix elements, including boundary-condition contributions, are up to several hundred keV [14].

Section IV contains some discussion of the SF procedure.

B. Level widths and spectroscopic factors

In addition to the energy discrepancy, the argument by SF that the $1/2^+, T=3/2$ levels of ^{11}B and ^{11}C have been misidentified also depends on the calculated spectroscopic factors for these levels being much less than those for the levels in ^{11}Be and ^{11}N, whereas approximate agreement was obtained for the $1/2^-$ and $5/2^+$ levels. The spectroscopic factors were based on calculated single-particle widths Γ_{SF} and experimental widths. Here we calculate single-particle widths Γ_m and Γ_r and compare the resultant spectroscopic factors with the values obtained in SF.

We consider first the single-particle widths for the $1/2^+, T=3/2$ levels in the $0^+, T=1, A=10$ proton channels (the only channels open). If the potential obtained by fitting the ^{11}Be ground-state energy ($V_0=59.1$ MeV) is used, as in the preceding section, the $1/2^+$ level in ^{11}B is at a channel energy $E_m=0.679$ MeV (Table I), much less than the experimental value of $E_{expt}=1.33$ MeV, and the corresponding width is small ($\Gamma_m=0.56$ MeV). This and other values similarly calculated are given in the upper part of Table III. The procedure adopted by SF, however, involves using a different potential to calculate the single-particle width, by choosing the potential depth to fit the experimental energy E_{expt} of the $1/2^+$ level in ^{11}B. To obtain $E_m=1.33$ MeV, one requires $V_0=54.9$ MeV, giving $\Gamma_m=2.15$ MeV. This is reasonably close to the value $\Gamma_{SF}=2.40$ MeV given in Table V of SF. These and other values are given in the lower part of Table III.

In Table IV, these values of Γ_m and similar values for the $1/2^-$ and $5/2^+$ levels are used to obtain values of the spectroscopic factors, using the procedure of SF, as in Table V of SF.

The calculated widths and deduced values of the spectroscopic factor $\mathcal{S}$ for the $0^+, T=1$ parent states, as given in Table IV, are not essentially different from those in Table V of SF (it seems that the SF value of $\mathcal{S}$ for the $1/2^+$ level of ^{11}C is in error and should be 0.17, not 0.26). For the ^{11}N levels, we have used the range of Γ values given in Table I of SF, rather than some mean value, and so obtain a range of $\mathcal{S}$ values. For the $1/2^+$ level, for example, we have $\mathcal{S}=0.29-1.05$, compared with SF's value $\mathcal{S}>0.55$. The calculated contribution to Γ_m for the $5/2^+$ level of ^{11}N from the $^{10}C(2^+)+p$ channel is 0.012 MeV, and is neglected in Table V.

In agreement with the findings of SF, the values $\mathcal{S}=0.29$ for ^{11}B and 0.16 for ^{11}C are smaller than model values, which range from 0.55 to 0.92 [15], and smaller than most experimental values for ^{11}Be (see Table III of SF and Sec.

TABLE III. Potential-model single-particle widths for $1/2^+, T=3/2, A=11$ levels and for $0^+, T=1, A=10$ proton channels. Upper part gives widths calculated using potentials that fit the ^{11}Be ground-state energy; lower part gives widths using potentials that fit experimental energies E_{expt} as shown. Widths are in MeV.

	^{11}B	^{11}C	^{11}N
Γ_{SF} (present)	0.39	0.61	0.84
Γ_m	0.56	0.92	1.30
Γ_r	0.87	1.31	1.85
E_{expt} (MeV)	1.33	1.73	1.45
Γ_{SF} (from Ref. [2])	2.40	2.40	1.28
Γ_m	2.15	2.55	1.37
Γ_r	2.31	2.69	1.42

TABLE IV. Computed spectroscopic factors for $T=3/2, A=11$ levels using SF procedure, with the single-particle width taken as Γ_m calculated at the experimental energy E_{expt}. All widths are in MeV.

J^π		^{11}Be		^{11}B			^{11}C		^{11}N
		n	n	p	Total	n	p	Total	p
$1/2^+$	Γ_m			2.15	0.72		2.55	1.70	1.37
	Expt.				0.21			0.27	0.4–1.44
	$\mathcal{S}$				0.29			0.16	0.29–1.05
$1/2^-$	Γ_m			0.81	0.27		0.99	0.66	0.93
	Expt.				0.20			0.49	0.25–0.84
	$\mathcal{S}$				0.74			0.74	0.27–0.90
$5/2^+$	Γ_m	0.153	0.12	0.56	0.27	0.05	0.64	0.44	0.62
	Expt.	0.100			0.25			0.20	0.40–0.60
	$\mathcal{S}$	0.65			0.93			0.45	0.64–0.96

IV) and for ^{11}N (see Table IV); however, one analysis [16] gave $\mathcal{S}=0.19\pm0.02$ for ^{11}Be, and values $\mathcal{S}\approx0.2$ [17] and $\mathcal{S}=0.1-0.2$ [13] have been given for ^{11}N.

