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Implications of the discrepancy between proton form factor measurements

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Recent polarization transfer measurements of the proton electromagnetic form factors yield very different results from previous Rosenbluth extractions. This inconsistency implies uncertainties in our knowledge of the form factors and raises questions about how to best combine data from these two techniques. If the discrepancy is due to missing corrections to the cross section data, as has been suggested, then the true form factors, related to the proton structure, differ from the form factors that parametrize the deviation from point scattering, and different applications will require the use of different form factors. We present two extractions of the form factors: a global fit to the world's cross section data, and a combined extraction from polarization transfer and cross section data. The former provides a parametrization of the elastic electron-proton cross section. The latter provides a consistent extraction of the underlying form factors, under the assumption that missing terms in the radiative correction explain the difference between the cross section and polarization transfer results.

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The proton electromagnetic form factors G_E and G_M parametrize deviations from a point particle in elastic electron-proton scattering, and are related to the charge and magnetization distribution of the proton. The form factors depend only on Q^2, the square of the four-momentum transfer, and until recently it was believed that the electric and magnetic form factors showed approximate scaling, i.e., nearly identical Q^2 dependence [1]. More recent Jefferson Lab measurements [2–4] utilized the polarization transfer technique to measure the ratio G_E/G_M and found that G_E decreases more rapidly than G_M at large Q^2. The polarization transfer measurements are more precise at high Q^2, and significantly less sensitive to systematic uncertainties than the Rosenbluth separation measurements. However, the two techniques disagree significantly even in the region where both yield precise results.

At the present time, it is not known why the techniques give different results. The systematic uncertainties of the polarization transfer measurements, primarily spin transport and backgrounds, have been carefully studied [5]. A detailed global analysis of the cross section measurements [6] does not show any inconsistencies in the cross section data sets, or yield any likely candidate to explain the discrepancy. To resolve the discrepancy, a systematic error in the cross section would have to have a significant dependence on the virtual photon polarization ε, $\varepsilon^{-1}=1+2(1+Q^2/4M_p^2)\tan^2(\theta_e/2)$, where M_p is the proton mass and θ_e is the electron scattering angle. Such a systematic error would have to yield a $(5-7)\%$ ε dependence in the cross section, roughly linear in ε, in order to resolve the discrepancy.

There appear to be two possibilities: either a fundamental flaw in the Rosenbluth or polarization transfer formalism, or an error in either the cross section or polarization transfer measurements. Recent works have suggested that additional radiative correction terms, related to two-photon exchange corrections, may lead to an error in determining the form factors from the measured cross sections [7–9]. If the two-photon exchange mechanism, or some other correction that is neglected in the cross section extraction, is the source of the discrepancy, then the form factors extracted from a Rosenbluth separation of cross section data will *not* represent the underlying structure of the proton, but they *will* parametrize the elastic electron-proton cross section in the usual one-photon approximation. Conversely, the true form factors will not yield the correct cross sections, and will thus give incorrect results if used as a parametrization of the elastic cross section in data analysis.

If the two-photon exchange term explains the discrepancy, then the polarization transfer result will relate to the true form factors, assuming that the two-photon exchange has a much smaller effect on the polarization transfer than on the Rosenbluth extractions. However, the existing polarization transfer experiments [2–4] have extracted the ratio G_E/G_M, rather than the individual form factors. To extract the form factors, these data must be combined with cross section measurements to determine the absolute magnitudes of G_E and G_M. If the two-photon exchange correction modifies the cross sections from those calculated from the underlying form factors, then it is not possible to consistently combine the two kinds of measurements without some assumption about the two-photon exchange correction.