III. MANY-CHANNEL *R*-MATRIX CALCULATION

We use the many-channel *R*-matrix model of Ref. [11] to calculate the $T=3/2$ level energies in ^{11}B, ^{11}C, and ^{11}N relative to those in ^{11}Be. Contributions to the Coulomb displacement energy ΔE_C come from the point-Coulomb interaction ΔH^c(Coul.) and the electromagnetic spin-orbit interaction ΔH^c(s.o.) as well as the boundary-condition contribution ΔL. The MWK shell-model interaction is used as in Ref. [11]. In each of the ^{11}B and ^{11}C cases, the Coulomb interaction, in addition to contributing directly to ΔH^c(Coul.), also produces isospin mixing with nearby $T=1/2$ levels, which leads to a further shift of the level energy. This shift, however, is not significant, as the Coulomb isospin-mixing matrix elements, calculated with harmonic-oscillator single-particle wave functions, are small. The greatest mixing occurs for the $1/2^-$ levels, where there is a $T=1/2$ level within about 50 keV of the $T=3/2$ level; in each of ^{11}B and ^{11}C, the Coulomb interaction produces about 2 % isospin mixing and a shift in energy of about 1 keV. The values of ΔH^c(Coul.) for the ^{11}N levels differ slightly from those given in Ref. [11], apparently due to the use of a different version of OXBASH.

The results are given in Table V for the conventional value of the channel radius $a=4.57$ fm. As before [11], because this type of calculation may not give absolute values of ΔE_C accurately, the values of E_m for the $1/2^-$ levels are adjusted to fit experimental energies, and the calculated relative values of E_r for different J^π are used to extract E_m for $J^\pi=1/2^+$ and $5/2^+$. Table V also gives the corresponding calculated values of Γ_m.

The deviations of the E_m values from the experimental energies are of the same order for the $1/2^+$ levels of ^{11}B and ^{11}C as for the $5/2^+$ levels, giving no support to the SF suggestion of misidentification. The calculated values of Γ_m for the $1/2^+$ levels of ^{11}B and ^{11}C are, however, much greater than the experimental widths, whereas there is reasonable agreement for the $1/2^-$ and $5/2^+$ levels, as well as for all the ^{11}N levels. This is similar to the result of the potential-model calculations in Sec. II, and agrees with the findings of SF regarding spectroscopic factors.

IV. DISCUSSION

SF give no justification for their procedure of adding contributions from the (0^+) and (2^+) channels, with the potentials for each channel being chosen to fit the experimental energy in ^{11}Be. It is to be noted that this leads to different energies of the (0^+) and (2^+) contributions in the other nuclides; for example, for the $1/2^+$ level of ^{11}N, these energies differ by more than 1 MeV (see Table II). SF also give no justification for using different potential depths for calculating energies and widths.

For the $1/2^+, T=3/2$ levels, it may be reasonable that a combination such as $0.80(0^+)+0.20(2^+)$, with the sum of the coefficients equal to one, should be used in Table IV of SF and Table II here, corresponding to one nucleon being in the (sd) shell; then the sum of the *s*-wave and *d*-wave spectroscopic factors over all $T=1, A=10$ parent states is one (neglecting the c.m. correction factor [18]). The same is true for the $5/2^+$ levels, but for the $1/2^-$ levels the argument for a combination such as $0.74(0^+)+0.26(2^+)$, as used by SF (with correction of a misprint in their Table IV), is not at all obvious. All the nucleons outside a ^{4}He core are in the *p* shell, and the sum of the *p*-wave spectroscopic factors over all $T=1, A=10$ parent states is 4.5. For the interaction used in Sec. III, one finds $\mathcal{S}(0_1^+)=0.76$, in good agreement with SF's 0.74, but $\mathcal{S}(2_1^+)=0.52$ and $\mathcal{S}(2_2^+)=1.09$. Thus the SF model does not seem suitable for the $1/2^-$ levels.