In this paper, we present two extractions of the proton form factors. From a global analysis of cross section measurements, we extract the "Rosenbluth form factors." From a combined analysis of cross section and polarization transfer data, with a "minimal" assumption about the nature of the two-photon exchange corrections, we extract the "polarization form factors." If two photon corrections are the source of the discrepancy, then the Rosenbluth form factors will parametrize the elastic cross section, and are therefore useful as input to analysis or simulations that require the electron-proton cross section. The polarization form factors will provide the true form factors, which relate to the underlying structure of the proton. These form factors are often described as the Fourier transformations of the charge and magnetization distributions of the proton in the Breit frame, although relativistic effects and the fact that each value of Q^2 corresponds to a different Breit frame lead to substantial theoretical difficulties in extracting charge and magnetization distributions [10].

The Rosenbluth form factors are determined from a global fit to elastic electron-proton cross section measurements. The details of the fitting procedure are described in Ref. [6]. For the present analysis we include more recent Jefferson Lab measurements of elastic scattering [11–13], as well as additional data sets to constrain the low Q^2 behavior [14–17] to the data sets used in Ref. [6,18]. In addition, we include all of the high Q^2 data, up to 30 GeV^2, while the previous analysis was limited to 8 GeV^2. The older data have updated radiative corrections, and the small-angle data from Walker *et al.* [1] are excluded, as described in Ref. [6]. The form factors are fit to the following form:

$$G_E(Q^2), G_M(Q^2)/\mu_p = [1 + p_2 Q^2 + p_4 Q^4 + \cdots + p_{2N} Q^{2N}]^{-1}, \tag{1}$$

where μ_p is the magnetic dipole moment of the proton and Q^2 values are in GeV^2. Reasonable fits are achieved for $N \geqslant 3$. Note that this is a different functional form than used in previous fits [6,19,20], which used polynomials in $q = \sqrt{Q^2}$. The polynomial in q is a very general form, with adequate flexibility to reproduce the data, but does not have the proper behavior as $Q^2 \to 0$.

The fit is quite insensitive to the order of the polynomial above $N=6$, except for G_E at large Q^2. For Q^2 above 6 GeV^2, fits with nearly identical χ^2 values can have G_E/G_M either rise or fall dramatically with Q^2. This is a result of the reduced sensitivity to G_E and the limited ε coverage for Q^2 values above 6 GeV^2. To avoid unreasonable behavior in the region where G_E is unconstrained by data, we keep the *ratio* G_E/G_M fixed for all Q^2 values above 6 GeV^2. This leads to a fit for G_E which is continuous, but not smooth, at $Q^2 = 6$ GeV^2. Because G_E has relatively little contribution to the total cross section at these momentum transfers, the cross section extracted is still quite smooth, and the value of G_E at large Q^2 values has little effect on the cross section, as long as the fit is constrained to avoid $|\mu_p G_E| \gg |G_M|$.

The normalization factor for each data set is allowed to vary along with the parameters of the fitting functions for G_E and G_M. The total χ^2 from the cross section measurements and normalization factors is

$$\chi^2_\sigma = \sum_{i=1}^{N_\sigma} \frac{(\sigma_i - \sigma_{\rm fit})^2}{(d\sigma_i)^2} + \sum_{j=1}^{N_{\rm expt}} \frac{(\eta_j - 1)^2}{(d\eta_j)^2}, \tag{2}$$

where σ_i and $d\sigma_i$ are the cross section and error (excluding normalization uncertainties) for each of the N_σ data points, η_j is the fitted normalization factor for the jth data set, and $d\eta_j$ is the normalization uncertainty for that data set. We fit to 470 data points ($N_\sigma=443$, $N_{expt}=27$) with 39 parameters (six parameters each for the electric and magnetic form factors, and 27 normalization parameters).

The result of the global fit to the cross section data is shown in Fig. 1. The fit yields a total χ^2 of 326.7 for 431 degrees of freedom, yielding a reduced χ^2, $\chi^2_\nu = \chi^2/N_{dof}$, of 0.758. This yields an unreasonably high confidence level, indicating that the quoted uncertainties of the measurements are too large. As was observed in the previous fit [6], the majority of the data sets, 20 out of 27, have values of $\chi^2_\nu < 1$, indicating that most of the experiments were overly conservative in estimating their uncertainties. Table I lists the parameters for the Rosenbluth form factors. The fit includes cross sections for Q^2 values from 0.005 to 30 GeV^2, and should be valid over this range, though the separation of G_E and G_M is only well constrained by the data for $Q^2 \lesssim 6$ GeV^2.

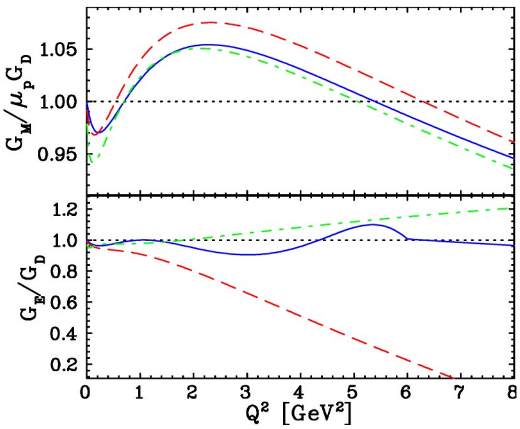

FIG. 1. (Color online) The "Rosenbluth form factors" (solid line) for G_E and G_M relative to the dipole form: $G_D = [1 + Q^2/M_D^2]^{-2}$, $M_D^2 = 0.71$ GeV^2. The dot-dashed line is the previous fit to Rosenbluth extracted form factors from Ref. [20], and the dashed curve is the fit to G_M from Ref. [19], with the form factor ratio constrained to give $\mu_p G_E/G_M = 1 - 0.13(Q^2 - 0.04)$.

The normalization factors were generally smaller than the quoted scale uncertainties of the experiments ($\chi^2 = 18.0$ for 27 normalization factors). The average normalization factor is 0.65%, and the rms normalization factor is 2.7%. The normalization factors are very close to those obtained in the previous global fit [6]. The average normalization factor differs by approximately 0.5%, and the individual normalization factors differ by less than 1% for 18 of the 20 experiments. Because the previous fit excluded data below $Q^2 = 0.6$ GeV^2, the agreement indicates that it is the self-consistency, rather than the form factor constraint at $Q^2 = 0$, that dominates the determination of the normalization factors.

We can test the self-consistency of the individual data sets by comparing the global fit to the results of single-experiment extractions of G_E and G_M. By comparing only the single-experiment extractions, we avoid the potentially large and correlated uncertainties that arise from the relative normalization of different data sets. Comparing the ratio

TABLE I. Fit parameters for the Rosenbluth form factors, using the parametrization of Eq. (1).

Parameter	G_E (Rosenbluth)	G_M/μ_p (Rosenbluth)
p_2	3.226	3.19
p_4	1.508	1.355
p_6	−0.3773	0.151
p_8	0.611	-1.14×10^{-2}
p_{10}	−0.1853	5.33×10^{-4}
p_{12}	1.596×10^{-2}	-9.00×10^{-6}

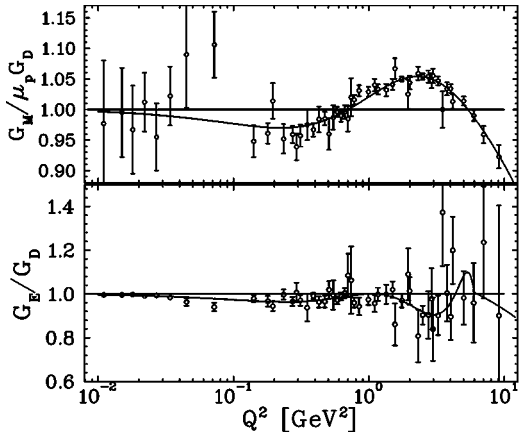

FIG. 2. G_M (top) and G_E (bottom) from direct Rosenbluth separation utilizing normalization factors from the global fit.