For the $1/2^+$ levels, SF use the mixture $0.80(0^+)+0.20(2^+)$. The value 0.80 comes from the average of the experimental values of the spectroscopic factor $\mathcal{S}$ given in Table III of SF. These values seem to be inaccurate and incomplete. Reference 11 of SF gives experimental values of $\mathcal{S}$ from 0.66 to 0.79 (rather than 0.84 as quoted by SF), and says that these values would be multiplied by a factor of 0.6–0.7 if allowance is made for ^{11}Be recoil and breakup effects. Reference 13 of SF gives $\mathcal{S}=0.74$ (rather than 0.80)

TABLE V. Calculated Coulomb displacement energies, and predicted energies and widths of ^{11}B, ^{11}C, and ^{11}N levels. Energies are channel energies in ^{11}Be and ^{11}N, excitation energies in ^{11}B and ^{11}C. $a=4.57$ fm. All energies and widths are in MeV.

J^π		^{11}Be	^{11}B	^{11}C	^{11}N
$1/2^+$	ΔH^c(Coul.)		1.951	4.397	7.330
	ΔH^c(s.o.)		0.0	0.0	0.0
	ΔL		−0.215	−0.396	−0.760
	ΔE_C		1.736	4.001	6.570
	E_r	−0.503[a]	12.459	11.960	1.409
	E_m	−0.503[a]	12.521	12.023	1.437
	E_{expt}	−0.503	12.557±0.016	12.16±0.04	1.27−1.63
	E_m-E_{expt}	0.0	−0.036	−0.14	0.17−(−0.19)
	Γ_m	0.0	0.46	1.14	1.13
	Γ_{expt}		0.21±0.02	0.27±0.05	0.4−1.44
$1/2^-$	ΔH^c(Coul.)		1.976	4.464	7.464
	ΔH^c(s.o.)		−0.020	−0.042	−0.065
	ΔL		−0.162	−0.356	−0.617
	ΔE_C		1.794	4.066	6.782
	E_r	−0.183[a]	12.837	12.345	1.941
	E_m	−0.183[a]	12.916[a]	12.51[a]	2.12[a]
	E_{expt}	−0.183	12.916±0.012	12.51±0.03	2.01−2.24
	E_m-E_{expt}	0.0	0.0	0.0	0.11−(−0.12)
	Γ_m	0.0	0.19	0.42	0.55
	Γ_{expt}		0.20±0.025	0.49±0.04	0.25−0.84
$5/2^+$	ΔH^c(Coul.)		1.939	4.354	7.240
	ΔH^c(s.o.)		−0.011	−0.027	−0.051
	ΔL		−0.072	−0.176	−0.300
	ΔE_C		1.856	4.151	6.889
	E_r	1.278	14.357	13.888	3.506
	E_m	1.275[a]	14.436	14.064	3.701
	E_{expt}	1.275	14.34±0.02	13.90±0.02	3.61−3.75
	E_m-E_{expt}	0.0	0.10	0.16	0.09−(−0.05)
	Γ_m	0.11	0.19	0.30	0.43
	Γ_{expt}	0.10±0.02	0.25±0.02	0.20±0.10	0.40−0.60

[a]Fitted value.

as a calculated value, and says that there is good agreement with experiment. Reference 14 of SF is essentially an expanded version of their reference 11, and it gives a similar range of experimental values $\mathcal{S}=0.65-0.80$; it also discusses various effects neglected in the calculations that would appreciably reduce the extracted values of $\mathcal{S}$. Table III of SF does not include the published value $\mathcal{S}=0.19\pm0.02$ [16].

There is therefore reason to believe that $\mathcal{S}$ may be smaller than the value 0.80 assumed by SF. If SF had used a mixture $0.60(0^+)+0.40(2^+)$, say, their deviations for ^{11}B and ^{11}C would have been −41 keV and −151 keV, rather than −113 keV and −320 keV as given in their Table IV, and if this mixture were used in our Table II, the deviations would be 2 keV and −31 keV. Thus the argument by SF that these deviations are large, indicating misidentification of these $1/2^+$ levels in ^{11}B and ^{11}C, appears to be of doubtful validity.