G_E/G_M from the fit to the individual experiments, taken from Refs. [11,13] and the reanalysis of older experiments presented in Ref. [6] yields $\chi^2=45.3$ for 50 data points ($\chi^2=17.8$ for the 20 points above $Q^2=1.5$ GeV^2).

We can estimate the uncertainties in the form factors by performing direct Rosenbluth separations in several Q^2 bins using the full data set, with normalization factors determined from the global fit. For each Q^2 bin, the data are scaled to the average Q^2 value of the data points in that bin, using the global fit as the scaling function. Q^2 bins were chosen so that there are at least three data points in the bin, the ε range covered is at least 0.3, and the correction for scaling each point to the average Q^2 value was $\lesssim 10\%$ (typically $<2\%$). The scaling was also done using the fits of Refs. [20] and [19], shown in Fig. 1. Varying the scaling procedure changed the ratios by $\leqslant 1\%$, except for the very highest (lowest) Q^2 points, where the change in $G_E(G_M)$ was as much as 3%, but was still much smaller than the uncertainty in the extracted form factor.

Figure 2 shows the fits to G_E and G_M, along with the direct Rosenbluth separation points, using the normalization factors from the fit. Except for the very low Q^2 values, typical uncertainties on G_M are $\approx 1\%$, increasing to $\sim 2\%$ for $Q^2=10$ GeV^2 (8% for $Q^2=30$ GeV^2). At low Q^2, the experimental uncertainties become quite large, but the constraint on the behavior as $Q^2 \rightarrow 0$ yields a much smaller uncertainty on the fit. For G_E, the uncertainties are (1–2)% at low Q^2, but are (5–10)% for intermediate Q^2 values (2–4 GeV^2), and grow rapidly as Q^2 increases. Note that the uncertainties in G_E and G_M are highly anticorrelated, due to the way the form factors are separated from the cross section measurements. This can be seen in the anticorrelation of the deviation of the points from the fits in Fig. 2. Thus, the uncertainty on the cross sections extracted from this parametrization is not just the sum of the uncertainties in the contributions from G_E and G_M. Up to $Q^2 \approx 4$ GeV^2, there is a large body of cross section measurements with point-to-point uncertainties of $\sim 1\%$. Because the normalization factors are determined in the fit, and the residual uncertainty in the normalization is small, the absolute cross sections should be known to better than 2%. Above $Q^2=4$ GeV^2, the number of data points decreases, and the uncertainties in the cross sections grow, reaching 10% at $Q^2=25$ GeV^2.

Even with the uncertainty related to the discrepancy between Rosenbluth and polarization transfer, this fit yields a precise parametrization of the elastic cross section in the one-photon exchange formalism. While these may not be the underlying form factors of the proton (e.g., if there are missing radiative correction terms), this is still the appropriate parametrization to use as input to a calculation or analysis that requires the elastic cross section. Using the form factors derived from the polarization transfer technique will not yield the correct cross section, even in a combined analysis of Rosenbluth and polarization transfer such as performed in Refs. [6,19]. More importantly, an *inconsistent* combination of cross section and polarization transfer results can magnify the error. Combining a parametrization of G_M from a Rosenbluth analysis with the form factor ratios measured in polarization transfer decreases G_E, and thus decreases the total cross section, relative to the best fit to the cross section data, without allowing a corresponding increase in G_M. This leads to form factors which give cross sections that are (4–10)% below the measured cross sections at large ε over a large Q^2 range ($0.1 < Q^2 < 15$ GeV^2).

While the Rosenbluth form factors yield the best parametrization for the cross section in the usual one-photon exchange picture, the ratio does not agree with the ratio extracted from the polarization transfer technique. For the larger Q^2 values, the polarization transfer technique is less sensitive to knowledge of the kinematics, radiative correction, and other systematic uncertainties that are important in the Rosenbluth separation.

If this discrepancy is related to a problem in the cross section data, then the polarization transfer will yield the true ratio of the form factors, but has to be combined with cross section data to obtain both G_E and G_M. We present in this section a combined analysis of the polarization transfer and cross section data, which will yield the polarization form factors.

In order to obtain a consistent extraction of the form factors, we must make an assumption about the nature of the discrepancy. We assume that the difference comes from a common systematic error in the cross section measurements. Analyses of this discrepancy [6,8] indicate that there must be an ε-dependent correction of (5–7)%, roughly linear in ε, for $1 < Q^2 < 6$ GeV^2.

In the combined analysis, we apply a linear, ε-dependent correction of 6% to all data sets. This is the minimal assumption necessary to make the two techniques consistent, to the extent that a correction that was not linear in ε, or which modified only some of the data sets, would have to be larger. A correction that is nearly linear in ε and fairly Q^2 independent is consistent with the form for the two-photon exchange term in the analysis of Ref. [7], although the size of the correction in Ref. [7] is only $\sim 2\%$, less than half the size necessary to explain the discrepancy.

We repeat the fit from earlier, but with cross sections modified by the linear ε dependence, and with the polarization transfer data included in the fit, as described in Ref. [6]. The correction to the cross section could either lower the cross section at large ε values, or increase it at small ε values:

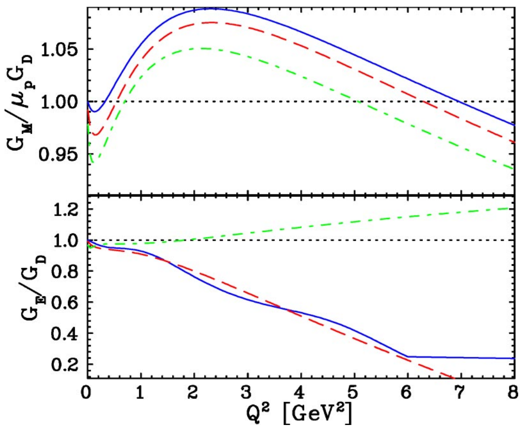

FIG. 3. (Color online) The "polarization form factors" (solid line) for G_E and G_M, relative to the dipole form. The dot-dashed line is the previous fit to Rosenbluth extracted form factors from Ref. [20], and dashed curve is the fit to G_M from Ref. [19], with the form factor ratio constrained to give $\mu_p G_E/G_M = 1-0.13(Q^2-0.04)$.

$$\sigma_{c1} = \sigma_0(1-0.06\varepsilon), \tag{3}$$

$$\sigma_{c2} = \sigma_0[1-0.06(\varepsilon-1)] = 0.94\sigma_{c1}. \tag{4}$$

The first correction is consistent with the form from Ref. [8], while the second is consistent with the behavior of Ref. [7]. The second form was chosen for the main fit because the correction is small at large ε (small θ_e), where comparisons of positron to electron scattering from SLAC [21] set fairly tight limits on the size of two-photon exchange.

The polarization form factors, from the combined fit to the cross section and the 26 polarization transfer data points from Refs. [2–4] is shown in Fig. 3. The fit yields a total χ^2 of 391.6 for 457 degrees of freedom, $\chi^2_\nu=0.857$, including the additional χ^2 contribution for the polarization transfer data [Eq. (8) of Ref. [6]]. Table II gives the fit parameters for the polarization form factors.

The fit was also performed with the correction of Eq. (3). This leads to an overall rescaling of all of the cross sections by 6%, relative to the correction of Eq. (4). However, this does not yield a simple rescaling of the form factors, because each data set has a normalization factor that is determined in the fit, and because the form factors are constrained to reproduce the charge and magnetic moment at $Q^2=0$. While a two-photon correction of this size for large ε values would appear to be ruled out by the SLAC positron-proton measurements [21], an ε-dependent systematic other than two-photon exchange could also resolve the discrepancy. However, this fit yields a much worse χ^2 value: 575.1 for 457 degrees of freedom, and so we choose to apply Eq. (4) for our combined fit.

TABLE II. Fit parameters for the polarization form factors, using the parametrization of Eq. (1).