Part of SF's argument that the $1/2^+, T=3/2$ level in ^{11}B has been misidentified is that $\sigma(1/2^+)/\sigma(1/2^-)$ is 1.1 in ^{9}Be(^{3}He,p)^{11}B($T=3/2$) [4], whereas it is about 0.24 in ^{9}Be(t,p)^{11}Be [5]. The ratio should be the same under equivalent kinematic conditions; however, it is not obvious that the conditions are equivalent, as the ^{3}He beam energy was 38 MeV [4] while the triton beam energy was 15 MeV [5]. An alternative way of comparing the results for the ^{9}Be(^{3}He,p)^{11}B [4] and ^{9}Be(t,p)^{11}Be [5] reactions is to consider the distorted-wave Born approximation fits to the data. Liu and Fortune [5] fitted their $1/2^+$ data with a normalizing factor $N=167.9$, after assuming unity for the ^{10}Be(g.s.) $\rightarrow$ ^{11}Be($1/2^+$) spectroscopic factor. They say that N ranges from 200 to 400 for other (t,p) reactions when realistic wave functions are used, suggesting a spectroscopic factor $\mathcal{S}$ of 0.4−0.8. Zwieglinski *et al.* [4], using a spectroscopic factor of 0.82 from Teeters and Kurath [19], required a normalizing factor $\epsilon=0.67$ to fit their $1/2^+$ data, suggesting $\mathcal{S}\approx0.55$. There seems to be reasonable agreement between the values of $\mathcal{S}$ from these two reactions. There is also agreement with

the $\mathcal{S}$ values discussed and assumed in the preceding paragraphs.

An argument can be made that the SF procedure overestimates values of the FWHM Γ_m. The *R*-matrix observed width Γ^0 (which should be close to Γ_m) in the one-level approximation is given by [12]

$$\Gamma^0 = \frac{2\gamma_p^2 P_p(E_r)}{1 + \sum_c \gamma_c^2 (dS_c/dE)_{E_r}}, \tag{3}$$

as only the proton channel is open; however, both open and closed channels contribute to the sum in the denominator, in which all terms are positive. As an example, for the $1/2^+$ level of ^{11}B, we include in this sum only the $^{10}\mathrm{Be}(0^+)+p$ open channel and the corresponding $^{10}\mathrm{B}(0^+, T=1)+n$ closed channel. For $\mathcal{S}=0.74$, we calculate $\gamma_p^2=0.620$ MeV and $\gamma_n^2=0.998$ MeV, giving

$$\Gamma^0 = \frac{0.815 \text{ MeV}}{1 + 0.095 + 0.601} = 0.48 \text{ MeV} \tag{4}$$

(inclusion of contributions from the 2^+ channels would lower this value to 0.47 MeV). If we had neglected the neutron channel altogether, as in the SF procedure, we would have obtained $\Gamma^0=0.74$ MeV.

Equation (4) uses $\mathcal{S}=0.74$; in order to fit the experimental width $\Gamma_{\text{expt}}=0.21$ MeV, one would require $\mathcal{S}=0.23$. Similarly, fitting $\Gamma_{\text{expt}}=0.27$ MeV for the $1/2^+$ level of ^{11}C gives $\mathcal{S}=0.11$. These values of $\mathcal{S}$ are smaller than all model values and most experimental values for ^{11}Be and ^{11}N (see Sec. II B). Before laying the blame on these calculated values, however, we consider the reliability of the experimental values.

For the $1/2^+$ level of ^{11}B, the value $\Gamma_{\text{expt}}=0.21$ MeV, which is taken from Table 11.15 of Ref. [1], is the mean of consistent values from three different papers: 202 ± 25 keV [4], 230 ± 65 keV [20], and 260 ± 50 keV [6], with the first of these essentially determining the value of Γ_{expt}. The width values for the $1/2^-, T=3/2$ level in ^{11}B from these three papers are 155 ± 25 keV [4], 235 ± 27 keV [20], and 390 ± 120 keV [6]; other values are 350 ± 50 keV [7], 260 ± 50 keV [21], and 238 ± 15 keV [22]. For each level, the width from Ref. [4] is smallest. There does not, however, seem much justification for arguing for an appreciable increase in Γ_{expt} for the $1/2^+$ level, especially as earlier work gave even smaller values: 145 ± 30 keV [23] and 150 ± 50 keV [3].

The situation is somewhat different for ^{11}C. The only reported observation of the $1/2^+, T=3/2$ level of ^{11}C is by Watson *et al.* [6], using three different reactions. Width values are given for two of the reactions: 290 ± 50 keV from $^{11}\mathrm{B}(^3\mathrm{He}, t)^{11}\mathrm{C}$ and 200 ± 100 keV from $^{9}\mathrm{Be}(^3\mathrm{He}, n)^{11}\mathrm{C}$, leading to $\Gamma_{\text{expt}}=0.27$ MeV. In their Fig. 2, Watson *et al.* [6] show the neutron spectrum from the $^{9}\mathrm{Be}(^3\mathrm{He}, n)^{11}\mathrm{C}$ reaction in the region of the $1/2^+$ and $1/2^-, T=3/2$ levels, before and after subtraction of a background. The peak heights are about 10% of the background, so the properties assigned to the levels would depend sensitively on the choice of background. The triton spectrum from the $^{11}\mathrm{B}(^3\mathrm{He}, t)^{11}\mathrm{C}$ reaction that Watson *et al.* show in their Fig. 1 has already had a background subtracted—the size of the background is unknown. It does not seem impossible that the width of the $1/2^+$ level of ^{11}C could be appreciably higher than the values given by Watson *et al.* From our calculations, the quantity least sensitive to the value of $\mathcal{S}$ is probably the ratio of the widths of the $1/2^+$ levels in ^{11}C and ^{11}B (2.36 from Table IV and 2.46 from Table V). Taking a ratio of 2.4, and $\Gamma_{\text{expt}}=0.21$ MeV for the $1/2^+$ level of ^{11}B, we would expect a width of about 0.50 MeV for the $1/2^+$ level of ^{11}C. This may not be inconsistent with the data of Watson *et al.*

Another argument can be given for an increase in the value of Γ_{expt} for the $1/2^+$ level of ^{11}C. Watson *et al.* also observed the $1/2^-$ level, with widths given as 370 ± 90 keV, 350 ± 100 keV, and 400 ± 100 keV; the mean of these is 373 ± 56 keV. Other width values for this level are 550 ± 50 keV [7] and 540 ± 60 keV [21], with a mean of 546 ± 38 keV. Estimates of the width are about 540 keV from Fig. 2 of Ref. [24] and about 600 keV from Fig. 8 of Ref. [25]. This suggests that the width from Watson *et al.* for the $1/2^-$ level might be increased by about 50%; a similar increase for the $1/2^+$ level would increase Γ_{expt} to about 0.40 MeV.

Even if the Γ_{expt} value for the $1/2^+$ level of ^{11}C is increased to, say, 0.50 MeV, there still remains the problem that these width values for the $1/2^+$ levels in ^{11}B and ^{11}C correspond to $\mathcal{S}=0.23$. So far we have assumed the conventional value for the channel radius, $a=4.57$ fm. For other values of a, the values of $\mathcal{S}$ obtained by fitting $\Gamma_{\text{expt}}=0.21$ MeV for the $1/2^+$ level of ^{11}B, using the approximation underlying Eq. (4), range from 0.22 for $a=4.0$ fm to 0.31 for $a=6.0$ fm. These values of $\mathcal{S}$ are still well below model values.

In Sec. III, we have assumed a one-level *R*-matrix approximation for each of the $1/2^+, 1/2^-$, and $5/2^+$ levels. In what might be considered a similar situation, involving the low-lying levels of ^{13}C and ^{13}N, fits to properties involving the $1/2^+$ first-excited states required a two-level approximation, whereas the properties involving the $1/2^-$ and $5/2^+$ states could be adequately fitted using a one-level approximation [26,27]. In these $A=13$ nuclei, the second $1/2^+$ level is predicted in shell-model calculations [19,28] to lie about 10 MeV above the first, while in the $T=3/2, A=11$ nuclei, the energy difference is about 7 MeV. It may be that the one-level approximation is inadequate for the $1/2^+, T=3/2, A=11$ levels, and that this is the reason for the wide spread of experimental values of $\mathcal{S}$ for these levels in ^{11}Be and ^{11}N, and for the small values of $\mathcal{S}$ obtained here for ^{11}B and ^{11}C.

Alternatively, or perhaps additionally, it might be that the small $\mathcal{S}$ values could be due in part to isospin mixing, which produces first-order changes in spectroscopic factors and second-order changes in energies. For the MWK shell-model interaction used here, the $1/2^+, T=1/2$ state nearest to the $1/2^+, T=3/2$ state is about 0.4 MeV lower, while the Teeters and Kurath [19] interaction gives a separation of about 0.1 MeV; however, such $1/2^+, T=1/2$ states have not been identified experimentally whether or not significant isospin mixing occurs.

V. SUMMARY

Both potential-model calculations and many-channel R-matrix calculations seem consistent with the energies presently assigned to the $1/2^+, T=3/2$ levels in ^{11}B and ^{11}C. The spectroscopic factors derived from the assigned widths of these levels are appreciably lower than model values and most experimental values for the analog levels in ^{11}Be and ^{11}N, but are in agreement with some of the latter.

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