Parameter	G_E (Polarization)	G_M/μ_p (Polarization)
p_2	2.94	3.00
p_4	3.04	1.39
p_6	−2.255	0.122
p_8	2.002	-8.34×10^{-3}
p_{10}	−0.5338	4.25×10^{-4}
p_{12}	4.875×10^{-2}	-7.79×10^{-6}

Note that the result of the combined fit (Table II will *not* reproduce the measured elastic cross section in the one-photon exchange formalism; it will reproduce the *modified* cross sections of Eq. (4). Therefore, the polarization form factors should not be used to model elastic electron-proton cross section measurements. However, if the minimal assumption that a correction consistent with the form of Eq. (4) explains the discrepancy, this should yield a consistent extraction of the underlying form factors of the proton.

Form factors extracted using the Rosenbluth technique provide a parametrization of the deviation of the elastic electron-proton cross section from the point-scattering cross section. If the cross section has additional corrections, such as two-photon exchange terms, that are not being taken into account, then the Rosenbluth extraction does not yield the true proton form factors that relate to the structure of the proton. In this case, G_E must be extracted from the polarization transfer measurements, which yield G_E/G_M, and the cross section data must be utilized to determine G_M. While we cannot know how to properly combine the polarization transfer and cross section data until we understand the cause of the discrepancy, the uncertainties in G_M that arise from this problem are much smaller than those in G_E. The same holds true if there is some other correction or combination of corrections to the cross section other than the two-photon exchange (e.g., Coulomb corrections [22]). It is of course possible that the discrepancy is due to a problem with the polarization transfer data or technique rather than the cross section data. If so, then the Rosenbluth form factors represent both the correct cross section and the correct nucleon structure. However, there do not appear to be any obvious candidates for problems in the technique, and the experiment should be less prone to systematic uncertainties than the Rosenbluth extractions.

We have presented two extractions of the proton electromagnetic form factors. The Rosenbluth form factors come from a global Rosenbluth extraction of the form factors from electron-proton elastic scattering measurements. The polarization form factors come from a combined fit to the cross section and polarization transfer data, under the assumption that the discrepancy between the techniques is caused by a linear, ε-dependent correction to the cross sections. The Rosenbluth form factors give a global parametrization of the elastic electron-proton scattering cross section in the one-photon exchange approximation. Even if there is a correction to the cross sections, neglected in the one-photon exchange formalism, this parametrization will yield the correct cross sections in the one-photon approach. Under the above assumption of an unknown correction to the cross sections, the polarization form factors yield the underlying form factors, but will not reproduce cross sections, and will therefore yield incorrect results if used as input for an analysis that requires the elastic cross section, as was observed in an analysis of quasielastic scattering from nuclei [11].

Additional data will help shed light on the origin of the discrepancy. An improved "Super-Rosenbluth" separation measurement [23] completed at Jefferson Lab in 2002 will yield a precise extraction of G_E/G_M, and will determine if the discrepancy can be explained by experimental problems in the Rosenbluth extractions. A new polarization transfer experiment [24], approved to run at Jefferson Lab, will provide an independent confirmation of the existing polarization transfer results, as well as extending the measurements to higher Q^2 values. If sufficiently improved calculations or direct measurements of the two-photon exchange corrections become available, we should be able to determine if they are responsible for the discrepancy, and if so, remove the current uncertainty in combining cross section and polarization transfer measurements.

If the discrepancy is explained by two-photon corrections or some other effect on the cross sections, and we have reliable calculations for these effects, then the cross section data can be combined with the polarization transfer data to extract the form factors without ambiguity. These form factors will represent the underlying structure of the proton and provide a useful parametrization of the elastic electron-proton cross section, as long as the effect is properly accounted for. Until the discrepancy is well understood, however, both sets of form factors are necessary, and it is important to use form factors that are (1) extracted consistently from the cross section and/or polarization transfer data, and (2) appropriate for the problem being addressed.

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